

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Notes: Domain and Range 2.0

Do Now:

- 1) Find the value(s) of  $x$  that will make the following function undefined.

$$f(x) = \frac{2}{x + 5}$$

$$\begin{aligned} x + 5 &= 0 \\ x &= -5 \end{aligned}$$

- 2) Find all the real values of  $x$  that will make the following function imaginary.

$$f(x) = \sqrt{2x - 20}$$

$$\begin{aligned} 2x - 20 &< 0 \\ +20 &+20 \end{aligned}$$

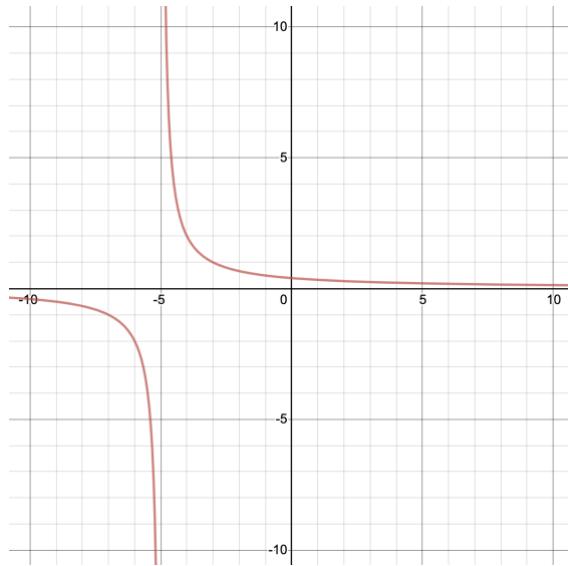
$$\begin{aligned} \frac{2x}{2} &< \frac{20}{2} \\ x &< 10 \end{aligned}$$

## What Should I Be Able to Do?

- I can algebraically find the domain and range of linear and square root equations.
- I can algebraically find the domain of rational equations.

Let's take a look at the graphs of the functions in our Do Now:

1)  $f(x) = \frac{2}{x+5}$



How does the graph of  $f(x) = \frac{2}{x+5}$  show us the domain of the function?

There is no point on the graph when  $x = -5$ .

How does the  $(x,y)$  table of  $f(x) = \frac{2}{x+5}$  support our findings?

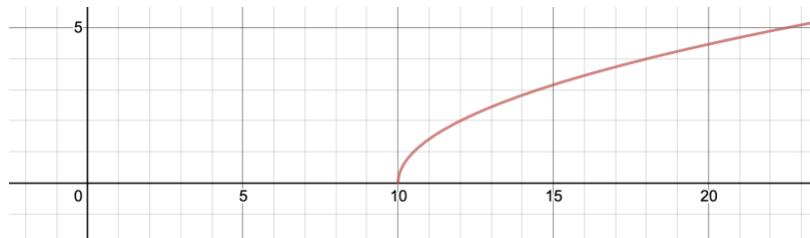
When  $x = -5$ , the  $x,y$  table has an ERROR message.

What is the range of  $f(x) = \frac{2}{x+5}$ ?

$$(-\infty, 0) \cup (0, \infty)$$

All real numbers such that  $y \neq 0$ .

2)  $f(x) = \sqrt{2x - 20}$



How does the graph of  $f(x) = \sqrt{2x - 20}$  show us the domain of the function?

There are no graphed points for  $x < 10$ .

How does the  $(x,y)$  table of  $f(x) = \sqrt{2x - 20}$  support our findings?

For all  $x < 10$ , the y values have an ERROR message.

What is the range of  $f(x) = \sqrt{2x - 20}$ ?

$$[0, \infty)$$

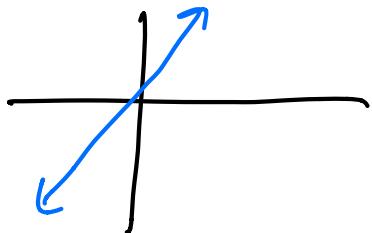
Determine the domain of the function  $f(x) = \frac{3x}{x+1}$ .

$$\begin{array}{c} x+1 \neq 0 \\ -1 -1 \\ x \neq -1 \end{array}$$

$$(-\infty, -1) \cup (-1, \infty)$$

All real numbers such that  $x \neq -1$ .

Determine the domain and range of the function  $f(x) = 2x + 1$ .



Domain: All real numbers

Range: All real numbers.

Determine the domain and range of the function  $f(x) = \sqrt{x-3}$ .

$$\begin{array}{rcl} x-3 \geq 0 & & \\ +3 & +3 & \end{array}$$

Domain: All real numbers such that  $x \geq 3$ .

$$x \geq 3$$

Range: All real numbers such that  $y \geq 0$ .

