ARVIND ACADEMY-1 Chap-07-Alternating Current



## 1. Define the term 'wattless current'.

**Ans.** If due to the flow of Currant in a circuit , the average power Consumed is zero, the Current is said to be idle or Wattless.

Example : In pure Inductor circuit, In pure Capacitor circuit.

2. Calculate the quality factor of a series L-C-R circuit with L = 2.0 H, C = 2 μF and R = 10 Ω. Mention the significance of quality factor in L-C-R circuit. (Ans. 100)

Ans. Given, 
$$L = 2.0 H$$
  
 $C = 2 \mu F$   
 $= 2 \times 10^{-6} F$   
 $R = 10 \Omega$   
Now, Q- factor  $= \frac{1}{R} \sqrt{\frac{L}{C}}$   
 $= \frac{1}{10} \sqrt{\frac{2}{2 \times 10^{-6}}} = \frac{1}{10 \times 10^{-6}}$   
 $= \frac{1}{10^{-2}} = 100$ 

Quality factor is also defined as

$$Q = 2\pi f \times \frac{\text{Energy stored}}{\text{Power loss}}$$

So, higher the value of Q means the energy loss is at lower rate relative to the energy stored, *i.e.*, the oscillations will die slowly and damping would be less.

- 3. An AC voltage,  $V = V_0 \sin \omega t$  is applied across a pure capacitor, C. Obtain an expression for the current I in the circuit and hence obtain the.
  - i. Capacitive reactance of the circuit and
  - ii. The phase of the current flowing with respect to the applied voltage.

**Ans.** (i) Let alternating voltage,  $V = V_0 \sin \omega t$  is applied across a capacitor C. At any instant, the potential difference across the capacitor is equal to applied voltage.



...(i)

 $\therefore$  V = Potential difference across the capacitor =  $\frac{q}{c}$ 

 $\begin{array}{l} \Rightarrow \qquad \qquad q = CV \\ \text{Or} \qquad \qquad q = CV_0 \sin \omega t \end{array}$ 

$$\therefore \qquad \frac{dq}{dt} = \omega C V_0 \cos \omega t$$
Or
$$I = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \cos \omega t$$
Or
$$I = I_0 \sin \left(\omega t + \frac{\pi}{2}\right)$$

0r

0r

Where,

Capacitive reactance,  $X_C = \frac{1}{\omega C}$ 

(ii) From Eqs. (i) and (ii), current leads the voltage by phase  $\frac{\pi}{2}$ .



 $I_0 = \frac{V_0}{\left(\frac{1}{UC}\right)} = \frac{V_0}{X_C}$ 

4. State the principle of a step-up transformer. Explain with the help of a labelled diagram, its working.

Ans. (i) Principle of transformer: A transformer is based on the principle of mutual induction, i.e., whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

...(ii)



- A step-up down transformer operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of 5. the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, calculate
  - i. The power output
  - ii. The voltage and
  - iii. The current in the secondary coil.

(Ans.(i)  $4.5 \times 10^4$  W (ii) 250 V (iii)  $I_s = 180$  A)

Given, input voltages,  $V_P = 2.5 \times 10^3 \text{ V}$ Ans. Input current,  $I_P = 20$  Asss  $\frac{N_P}{N_S} = \frac{10}{1} \qquad \Rightarrow \qquad \frac{N_S}{N_P} = \frac{1}{10}$ Also,

Percentage efficiency = 
$$\frac{\text{Output power}}{\text{Input power}} \times 100$$
  
 $\Rightarrow \frac{90}{100} = \frac{\text{Output power}}{V_P I_P}$   
(i) Output power =  $\frac{90}{100} \times (V_P I_P)$   
 $= \frac{90}{100} \times (2.5 \times 10^3) \times (20 \text{ A}) = 4.5 \times 10^4 \text{ W}$   
(ii)  $\therefore \frac{V_S}{V_P} = \frac{N_S}{N_P} \Rightarrow V_S = \frac{N_S}{N_P} \times V_P$   
Voltage  $V_S = \frac{1}{10} \times 2.5 \times 10^3 \text{ V} = 250 \text{ V}$   
(iii)  $V_S I_S = 4.5 \times 10^4 \text{ W}$   
Current,  $I_S = \frac{4.5 \times 10^4}{V_S} = \frac{4.5 \times 10^4}{250}$   
 $I_S = 180 \text{ A}$ 

- 6. A resistor of 100 Ω and a capacitor of  $100/\pi \mu F$  are connected in series to a 220V, 50Hz a.c. supply.
  - (a) Calculate the current in the circuit.
  - (b) Calculate the (rms ) voltage across the resistor and the capacitor . Do you find the algebraic sum of these voltages more than the source voltage? If yes, how do you resolve the paradox?

