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SMART GUIDANCE

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Physics (Q. & Ans.)
Class- XI

Chapter: 07
System of Particles and Rotational Motion

1. **When do we call a body rigid?**
   Ans. When the separation between any two masses constituting the body does not vary, the body is said to be rigid.

2. **Why do we place handles at maximum possible distance from the hinges in a door?**
   Ans. To develop torque with less force being applied.

3. **When a sphere, ring, disc and a spherical shell are coming down on an inclined plane, rolling without slipping, which will reach the ground first?**
   Ans. Solid sphere, since acceleration is more.

4. **For uniform circular motion, does the direction of the centripetal force depend on the sense of rotation (*i.e.,* clockwise or anticlockwise)?**
   Ans. No.

5. **Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?**
   Ans. All the four structures have centre of mass at their applied geometrical centre. No., the COM does not necessarily lie inside the body.

6. **What are the factors on which moment of inertia of a body depends?**
   Ans. Moment of inertial of a body depends on position and orientation of axis of rotation. It also depends on shape, size of the body and also on the distribution of mass of the body about the given axis.

7. **Write an expression for the moment of inertia of a rod of mass 'M' and length 'l' about an axis perpendicular to it through one end.**
   Ans. $I = \frac{Ml^2}{3}$

8. **Write an expression for the moment of inertia of a circular disc of mass M and radius R**
   i. **About an axis passing through its centre, and perpendicular to its plane.**
   ii. **About its diameter.**
   iii. **About a tangent to its own plane.**
   iv. **About a tangent perpendicular to the plane of the disc.**

   Ans. (i) $I = \frac{MR^2}{2}$

   (ii) $I_d = \frac{MR^2}{4}$

(iii) $I_{tp} = \frac{MR^2}{4} + MR^2 = \frac{5}{4} MR^2$

(iv) $I_{t\perp} = \frac{MR^2}{2} + MR^2 = \frac{3}{2} MR^2$

9. State and prove the parallel axis theorem.

Ans. The moment of inertia about an axis parallel to the axis through the centre of mass is the sum of the moment of inertia about the axis through centre of mass and the product of the mass M and the square of the separation between the axis, *i.e.* $I = I_{c.m.} + Ma^2$.

Moment of inertia is the product of mass and the square of the separation from the axis. The moment of inertia of the system of n masses shown in the figure about the parallel is,

$$I = m_1(x_1 + a)^2 + m_2(a - x_2)^2 + m_3(x_3 - a)^2 + \ldots\ldots \text{ for } n \text{ masses.}$$

In general, $I = \sum_{i=1}^{n} m(\pm x_i \pm a_i)^2$

If all the n masses are considered equal.

$$I = \sum_{i=1}^{n} mx_i^2 + \sum_{i=1}^{n} ma^2 + 2a \sum_{i=1}^{n} mx_i$$

$I = I_{c.m.} + Ma^2$ $\quad \because \sum mx_i = 0$ if the centre of mass is considered as origin.

10. State and prove the perpendicular axis theorem.

Ans. According to perpendicular axis theorem, the sum of the moment of inertia about x and y axes is equal to the moment of inertia about z-axis. The mass m has co-ordinates $(x, y,)$.

The moment of inertia about x-axis,

$$I_x = my^2$$

about y-axis, $I_y = mx^2$

$$I_x + I_y = m(x^2 + y^2)$$

$$= m\left(\sqrt{x^2 + y^2}\right)^2$$

$$I_x + I_y = m(\perp^r \text{ distance from z} - \text{axis})^2$$

$$I_x + I_y = I_z$$

11. Three point masses of 1 kg, 2 kg and 3 kg lie at (1, 2), (0, −1) and (2, −3) respectively. Calculate the co-ordinates of centre of mass of the system.

Ans. $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 3$ kg

$x_1 = 1,\ y_1 = 2,\ x_2 = 0,\ y_2 = -1,\ x_3 = 2,\ y_3 = -3$

12. The distance between the centres of carbon and oxygen atoms in carbon monoxide gas molecules is 1.13 Å. Locate the centre of mass of the molecules relative to carbon atom.

Ans. Here, distance between carbon and oxygen atoms = 1.13 Å.

