Application of Derivative Board Question 2017

1 Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.

2017, 2 Marks.

2 The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

3 Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$.

1 Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.

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Sol Just to Revise ... what is Rolle's theorem

Def :- Let $f : [a,b] \to R$ be continuous on $[a,b]$ & differentiable on (a,b) such that $f(a) = f(b)$, where a & b are some real numbers. Then there exists some c in (a,b) such that $f'(c) = 0$

Given fn is $f(x) = x^3 - 3x$

Here
(i) $f(x)$ is a Polynomial fn so it is continuous everywhere so $f(x)$ is continuous on $[-\sqrt{3}, 0]$ ✓

(ii) $f(-\sqrt{3}) = f(0) = 0$

(iii) $f'(x) = 3x^2 - 3$ It is again a Polynomial fn. so continuous on $(-\sqrt{3}, 0)$ ✓

This satisfies all the three conditions of Rolle's Theorem so there exist at least one $c \in (-\sqrt{3}, 0)$ where $f'(c) = 0$

$\Rightarrow f'(x) = 3x^2 - 3$, for c $f'(c) = 3c^2 - 3 = 0$

$c = \pm 1 \Rightarrow c = -1 \in (-\sqrt{3}, 0)$ Ans. $c = -1$

2 The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its

2017, 2 Marks

(2) The volume of a cube is increasing at the rate of 9 cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

2017, 2 Marks

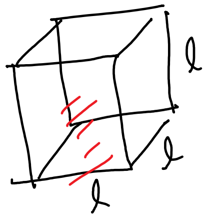
Sol: let volume of a cube is $= V$

given $\frac{dV}{dt} = 9\,cm^3/s$, $\frac{dS}{dt} = ?$, $l = 10$ cm

for cube $V = l^3$

$$\frac{dV}{dt} = \frac{d}{dt}(l^3) = 9$$

$$3l^2 \frac{dl}{dt} = 9 \Rightarrow \boxed{\frac{dl}{dt} = 3/l^2} \quad \text{---} (1)$$



For cube, $S = 6l^2$

$$\text{Now } \frac{dS}{dt} = \frac{d}{dt}(6l^2)$$

$$\Rightarrow 6 \times 2l \frac{dl}{dt} = 12l \frac{dl}{dt} \quad \text{---} (2)$$

from (1) $\left(\frac{dl}{dt}\right) = 3/l^2$ Put this Value in eqn (2)

($l = 10$ cm given)

$$\therefore \frac{dS}{dt} = 12l \times 3/l^2 = \frac{36}{l} = \frac{36}{10} = 3.6\,cm^2/s$$

Ans.

(3) Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on $\mathbb{R}$.

2017, 2 Marks

Sol: Given fn $f(x) = x^3 - 3x^2 + 6x - 100$

$$\therefore f'(x) = 3x^2 - 6x + 6 = 3(x^2 - 2x + 3)$$

$$f'(x) = 3\{(x-1)^2 + 2\}$$

$\Rightarrow f'(x) > 0$ for all $x \in \mathbb{R}$

So $f(x)$ is increasing on $\mathbb{R}$. Ans

So $f(x)$ is increasing on R. Ans