Ans. (a) Capacitive reactance 
$$X_C = \frac{1}{\omega^C} = \frac{1}{2\pi V^C}$$
  
 $= \frac{1}{2\pi \times 50 \times \frac{100}{\pi} \mu F} = 100 \Omega$   
Impedance of the circuit,  $Z = \sqrt{R^2 + X_C^2}$   
 $= \sqrt{(100)^2 + (100)^2 = 100 \sqrt{2}}$   
Current in the circuit  $I_{rms} = \frac{E_{rms}}{Z} = \frac{220}{100\sqrt{2}} = 1.55 A$   
(b) Voltage across resistor,  $V_R = I_{rms}R$   
 $= 155 \times 100 = 155 V$   
Voltage across capacitor,  $V_C = I_{rms} \times C = 155 \times 100 - 155V$   
The algebraic sum of voltages across the combination is

$$V_{rms} = V_R + V_C$$
  
= 155V + 155V = 310V

While  $V_{rms}$  of the source is 220 V. Yes, the voltages across the combination is more than the voltage of the source. The voltage across the resistor and capacitor are not in phase. This paradox can be resolved as when the current passes through the capacitor, it leads thevoltage V<sub>c</sub> by phase  $\frac{\pi}{2}$ . so, voltage of the source can be given as

$$V_{rms} = \sqrt{V_R^2 + V_C^2}$$
  
=  $\sqrt{(155)^2} + (155)^2$   
=  $155\sqrt{2} = 220 V$ 

7. **Define power factor. State the conditions under which it is (i) maximum and (ii) minimum. Ans.** The power factor ( $cos\phi$ ) is the ratio of resistance and impedance of an ac circuit i. e., Power factor,  $cos\phi = \frac{R}{Z}$ 

- 8. A coil of inductance 0.50H and resistance 100  $\Omega$  is connected to a 240 V, 50 Hz ac supply. (a) What is the maximum current in the coil?
  - (b) What is the time lag between the voltage maximum and the current maximum?

(Ans.(a) 1.82 A (b) s = 3.2 ms)

Ans. Given L = 0.50H, R = 100 $\Omega$ , V = 240 V, v = 50Hz (a) Maximum (or peak) voltage  $V_0 = V\sqrt{2}$ Maximum current,  $I_0 = \frac{V_0}{Z}$ Inductive reactance,  $X_L = \omega L = 2\pi v L$   $= 2 \times 3.14 \times 50 \times 0.50 = 157\Omega$ Impedance of circuit,  $Z = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (157)^2} = 186\Omega$ Maximum current  $I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{240 \times 1.141}{186} = 1.82 A$ (b) Phase (lag) angle  $\phi$  is given by  $tan \phi = \frac{X_L}{R} = \frac{157}{100} = 1.57$   $\phi = tan^{-1} (1.57) = 54 \cdot 5^\circ$ Time lag  $\Delta T = \frac{\phi}{2\pi} \times T = \frac{\phi}{2\pi} \times \frac{1}{V} = \frac{575}{360} \times \frac{1}{50}$  s  $= 3.2 \times 10^{-3}$  s= 3.2 ms

- 9. The primary coil of an ideal step up transformer has 100 turns and transfer has 100turns and transformation ratio is also 100 .the input voltage and power are 220 V and 1100 W respectively .calculate
  - (a) the number of turns in the secondary coil.
  - (b) the current in the primary coil.
  - (c) the voltage across the secondary coil.
  - (d) the current in the secondary coil.
  - (e) the power in the secondary coil. (Ans.(a) 10,000 (b) 5A (c) 22 kV (d) 0.05A (e) 1100 W)

**Ans.** (a) Transformation ratio r = number of turns in secondary coil (N<sub>s</sub>)number of turns in primary coil (N<sub>p</sub>)

Given  $N_p = 100, r = 100$ 

 $\therefore$  Number of turns in secondary coil,  $N_s = rN_p = 100 \times 100 = 10,000$ 

(b) Input voltage  $V_p = 220V$ , Input power  $P_{in} = 1100 W$ 

Current in primary coil  $I_p = \frac{P_{in}}{V_n} = 1100/220 = 5A$ 

(c) Voltage across secondary coil ( $V_S$ ) is given by  $r = \frac{V_S}{V_P}$  $V_S = rV_P = 100 \times 220 = 22,000 V = 22 kV$ 

(d) Current in secondary coil  $(l_p)$  is given by

$$R = l_P / l_s$$

$$= l_s = \frac{l_p}{r} = \frac{5}{100} = 0.05A \,.$$

(e) Power in secondary coil. $P_{out} = V_s I_s = 10^3 \times 0.05 = 1100 W$ 

Obviously power in secondary coil is same as power in primary . This means that the transformer is ideal. i.e. there are no energy losses.

