# MECHANICS

## 1. SCALARS AND VECTORS :-

**Scalar:** Physical quantity that has only magnitude

**Vector:** Physical quantity that has magnitude and direction and obeys vector law of additⁿ

### TYPES OF VECTORS

**Zero Vector:** Vector having zero magnitude and undefined direction.

**Unit Vector:** Vector having unit magnitude and given direction.
Unit vectors along x y z are $\hat{i}, \hat{j}, \hat{k}$ ...

**Resultant Vector:** vector having same effect as produced by many vectors.

**Negative vector:** Vectors having same magnitude but different direction.

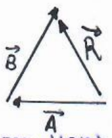
**Equal vector:** Vectors having same direction and same magnitude.

**Position vector:-** Vector that gives position of a particle at a point w.r.t origin of a chosen co-ordinate axis.

**Unit vector** = $\hat{u}_m = \dfrac{\vec{M}}{|\vec{M}|}$

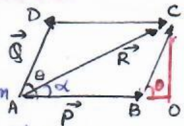
### TRIANGLE LAW OF ADDITION :-

If two vectors describing the same physical quantity are represented in magnitude and direction by two sides of triangle in order, then the third side represents the resultant vector taken in opposite sense. $\vec{A} + \vec{B} = \vec{R}$



### PARALLELOGRAM LAW OF ADDITION :-

If two vectors of same type originating from a point are represented by two adjacent sides of parallelogram then the resultant vector is given in magnitude and direction by the diagonal of parallelogram originating from same point.



**Proof:-** $AC^2 = AO^2 + OC^2$

$= (AB + BO)^2 + (BC^2 - OB^2)$

$= AB^2 + BC^2 + 2(AB)(BO)$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

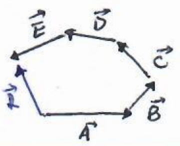
$\therefore \; R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\tan\alpha = \dfrac{OC}{AO} = \dfrac{OC}{AB + BO} = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

### POLYGON LAW OF ADDITION

$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = \vec{R}$

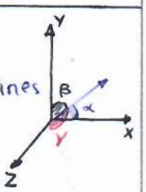


### DIRECTION COSINES

$A_x = A\cos\alpha$, $A_y = A\cos\beta$, $A_z = A\cos\gamma$

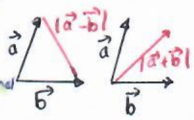
$\cos\alpha, \cos\beta, \cos\gamma$ are called direction cosines

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



### DIAGONALS OF PARALLELOGRAM

If the angle betⁿ $\vec{a}$ and $\vec{b}$ is acute then minor diagonal is of $|\vec{a} - \vec{b}|$ and major diagonal of $|\vec{a} + \vec{b}|$



### ADDITION OF VECTORS.

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

### DOT PRODUCT :-

→ Vector . Vector = Scalar.

→ $\vec{P} \cdot \vec{Q} = PQ\cos\theta$ → $\vec{A} \cdot \vec{B} = 0$ then $\vec{A} \perp \vec{B}$

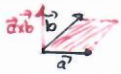
→ $\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}$ and $\vec{P} \cdot (\vec{Q} + \vec{R}) = \vec{P} \cdot \vec{Q} + \vec{P} \cdot \vec{R}$

→ $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

→ $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

→ $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$.

### CROSS PRODUCT :-



→ Vector x Vector = Vector

→ $\vec{P} \times \vec{Q} = PQ\sin\theta\,\hat{u}$

→ Direction of new vector is acc. to Right Hand Rule

→ $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

→ $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$

→ $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ → $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

→ $m\vec{A} \times n\vec{B} = (mn)(\vec{A} \times \vec{B})$

→ $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$ → $\vec{A} \times \vec{B}$ = area of $\|^{gm}$

* Volume of parallelopiped = $(\vec{a} \times \vec{b}) \cdot \vec{c}$

### FINDING COMPONENT OF VECTOR ALONG GIVEN VECTOR

$\vec{A}$ is given vector, and $\vec{B}$ is given vector whose component is to be found.

$|\vec{A}_c| = (\vec{A} \cdot \hat{b})\,\hat{b}$

e.g. $3\hat{i} + 4\hat{j}$ along $\hat{i} + \hat{j}$

$|\vec{A}_c| = 3\hat{i} + 4\hat{j} \cdot \left(\dfrac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

$|\vec{A}_c| = \dfrac{7\sqrt{2}}{2}$ $\therefore \; \vec{A}_c = \dfrac{7\sqrt{2}}{2}\left(\dfrac{\hat{i} + \hat{j}}{\sqrt{2}}\right)$

$\vec{A}_c = \dfrac{7}{2}(\hat{i} + \hat{j})$