Name: ______________________ Date: ________

Notes: Complex Numbers

What Should I Be Able to Do?

- I can define the set of complex numbers.

- I can add and subtract complex numbers.

- I can multiply complex numbers.

- I can explain and show why it is mathematically incorrect to multiply square roots that have negative radicands.

- I can divide complex numbers.
- I understand the similarities and differences between operations involving complex numbers and operations involving radicals.

Complex Numbers: The set of numbers in the form of

$$a + bi$$

where a and b are real numbers and i is the imaginary unit, $i = \sqrt{-1}$.

- The **real part** of this complex number is a.
- The **imaginary part** of this complex number is b.

a) What can be said about a complex number where $a = 0$?

$0 + bi$

bi

When $a = 0$, the simplified complex number bi is an imaginary number.

b) What can be said about a complex number where $b = 0$?

$a + 0i$

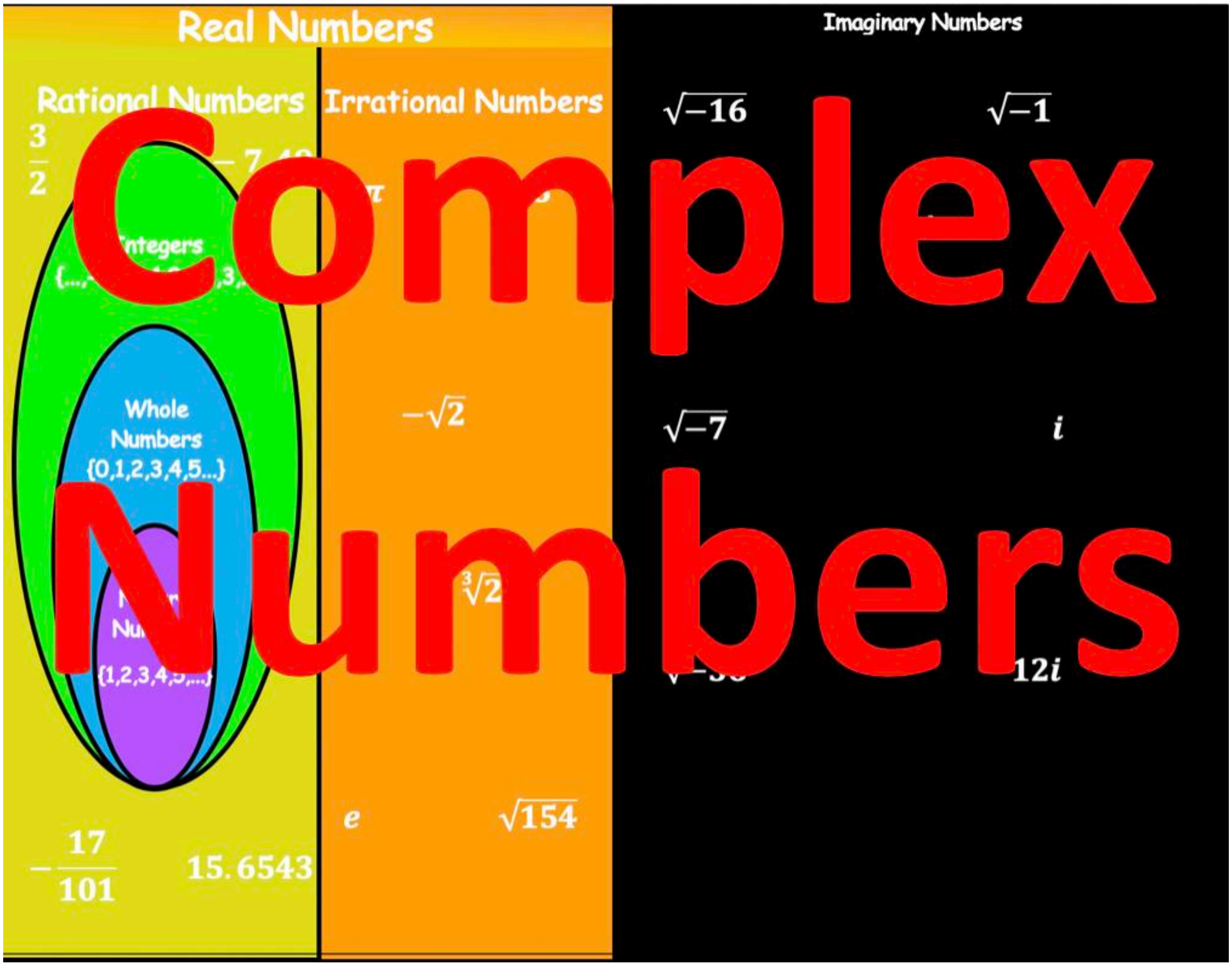
a

When $b = 0$, the simplified complex number a is a real number.

c) Draw a conclusion using your responses in parts a and b.

The set of complex numbers includes all real and imaginary numbers.

The set of complex numbers encompasses both the set of real numbers and the set of imaginary numbers!



Do Now: Simplify each expression.

1) $(3+5\sqrt{6})-(7-2\sqrt{6})$

$3+5\sqrt{6}-7+2\sqrt{6}$

$\boxed{-4+7\sqrt{6}}$

2) $(1-\sqrt{18})(3+\sqrt{72})$

$(1-3\sqrt{2})(3+6\sqrt{2})$

$3+6\sqrt{2}-9\sqrt{2}-18(2)$

$3+6\sqrt{2}-9\sqrt{2}-36$

$\boxed{-33-3\sqrt{2}}$

