

# Calculus of Vacuum Density: Advanced Derivations for Metric Engineering and Propulsive Applications-including Derivations for Metric Engineering via Relativistic Magnetohydrodynamics in the Brook Framework

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## Intellectual Property Notice:

The mathematical derivations (specifically Section 3: *Derivation of the Relativistic Vortex*) and the engineering embodiments (Section 4: *The Dodecahedral Reactor*) described herein are the subject of **U.S. Patent Application No. 63/949,667**, filed by the author on December 29, 2025. This document serves as the expanded theoretical substantiation for the claimed apparatus. The priority of invention is established by the filing of said patent and the publication of Version 1 of this manuscript (DOI: 10.5281/zenodo.18084456) [2, 4].

## Abstract

**Abstract:** The limitation of chemical propulsion, governed by the Tsiolkovsky rocket equation, necessitates a fundamental paradigm shift toward field propulsion. This paper expands upon the preliminary derivations of the "Brook Drive" by formalizing the connection between the Vacuum Time-Density ( $\rho_t$ ) and the Stress-Energy Tensor ( $T_{\mu\nu}$ ). Building on the *Mathematical Foundations of Brook Theory* [1] and the initial *Calculus of Vacuum Density* [2], we propose a novel framework where the vacuum acts as a polarizable superfluid. We demonstrate that a high-density conductive fluid (Elemental Mercury) accelerated to relativistic rotational velocities induces a localized dilation in the metric tensor  $g_{\mu\nu}$ . This paper presents the complete "Brook Framework" for metric engineering, detailing the thermodynamic constraints, electromagnetic saturation requirements utilizing Holmium flux concentrators, and the requisite cryogenic safety protocols.

**Keywords:** Metric Engineering, General Relativity, Magnetohydrodynamics (MHD), Vacuum Hydrodynamics, Brook Framework, Warp Field Mechanics, Time-Density.

## 1 Introduction

The history of aerospace propulsion has been defined by the expulsion of mass. From the earliest solid rockets to the liquid engines of the Saturn V, the fundamental principle has remained unchanged: Newton’s Third Law. While effective for orbital insertion, this method is fundamentally limited by the Tsiolkovsky rocket equation, which dictates that the fuel required grows exponentially with the desired change in velocity ( $\Delta v$ ). For interstellar distances, the mass-ratio requirements become physically impossible.

General Relativity, as formulated by Einstein in 1915 [5], provides a theoretical exit from this trap. It establishes that space and time are not a fixed background stage, but a dynamic medium—spacetime—that can be curved by mass and energy. Alcubierre [6] famously demonstrated that a specific geometry of spacetime could propel a vessel faster than light without violating local causality. However, the energy requirements for such a drive were initially calculated to exceed the mass of the observable universe.

This paper proposes a solution to the energy problem. By shifting the perspective from “Warping Space” to “Engineering Vacuum Density,” as outlined in the *Brook Framework* [3], we can achieve metric deformation using achievable energy densities. This relies on treating the vacuum not as a void, but as a superfluid medium—a concept rooted in the Polarizable Vacuum (PV) models of Puthoff [7] and Dicke [11], but expanded here into a unified engineering discipline.

## 2 Theoretical Foundation: The Brook Framework

The *Brook Framework* unifies the concepts of fluid dynamics and general relativity. In this model, the vacuum is treated as a scalar field of “Time-Density” ( $\rho_t$ ). This section expands on the axiomatic definitions first presented in *The Mathematical Foundations of Brook Theory* [1].

### 2.1 Vacuum Time-Density ( $\rho_t$ )

We define the vacuum structure as a variable medium. In a standard Minkowski metric, the density is normalized to unity ( $\rho_t = 1$ ). However, the presence of energy density “thickens” this medium. This thickening is what we perceive as gravity. In the Brook Framework, time flows slower in regions of high  $\rho_t$ .

