MECHANICS

1. SCALARS AND VECTORS :-

Scalar: Physical quantity that has only magnitude

Vector: Physical quantity that has magnitude and direction and obeys vector law of additn

TYPES OF VECTORS

Zero Vector: Vector having zero magnitude and undefined direction.

Unit Vector: Vector having unit magnitude and given direction. Unit vectors along x y z are $\hat{i}, \hat{j}, \hat{k}$ ...

Resultant Vector: vector having same effect as produced by many vectors.

Negative vector: Vectors having same magnitude but different direction.

Equal vector: Vectors having same direction and same magnitude.

Position vector:- Vector that gives position of a particle at a point w.r.t origin of a chosen co-ordinate axis.

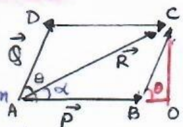
Unit vector = $\hat{u}_m = \dfrac{\vec{M}}{|\vec{M}|}$

TRIANGLE LAW OF ADDITION :-

If two vectors describing the same physical quantity are represented in magnitude and direction by two sides of triangle in order, then the third side represents the resultant vector taken in opposite sense. $\vec{A}+\vec{B}=\vec{R}$

PARALLELOGRAM LAW OF ADDITION :-

If two vectors of same type originating from a point are represented by two adjacent sides of parallelogram then the resultant vector is given in magnitude and direction by the diagonal of parallelogram originating from same point.



Proof:- $AC^2 = AO^2 + OC^2$

$= (AB+BO)^2 + (BC^2 - OB^2)$

$= AB^2 + BC^2 + 2(AB)(BO)$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

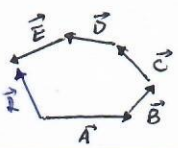
$\therefore R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\tan\alpha = \dfrac{OC}{AO} = \dfrac{OC}{AB+BO} = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

$\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

POLYGON LAW OF ADDITION

$\vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E} = \vec{R}$

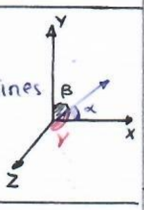


DIRECTION COSINES

$A_x = A\cos\alpha$, $A_y = A\cos\beta$, $A_z = A\cos\gamma$

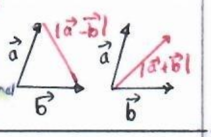
$\cos\alpha, \cos\beta, \cos\gamma$ are called direction cosines

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$



DIAGONALS OF PARALLELOGRAM

If the angle betn $\vec{a}$ and $\vec{b}$ is acute then minor diagonal is of $|\vec{a}-\vec{b}|$ and major diagonal of $|\vec{a}+\vec{b}|$



ADDITION OF VECTORS

$\vec{A}+\vec{B} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_z+B_z)\hat{k}$

DOT PRODUCT :-

→ Vector . Vector = Scalar

→ $\vec{P}\cdot\vec{Q} = PQ\cos\theta$ → $\vec{A}\cdot\vec{B}=0$ then $\vec{A}\perp\vec{B}$

→ $\vec{P}\cdot\vec{Q} = \vec{Q}\cdot\vec{P}$ and $\vec{P}\cdot(\vec{Q}+\vec{R}) = \vec{P}\cdot\vec{Q}+\vec{P}\cdot\vec{R}$

→ $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

→ $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

→ $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

CROSS PRODUCT :-

→ Vector x Vector = Vector

→ $\vec{P}\times\vec{Q} = PQ\sin\theta\,\hat{u}$

→ Direction of new vector is acc. to Right Hand Rule

→ $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

→ $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$

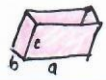
$\hat{j}\times\hat{i} = -\hat{k}$, $\hat{k}\times\hat{j} = -\hat{i}$, $\hat{i}\times\hat{k} = -\hat{j}$

→ $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ → $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

→ $m\vec{A}\times n\vec{B} = (mn)(\vec{A}\times\vec{B})$

→ $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ → $\vec{A}\times\vec{B}$ = area of $||^{gm}$

→ Volume of parallelopipede $= (\vec{a}\times\vec{b})\cdot\vec{c}$



FINDING COMPONENT OF VECTOR ALONG GIVEN VECTOR

$\vec{A}$ is given vector, and $\vec{B}$ is given vector whose component is to be found.