3) $(3+5i)-(7-2i)$

$3+5i-7+2i$

$\boxed{-4+7i}$

4) $(1-i^{17})(3+i^{19})$

$((1-i)(3+(-i)))$

$((1-i)(3-i))$

$3-i-3i+i^2$

$3-i-3i-1$

$\boxed{2-4i}$

5) $\frac{3}{(2-\sqrt{10})}\frac{(2+\sqrt{10})}{(2+\sqrt{10})}$

$\frac{6+3\sqrt{10}}{4+2\sqrt{10}-2\sqrt{10}-10}$

$\frac{6+3\sqrt{10}}{-6}$

$\frac{2+\sqrt{10}}{-2}$

$\boxed{\frac{-2-\sqrt{10}}{2}}$

6) $\frac{(3)}{(2-i)}\frac{(2+i)}{(2+i)}$

$\frac{6+3i}{4+2i-2i-i^2}$

$\frac{6+3i}{4-(-1)}$

$\frac{6+3i}{5}=\boxed{\frac{6}{5}+\frac{3}{5}i}$

7)
Why do operations with radicals and operations with complex numbers follow similar rules?

Because i represents a radical number, $i=\sqrt{-1}$.

Adding and Subtracting Complex Numbers

Simplify each of the following expressions in $a + bi$ form.

> **Step 1:** Simplify any terms (if necessary).
>
> **Step 2:** Add LIKE TERMS
> (Add the real parts and then add the imaginary parts)

1) $(11 - 6i) - (7 + 9i)$

$11 - 6i - 7 - 9i$

$\boxed{4 - 15i}$

2) $(11 - 6i^7) - (7 + 9i^{13})$

$(11 - 6(-i)) - (7 + 9i)$

$(11 + 6i) - (7 + 9i)$

$11 + 6i - 7 - 9i$

$\boxed{4 - 3i}$

3) $(3 + \sqrt{-4}) + (7 - \sqrt{-25})$

$(3 + 2i) + (7 - 5i)$

$\boxed{10 - 3i}$

4) $(19 - 10\sqrt{-125}) - (-6 + 3\sqrt{-48})$

$(19 - 10i\sqrt{25}\sqrt{5}) - (-6 + 3i\sqrt{16}\sqrt{3})$

$(19 - 10i(5)\sqrt{5}) - (-6 + 3i(4)\sqrt{3})$

$(19 - 50i\sqrt{5}) - (-6 + 12i\sqrt{3})$

$19 - 50i\sqrt{5} + 6 - 12i\sqrt{3}$

$\boxed{25 - 50i\sqrt{5} - 12i\sqrt{3}}$ OR

$\boxed{25 - (50\sqrt{5} + 12\sqrt{3})i}$

Multiplying Complex Numbers

Simplify each of the following expressions in $a + bi$ form.