Mass of carbon atom $m_1 = 14$

Mass of oxygen atom $m_2 = 16$

Let COM of molecule be at a distance x from carbon atom.

$\therefore$ $r_1 = x$ Å, $r_2 = (1.13 - x)$Å

As $m_1 r_1 = m_2 r_2$

$\therefore$ $14x = 16(1.13 - x)$

Or $30x = 16 \times 1.13$,

$$x = \frac{16 \times 1.13}{30} = 0.602 \text{ Å}$$

13. **From a uniform circular disc of diameter d, a circular hole of diameter d/6 and having its centre at a distance of d/4 from the centre of the disc is scooped out. Find the centre of mass of the remaining portion.**

Ans. Let the mass per unit area of the disc be m.

Then,

Mass of the disc, $M = \pi\left(\frac{d}{2}\right)^2 m = \frac{\pi m d^2}{4}$

Mass of the scooped out portion of the disc,

$$M' = \pi\left(\frac{d}{12}\right)^2 m = \frac{\pi m d^2}{144}$$

Let us take the centre of the disc O as origin. The masses M and M' and are supposed to be concentrated at their respective centres of the disc and scooped out portion. Since the portion is removed, its mass M' will be represented by negative sign.

Then x-coordinate of the COM of the remaining portion of the disc is given by,

$$x = \frac{Mx_1 - M'x_2}{M - M'}$$

$$= \frac{M \times 0 - M' \times d/4}{\frac{\pi m d^2}{4} - \frac{\pi m d^2}{144}}$$

$$= \frac{-M'd}{4} \times \frac{144}{35\,\pi m d^2}$$

$$= \frac{-\pi m d^2}{144} \cdot \frac{d}{4} \cdot \frac{144}{35\,\pi m d^2}$$

$$= \frac{-d}{140}$$

i.e. the COM of the remaining portion of the disc is at a distance of $\frac{d}{140}$ towards left of the origin O.

14. Two particles of masses 1 kg and 3 kg are located at $(2\hat{i} + 5\hat{j} + 13\hat{k})$ and $(-6\hat{i} + 4\hat{j} - 2\hat{k})$ metre respectively. Find the position of their centre of mass.

Ans. $m_1 = 1$ kg, $m_2 = 3$ kg,

$x_1 = 2,\ y_1 = 5,\ z_1 = 13,$

$x_2 = -6,\ y_2 = +4,\ z_2 = -2$

$$\therefore \quad x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{1(2) + 3(-6)}{1+3} = \frac{-16}{4} = -4$$

$$y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$= \frac{1(5) + 3(4)}{1+3} = \frac{17}{4}$$

$$z = \frac{m_1z_1 + m_2z_2}{m_1 + m_2}$$

$$= \frac{1(13) + 3(-2)}{1+3} = \frac{7}{4}$$

Hence, position of COM is $\left[-4\hat{i} + \frac{17}{4}\hat{j} - \frac{7}{4}\hat{k}\right]$

15. A disc of mass 5 kg and radius 50 cm rolls on the ground at the rate of 10 ms^{-1}. Calculate the K.E. of the disc. $\left(\text{Given : } I = \frac{1}{2}MR^2\right)$

Ans. Here, mass of the disc, M = 5 kg, Radius of the disc R = 50 cm = 1/2 m.

Linear velocity of the disc, $v = 10\ ms^{-1}$.

As $\quad v = R\omega \qquad \therefore\ 10 = \frac{1}{2}\,\omega$

Or $\omega = 10 \times 2 = 20$ radian/sec.

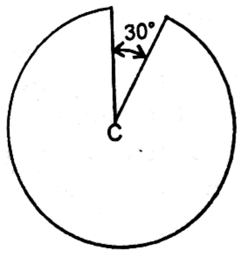
Also, moment of inertia of disc about or axis through its centre.

$$I = \frac{1}{2}MR^2.$$

K.E. of the disc

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$
$$= \frac{1}{2} \frac{MR^2}{2} \omega^2 + \frac{1}{2} Mv^2$$
$$= \frac{1}{4} \times 5 \times \left(\frac{1}{2}\right)^2 \times (20)^2 + \frac{1}{2} \times 5 \times (10)^2$$
$$= 375 \text{ J.}$$

