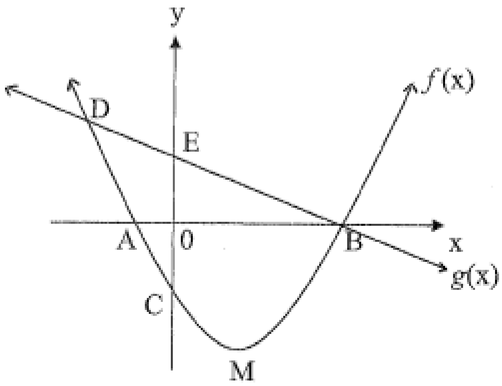
QUESTION SIX:

6.1. $f(x) = 2x^2 - 4x - 6$
$g(x) = -2x + 6$; M is the turning point of $f(x)$.



6.1.1. Calculate the coordinates of the intercepts A, B and C. (5)

6.1.2. Calculate the coordinates of the turning point M. (3)

6.1.3. If both graphs intersect at B and D, calculate the coordinates of D. (3)

6.1.4. Use the graph to solve the inequality $f(x) \cdot g(x) < 0$ (3)

6.1.5. For which value(s) of x is $f(x)$ strictly increasing? (2)

6.2. With reference to the above graph calculate the average gradient between the points B and D of f. (3) [19]

QUESTION SEVEN:

The graph of $f(x) = \frac{-1}{x-3} + 2$

7.1. Determine the y – intercept of f. (2)

7.2. Write down the **equations** of the asymptotes of f. (2)

7.3. For what value(s) of x is $f(x) = 0$? (3)

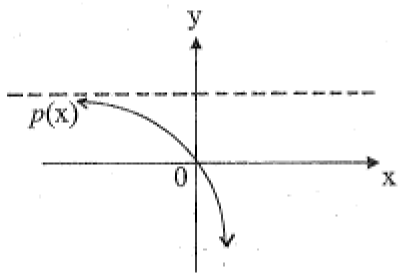
7.4. Draw a neat sketch of $f(x)$. (5)

7.5. Write down **one** equation of the line of symmetry of $f(x)$. (2) [14]

QUESTION EIGHT:

$p(x) = t \cdot 3^x + 1$; (t is a constant)

passes through the point (0 ; 0)

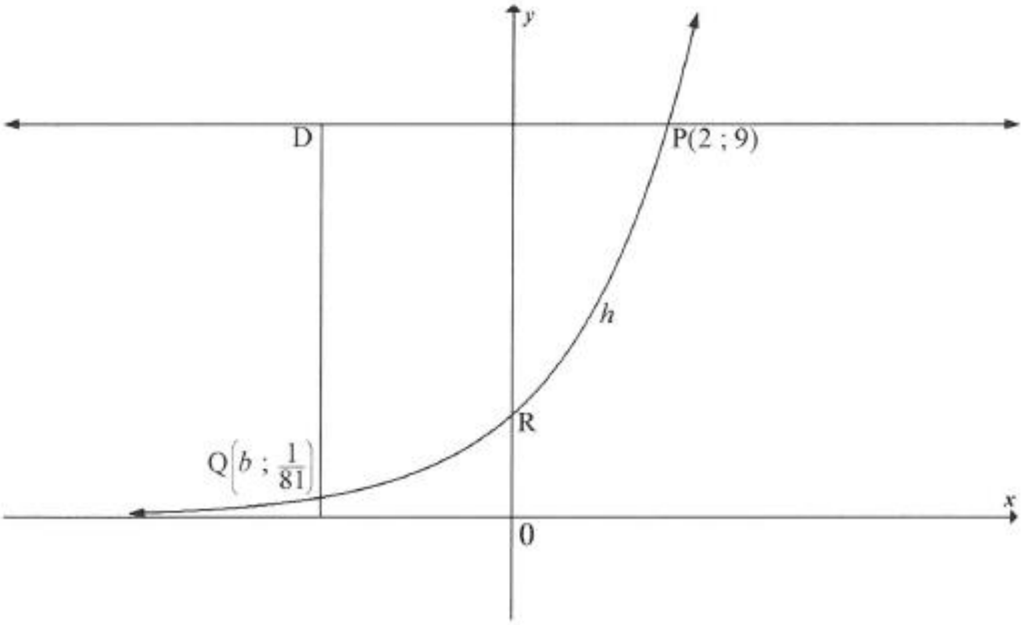


8.1. Show that $t = -1$. (2)

8.2. If p is shifted 4 units to the left to give a new function h, write down the equation of h in the form y = ... (3) [5]

QUESTION 4

Sketched below is the graph of $h(x) = a^x$, $a > 0$. R is the y-intercept of h.
The points P(2 ; 9) and $Q\left(b\,;\,\frac{1}{81}\right)$ lie on h.