Determine the domain of the function  $f(x) = \frac{1}{\sqrt{x-3}}$ .

$$\sqrt{x-3} \neq 0$$

but also

$$x-3 \geq 0$$

Combine both stipulations to get

$$\begin{array}{rcl} x-3 > 0 & & x > 3 \\ +3 & +3 & \end{array}$$

Domain: All real numbers such that  $x > 3$ .

Determine the domain of the function  $f(x) = \frac{\sqrt{x-2}}{x-7}$ .

$$x-2 \geq 0$$
$$+2 \quad +2$$

$$x \geq 2$$

$$x-7 \neq 0$$
$$+7 \quad +7$$

$$x \neq 7$$

Must satisfy both stipulations

$$[2, 7) \cup (7, \infty)$$

Determine the domain of the function  $f(x) = \frac{\sqrt{x+5}}{\sqrt{x-4}}$ .

$$x+5 \geq 0$$
$$-5 \quad -5$$

$$x \geq -5$$

$$x-4 > 0$$
$$+4 \quad +4$$

$$x > 4$$

Must satisfy both stipulations

$$(4, \infty)$$

## Checkpoint:

Find the domain and range of each function.

1)  $f(x) = \sqrt{x + 13.5}$

$$x + 13.5 \geq 0$$

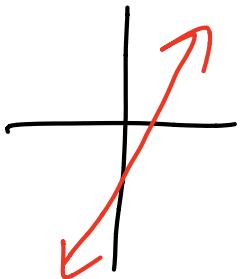
$$x \geq -13.5$$

D:  $x \geq -13.5$   
 R:  $y \geq 0$

2)  $2y - 14x = -\frac{8}{7} + 19x$   
 $+14x \quad +14x$

$$\frac{2y}{2} = \frac{-\frac{8}{7} + 23x}{2}$$

$$y = \frac{23}{2}x - \frac{8}{14}$$



D: All real numbers  
 R: All real numbers

Find the domain of each function.

3)  $f(x) = \frac{x-7}{x+14}$

$$x + 14 \neq 0$$

$$-14 \quad -14$$

$$x \neq -14$$

All real numbers such that  $x \neq -14$

$$(-\infty, -14) \cup (-14, \infty)$$

or

5)  $f(x) = \frac{\sqrt{x}}{x-6}$

$$\sqrt{x} \rightarrow x \geq 0$$

$$x - 6 \neq 0$$

$$x \neq 6$$

Put these together!

$[0, 6) \cup (6, \infty)$

4)  $f(x) = \frac{3x-4}{\sqrt{4x-23}}$

$$4x - 23 > 0$$

$$4x > 23$$

$$x > 5.75$$

All real numbers such that  $x > 5.75$

or  $(5.75, \infty)$

6)  $f(x) = \frac{\sqrt{x-2}}{\sqrt{x+1}}$

$$x - 2 \geq 0$$

$$x \geq 2$$

$$x + 1 > 0$$

$$x > -1$$

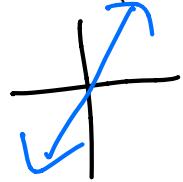
Put these together!

$x \geq 2$

# Success Criteria

- I can algebraically find the domain and range of linear and square root equations.  
Find the domain and range of each function.

1)  $f(x) = \frac{14}{3}x - 1$



D:  $\mathbb{R}$   
R:  $\mathbb{R}$

2)  $f(x) = \sqrt{3x + 20}$

$$3x + 20 \geq 0$$

$$3x \geq -20$$

$$x \geq -\frac{20}{3}$$

D:  $x \geq -\frac{20}{3}$   
R:  $y \geq 0$

- I can algebraically find the domain of rational equations.  
Find the domain of each function.

1)  $f(x) = \frac{3x}{5x - 6}$

$$5x - 6 \neq 0$$

$$5x \neq 6$$

$$x \neq \frac{6}{5}$$

D: All real numbers  
excluding  $\frac{6}{5}$ .

2)  $f(x) = \frac{x-1}{\sqrt{6x+15}}$

$$6x + 15 > 0$$

$$6x > -15$$

$$x > -\frac{15}{6}$$

D:  $(-\frac{15}{6}, \infty)$

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Classwork: Domain and Range 2.0

Find the domain and range of each function.