1) $-5i(2+8i)$

$-10i - 40i^2$

$-10i - 40(-1)$

$-10i + 40$

$\boxed{40 - 10i}$

2) $(10-i)(3-4i)$

$30 - 40i - 3i + 4i^2$

$30 - 43i + 4(-1)$

$30 - 43i - 4$

$\boxed{26 - 43i}$

3) $(\sqrt{-4})(\sqrt{-9})$

Correct Method:

$(\sqrt{-4})(\sqrt{-9})$

$2i(3i)$

$6i^2$

$6(-1)$

$\boxed{-6}$

Incorrect Method:

$\sqrt{-4}\,\sqrt{-9}$

$\sqrt{(-4)(-9)}$

$\sqrt{36}$

$\cancel{\boxed{6}}$

When multiplying squares roots of negative numbers, why can you not multiply the radicands first?

Because it cancels out the imaginary part of the numbers.

4) $(9+\sqrt{-64})(5-\sqrt{-400})$

$(9+8i)(5-20i)$

$45 - 180i + 40i - 160i^2$

$45 - 180i + 40i - 160(-1)$

$\boxed{205 - 140i}$

5) $(1-i)^2$

$(1-i)(1-i)$

$1 - i - i + i^2$

$1 - i - i + (-1)$

$\boxed{-2i}$

Dividing Complex Numbers

Simplify each of the following expressions in $a + bi$ form.

1) $\frac{(7-8i)}{(5i)}\cdot\frac{i}{i}$

$$\frac{7i-8i^2}{5i^2}$$

$$\frac{7i-8(-1)}{5(-1)}$$

$$\frac{7i+8}{-5}$$

$$\frac{-8-7i}{5}$$

$$= \boxed{\frac{-8}{5}-\frac{7}{5}i}$$

2) $\frac{(8+i)}{(3-2i)}\cdot\frac{(3+2i)}{(3+2i)}$

$$\frac{24+16i+3i+2i^2}{9+6i-6i-4i^2}$$

$$\frac{24+19i+2(-1)}{9-4(-1)}$$

$$\frac{22+19i}{13} = \boxed{\frac{22}{13}+\frac{19}{13}i}$$

3) $\frac{3-\sqrt{-9}}{4+\sqrt{-25}}$

$$\frac{(3-3i)}{(4+5i)}\cdot\frac{(4-5i)}{(4-5i)}$$

$$\frac{12-15i-12i+15i^2}{16-20i+20i-25i^2}$$

$$\frac{12-27i+15(-1)}{16-25(-1)}$$

$$\frac{-3-27i}{41}$$

$$\frac{-3-27i}{41}$$

$$\boxed{\frac{-3}{41}-\frac{27}{41}i}$$

Complex Conjugate: The complex conjugate of the number $a + bi$ is $a - bi$. Likewise, the complex conjugate of $a - bi$ is $a + bi$.

Success Criteria

- I can define the set of complex numbers.

The set of numbers in the form of $a+bi$ where a and b are real numbers and i is the imaginary unit, $i=\sqrt{-1}$.

- I can add and subtract complex numbers.

Simplify each of the following expressions in $a + bi$ form.

1) $\left(14 - \frac{7}{3}i\right) + (-9 + 9i)$

$(14-9)+\left(-\frac{7}{3}i+9i\right)$

$\boxed{5+\frac{20}{3}i}$

2) $(2 - 3\sqrt{-245}) - (-5 - \sqrt{-80})$

$(2-3i\sqrt{49}\sqrt{5})-(-5-i\sqrt{16}\sqrt{5})$

$(2-3i(7)\sqrt{5})-(-5-i4\sqrt{5})$

$(2-21i\sqrt{5})+(+5+4i\sqrt{5})$

$\boxed{7-17i\sqrt{5}}$

- I can multiply complex numbers.