16. From a complete ring of mass M and radius R, a 30° sector is removed. What is the moment of inertia of the incomplete ring about an axis passing through the centre of the ring and perpendicular to centre of the ring and perpendicular to the plan of the ring? **(Ans.** $\frac{11}{12} MR^2$ **)**



Ans. Mass of incomplete ring = $M - \frac{M}{dt} \times \frac{\pi}{6}$

$$= M - \frac{M}{12} = \frac{11}{12} M$$

Moment of inertia of incomplete ring $= \left(\frac{11M}{12}\right) R^2 = \frac{11}{12} MR^2$

17. A disc of mass M and radius r is rotating with an angular velocity ω. If gently, two masses M are placed at a distance r/2 on either side of the axis , what will be its angular velocity ?

Ans.

Angular momentum initially

$$= \frac{Mr^2}{2} . \omega \qquad [\because L = I\omega]$$

When the masses are placed,

$$I = \frac{Mr^2}{2} + \left[m \left(\frac{r}{2}\right)^2 \right] \times 2$$
$$= \frac{r^2}{2} [M + m]$$

As I increases, ω decreases because angular momentum is to be conserved.

New angular velocity, ω'

$$= \frac{\frac{Mr^2}{2}\omega}{\frac{r^2}{2}[M+m]} = \frac{M\omega}{(M+m)}$$

18. Find the torque of a force $7\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ about the origin which acts on a particle whose position vector is $\hat{\imath} + \hat{\jmath} + \hat{k}$. **(Ans.** $-8\hat{\imath} - \hat{2}\jmath - 10\hat{k}$**)**

Ans. $\vec{F} = \widehat{7\imath} \widehat{-3\jmath} \widehat{-5k}$

$\vec{r} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (\hat{\imath} + \hat{\jmath} - \hat{k}) \times (7\hat{\imath} - \hat{3}\jmath - 5\hat{k})$

$= -8\hat{\imath} - \hat{2}\jmath - 10\hat{k}$

19. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rads^{-1} . The radius of the cylinder is 0.25 m. What is the K.E. associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Ans.

m = 20 kg, ω = 100 rads^{-1}, r = 0.25 m

Moment of inertia

$$I = \frac{mr^2}{2} = \frac{20 \times (0.25)^2}{2}$$

$$= 0.625 \text{ kgm}^2$$

$\therefore$ Rotational Kinetic Energy (R.K.E.)

$$= \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.625 \times 10^4$$

$$= 0.3125 \times 10^4 \text{ J}$$

Angular momentum = $L = I\omega = 0.625 \times 100$

$$= 62.5 \text{ Nms}$$

20. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to the 10 π radiaus^{-1} .Which of the two will start to roll earlier? The coefficient of kinetic friction $\mu_k = 0.2$.

Ans.

In pure rolling,

(i) the lower-most point will be at momentary rest.

(ii) the COM will be having a velocity $v = r\omega$, where r is the radius and ω is the angular velocity.

Friction brings necessary acceleration,

$$\therefore \quad \mu mg = ma$$

$$a = \mu g$$

Torque due to friction = $\tau = \mu mgr = -I\alpha$

Rolling begins when $v = r\omega$

(i.e.,) $v = 0 + at = \mu gt = r\omega$

Here, $\omega_i = 10\pi$

$$\therefore \quad \omega = \omega_i + \alpha t = 10\pi - \frac{\mu mgr}{I}t$$

$$\frac{v}{r} = \frac{\mu gt}{r} = 10\pi - \frac{\mu mgr}{I}t$$

Solving, we get

$$t\mu g\left(1 + \frac{mr^2}{I}\right) = 10\pi r$$

$$\Rightarrow \quad t = \frac{10\pi r}{\mu g\left(1 + \frac{mr^2}{I}\right)}$$

$$= \frac{r\omega_i}{\mu g\left(1 + \frac{mr^2}{I}\right)}$$

For a disc : $I = \dfrac{mr^2}{2}$

$\therefore$ Time = 0.53 sec.

For a ring, $I = mr^2$

$\therefore$ Time = 0.80 sec.