1)  $f(x) = 2\sqrt{\frac{3}{2}x + 5}$

$$\frac{3}{2}x + 5 \geq 0$$

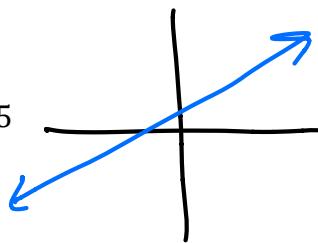
$$\frac{3}{2}x \geq -5$$

$$x \geq -\frac{10}{3}$$

$$D: x \geq -\frac{10}{3}$$

$$R: y \geq 0$$

2)  $x = \frac{1}{3}y + 5$



$D: \mathbb{R}$   
 $R: \mathbb{R}$

Find the domain of each function.

3)  $f(x) = \frac{x-7}{x}$

$$x \neq 0$$

$$D: (-\infty, 0) \cup (0, \infty)$$

4)  $f(x) = \frac{2x+1}{\sqrt{x-15}}$

$$x-15 > 0$$

$$x > 15$$

$$D: x > 15$$

5)  $f(x) = \frac{\sqrt{x}}{\sqrt{x-7}}$

$$x \geq 0$$

$$\begin{aligned} x-7 &> 0 \\ x &> 7 \end{aligned}$$

$$x > 7$$

6)  $f(x) = \frac{\sqrt{x+8}}{2x-\frac{3}{4}}$

$$x+8 \geq 0$$

$$x \geq -8$$

$$2x - 0.75 \neq 0$$

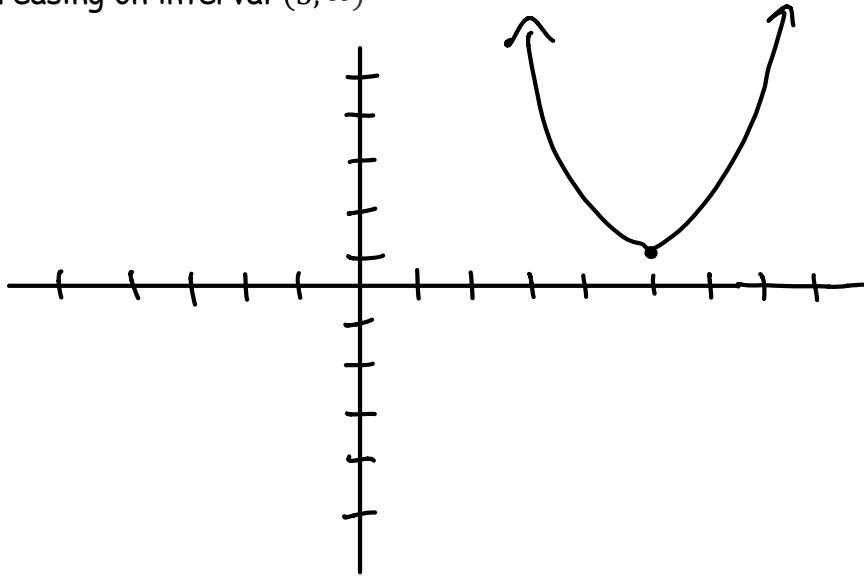
$$2x \neq 0.75$$

$$x \neq 0.375$$

$$[-8, 0.375) \cup (0.375, \infty)$$

7) Sketch the graph of  $f(x)$  using the following information.

- $f(x)$  is decreasing on interval  $(-\infty, 5)$
- $f(5) = 1$
- $f(x)$  is increasing on interval  $(5, \infty)$



Completely simplify each expression.

$$8) \left( \frac{74x^{-15}y^5z^{-1/6}}{4y^{-8}z^{8/6}} \right)^{-2}$$

$$\left( \frac{4y^{-8}z^{8/6}}{74x^{-15}y^5z^{-1/6}} \right)^2$$

$$\left( \frac{4}{74} \cdot \frac{1}{x^{-15}} \cdot \frac{y^{-8}}{y^5} \cdot \frac{z^{8/6}}{z^{-1/6}} \right)^2$$

$$\left( \frac{2}{37} x^{15} y^{-13} z^{3/2} \right)^2$$

$$\frac{4}{1369} x^{30} y^{-26} z^3$$

$$\boxed{\frac{4x^{30}z^3}{1369y^{26}}}$$

$$9) \sqrt[3]{-\frac{1}{343}x^{20}y^{33}z^{70}}$$

$$\sqrt[3]{-\frac{1}{343}} \quad \sqrt[3]{x^{20}} \quad \sqrt[3]{y^{33}} \quad \sqrt[3]{z^{70}}$$

$$-\frac{1}{7} \sqrt[3]{x^{18}} \sqrt[3]{x^2} \quad y''' \sqrt[3]{z^{69}} \sqrt[3]{z^2}$$