Simplify each of the following expressions in $a + bi$ form.

1) $\left(\frac{1}{2} + 24i\right)\left(\frac{2}{3} - 2i\right)$

$\frac{2}{6}-i+16i-48i^2$

$\frac{1}{3}-15i-48(-1)$

$\frac{1}{3}-15i+48$

$\boxed{\frac{145}{3}-15i}$

2) $(1 - \sqrt{-169})(3 + \sqrt{-1})$

$(1-13i)(3+i)$

$3+i-39i-13i^2$

$3+i-39i+13$

$\boxed{16-38i}$

- I can explain and show why it is mathematically incorrect to multiply negative radicands that are inside of square roots.

When multiplying squares roots of negative numbers, why can you not multiply the radicands first?

Because if you multiply $\sqrt{-a}\cdot\sqrt{-b}$ by just multiplying the radicands, you obtain $\sqrt{ab}$ which does not take into effect the imaginary parts.

- I can divide complex numbers.

Simplify each of the following expressions in $a + bi$ form.

1) $\frac{(12+10i)}{(9i)} \cdot \frac{i}{i}$

$$\frac{12i+10i^2}{9i^2}$$

$$\frac{12i-10}{-9}$$

$$\frac{-12i+10}{9}$$

$$\frac{10-12i}{9}$$

$$\boxed{\frac{10}{9} - \frac{4}{3}i}$$

2) $\frac{(-2+i)^2}{4-3i}$

$(-2+i)(-2+i)$

$4-2i-2i+i^2$

$4-2i-2i-1$

$3-4i$

$$\frac{3-4i}{4-3i} \cdot \frac{(4+3i)}{(4+3i)}$$

$$\frac{12+9i-16i-12i^2}{16+12i-12i-9i^2}$$

$$\frac{12-7i-12(-1)}{16-9(-1)}$$

$$\frac{24-7i}{25}$$

$$\boxed{\frac{24}{25} - \frac{7}{25}i}$$

- I understand the similarities and differences between operations involving complex numbers and operations involving radicals.

Similarities:
- Adding/Subtracting Like Terms
- Multiply by Conjugate in numerator and denominator

Differences:
- Must write complex numbers with an i before doing any operations
- Must write in $a+bi$ form

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Classwork: Complex Numbers

Simplify each of the following expressions in $a + bi$ form.

1) $(17 - i^{79}) - (-28 + \sqrt{-225})$

$(17-(-i))-(-28+15i)$

$(17+i)+(28-15i)$

$\boxed{45-14i}$

2) $\left(\frac{3}{4} + 7i\right)(5 - \frac{1}{2}i)$

$3.75-0.375i+35i-3.5i^2$

$3.75+34.625i-3.5(-1)$

$\boxed{7.25+34.625i}$

3) $\frac{(20-2i)}{(-6i)}\frac{i}{i}$

$\frac{20-2i^2}{-6i^2}$

$\frac{20i-2(-1)}{-6(-1)} = \frac{2+20i}{6}$

$\frac{1+10i}{3} = \boxed{\frac{1}{3}+\frac{10}{3}i}$

4) $(1 - i)^3$

$(1-i)(1-i)(1-i)$

$(1-i-i+i^2)(1-i)$

$(-2i)(1-i)$

$-2i+2i^2$

$-2i-2$

$\boxed{-2-2i}$

5) $\frac{(3-4i)}{(5-2i)}\frac{(5+2i)}{(5+2i)}$

$\frac{15+6i-20i-8i^2}{25+10i-10i-4i^2}$

$\frac{15+6i-20i-8(-1)}{25-4(-1)}$

$\frac{23-14i}{29} = \boxed{\frac{23}{29}-\frac{14}{29}i}$

6) $(\sqrt{-4} - 2)^2 - (3 - 8i)$

$(2i-2)^2-(3-8i)$

$(2i-2)(2i-2)-(3-8i)$

$(4i^2-4i-4i+4)-(3-8i)$

$(-4-4i-4i+4)-(3-8i)$

$-8i+(-3+8i)$

$\boxed{-3}$

7) Which statement is *not* always true? (Select all that apply)

(1) The product two integers is a whole number. $-1(5)=-5$

(2) The product of two rational numbers is rational.