$|\vec{A_c}| = (\vec{A}\cdot\hat{b})\,\hat{b}$

e.g. $3\hat{i}+4\hat{j}$ along $\hat{i}+\hat{j}$

$|\vec{A_c}| = 3\hat{i}+4\hat{j}\cdot\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$|\vec{A_c}| = \dfrac{7\sqrt{2}}{2}$ $\therefore \vec{A_c} = \dfrac{7\sqrt{2}}{2}\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$\vec{A_c} = \dfrac{7}{2}(\hat{i}+\hat{j})$

KINEMATICS - 1D

* DISTANCE & DISPLACEMENT

Distance	Displacement
actual length of path travelled	minimum distance betn initial & final positn of particle.
→ Scalar quantity	→ Vector Quantity
→ Unit : m	→ Unit : m

Distance ≥ Displacement.

* AVERAGE SPEED & INSTANTANEOUS SPEED

$v_{avg} = \dfrac{\text{total distance}}{\text{time}}$ $\quad v_{inst} = \dfrac{dx}{dt}$

• Unit : m/s • Scalar Quantity

* AVERAGE VELOCITY & INSTANTANEOUS VEL.

$\vec{v}_{avg} = \dfrac{\vec{x}_2-\vec{x}_1}{t_2-t_1}$ $\quad \vec{v}_{inst} = \dfrac{dx}{dt}$

$\vec{v}_{avg} = \dfrac{\Delta\vec{x}}{\Delta t}$

• Unit = m/s • Vector quantity

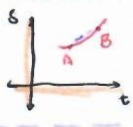
* AVERAGE ACCELERATION & INSTAN. ACCELERATn

$\vec{a}_{avg} = \dfrac{\vec{v}_2-\vec{v}_1}{t_2-t_1}$ $\quad \vec{a}_{inst} = \dfrac{d\vec{v}}{dt} = \dfrac{d\vec{v}}{dx}\cdot\dfrac{dx}{dt}$

$\vec{a}_{avg} = \dfrac{\Delta\vec{v}}{\Delta t}$ $\quad \vec{a}_{inst} = v\dfrac{d\vec{v}}{dx}$

• Unit : m/s² • Vector quantity

* POSITION - TIME GRAPH

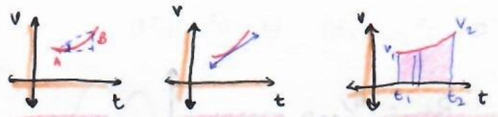


avg. velocity = slope of chord AB

inst velocity = slope of tangent $= \dfrac{ds}{dt}$

• Slope +ve : v +ve
• slope -ve : v < 0
• Slope 0 : v = 0

* VELOCITY - TIME GRAPH



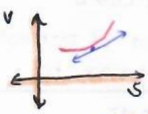
avg. accn	inst. accn	displacement
Slope of chord	Slope of tangent	Area under curve
$a_{vg} = \dfrac{v_2-v_1}{t_2-t_1}$	$a_{inst} = \dfrac{dv}{dt}$	$s = \int_{t_1}^{t_2} v\,dt$

* ACCELERATION - TIME GRAPH

Slope of tangent — Jerk $= \dfrac{d\vec{a}}{dt}$

Area under curve — change in velocity — $(v-u) = \int_{t_1}^{t_2} a\,dt$

* VELOCITY - DISPLACEMENT GRAPH



$v\cdot\dfrac{dv}{ds} = a \quad \therefore \dfrac{a}{v} = \dfrac{dv}{ds}$

Slope $= \dfrac{dv}{ds} = \dfrac{a}{v}$

Slope of tangent $= a/v$

* EQUATIONS OF MOTION

$\vec{v} = \vec{u}+\vec{a}t$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

→ accn must be constant

→ applied vectorially

* GALILEO TRICK

t = T → s = x

t = 2T → s = 3x

t = 3T → s = 5x

1 : 3 : 5 : 7 : ...

→ When body falls down only under the influence of gravity it covers distances in ratio of odd numbers in equal time intervals.

* VERTICAL MOTION

Time of ascent = Time of descent

$0 = u - gt \Rightarrow t = \dfrac{u}{g}$

Total time = $2u/g$.