$$-\frac{1}{7} x^6 \sqrt[3]{x^2} \quad y''' z^{23} \sqrt[3]{z}$$

$$-\frac{1}{7} x^6 y''' z^{23} \sqrt[3]{x^2 z}$$

$$10) \frac{(\frac{1}{64})^{-2/3} - (2)^{3/2}}{(\frac{1}{16})^{3/4} - (2)^{5/2}}$$

$$(64)^{2/3} = \sqrt[3]{64}^2 = 16$$

$$\sqrt{2^3} = \sqrt{8} = 2\sqrt{2}$$

$$\sqrt[4]{16}^3 = 2^3 = 8$$

$$\sqrt{2^5} = \sqrt{32} = 4\sqrt{2}$$

$$\frac{16-2\sqrt{2}}{8-4\sqrt{2}} = \frac{2(8-\sqrt{2})}{2(4-2\sqrt{2})} = \frac{8-\sqrt{2}}{4-2\sqrt{2}} \cdot \frac{(4+2\sqrt{2})}{(4+2\sqrt{2})}$$

$$\frac{32+16\sqrt{2}-4\sqrt{2}-2(2)}{16+8\sqrt{2}-8\sqrt{2}-4(2)} = \frac{28+12\sqrt{2}}{8} = \boxed{\frac{7+3\sqrt{2}}{2}}$$

Solve each of the following equations.

$$11) \frac{-3x^{\frac{3}{2}}}{-3} = \frac{-24}{-3}$$

$$(x^{\frac{3}{2}})^{\frac{2}{3}} = (8)^{\frac{2}{3}}$$

$$x = (\sqrt[3]{8})^2$$

$$x = (2)^2$$

$$\boxed{x = 4}$$

Check:

$$-3(4)^{\frac{3}{2}} = -24$$

$$\sqrt{-24} = -24$$

$$12) -\frac{4}{5}(x+9)^{\frac{5}{3}} + 1 = -79,999$$

$$\left(-\frac{5}{4}\right) \left[-\frac{4}{5}(x+9)^{\frac{5}{3}}\right] = \left[-80,000\right] \left(-\frac{5}{4}\right)$$

$$(x+9)^{\frac{5}{3}} = (100,000)^{\frac{3}{5}}$$

$$x+9 = \sqrt[5]{10,000}$$

Check:

$$x+9 = (10)^3 - \frac{4}{5}(991+9)^{\frac{5}{3}} + 1 =$$

$$x+9 = 1000$$

$$\boxed{x = 991}$$

13) Solve the following system of equations.

$$\begin{array}{r} 12x + 28y + 4z = -24 \\ + -5x - 6y - 4z = 33 \\ \hline 7x + 22y = 9 \end{array}$$

$$\begin{bmatrix} x & y & z \\ \begin{array}{l} 3x + 7y + z = -6 \\ -5x - 6y - 4z = 33 \\ 4x - 3y + 9z = -71 \end{array} & \begin{array}{l} x(-9) \\ -27x - 63y - 9z = 54 \\ + 4x - 3y + 9z = -71 \\ \hline -23x - 66y = -17 \end{array} \end{bmatrix}$$

$$3(7x + 22y = 9)$$

$$-23x - 66y = -17$$

$$\begin{array}{r} 21x + 66y = 27 \\ + -23x - 66y = -17 \\ \hline -2x = 10 \end{array}$$

