Matrices 2017 Board Questions

Q1. If for any 2×2 square Matrix A, $A(\text{adj}A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$. 2017, 1 Mark

Total 13 Marks

Q2. If A is a skew-symmetric matrix of order 3, then prove that $\det A = 0$ 2017, 2 Marks.

Q3. Find Matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ 2017, 4 Marks

For Video Lectures
Subscribe to
ARVIND ACADEMY
Youtube Channel

https://www.youtube.com/watch?v=vBCyXbcZ57E

Q4. Determine the product

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

and use it to solve the system of equations

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

2017, 6 Marks

Q1. If for any 2×2 square Matrix A, $A(\text{adj}A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$. 2017 1 Mark

Sol We know the property $\boxed{A(\text{adj}A) = |A| I_n}$

$$\Rightarrow |A| I_n = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow |A| I_n = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \boxed{|A| = 8 \text{ Ans.}}$$

$A' = A$ Symmetric

Q2. If A is a skew-symmetric matrix of order 3, then prove that $\det A = 0$ 2017, 2 Marks.

Sol def:- A square Matrix $A = [a_{ij}]$ is said to be skew symmetric Matrix if $\boxed{A' = -A}$. that is $a_{ij} = -a_{ji} \Rightarrow a_{ij} + a_{ji} = 0$

$i = i$, a_{11}, a_{22}, a_{33}

If we put $i = j$ here then $a_{ii} + a_{ii} = 0 \Rightarrow 2a_{ii} = 0 \Rightarrow \boxed{a_{ii} = 0}$

This means that all the diagonal element of a skew symmetric matrix are zero.

$a_{12} = -a_{21}$

for example. $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \end{bmatrix}$ skew symmetric Matrix symmetric matrix

for example $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ skew symmetric Matrix

. Expanding along R_1

$$\det A = |A| = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} = 0[(0\times0)-(c)(-c)] - a[(-a)\times0-(-b)\times c] + b[(-a)(-c)-0(-b)]$$

$$= -abc + abc = 0 \quad \text{Proved}$$

Q3. Find Matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$ 2017, 4 Marks

Sol Let Matrix A is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For multiplication

$$\boxed{A_{m\times n} \times B_{n\times q} = C_{m\times q}}$$

$3\times2 \quad 2\times2 = C_{3\times2}$

$A = 2\times2$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

comparing both sides. $a=1$, $b=-2$

$2a-c=-1$, $2b-d=-8$,

after Solving

$c=3$, $d=-4$

So $\boxed{A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}}$ Ans.

4

Q4. Determine the product

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

and use it to solve the system of equations

$x-y+z=4$

2017, 6 Marks

and use it to solve ...

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

2017, 6 Marks

Sol. Product of Matrices

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = \underline{\underline{8 I_3}}$$

Here

$$\overset{A}{\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}} \overset{B}{\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}} = 8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8 I_3$$

We know that $AB = 8I \Rightarrow B^{-1} = \frac{1}{8} A$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Now according to system of equations

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \vdots \\ Y \\ Z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} \cdots & & \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} \vdots \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\Rightarrow$ $\boxed{X = 3,\ Y = -2,\ Z = -1 \text{ Ans.}}$

6 M