



Structural Sample - Assessment Lab

Pure Mathematics - Sample Paper

Method Room Assessment Lab

Time allowed: 2 hours	Total marks: 100
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This paper deliberately uses a different question count (12, not 14), a different topic mix, and several cross-topic combined questions, to pilot structural variety across the Method Room mock paper library. Answer ALL questions.

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1.

$$f(x) = x^3 - 2x^2 - 5x + 6.$$

- (a) Show that $(x - 1)$ is a factor of $f(x)$. **(2 marks)**
(b) Hence factorise $f(x)$ completely, and solve $f(x) = 0$. **(4 marks)**

(Total for Question 1 is 6 marks)

2.

- (a) Prove that $n^3 - n$ is divisible by 6 for every positive integer n . **(5 marks)**

(Total for Question 2 is 5 marks)

3.

The circle C has equation $x^2 + y^2 - 6x + 2y - 15 = 0$. The line l has equation $y = x + k$, where k is a constant.

- (a) Find the centre and radius of C . **(3 marks)**
(b) Find the two values of k for which l is a tangent to C , giving your answers in the form $p \pm q\sqrt{2}$. **(6 marks)**

(Total for Question 3 is 9 marks)

4.

- (a) Express $5 \sin x + 12 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places. **(4 marks)**
(b) Hence solve, for $0 \leq x < 360^\circ$, the equation $5 \sin x + 12 \cos x = 6$, giving your answers to 1 decimal place. **(5 marks)**

(Total for Question 4 is 9 marks)

5.

- (a) Find the binomial expansion of $(1 + 2x)^{1/3}$ in ascending powers of x , up to and including the term in x^3 , simplifying each coefficient. State the range of values of x for which the expansion is valid. **(6 marks)**
(b) By substituting a suitable value of x , use your expansion to find an approximation for $1.02^{1/3}$, giving your answer to 6 decimal places. **(2 marks)**

(Total for Question 5 is 8 marks)

6.

A curve has parametric equations $x = 3t^2$, $y = 4t^3$.

- (a) Find dy/dx in terms of t . **(3 marks)**
(b) Find an equation of the tangent to the curve at the point where $t = 1$. **(4 marks)**
(c) Find the exact area between the curve, the x -axis, and the lines corresponding to $t = 0$ and $t = 2$, using $\int y \, (dx/dt) \, dt$. **(6 marks)**

(Total for Question 6 is 13 marks)

7.

A quantity y is modelled by $y = Ab^x$, where A and b are constants. It is known that $y = 50$ when $x = 2$, and $y = 400$ when $x = 5$.

- (a) Using $\ln y = \ln A + x \ln b$, form and solve a pair of simultaneous equations to find A and b . **(5 marks)**

(b) State one limitation of using only two data points to determine a model of this kind. **(2 marks)**

(Total for Question 7 is 7 marks)

8.

A curve is defined by the equation $x^2 + 3xy - y^2 = 9$.

(a) Find dy/dx in terms of x and y , using implicit differentiation. **(5 marks)**

(b) Show that $(3, 0)$ lies on the curve, and find the gradient of the curve at this point. **(4 marks)**

(Total for Question 8 is 9 marks)

9.

Relative to a fixed origin O , points A , B and C have position vectors $a = i + 2j - k$, $b = 4i + 3j + k$, and $c = 6i + 7j + 2k$. The point D is such that $ABCD$ is a parallelogram (in that vertex order).

(a) Find the position vector of D . **(4 marks)**

(b) Show, using vectors, that $ABCD$ is indeed a parallelogram. **(3 marks)**

(c) Find $|AC \rightarrow|$, the length of the diagonal AC . **(2 marks)**

(Total for Question 9 is 9 marks)

10.

$f(x) = x^3 - 4x - 9$.

(a) Show that $f(x) = 0$ has a root between $x = 2$ and $x = 3$. **(2 marks)**

(b) Using $x_0 = 2.5$ and the Newton-Raphson formula $x_{n+1} = x_n - f(x_n)/f'(x_n)$, find x_1 , x_2 and x_3 , giving your answers to 5 decimal places. **(7 marks)**

(Total for Question 10 is 9 marks)

11.

(a) Solve the equation $|2x - 5| = x + 4$. **(5 marks)**

(b) Sketch, on the same diagram, the graphs of $y = |2x - 5|$ and $y = x + 4$, marking the points found in part (a). **(3 marks)**

(Total for Question 11 is 8 marks)

12.

Water drains from a tank so that the volume, V litres, remaining after t minutes satisfies $dV/dt = -0.5\sqrt{V}$. Initially the tank contains 100 litres.

(a) By separating variables, show that $2\sqrt{V} = 20 - 0.5t$. **(5 marks)**

(b) Find the time taken for the tank to empty completely. **(3 marks)**

(Total for Question 12 is 8 marks)

END OF PAPER