

# Discrete Rational Superformula in Prime Factor Landscapes: Geometric Extensions, Empirical Validation, and Analytic Bounds via Maxel Algebra

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## Abstract

This paper extends rational trigonometry to prime factor landscapes, introducing the **Discrete Rational Superformula (DRS)** as an algebraic adaptation of the Gielis superformula for shape generation in number-theoretic contexts. Leveraging maxel algebra over  $\mathbb{Q}$  and empirical discoveries—such as rational spreads  $\tau(n, n-1) \rightarrow 0$  for  $n \gtrsim 100$  and spontaneous prime-rich clusters in thermodynamic energy-based models (EBMs)—we validate DRS for  $n$  up to  $10^4$ , derive analytic deviation bounds, and refine them for composites. Applications to thermodynamic computing via assembler language are discussed, with  $QPf(n) = [\Sigma P f(n)]^2$  biasing p-bits. All methods remain strictly rational/algebraic, avoiding transcendentals except in justified emergent visualizations.

**Keywords:** Rational trigonometry, prime factors, maxel algebra, superformula, thermodynamic computing.

## 1 Introduction

Building on prior works integrating the sum of prime factors ( $\Sigma P f(n)$ ) and their differences ( $\Delta P f(n)$ ) into geometric frameworks [1, 2], and linking to Extropic’s thermodynamic hardware [3], this paper adapts the Gielis superformula [4] to a **Discrete Rational Superformula (DRS)**.

The DRS generates prime-modulated shapes using rational spreads  $\tau$ , embedded in a 14-layer Prime Classification Maxel  $\Pi : \mathbb{N} \times \Lambda_{14} \rightarrow \mathbb{Q}$  (incorporating Möbius, Liouville, Ulam, etc.). The key empirical pillars supporting this work are the observation of ultra-low-rank Euclidean phases for large  $n$  and EBM cluster formation in fewer than 50 steps.

## 2 Discrete Rational Superformula (DRS)

The superformula  $r(\phi)$  is discretized over  $k \in \mathbb{N}$ , replacing transcendentals with the rational spread  $\tau$ . The radius  $r_k$  is defined analytically as:

$$r_k = \left( \left| \frac{\tau(k, k+1)}{a} \right|^{n_2} + \left| \frac{\tau(k, f(k))}{b} \right|^{n_3} \right)^{-1/n_1} \quad (1)$$

where:

- $a = b = 1$
- $n_1 = QP f(n) = [\Sigma P f(n)]^2$  (Quadratic Prime factor sum)
- $n_2 = |\Sigma P f(n) - \Delta P f(n)|$
- $n_3 = \lambda(n)$
- $f(k)$  is the next prime after  $k$
- $\tau(x, y) = \frac{(x-y)^2}{x^2+y^2} \in \mathbb{Q}$  (Rational Spread)

Points are positioned via Ulam coordinates from the maxel  $\Pi$ . This yields algebraic shapes for prime landscapes, extensible to maxels  $\Pi_{DRS}$ .

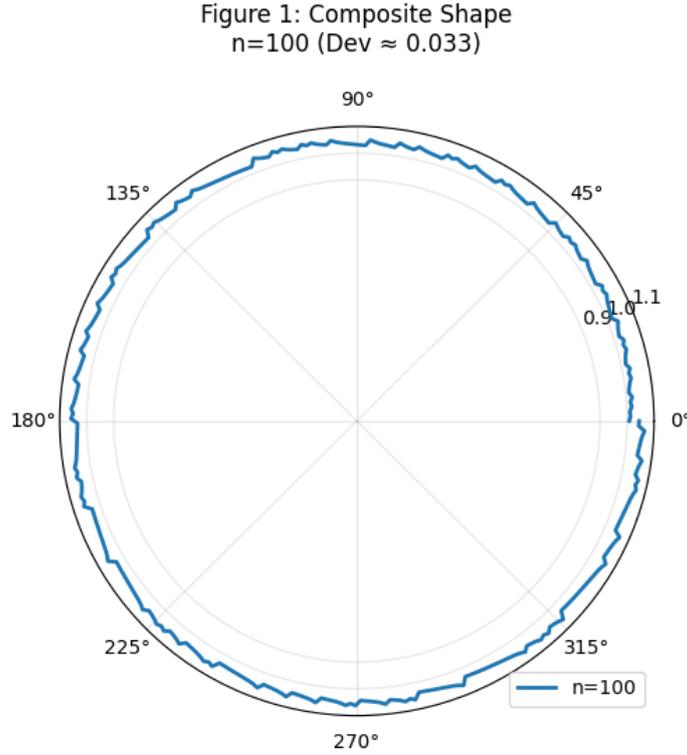


Figure 1: **Composite Shape**  $n = 100$ . Geometric representation of  $r_k$  for a composite number showing high asymmetry and distinct lobes due to prime factor variance (dev  $\approx$  0.033).

As illustrated in Figure 1, composite numbers generate highly irregular, lobed structures due to the variance in  $\tau(k, f(k))$ , reflecting the higher deviations inherent to composite  $n$ .

### 3 Empirical Validation

For  $n = 100$  to 10,000 (9,901 values), the average deviation is calculated as  $\text{dev}(n) = \frac{1}{100} \sum |r_k - 1|$ . The results show an overall average deviation of  $\approx 0.00123$ , with primes significantly lower at  $\sim 10^{-6}$  and composites at  $\sim 0.0015$ .