$$x = -5$$

$$7(-5) + 22y = 9$$

$$-35 + 22y = 9$$

$$22y = 44$$

$$y = 2$$

$$\boxed{\begin{array}{l} x = -5 \\ y = 2 \\ z = -5 \end{array}}$$

$$3(-5) + 7(2) + 2 = -6$$

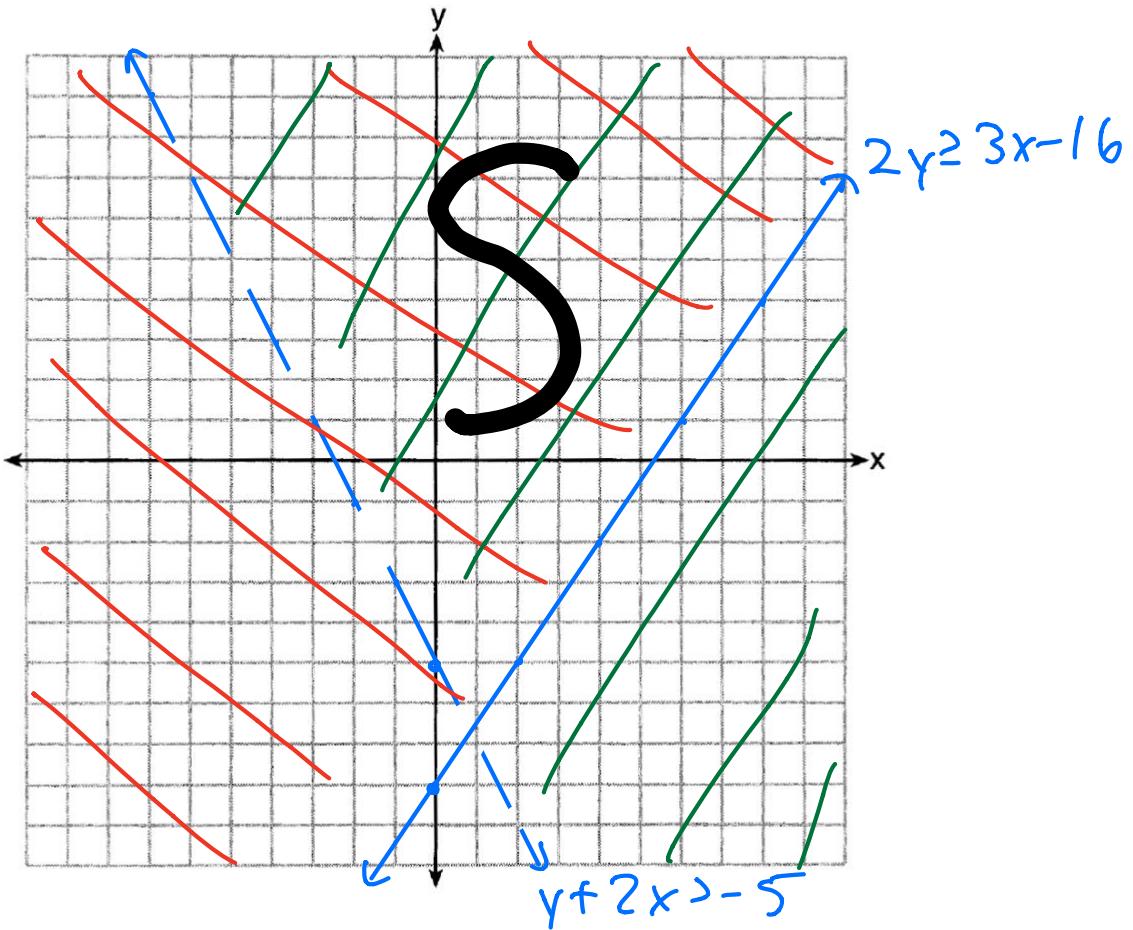
$$\begin{array}{r} -15 + 14 + 2 = -6 \\ -1 + 2 = -6 \end{array} \quad z = -5$$

14)

Graph the following system of inequalities on the set of axes below:

$$\begin{aligned}2y &\geq 3x - 16 \\ \frac{2y}{2} &\geq \frac{3x - 16}{2} \\ y + 2x &> -5 \\ -2x &- 2x\end{aligned}$$

$$\begin{aligned}y &\geq \frac{3}{2}x - 8 \\ y &> -2x - 5\end{aligned}$$



Based upon your graph, explain why  $(6,1)$  is a solution to this system and why  $(-6,7)$  is *not* a solution to this system.

$(6,1)$  is a solution because it is in the double-shaded region on a solid line.

$(-6,7)$  is not a solution because it is on a dotted line.

15)

Given that  $f(x) = 2x + 1$ , find  $g(x)$  if  $g(x) = 2[f(x)]^2 - 1$ .

$$\begin{aligned} & 2(2x+1)^2 - 1 \\ & 2(2x+1)(2x+1) - 1 \\ & 2(4x^2 + 2x + 2x + 1) - 1 \\ & 2(4x^2 + 4x + 1) - 1 \\ & \underline{8x^2 + 8x + 2 - 1} \\ & \boxed{8x^2 + 8x + 1} \end{aligned}$$

16) Completely simplify the following expression.

$$\begin{aligned} & -5i^{102} + 6.25i^{41} + \frac{17}{3}i^{28} - i^{1,123} \\ & -5(-1) + 6.25(i) + \frac{17}{3}(1) - (-i) \\ & 5 + 6.25i + \frac{17}{3} + i \\ & \boxed{\frac{33}{3} + 7.25i} \end{aligned}$$

17) Solve for  $f$  in the equation below.

$$\frac{a+b}{c} = \frac{d+e}{f}$$
$$\frac{f(a+b)}{atb} = \frac{c(d+e)}{atb}$$
$$\boxed{f = \frac{c(d+e)}{a+b}}$$