FLUID MECHANICS

FLUID

- A substance that can flow is fluid.
- Shear modulus of fluid is almost zero.
- They have very low rigidity and start flowing when shear stress is applied.
- Includes gases, liquids and plasma.

Ideal fluid

- Incompressible: density is constant throughout
- flow is irrotational: no turbulences in flow
- non viscous: no viscosity
- steady flow: velocity of fluid is constant at each point.

Properties of fluid

- They don't resist deformation, get permanently deformed.
- They have ability to flow.
- They have the ability to take shape of container.

FLUID MECHANICS

HYDROSTATICS — study of fluid at rest

HYDRODYNAMICS — study of fluids in motion

DENSITY

density, $\rho = \lim_{\Delta v \to 0} \frac{\Delta m}{\Delta v} = \frac{dm}{dv}$ • scalar quantity • Unit: kg/m^3

relative density $= \frac{\text{density of substance}}{\text{density of ref. substance}}$

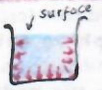
specific gravity $= \frac{\text{density of substance}}{\text{density of water}}$

important values:

$\rho_{water} = 1000\ kg/m^3 = 1\ g/cc$

$\rho_{mercury} = 13600\ kg/m^3 = 13.6\ g/cc$

PRESSURE



- Fluid at rest exerts a force on surface of container normally.
- Fluid pressure is the normal force exerted by a fluid at rest per unit surface area.

$P = \frac{F}{A}$ • Pressure is scalar. • Unit: Pascal, bar, torr, atm.

important conversions:

1 atm $= 1.013\times10^5$ Pa 760 Torr = 1 bar

1 bar $= 10^5$ Pa

FLUID PRESSURE

$P = \frac{F}{A} = \frac{mg}{A}$ $m = V\times\rho$, $m = Ah\times\rho$

$P = \frac{Ah\rho g}{A}$ $P = \rho g h$

$P = \rho g h + P_{atm}$

- As depth increases, pressure also increases. (linearly)
- In a same layer (horizontal) pressure remain constant.
- Pressure is independent of size & shape of container.

Pressure at bottom of each is same.

HYDROSTATIC PARADOX

Pressure at the base of all container is same, instead one would think that C has great pressure.

PRESSURE EXERTED ON WALLS OF VESSEL

$dA = dy\,w$ $P = y\rho g$

$\int dF = \int P\,dA$

$F = w\rho g \int y\,dy$ $\frac{F}{A} = \frac{h}{2}\rho g$

$F = \frac{w\rho g y^2}{2}$ $P_{avg} = \frac{\rho g h}{2}$

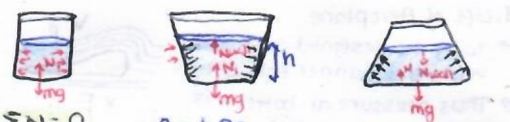
Avg. pressure exerted on wall is equal to pressure at centre of wall.

ABSOLUTE PRESSURE & GAUGE PRESSURE

- Absolute pressure is the total pressure at that point.
- Gauge pressure is the difference between absolute and atmospheric pressure.

$P - P_0 = h\rho g$

FORCE BY SIDE WALLS OF CONTAINER



$\Sigma N = 0$ $P = h\rho g$ $N_1 = mg + N_{wall}$

F at bottom $= N_1$ $N_{wall} = N_1 - mg$

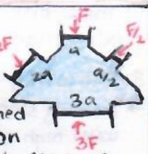
$N_1 = h\rho g A$ $N_{wall} = h\rho g A - mg$

$mg = N_1 + N_{wall}$

$N_{wall} = mg - h\rho g A$

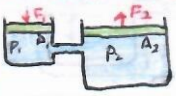
PASCAL'S LAW

"Pressure applied at any point of an enclosed fluid at rest is transmitted equally and undiminished to every point of fluid and also on the walls of container, provided effect of gravity is neglected."



→ In short, change in pressure is constant.

APPLICATIONS OF PASCALS LAW



Hydraulic lift

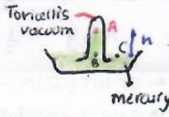
$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

Hydraulic Brake system

MERCURY BAROMETER

→ invented by Toricelli

→ measures atmospheric pressure.



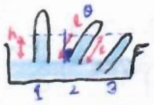
$P_B = P_C$ → $P_{atm} = h\rho g$

$= 76\times10^{-2}\times13.6\times10^3\times9.81$

$P_{atm} = 101325$ Pa

$P_{atm} = 1.013\times10^5$ Pa $= 760$ mm of Hg.