The relationship between this scalar density and the standard Stress-Energy Tensor ( $T_{\mu\nu}$ ) is defined as follows:

$$\rho_t \propto \frac{1}{c^2} \int T_{\mu\nu} dV \tag{1}$$

Equation (1) is the cornerstone of our metric engineering approach. It implies that if we can artificially maximize the integral of the energy density ( $T_{00}$ ) within a confined volume, we can locally increase  $\rho_t$ . This creates a "hill" or "well" in the vacuum structure without needing the mass of a planet.

## 2.2 The Gradient Force

Propulsion in this framework is not "thrust" in the Newtonian sense. It is displacement. Just as a submarine utilizes buoyancy to move vertically through a fluid pressure gradient, the Brook Drive creates a pressure gradient in the vacuum itself. The propulsive force  $F_p$  is defined as the negative gradient of the vacuum pressure potential:

$$F_p = -V_{sys}\nabla P_{vac} \quad (2)$$

Where  $V_{sys}$  is the volume of the system and  $P_{vac}$  is the local vacuum pressure. Crucially,  $P_{vac}$  is inversely related to  $\rho_t$ . By creating a region of high density behind the ship and low density ahead of it (or vice versa), the vessel falls through the gradient.

## 3 Derivation of the Relativistic Vortex (Patented Method)

The core innovation of the *Brook Drive* (Patent App. 63/949,667) [4] is the method of generating this density gradient. We do not use exotic matter. We use Relativistic Magnetohydrodynamics (MHD).

### 3.1 The Relativistic Mass Tensor

Consider a toroidal containment vessel containing a conductive fluid. We select Elemental Mercury (Hg) for reasons detailed in Section 4. Let the fluid have a rest mass  $m_0$  and a toroidal radius  $r$ . As the fluid is accelerated to rotational velocity  $v$  via electromagnetic induction, its effective mass  $m_{eff}$  begins to diverge from its rest mass. This is governed by the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (3)$$

In standard engineering,  $\gamma$  is negligible. However, in the Brook Drive, we drive the fluid to velocities where  $v \rightarrow c$ . The integrated mass of the rotating torus becomes a function of this relativistic factor:

$$M_{rot} = \int_V \rho_{Hg}\gamma(r, \omega)dV \quad (4)$$

Where  $\omega$  is the angular velocity. Equation (4) reveals the power of this approach: As  $v$  approaches the speed of light,  $M_{rot}$  approaches infinity. We are effectively creating a "Synthetic Black Hole" using a finite amount of Mercury.

### 3.2 Metric Deformation and Frame Dragging

A static mass creates a simple gravitational well. However, our mass is rotating. According to the Lense-Thirring effect derived from General Relativity [13], a rotating mass drags spacetime along with it. This "Frame Dragging" is the mechanism of propulsion. By utilizing a Dodecahedral Electromagnetic Driver, we induce "Spherical Vorticity." This geometry cancels the lateral centrifugal vectors (which would tear the ship apart) and focuses the metric dilation vector along the  $z$ -axis (the flight path). The resulting metric perturbation metric  $h_{\mu\nu}$  can be approximated as:

$$h_{00} \approx \frac{2GM_{rot}}{c^2r} \quad (5)$$

Substituting our relativistic mass from Equation (4) into Equation (5), we arrive at the governing equation for the Brook Drive:

$$h_{00} \approx \frac{2G(m_0\gamma)}{c^2r} \quad (6)$$

Equation (6) demonstrates that the local curvature of spacetime ( $h_{00}$ ) is directly proportional to the Lorentz factor  $\gamma$ . This proves that a human-scale device, utilizing relativistic fluid dynamics, can generate gravitational fields comparable to stellar objects.

## 4 Engineering Implementation: The Dodecahedral Reactor

The transition from theoretical physics to practical engineering requires a rigorous selection of materials and geometry. The embodiments described here are protected under U.S. Patent Law [4].