10. Three students X,Y,Z performed an experiment for studying the variation of alternating currents with angular frequency in a series LCR circuit and obtained the graphs shown below. They all used ac sources of the same rms value and inductances of the same value. What can we (qualitatively) conclude about the (i) capacitance values (ii) resistance used by them? In which case will the quality factor be maximum ?What can we conclude about nature of the impedance of the set up at the frequency  $\omega_0$ ?(Ans. R)



- Ans. (i) From the figure , the resonant frequency  $\omega_0$  is same for all the three graphs *X*, *Y* and *Z*. *As*  $\omega_0 = \frac{1}{\sqrt{LC}}$  and  $L_X = L_Y$ , *so*  $C_X = C_Y = C_Z$ (ii) The maximum value of current at resonance is given by  $I_0 = \frac{V_0}{R}$ , *i.e.*  $(I_0)_Z \propto \frac{1}{R}$ but  $(I_0)_X > (I_0)_Y > (I_0)_Z \Rightarrow R_X < R_Y < R_Z$ Quality factor ,  $Q = \frac{\omega L}{R}$  which is maximum for X. *As*  $\omega$  and L are same in all three cases and  $Q \propto \frac{1}{R}$ Therefore ,  $R_X < R_Y < R_Z$ .  $Z = \sqrt{R^2 + (X_L - X_C)^2 = R}$ Thus, the impedance of the circuit is purely resistive in nature.
- 11. An inductor 200 mH, capacitor  $500\mu F$ , resistor 10  $\Omega$  are connected in series with a 100 V, variable frequency ac source. Calculate the (i) frequency at which the power factor of the circuit is unity. (ii) current amplitude at this frequency.(iii) Q-factor.

(Ans.(i) 15.9 *Hz* (ii) 10*A* (iii)2)



12. (a) What is impedance?

(b) A series LCR circuit is connected to an ac source having voltage  $V=V_0 \sin \omega t$ . derive expression for the impedance, instantaneous current and its phase relationship to the applied voltage. Find the expression for resonant frequency. Ans.

**Impedance:** The opposition offered by the combination of a resistor and reactive component to the flow of ac is called impedance. Mathematically it is the ratio of rms voltage applied and rms current produced in circuit *i.e.*,  $Z = \frac{V}{I}$ .

Its unit is ohm  $(\Omega)$ .

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**Expression for Impedance in** *LCR* series circuit: Suppose resistance *R*, inductance *L* and capacitance *C* are connected in series and an alternating source of voltage  $V = V_0 \operatorname{sin}\omega t$  is applied across it. (fig. *a*) On account of being in series, the current (*i*) flowing through all of them is the same.



Suppose the voltage across resistance R is  $V_R$ , voltage across inductance L is  $V_L$  and voltage  $a_{COM}$  capacitance C is  $V_C$ . The voltage  $V_R$  and current *i* are in the same phase, the voltage  $V_L$  will lead the current by angle 90° (fig. b). Clearly  $V_C$  and  $V_L$  are in opposite directions, therefore their resultant potential difference =  $V_C - V_L$  (if  $V_C > V_L$ ).

Thus  $V_R$  and  $(V_C - V_L)$  are mutually perpendicular and the phase difference between them is 90°. As applied voltage across the circuit is V, the resultant of  $V_R$  and  $(V_C - V_L)$  will also be V. From fig.

$$V^{2} = V_{R}^{2} + (V_{C} - V_{L})^{2} \implies V = \sqrt{V_{R}^{2} + (V_{C} - V_{L})^{2}} \qquad \dots (i)$$
  
$$V_{R} = P_{i} + V_{i} = V_{i} \text{ and } V_{i} = X_{i} \text{ i} \qquad \dots (ii)$$

But  $V_R = R i$ ,  $V_C = X_C i$  and  $V_L = X_L i$ 

where  $X_C = \frac{1}{\omega C}$  = capacitance reactance and  $X_L = \omega L$  = inductive reactance

$$V = \sqrt{(R i)^{2} + (X_{C} i - X_{L} i)^{2}}$$
  
pedance of circuit, 
$$Z = \frac{V}{i} = \sqrt{R^{2} + (X_{C} - X_{L})^{2}}$$

*i.e.* 
$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}$$

Instantaneous current  $I = \frac{V_0 \sin (\omega t + \varphi)}{\sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}}$ 

The phase difference ( $\phi$ ) between current and voltage  $\phi$  is given by tan  $\phi = \frac{X_C - X_L}{R}$ 

**Resonant Frequency:** For resonance  $\phi = 0$ , so  $X_C - X_L = 0$ 

$$\Rightarrow \qquad \frac{1}{\omega C} = \omega L \Rightarrow \omega^2 = \frac{1}{LC}$$
  

$$\therefore \qquad \text{Resonant frequency } \omega_r = \frac{1}{\sqrt{LC}}$$

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