TILTED BAROMETER

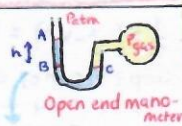


Vertical height will always remain constant, instead length of tube filled will change.

For tube 2, $P_{atm} = \rho g l\cos\theta$

MANOMETER

- used to measure for a gas
- consists of high density liquid to show large diffn in arms of manometer.



Open end manometer

Here, $P_B = P_C$

$P_{gas} = P_A + \rho g h$

$P_{gas} = P_0 + \rho g h$

$P_{gas} = \rho g h$

Closed Manometer

VARIATION IN ACCELERATED SYSTEM

1. Vertically accelerated system.

$P_B - P_A = \rho g_{eff} h$

$P_B - P_A = \rho(g+a_0)h$

$P_B - P_A = \rho g_{eff} h$

$P_B - P_A = \rho(g-a_0)h$

2. Horizontally accelerated system

$\tan\theta = \frac{a_0}{g}$ Here $h =$

NOTE: The open surface is always normal to the net acceleratn of system.

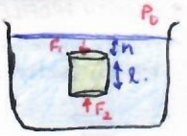
TRICK: Take layers parallel to surface and h ⊥ to surface (travel ⊥ to surface)

3. Closed accelerated system.

- Take layers ⊥ to effective acceleratn.

ARCHIMEDES PRINCIPLE

"A body whether partially or completely submerged in a fluid is buoyant upward by a force due to fluid pressure. This is equal to weight of displaced liquid"



Buoyant force

$F_B = F_2 - F_1$

$F_B = (P_0 + h\rho g + l\rho g)A - (P_0 + h\rho g)A$

$F_B = Al\rho g$

$F_B = V\rho g$ $F_{Buoyant} = V\rho g$ (V: Volume of submerged body; ρ: density of liquid)

LAW OF FLOATATION

$V\rho g = Mg$

- Horizontal components of buoyant force get cancelled and net vertical force appears.

APPARENT WEIGHT IN LIQUID

- Reading is always Tension/g

$T + F_B = Mg$

$T = Mg + V\rho g$ $T/g = M\left(1 + \frac{\rho_l}{\rho_s}\right)$

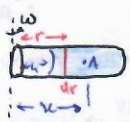
$T = Mg + \frac{M\rho_l g}{\rho_s}$

PRESSURE IN ROTATING TUBE

$dP = dr\,\rho\omega^2 r$

$\int dP = \int \rho\omega^2 r\,dr$ $P = \frac{\rho}{2}\omega^2 L^2$

$P_A = P_0 + \rho\omega^2 \frac{r^2}{2}\Big|_{L_1}^{x}$



$P_A = P_0 + \frac{\rho\omega^2}{2}(x^2 - L_1)^2$

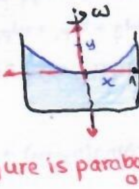
TRICK:

$P_A = P_0$ + Pressure at A if tube was full of water − Pressure exerted by empty part if filled with water.

ROTATING FLUID

$\rho g y = \frac{\rho}{2}\omega^2 x^2$, P at A.

$y = \frac{\omega^2 x^2}{2g}$ → Eqn of parabola. In 3D, figure is paraboloid



Volume of cone $= \frac{1}{3}\times$ Volume cylinder

Volume of paraboloid $= \frac{1}{2}\times$ cylinder.

FLUID DYNAMICS

Steady flow: At a point, velocity of fluid remains constant over time.

Incompressible flow: Fluid density remains constant.

Non-viscous flow: Neglecting viscous forces.

Flow line: path of an individual particle in fluid

Flow tube: imaginary bundle of flow lines bound by imaginary wall. Fluids in adjacent flow tube cannot mix.

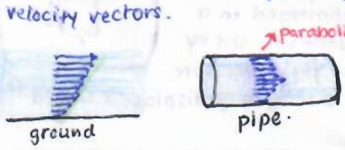
STREAMLINES

- curve whose tangent at any point in the flow is in directn of flow velocity at that point.
- Streamlines & Flowlines are identical for steady flow.
- Dense stream lines: velocity is more.

- Sparse stream lines: velocity is less.

* VELOCITY PROFILE OF FLOWING FLUID *

→ velocity profile is drawn by joining the tips of velocity vectors.



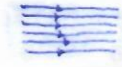
* STREAMLINE FLOW / LAMINAR FLOW *

→ smooth flow of liquid, with velocity smaller than certain critical velocity.