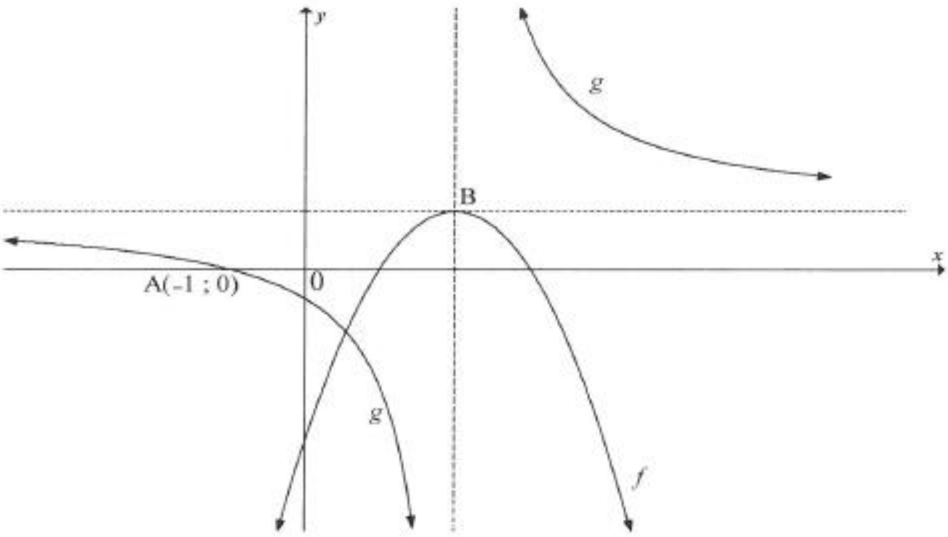


4.1	Write down the equation of the asymptote of h.	(1)
4.2	Determine the coordinates of R.	(1)
4.3	Calculate the value of a.	(2)
4.4	D is a point such that DQ $\parallel$ y-axis and DP $\parallel$ x-axis. Calculate the length of DP.	(4)
4.5	Determine the values of k for which the equation $h(x+2)+k=0$ will have a root that is less than -6.	(3)

QUESTION 5

Sketched below is the parabola f with equation $f(x) = -x^2 + 4x - 3$ and a hyperbola g with equation $(x - p)(y + t) = 3$.

- B, the turning point of f, lies at the point of intersection of the asymptotes of g.
- $A(-1 ; 0)$ is the x-intercept of g.



5.1	Show that the coordinates of B are $(2 ; 1)$	(2)
5.2	Write down the range of f.	(1)
5.3	For which value(s) of x will $g(x) \geq 0$?	(2)
5.4	Determine the equation of the vertical asymptote of the graph of h if $h(x) = g(x + 4)$	(1)
5.5	Determine the values of p and t.	(4)
5.6	Write down the values of x for which $f(x).g'(x) \geq 0$	(4)

QUESTION 6

Given: $f(x) = -x + 3$ and $g(x) = \log_2 x$

6.1	On the same set of axes, sketch the graphs of f and g, clearly showing ALL intercepts with the axes.	(4)
6.2	Write down the equation of $g^{-1}(x)$, the inverse of g, in the form $y = \ldots$	(2)
6.3	Explain how you will use QUESTION 6.1 and/or QUESTION 6.2 to solve the equation $\log_2(3 - x) = x$.	(3)
6.4	Write down the solution to $\log_2(3 - x) = x$.	(1)
		[10]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=3x^2$ (5)

8.2 John determines $g'(a)$, the derivative of a certain function g at $x=a$, and arrives at the answer: $\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$
Write down the equation of g and the value of a. (2)

8.3 Determine $\frac{dy}{dx}$ if $y=\sqrt{x^3}-\frac{5}{x^3}$ (4)

8.4 $g(x)=-8x+20$ is a tangent to $f(x)=x^3+ax^2+bx+18$ at $x=1$. Calculate the values of a and b. (5)

[16]

QUESTION 9

For a certain function f, the first derivative is given as $f'(x)=3x^2+8x-3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

9.2 For which values of x is f concave down? (3)

9.3 Determine the values of x for which f is strictly increasing. (2)

9.4 If it is further given that $f(x)=ax^3+bx^2+cx+d$ and $f(0)=-18$, determine the equation of f. (5)

[13]

