Inverse Trig functions
(Board Questions) 4 Marks

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1. If $\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$, then find the value of x. **2017, 4 Marks**

2. *Solve for x :* $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ **2016, 4 Marks**

OR

3. $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x\,;\ |2x|<\frac{1}{\sqrt{3}}$

4. *Evaluate : tan* $\left\{2\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\}$ **2015, 4 Marks**

5. *Prove that* $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\quad \frac{-1}{\sqrt{2}}\le x\le 1$ **2014, 4 Marks**

OR

6. *If* $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$, *find the value of x*

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1. If $\tan^{-1}\frac{x-3}{x-4}+\tan^{-1}\frac{x+3}{x+4}=\frac{\pi}{4}$, then find the value of x.

2017

2017, 4 Marks

Sol applying formula

Formula

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right),\ xy<1$

$\Rightarrow \tan^{-1}\left[\frac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\left(\frac{x-3}{x-4}\right)\left(\frac{x+3}{x+4}\right)}\right]=\frac{\pi}{4}$

if $\left(\frac{x-3}{x-4}\right)\left(\frac{x+3}{x+4}\right)<1$

$=\frac{x^2-9}{x^2-16}<1$

$\tan^{-1}\left[\frac{\frac{(x-3)(x+4)+(x+3)(x-4)}{x^2-4^2}}{1-\frac{x^2-3^2}{x^2-4^2}}\right]=\pi/4$

$\tan^{-1}\frac{(x-3)(x+4)+(x+3)(x-4)}{(x^2-4^2)-(x^2-3^2)}=\pi/4$

$\sqrt{\frac{18}{2}}=\sqrt{9}=3$

$\frac{(x^2+4x-3x-12)+(x^2-4x+3x-12)}{\cancel{x^2}-16-\cancel{x^2}+9}=\tan\pi/4$

$\frac{2x^2-24}{-7}=1 \Rightarrow 2x^2-24=-7 \Rightarrow 2x^2=17 \Rightarrow x^2=\frac{17}{2} \Rightarrow \boxed{x=\pm\sqrt{\frac{17}{2}}}$

Ans.

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2. *Solve for x :* $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

2016, 4 Marks

Sol Rearranging

$\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

Formula

$xy<1,\ \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy},\ xy<1$

$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy},$ if $xy>-1$

applying formula –

$\Rightarrow \tan^{-1}\left[\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}\right]=\tan^{-1}\left[\frac{3x-x}{1-(3x)(x)}\right]$

$$\Rightarrow \frac{2x}{1-(x^2-1)} = \frac{2x}{1+3x^2} \Rightarrow 2x+6x^3 = 4x-2x^3 \Rightarrow 8x^3-2x=0$$

$$\Rightarrow 2x(4x^2-1)=0$$

$3x^2 = 0$

$3\times\frac{1}{4} = 3/4$

$$\Rightarrow x=0 \text{ or } 4x^2-1=0$$

$$\Rightarrow x=0 \text{ or } x=\pm\frac{1}{2}$$

$$\Rightarrow \boxed{x=0, +\frac{1}{2}, -\frac{1}{2}}$$ Ans.

2016, 4 Marks

3 $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right) - \tan^{-1}\left(\frac{4x}{1-4x^2}\right) = \tan^{-1}2x\,;\ |2x| < \frac{1}{\sqrt{3}}$

Sol Taking LHS

$$\tan^{-1}\left(\frac{3(2x)-(2x)^3}{1-3(2x)^2}\right) - \tan^{-1}\left(\frac{2(2x)}{1-(2x)^2}\right)$$

let $2x = \tan\theta \Rightarrow \theta = \tan^{-1}2x$

$$\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right) - \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta) - \tan^{-1}(\tan 2\theta)$$

$$\Rightarrow 3\theta - 2\theta = \theta \Rightarrow \boxed{\tan^{-1}2x}$$ RHS.

2015, 4 Marks

4 *Evaluate : tan* $\left\{2\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\}$

Sol $\tan\left\{2\tan^{-1}(1/5) + \pi/4\right\}$

We know that (formula)

$2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

$$= \tan\left\{\tan^{-1}\frac{2\times 1/5}{1-(1/5)^2} + \tan^{-1}1\right\}$$

$\frac{5}{12}\times 1 = \frac{5}{12} < 1$

$$= \tan\left\{\tan^{-1}\frac{2}{5}\times\frac{25}{24} + \tan^{-1}1\right\}$$

$$= \tan\left\{\tan^{-1}5/12 + \tan^{-1}1\right\} = \tan\left\{\tan^{-1}\frac{5/12+1}{1-5/12\times 1}\right\}$$

$$= \tan\left\{\tan^{-1}\frac{17}{12}\times\frac{12}{7}\right\} = \tan\left\{\tan^{-1}\frac{17}{7}\right\} = \boxed{\frac{17}{7}}$$ Ans

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5 *Prove that* $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, \quad \frac{-1}{\sqrt{2}} \le x \le 1$

2014, 4 Marks

Sol In LHS Put $x = \cos 2\theta$

$$\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right] = \tan^{-1}\left[\frac{\sqrt{\cancel{1}+2\cos^2\theta-\cancel{1}}-\sqrt{\cancel{1}-\cancel{1}+2\sin^2\theta}}{\sqrt{\cancel{1}+2\cos^2\theta-\cancel{1}}+\sqrt{\cancel{1}-\cancel{1}+2\sin^2\theta}}\right]$$

$\sin\theta$ #

$$\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$$

$$= \tan^{-1}\left[\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right] = \tan^{-1}\left[\frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\right]$$

$x = \cos 2\theta$

$2\theta = \cos^{-1} x$

$\theta = \frac{1}{2}\cos^{-1} x$

$$= \tan^{-1}\left[\frac{1-\tan\theta}{1+1\cdot\tan\theta}\right] = \tan^{-1}\left[\frac{\tan\pi/4 - \tan\theta}{1+\tan\pi/4\cdot\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\pi/4-\theta\right)\right] = \pi/4 - \theta = \boxed{\pi/4 - \frac{1}{2}\cos^{-1} x} \quad \text{Ans.}$$

$xy = \frac{x^2-4}{x^2-16} < 1$ **2014, 4 Marks** $xy < 1$

6) *If* $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$,   ***find the value of x***

$x = \pm\sqrt{2}$

Sol applying formula

$$\tan^{-1}\left[\frac{\left(\frac{x-2}{x-4}\right)+\left(\frac{x+2}{x+4}\right)}{1-\left(\frac{x-2}{x-4}\right)\left(\frac{x+2}{x+4}\right)}\right] = \tan^{-1}\left[\frac{\frac{(x-2)(x+4)+(x+2)(x-4)}{(x-4)(x+4)}}{\frac{(x-4)(x+4)-(x-2)(x+2)}{(x-4)(x+4)}}\right]$$

$$= \tan^{-1}\left[\frac{(x^2+4x-2x-8)+(x^2-4x+2x-8)}{(x^2-16)-(x^2-4)}\right]$$

$$= \tan^{-1}\left[\frac{2x^2-16}{-12}\right] = \pi/4 \quad \text{(As per Ques.)}$$

$$\frac{2x^2-16}{-12} = \tan\pi/4 \Rightarrow \boxed{\frac{2x^2-16}{-12} = 1}$$

$$\Rightarrow 2x^2-16 = -12 \Rightarrow 2x^2 = 4 \Rightarrow x^2 = 2 \Rightarrow \boxed{x = \pm\sqrt{2}} \quad \text{Ans.}$$