(3) The product of two irrational numbers is irrational. $\pi\left(\frac{1}{\pi}\right)=1$

(4) The product of two real numbers is a real number.

8)

Which expression is equivalent to $(3k-2i)^2$, where i is the imaginary unit?

(1) $9k^2 - 4$

(2) $9k^2 + 4$

(3) $9k^2 - 12ki - 4$

(4) $9k^2 - 12ki + 4$

$(3k-2i)(3k-2i)$

$9k^2 - 6ki - 6ki + 4i^2$

$9k^2 - 12ki - 4$

9)

Elizabeth tried to find the product of $(2+4i)$ and $(3-i)$, and her work is shown below.

$$\begin{aligned} &(2+4i)(3-i) \\ &= 6 - 2i + 12i - 4i^2 \\ &= 6 + 10i - 4i^2 \\ &= 6 + 10i - 4(1) \\ &= 6 + 10i - 4 \\ &= 2 + 10i \end{aligned}$$

Identify the error in the process shown and determine the correct product of $(2+4i)$ and $(3-i)$.

$i^2 = -1$ but Elizabeth substituted 1 in for i^2.

$(2+4i)(3-i)$

$6 - 2i + 12i - 4i^2$

$6 + 10i - 4i^2$

$6 + 10i - 4(-1)$

$6 + 10i + 4$

$\boxed{10 + 10i}$

10)

The expression $6xi^3(-4xi+5)$ is equivalent to

(1) $2x - 5i$

(2) $-24x^2 - 30xi$

(3) $-24x^2 + 30x - i$

(4) $26x - 24x^2i - 5i$

$-24x^2i^4 + 30xi^3$

$-24x^2 + 30x(-i)$

$-24x^2 - 30xi$

11) Solve for x: $(2-8i)-(-2+xi)=4-6i$

$2-8i+2-xi=4-6i$

$4-8i-xi=4-6i$

$-4 \qquad -4$

$-8i-xi=-6i$

$+8i \qquad +8i$

$\frac{-xi}{-i}=\frac{2i}{-i}$

$\boxed{x=-2}$

12) Show that the product of $a+bi$ and its conjugate is a real number.

$(a+bi)(a-bi)$

$a^2-abi+abi-b^2i^2$

$a^2-b^2(-1)$

a^2+b^2

13) Solve for x in the equation $\left(\frac{1}{512}\right)^{-2/3}=\left[(8-2x)^{-3/2}\right]^{-2/3}$.

$64=8-2x$

$-8 \quad -8$

$\frac{56}{-2}=\frac{-2x}{-2}$

$\boxed{x=-28}$

Check:

$\frac{1}{512}=(8-2(-28))^{-3/2}$

$\frac{1}{512}=\frac{1}{512}$ ✓

14) State the conjugate of $13-\sqrt{-48}$ expressed in simplest $a+bi$ form.

$13+i\sqrt{16}\sqrt{3}$

$\boxed{13+4i\sqrt{3}}$

15) Impedance measures the opposition of an electrical circuit to the flow of electricity. The total impedance in a particular circuit is given by the formula $Z_r=\frac{Z_1Z_2}{Z_1+Z_2}$. What is the total impedance of a circuit, Z_r, if $Z_1=1+2i$ and $Z_2=1-2i$?

$\frac{(1+2i)(1-2i)}{(1+2i)+(1-2i)}=\frac{1-2i+2i-4i^2}{2}$

$\frac{1-4(-1)}{2}=\frac{1+4}{2}$

$\boxed{\frac{5}{2}}$