→ velocity of flow at given point is constant.

→ streamlines are always parallel.

→ Through a pipe, streamlines are parallel to axis of tube.



* TURBULENT FLOW *

→ irregular and unsteady flow of fluid when its velocity increases beyond critical velocity.

→ velocity of fluid at a point is not constant.

→ fluid may experience rotational motion.

→ direction of velocity is random.

* EQUATION OF CONTINUITY *

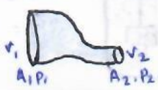
"Volume rate of flow of an incompressible fluid for a steady flow is same throughout the flow".

$dm_1 = dm_2$

$\therefore \rho \times volume_1 = \rho \times volume_2$

$\therefore A_1 v_1 \Delta t = A_2 v_2 \Delta t$

$\therefore A_1 v_1 = A_2 v_2 = constant.$



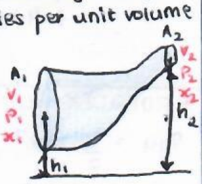
$Q = Av$, volume rate of flow = Q. $Av = \frac{dV}{dt}$.

$\frac{dV}{dt}$ ⇒ Volume flux $\frac{\rho\, dV}{dt}$ ⇒ mass flux.

In short, inflow = outflow.

* BERNOULLIS EQUATION *

"The work done per unit volume of a fluid by the surrounding fluid is equal to sum of change of kinetic and potential energies per unit volume that occur during flow"



By work, energy theorem

$W_g + W_{P_1} + W_{P_2} = \Delta K.E$

$-\Delta mg(h_2-h_1) + P_1 A_1 x_1 + P_2 A_2 x_2 = \frac{1}{2}\Delta m(v_2^2 - v_1^2).$

$\therefore -\Delta mg(h_2-h_1) + P_1 \Delta V - P_2 \Delta V = \frac{1}{2}\Delta m(v_2^2 - v_1^2)$

$\therefore -\rho g(h_2-h_1) + P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2).$

$\therefore P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$

$\therefore P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$

$\therefore P + \frac{1}{2}\rho v^2 + \rho g h = constant$

(pressure energy per volume; K.E per unit volume; P.E per unit volume)

$\frac{P}{\rho g} + \frac{v^2}{2g} + h = constant.$

(pressure head; velocity head; Potential head)

* In short :- energy conservatn in fluid.

* PRESSURE-VELOCITY TRADE OFF *

For fluid at same horizontal level, by B.T,

$P + \frac{1}{2}\rho v^2 = constant.$

pressure ↑ velocity ↓

pressure ↓ velocity ↑

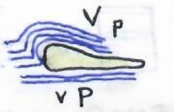
Note: When fluid falls freely, all particles of fluid have same pressure, i.e Patm.

* APPLICATIONS OF BERNOULIS THEOREM *

1. Lift of Aeroplane

→ wing is designed such that velocity at upper face is high.

→ Thus pressure at bottom is high and that lifts aeroplane.



2. Atomizer spray.

→ Through a piston, air is blown at a high speed. Thus, low pressure is created at tip of tube.

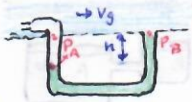
→ Thus, the liquid in tube rises and moves with high speed.

3. Blowing off of Roofs.

→ During storms, the wind blows with high speed on roof tops, thus low pressure is created. Inside the roof, there is pressure Patm & thus the roof blows off.

* PITOT TUBE *

→ used to calculate speed of aeroplane.



$P_A = P_B + \rho_l g h$

$\therefore P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_g^2$

$P_A + 0 = P_B + \frac{1}{2}\rho g v_g^2$

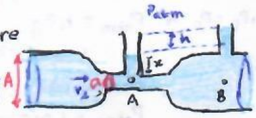
$P_A - P_B = \frac{1}{2}\rho g v_g^2$

$h\rho_l g = \frac{1}{2}\rho g v_g^2$

$v_g = \sqrt{\frac{2h\rho g}{\rho_{gas}}}$

* VENTURIMETER *

→ Device used to measure flow velocity of liquids in pipelines.



$Av_1 = av_2 \Rightarrow v_2 = \left(\frac{A}{a}\right)v_1$

$P_A = P_0 + x\rho g$ $P_B = P_0 + (x+h)\rho g.$

$P_B - P_A = h\rho g.$

By B.T,

$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$

$\frac{1}{2}\rho(v_2^2 - v_1^2) = h\rho g$

$v_2^2 - v_1^2 = 2hg.$

$-v_1^2 + \left(\frac{A}{a}\right)^2 v_1^2 = 2hg.$

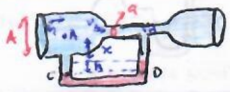
$v_1 = \sqrt{\frac{2gh}{(A/a)^2 - 1}}$

Step 1: Eqn of continuity

Step 2: Play with pressure

Step 3: Bernoullis theorem.

