Determinants Board Questions

(1) Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

2017, Marks 4

(2) Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

2016, Marks 6

(3) Using properties of determinants, prove that

$$\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix} = 2(a-b)(b-c)(c-a)(a+b+c).$$

2015, Marks 4

(4) Using properties of determinants, prove that

prove, $$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

2014, Marks 4

(5) Using properties of determinants, prove the following :

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

2013, Marks 4

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(1) Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

2017, Marks 4

Sol

$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$

$$\text{LHS} \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+1)(a-1) & (a-1) & 0 \\ 2(a-1) & (a-1) & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking common $(a-1)$ from R_1 & R_2

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^2 \left[1\left((a+1)-2\right)\right]$$

$$= \boxed{(a-1)^3 \quad \text{Proved}} \;\#$$

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(2) Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

2016, Marks 6

$R_1 \to zR_1$
$R_2 \to xR_2$
$R_3 \to yR_3$

Sol

$$\frac{xyz}{xyz}$$

$$= \frac{1}{xyz} \begin{vmatrix} z(x+y)^2 & z^2x & z^2y \\ zx^2 & x(z+y)^2 & x^2y \\ zy^2 & xy^2 & y(z+x)^2 \end{vmatrix}$$

Taking z, x, y common from C_1, C_2, C_3 resp.

$$= \frac{\cancel{xyz}}{\cancel{xyz}} \begin{vmatrix} (x+y)^2 & z^2 & z^2 \\ x^2 & (z+y)^2 & x^2 \\ y^2 & y^2 & (z+x)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_3$

$$\left| (x+y)^2 - z^2 \quad 0 \quad z^2 \right|$$

$$\begin{vmatrix} \ & y^2 & y^2 & (z+x)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (x+y)^2-z^2 & 0 & z^2 \\ 0 & (z+y)^2-x^2 & x^2 \\ y^2-(z+x)^2 & y^2-(z+x)^2 & (z+x)^2 \end{vmatrix} \qquad \begin{matrix} C_1 \to C_1 - C_3 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= \begin{vmatrix} (x+y+z)(x+y-z) & 0 & z^2 \\ 0 & (z+y-x)(z+y+x) & x^2 \\ (y-z-x)(y+z+x) & (y-z-x)(y+z+x) & (z+x)^2 \end{vmatrix}$$

$$= \underline{\underline{(x+y+z)^2}} \begin{vmatrix} (x+y-z) & 0 & z^2 \\ 0 & (z+y-x) & x^2 \\ (y-z-x) & (y-z-x) & (z+x)^2 \end{vmatrix}$$

$R_3 \longrightarrow$

$C_1 \downarrow$

$$= (x+y+z)^2 \begin{vmatrix} (x+y-z) & 0 & z^2 \\ 0 & (z+y-x) & x^2 \\ -2x & -2x & 2xz \end{vmatrix} \qquad R_3 \to R_3 - R_2 - R_1$$

$$= (x+y+z)^2 \begin{vmatrix} x+y & z^2/x & z^2 \\ x^2/z & z+y & x^2 \\ 0 & 0 & 2zx \end{vmatrix} \qquad \begin{matrix} C_1 \to C_1 + \frac{1}{z} C_3 \\ C_2 \to C_2 + \frac{1}{x} C_3 \end{matrix}$$

Expanding along R_3

$$= 2xz(x+y+z)^2 \left[(x+y)(z+y) - \left(\frac{x^2}{z}\right)\left(\frac{z^2}{x}\right) \right]$$

$$= 2xz\,(x+y+z)^2 \left[\cancel{xz} + xy + yz + y^2 - \cancel{xz} \right]$$

$$= \boxed{2xyz\,(x+y+z)^3}$$

Proved.