Maximum height (H),

$v^2 = u^2 - 2gH$

$H = \dfrac{u^2}{2g}$

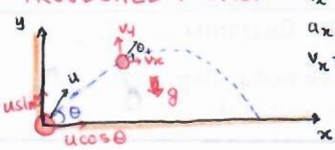
KINEMATICS - 2D

* PROJECTILE MOTION

$u_x = u\cos\theta \quad u_y = u\sin\theta$

$a_x = 0 \quad a_y = -g$

$v_x = v\cos\theta, \; v_y = v\sin\theta$



Time of flight,

$y = u_yT + \frac{1}{2}a_yT^2$

$0 = u\sin\theta T - \frac{1}{2}gT^2$

$T = \dfrac{2u\sin\theta}{g}$

Max Height

$v_y^2 = u_y^2 + 2a_yH$

$0 = u^2\sin^2\theta - 2gH$

$\therefore H = \dfrac{u^2\sin^2\theta}{2g}$

Range, (R)

$S_x = u_xt + \frac{1}{2}a_xt^2$

$R = u\cos\theta\left(\dfrac{2u\sin\theta}{g}\right) + 0$

$R = \dfrac{u^2}{g}2\sin(2\theta)$

maximum range

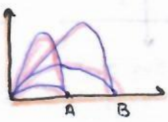
$\theta = 45°$

$H = \dfrac{R}{4}$ at $45°$

* R/H TRICK

$R = \frac{u^2}{g}\sin(2\theta)$ $\quad H = \frac{u^2}{2g}\sin^2\theta$

$\boxed{R/H = 4/\tan\theta}$



* Range of projectile is same for any two angles, θ and $(90-\theta)$ with same velocity

* EQUATION OF TRAJECTORY

$x = (u\cos\theta)t \quad y = u_y t + \frac{1}{2}a_y t^2$

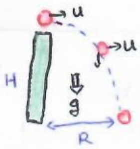
$t = \frac{x}{u\cos\theta} \rightarrow y = u\sin\theta\, t - \frac{1}{2}gt^2$

$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$

$\boxed{y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}}$

Equat^n of parabola
$y = ax - bx$

* PROJECTILE FROM HEIGHT



$y = u_y t + \frac{1}{2}a_y t^2 \quad x = u_x t + 0$

$H = +\frac{1}{2}gt^2 \quad R = u\left(\sqrt{\frac{2H}{g}}\right)$

$\boxed{t = \sqrt{\frac{2H}{g}}} \quad \boxed{R = u\sqrt{\frac{2H}{g}}}$

* PROJECTILE ON AN INCLINE

→ Take incline as x axis
→ ⊥ to incline as Y axis.

$a_x = -g\sin\alpha \quad a_y = -g\cos\alpha$

$y = u_y t + \frac{1}{2}a_y t^2 \quad x = u_x t + \frac{1}{2}a_x t^2$

$0 = u\sin\theta T - \frac{1}{2}g\cos\alpha T^2 . \quad R = u\cos\theta\, t - \frac{1}{2}g\sin\alpha t^2$

$T = \frac{2u\sin\theta}{g\cos\alpha} = \frac{2u_y}{g_y} \quad$ R, R_{max} Solve and find acc. question.

• Don't memorise, just go by steps.

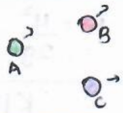
RELATIVE MOTION

* RELATIVE MOTION

$\vec{S}_{AB} = \vec{S}_{AG} - \vec{S}_{BG}$

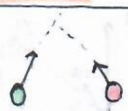
$\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}$

$\vec{a}_{AB} = \vec{a}_{AG} - \vec{a}_{BG} \quad \boxed{\vec{V}_{AB} = \vec{V}_{AG} + \vec{V}_{GB}}$



* CONDITION OF COLLISION

TRICK: Make one body stop and then calculate.



* RAIN-MAN PROBLEM

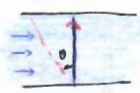
$V_r = x\hat{i} - y\hat{j}$

$V_m = x_m\hat{i}$

$V_{rm} = (x - x_m)\hat{i} - y\hat{j}$

$\boxed{\tan\theta = \frac{x - x_m}{y}}$

* RIVER-BOAT PROBLEMS



Shortest distance
V_{mR} inclined

Shortest time.
$V_{mR} \perp$ to river flow