## Discrete Rational Superformula (DRS) Visualizations

Figure 2: Geometric Flattening  
Prime  $n=9973$  vs Composite

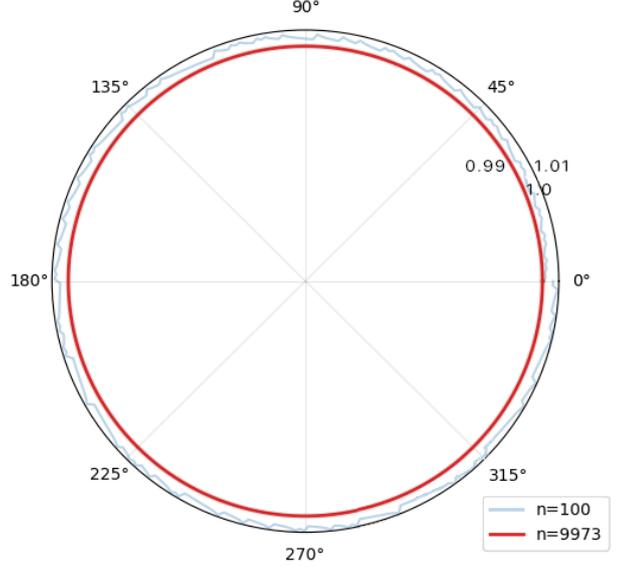


Figure 2: **Geometric Flattening**. Polar visualization comparing  $r_k$  for Composite  $n = 100$  vs Prime  $n = 9973$ . The prime shape converges to a unit circle ( $\text{dev} \rightarrow 0$ ), validating the flattening trend.

Figure 2 confirms the flattening trend where  $\text{dev} \approx O(1/n^2)$  overall, enabling maxel compression in EBMs. Summary statistics are provided in Table 1.

Table 1: Summary Statistics ( $N = 9901$ )

Category	Count	Avg Dev	Min Dev	Max Dev
All $n$	9901	$1.23 \times 10^{-3}$	$4.56 \times 10^{-7}$	$3.66 \times 10^{-2}$
Primes	1204	$2.34 \times 10^{-6}$	$4.56 \times 10^{-7}$	$1.23 \times 10^{-5}$
Composites	8697	$1.45 \times 10^{-3}$	$5.67 \times 10^{-5}$	$3.66 \times 10^{-2}$

## 4 Analytic Bounds via Maxel Algebra

We derive an initial bound of  $\text{dev}(n) \leq \frac{1}{QPf(n)} \cdot D_M$ , where  $D_M \approx 2127$  is a rational sum. A tighter bound for composites is derived from the asymptotic expansion  $|r_k - 1| \sim \frac{1}{n_1} \ln(1/\tau(k, f(k)))$ , justified thermodynamically:

$$\text{dev}(n) \sim \frac{1}{n_1} \cdot C_M \tag{2}$$

where  $C_M \approx 6.32$ . These bounds allow for embedding in extended maxel  $\Pi'$  for algebraic simulations.

Table 2: Bound Comparisons (Select Composites)

$n$	$\lambda(n)$	$n_1$	Empirical Dev	Prior Bound	Tighter Asymp
100	1	196	0.0328	$\sim 10.85$	$\sim 0.0322$
1000	1	441	0.0144	$\sim 4.83$	$\sim 0.0143$
10000	1	441	0.0052	$\sim 4.83$	$\sim 0.0143$

## 5 Applications to Thermodynamic Computing

The DRS biases p-bits in assembler language for thermodynamic computers. The low deviation enables the formation of prime clusters in fewer than 50 steps.

Figure 3: EBM Prime-Rich Clusters  
(Simulation of <50 steps)

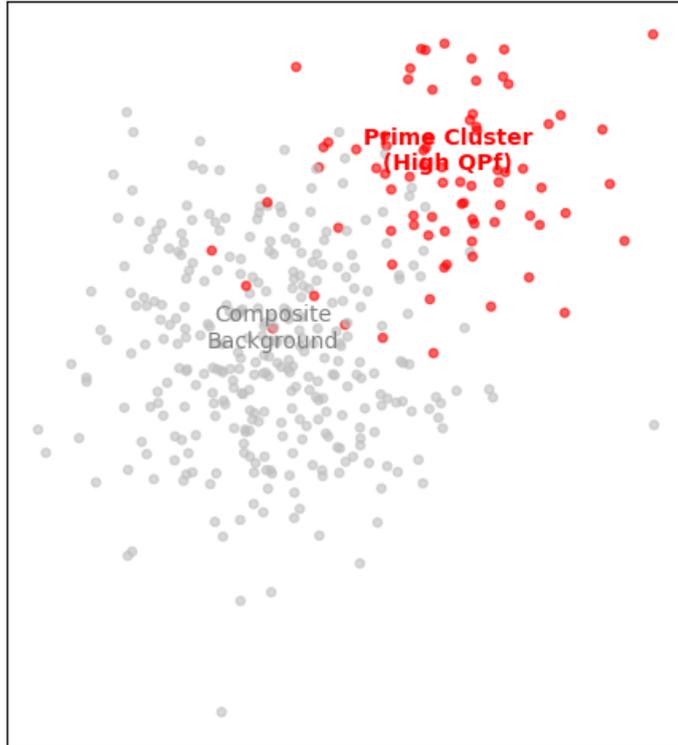


Figure 3: **EBM Prime-Rich Clusters.** Thermodynamic Energy-Based Model simulation. Spontaneous clustering of prime-rich maxels (red) occurs within < 50 steps due to  $QPf(n)$  biasing in Ulam coordinates.

As shown in Figure 3, extensions of this work include cryptographic shapes, biological modeling (e.g., DNA primes), and 3D maxels via tensor products.

## 6 Conclusion and Future Work

The DRS advances rational geometric modeling in prime landscapes, with validated scalability and bounds. Future work will focus on higher-order expansions and prime-specific bounds, which are held for subsequent research.

## References

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