3D Board Questions 2017

1. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2\cdot5y + 5z = 20$. — 2017, 1 Mark

2. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, – 2) is 4. Find its z-coordinate. — 2017, 2 Marks

3. Find the value of λ, if four points with position vectors $3\hat{i} + 6\hat{j} + 9\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are coplanar. — 2017, 4 Marks

4. Find the coordinates of the point where the line through the points (3, – 4, – 5) and (2, – 3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, – 3) and (0, 4, 3). — 2017, 6 Marks

5. A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$. — 2017, 6 Marks

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1. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2\cdot5y + 5z = 20$. — 2017, 1 Mark

Sol

distance b/w two parallel planes $= \left|\frac{d_1 - d_2}{\sqrt{a^2+b^2+c^2}}\right|$

eq^n are $2x - y + 2z = 5$ —— (1)

and $5x - 2.5y + 5z = 20$

Can be rewritten as (divided by 2.5)

$2x - y + 2z = 8$ —— (2)

distance b/w Planes $= \left|\frac{d_1 - d_2}{\sqrt{a^2+b^2+c^2}}\right| = \left|\frac{5-8}{\sqrt{(2)^2+(-1)^2+(2)^2}}\right|$

$= \left|\frac{3}{\sqrt{9}}\right| = \boxed{1 \text{ Ans.}}$

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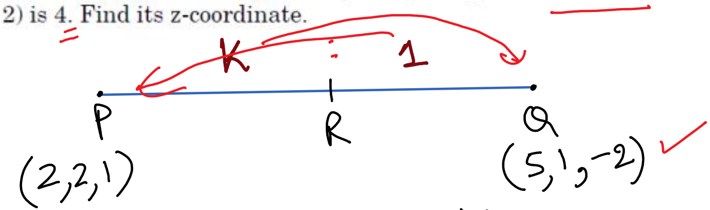
2. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, – 2) is 4. Find its z-coordinate. — 2017, 2 Marks

Sol



R divides PQ in the ratio k:1

Coordinates of R $\left(\frac{5k+2}{k+1}, \frac{k+2}{k+1}, \frac{-2k+1}{k+1}\right)$

Here x coordinate is 4.

$\therefore \frac{5k+2}{k+1} = 4 \Rightarrow 5k+2 = 4k+4 \Rightarrow \boxed{k=2}$

So z coordinate $= \frac{-2(2)+1}{2+1} = \boxed{-1 \text{ Ans.}}$

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(3) Find the value of λ, if four points with position vectors $3\hat{i}+6\hat{j}+9\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k}$ are coplanar.

2017, 4 Marks

A B C D

Sol: let Position vector of $A = 3\hat{i}+6\hat{j}+9\hat{k}$

" " " $B = \hat{i}+2\hat{j}+3\hat{k}$

" " " $C = 2\hat{i}+3\hat{j}+\hat{k}$

" " " $D = 4\hat{i}+6\hat{j}+\lambda\hat{k}$

$\overrightarrow{AB} = -2\hat{i}-4\hat{j}-6\hat{k}$

$\overrightarrow{AC} = -\hat{i}-3\hat{j}-8\hat{k}$

$\overrightarrow{AD} = \hat{i}+(\lambda-9)\hat{k}$

So if these three vectors are coplanar then

$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = 0$

S.T.P = Scalar Triple Product

$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$

If $\vec{a}, \vec{b}$ & $\vec{c}$ are coplanar vectors then the necessary & sufficient cond^n is

$[\vec{a}\ \vec{b}\ \vec{c}] = 0 = \vec{a}\cdot(\vec{b}\times\vec{c})$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overrightarrow{AB}\cdot(\overrightarrow{AC}\times\overrightarrow{AD}) = \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & (\lambda-9) \end{vmatrix} = 0$$

expanding along R_1

$\Rightarrow -2[(-3)(\lambda-9)-(-8)(0)]-(-4)[(-1)(\lambda-9)-(1)(-8)]+(-6)[(-1)\times 0-(-3)(1)] = 0$

$\Rightarrow -2[-3\lambda+27]+4[-\lambda+9+8]-6[+3] = 0$

$\Rightarrow 6\lambda-54-4\lambda+68-18 = 0$

$\Rightarrow 2\lambda-4 = 0 \Rightarrow \boxed{\lambda = 2 \text{ Ans}}$

4 Marks

(4) Find the coordinates of the point where the line through the points $(3, -4, -5)$ and $(2, -3, 1)$, crosses the plane determined by the points $(1, 2, 3)$, $(4, 2, -3)$ and $(0, 4, 3)$.