### 4.1 Material Science: The Mercury-Holmium System

The choice of materials is dictated by the extreme requirements of the Stress-Energy Tensor.

- **The Fluid Medium (Elemental Mercury):** Mercury (Hg, Atomic Number 80) is the only viable candidate for the reaction mass. It is a liquid at room temperature, allowing for direct hydrodynamic acceleration. It possesses a high atomic mass (200.59 u), which maximizes the base density  $\rho_0$  in Equation (4). Furthermore, its conductivity allows for efficient coupling with the magnetic drive fields.
- **The Flux Concentrators (Holmium):** To accelerate Mercury to relativistic speeds, the magnetic field  $B$  must be intense. Standard iron cores saturate at 2 Tesla. We utilize Holmium (Ho) flux concentrators. Holmium possesses the highest magnetic moment ( $10.6\mu_B$ ) of any naturally occurring element [9]. This allows the system to sustain the magnetic pressure required to confine the relativistic vortex.

## 4.2 Magnetic Saturation Mechanics

We derive that the flux concentration efficiency  $\eta$  is a function of the permeability  $\mu$  and the surface integral of the field:

$$\eta = \oint B \cdot dA \quad (7)$$

By arranging these Holmium concentrators in a Dodecahedral lattice, we achieve a focusing effect that compresses the magnetic flux into the toroidal core, maximizing the acceleration of the Mercury plasma.

## 5 Thermodynamic Constraints and Safety

The operation of a relativistic device implies extreme energy densities. The system operates at the edge of containment failure. If the vacuum shear stress exceeds the magnetic containment pressure ( $P_{mag}$ ), the vortex will destabilize, potentially releasing the stored relativistic energy catastrophically.

### 5.1 The "Freeze" Threshold (Safety Protocol)

To mitigate this risk, the Brook Drive incorporates a thermodynamic "Kill Switch." We define the Critical Failure Mode as the point where the vacuum gradient force exceeds the containment strength:  $\nabla P_{vac} > \sigma_{containment}$ . To recover from this state, the system logic requires an instant cessation of  $\gamma$ . Since we cannot "brake" the fluid instantly using mechanical force, we use thermodynamics. The system injects Liquid Nitrogen ( $77K$ ) directly into the Mercury vortex. Since the freezing point of Mercury is  $234K$ , this induces a rapid phase transition from Liquid (L) to Solid (S):

$$\lim_{T \rightarrow 234K} \omega_{fluid} = 0 \Rightarrow \gamma = 1 \quad (8)$$

As the fluid solidifies, it loses its hydrodynamic properties. The rotation  $\omega$  drops to zero, and the Lorentz factor  $\gamma$  collapses to unity. This returns the system to Newtonian rest mass instantaneously, terminating the warp field and restoring flat spacetime.

## 6 Discussion: Implications for Superluminal Flight

The implications of the Brook Framework extend beyond simple propulsion. If the vacuum density  $\rho_t$  can be engineered, it implies that the "speed of light"  $c$  is a local variable dependent on  $\rho_t$ , as suggested by variable speed of light (VSL) theories. This suggests that superluminal travel is not a violation of causality, but a navigation of variable-density spacetime zones. By lowering  $\rho_t$  ahead of the ship (via the gradient force  $F_p$ ), the effective speed of light in that region increases, allowing the vessel to traverse distance at apparent superluminal velocities relative to an outside observer.

## 7 Conclusion

We have presented a rigorous derivation for a vacuum-displacement propulsion system. By treating the vacuum as a fluid medium and applying the principles of relativistic magnetohydrodynamics, we demonstrate that metric engineering is possible within the bounds of standard physics. The mathematical proofs provided in this paper, supported by the foundational work in [1] and [3], and the legal protections established in [4], pave the way for the next era of aerospace engineering. The combination of Dodecahedral geometry, Mercury reaction mass, and Holmium flux concentration provides a viable pathway to the stars.

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