* EXAMPLE *



$Av_1 = av_2 \rightarrow v_2 = \frac{A}{a}v_1$

$P_C = P_D$

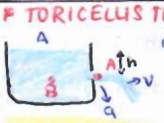
$P_A + (x+h)\rho_w g = P_B + x\rho_w g + h\rho_{Hg} g.$

$P_A + h\rho_w g = P_B + h\rho_{Hg} g.$

$P_A + \frac{\rho_w}{2} v_1^2 = P_B + \frac{\rho_w}{2} v_2^2.$

$h\rho_w g - \frac{\rho_w}{2} v_1^2 = h\rho_{Hg} g - \frac{\rho_w}{2} v_2^2$ → and then solve.

* TORICELLIS THEOREM *



$a \lll A$ $av = Av_B$ ∴ $v_B \approx 0$.

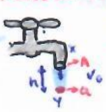
By B.T,

$P_{atm} + \frac{1}{2}\rho v^2 = P_{atm} + h\rho g + \frac{\rho}{2}v_B^2$

$v = \sqrt{2gh}$

Maximum Range at $h = H/2$

* CROSS-SECTION OF FREELY FALLING LIQUID *



$P_1 + \frac{1}{2}\rho v_0^2 + \rho g h = P_1 + \frac{1}{2}\rho v^2$

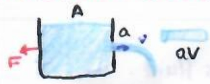
$v = \sqrt{v_0^2 + 2gh}$

$Av_0 = av$

$Av_0 = a\sqrt{v_0^2 + 2gh}$

$a = \frac{Av_0}{\sqrt{v_0^2 + 2gh}}$

* REACTION FORCE DUE TO EJECTN OF FLUID *



$F_{thrust} = \frac{dm}{dt} u_{rel.}$

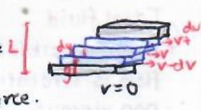
$= \rho a v\, u_{rel}$

$F_{thrust} = \rho a v^2$

$F_{th} = \rho a v^2$, (v = velocity)

* VISCOUS FORCE *

Viscosity is property of fluid, by virtue of which, the relative motn betn different layers of fluid experience dragging force.



→ In liquids due to short range molecular cohesive forces.

→ In gases due to collision betn fast moving molecules.

$F_v = \eta A \frac{dv}{dx}$ → A = area of layer; → η = coefficient of viscosity

→ Liquids: density ↑, viscosity ↑

→ Gases: density ↑, viscosity ↓

* COEFFICIENT OF VISCOSITY *

η → depends on fluid → depends on temp.

SI unit: Poiseuille (PL) (Ns/m²)

CGS: Poise

1 PL = 10 Poise

→ $\eta_{liq.} >> \eta_{gas}$.

→ Liquids: Temp ↑ η ↓ → less intermol. cohesive forces.

→ Gases: Temp ↑ η ↑ → increased collision.

* ADHESIVE AND COHESIVE FORCES *

Attractive forces b/w diffn materials

Attractive forces b/w same material

* STOKES LAW *

Viscous force (F_v) acting on a small sphere falling through a viscous medium is directly proportional to the radius of sphere(r), its velocity (v) through fluid, and coeff. of viscosity (η) of fluid.

$F_v = 6\pi\eta r v$ → For sphere.

* TERMINAL VELOCITY *

* As speed increases, resistance from fluid increases, at a particular time the forces acting are balanced and accn becomes zero. The body moves with constant terminal velocity.

$W = F_v + F_B$

$F_v = W - F_B$

$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho_s g - \frac{4}{3}\pi r^3 \rho_l g$

$v = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_s - \rho_l)$

Don't Learn, Derive!

* CRITICAL VELOCITY AND REYNOLDS NUMBER *

→ The velocity beyond which a streamline flow becomes turbulent is called critical velocity.

$V_c = R_N \frac{\eta}{r\rho}$ → radius of tube = r

→ R_N → Reynolds Number → pure number (No dimn).

याद: तीन 2 sound.

* POISEUILLE'S EQUATION *

→ used to measure flow rate in viscous fluid.

$Q = \frac{\pi r^4}{8\eta l}(\Delta P).$