2017, 6 Marks

Sol eqn of a line passing through $(3,-4,-5)$ & $(2,-3,1)$

$$\frac{x-3}{-3+2} = \frac{y-(-4)}{(-3)-(-4)} = \frac{z-(-5)}{(1)-(-5)}$$

Formula

line passing through two pts. (x_1,y_1,z_1) (x_2,y_2,z_2)

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = K \quad \text{——— (1)}$$

$\boxed{x = -K+3,\ y = K-4,\ z = 6K-5}$

eq of plane passing through $(1,2,3)$, $(4,2,-3)$ & $(0,4,3)$

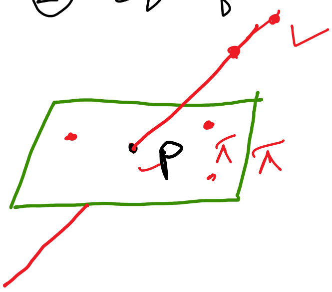
$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$$

Formula (Plane) Passing thru 3 Points

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(12) - (y-2)(-6) + (z-3)(6) = 0$$

$$\Rightarrow 2x + y + z - 7 = 0 \quad \text{——— (2) eq of Plane}$$



Let any point on line (1) is $P(-K+3, K-4, 6K-5)$

but this Point P also lies on the Plane so it will also satisfy the eqn of plane (2)

Putting the coordinates of P in eqn (2)

$$2(-K+3) + (K-4) + (6K-5) - 7 = 0$$

$$5K = 10 \Rightarrow K = 2\ \#$$

$\therefore$ P is $(-2+3,\ 2-4,\ 6\times2-5) \equiv$ P $\boxed{(1,-2,7)}$ Ans.

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(5) A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

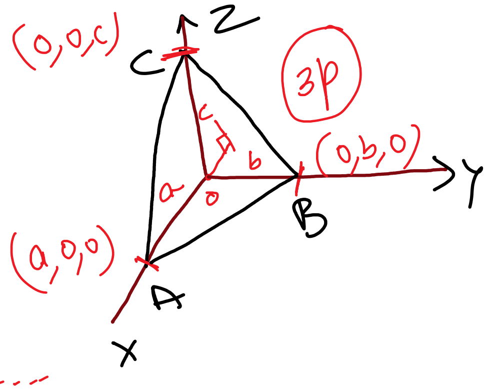
2017, 6Marks

Sol Let the eqn of plane (In intercept form) triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

Sol let the eq^n of plane (In Intercept form)

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

a, b, c are x, y, z intercepts



It cut the coordinate axes at A, B, C

$A(a,0,0)$, $B(0,b,0)$, $C(0,0,c)$

Now let the centroid of ΔABC be $\left(a/3, b/3, c/3\right)$ $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \ldots\right)$

given the distance of plane from origin is 3p

$$\therefore \left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = 3p \quad \text{--- (1)}$$

$$\Rightarrow \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$ax+by+cz+d=0$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2}$$

Now for locus of centroid

For centroid $x_1 = a/3$, $y_1 = b/3$, $z_1 = c/3$

$a = 3x_1$, $b = 3y_1$, $c = 3z_1$

Put the value of a, b, c in eq^n (1)

$$\frac{1}{\cancel{9}x_1^2} + \frac{1}{\cancel{9}y_1^2} + \frac{1}{\cancel{9}z_1^2} = \frac{1}{\cancel{9}p^2}$$

Now for locus generalising the prs. $x_1 = x$, $y_1 = y$, $z_1 = z$

$$\boxed{\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2} \quad \text{Proved}}$$