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③ Using properties of determinants, prove that

$$\begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix} = 2(a-b)(b-c)(c-a)(a+b+c).$$

2015, Marks 4

Sol We can write this

$$= - \begin{vmatrix} 2 & a^3 & a \\ 2 & b^3 & b \\ 2 & c^3 & c \end{vmatrix} = (-1)(-1) \begin{vmatrix} 2 & a & a^3 \\ 2 & b & b^3 \\ 2 & c & c^3 \end{vmatrix}$$

$$= +1 \begin{vmatrix} 2 & a & a^3 \\ 2 & b & b^3 \\ 2 & c & c^3 \end{vmatrix} = 2 \begin{array}{c} \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} \end{array} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

$$a^3 \qquad R_2 \to R_2 - R_1$$

$$\begin{vmatrix} 2 & c & c^3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix} \qquad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= 2\begin{vmatrix} 1 & a & a^3 \\ 0 & (b-a) & (b-a)(b^2+a^2+ab) \\ 0 & (c-a) & (c-a)(c^2+a^2+ac) \end{vmatrix}$$

$$= 2(b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & (b^2+a^2+ab) \\ 0 & 1 & (c^2+a^2+ac) \end{vmatrix}$$

$$= 2(b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 0 & b^2-c^2+ab-ac \\ 0 & 1 & c^2+a^2+ac \end{vmatrix} \qquad R_2 \to R_2 - R_3$$

$$= 2(b-a)(c-a)\begin{array}{c} C_1 \\ \begin{vmatrix} 1 & a & a^3 \\ 0 & 0 & (b-c)(b+c+a) \\ 0 & 1 & a^2+c^2+ac \end{vmatrix} \end{array}$$

Expanding along C_1

$$= 2(b-a)(c-a)[1(0-(b-c)(a+b+c))]$$

$$= -2(b-a)(c-a)(b-c)(a+b+c)$$

$$= \boxed{2(a-b)(b-c)(c-a)(a+b+c)} \qquad H.P.$$

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4. Using properties of determinants, prove that

$$\text{prove, } \begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$$

2014, Marks 4

Sol Applying property

$$= \begin{vmatrix} x & x & x \\ 5x & 4x & 2x \\ 10x & 8x & 3x \end{vmatrix} + \begin{vmatrix} y & x & x \\ 4y & 4x & 2x \\ 8y & 8x & 3x \end{vmatrix}$$

$$= x^3\begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2\begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 2 \\ 8 & 8 & 3 \end{vmatrix} \quad \text{Zero}$$

$$= x^3 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 2 \\ 10 & 8 & 3 \end{vmatrix} + yx^2 \begin{vmatrix} \cdots & \cdots & \cdots \\ 4 & 4 & \cdots \\ 8 & 8 & 3 \end{vmatrix}$$

For 1st Det. $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

For 2nd Det C_1 & C_2 same value = 0

$$= x^3 \begin{vmatrix} 1 & 0 & 0 \\ 5 & -1 & -3 \\ 10 & -2 & -7 \end{vmatrix} + yx^2 \times 0$$

$R_1 \to$

$$= x^3 \cdot 1\left[(-1)(-7) - (-2)(-3)\right]$$

$$= x^3 (7-6) = \boxed{x^3}$$

H.P.

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5. Using properties of determinants, prove the following :

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

2013, Marks 4

Sol

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 3x+3y & x+y & x+2y \\ 3x+3y & x & x+y \\ 3x+3y & x+2y & x \end{vmatrix}$$

$$= 3(x+y) \begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= 3(x+y) \begin{vmatrix} 1 & x+y & x+2y \\ 0 & -y & -y \\ 0 & +y & -2y \end{vmatrix}$$

Taking y common from R_2 & R_3

$C_1 \downarrow$

$$= 3(x+y)(y)(y) \begin{vmatrix} 1 & x+y & x+2y \\ 0 & -1 & -1 \\ 0 & +1 & -2 \end{vmatrix}$$

Expanding along C_1

$$= 3(x+y)(y^2)\left[2-(-1)\right] = \boxed{9y^2(x+y)}$$

H.P.