QUESTION 5

Given: $f(x)=\frac{-3}{x+2}+1$ and $g(x)=2^{-x}-4$

5.1 Determine $f(-3)$. (1)

5.2 Determine x if $g(x)=4$. (2)

5.3 Write down the asymptotes of f. (2)

5.4 Write down the range of g. (1)

5.5 Determine the coordinates of the x- and y- intercepts of f. (5)

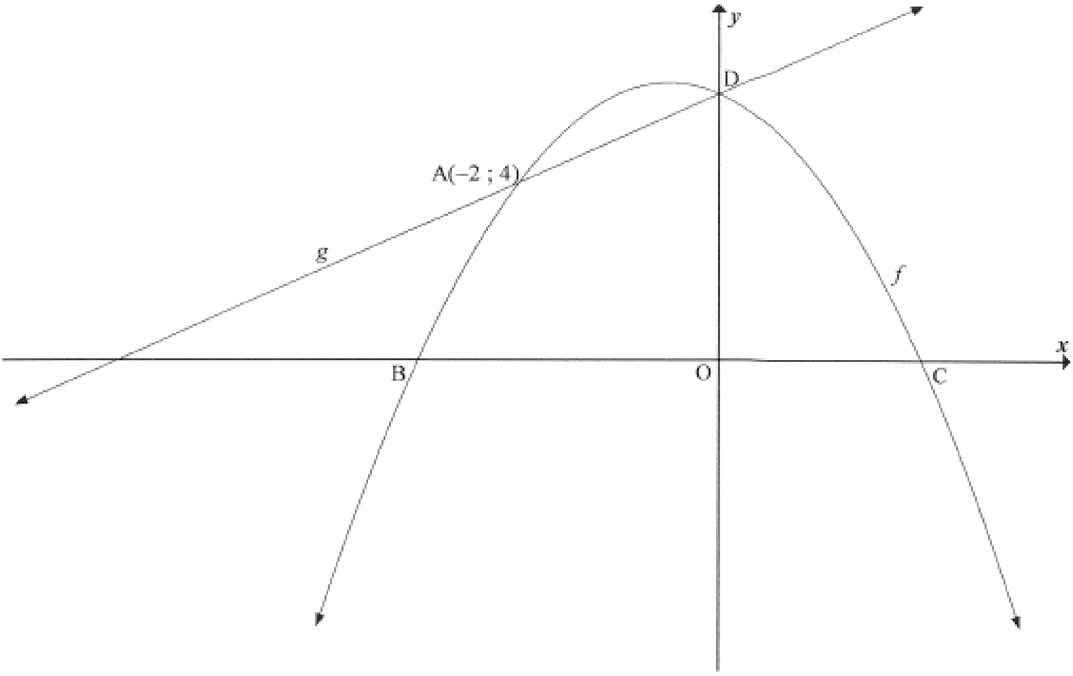
5.6 Determine the equation of the axis of symmetry of f which has a negative gradient. Leave your answer in the form $y=mx+c$. (2)

5.7 Sketch the graphs of f and g on the same system of axes. Clearly show ALL intercepts with the axes and any asymptotes. (6)

5.8 If it is given that $f(-1)=g(-1)$, determine the values of x for which $g(x)\geq f(x)$. (2)

[21]

The diagram below shows the graphs of $f(x) = -x^2 - x + 6$ and $g(x) = mx + c$. $A(-2\ ;\ 4)$ is the point of intersection of the graphs.



6.1	Determine the x-intercepts f.	(4)
6.2	Write down the equation of the axis of symmetry of f.	(2)
6.3	Determine the range of f.	(3)
6.4	Write down the equation of g in the form $g(x) = mx + c$.	(3)
6.5	Write down the average gradient between points A and D.	(1)
6.6	Determine the equation of h, if h is the reflection of f about the x-axis and then translated 3 units to the right. Leave your answer in the form $h(x) = a(x+p)^2 + q$.	(3)
6.7	Write down the values of x for which $f(x) > 0$.	(2)
6.8	If $f(p) = f(r) = 4$, calculate the value of $p - r$ if $r < 0$.	(4)
		[22]

ANSWERS

QUESTION SIX :

6.1.1. $2x^2 - 4x - 6 = 0$ ✓A

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$ ✓CA

$A(-1;0)$ ✓CA ; $B(3;0)$ ✓CA

$C(0;-6)$ ✓A (5)

6.1.2. $x = \frac{-b}{2a}$ ✓A OR $x = 2$ ✓✓ (answer only)

$= \frac{-(-4)}{2(2)}$

$= 2$ ✓A

$M(2;-6)$ ✓CA (3)

6.1.3. $2x^2 - 4x - 6 = -2x + 6$ ✓A

$2x^2 - 4x + 2x - 6 - 6 = 0$

$2x^2 - 2x - 12 = 0$

$2(x^2 - x - 6) = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$ ✓CA

$D(-1;8)$ ✓CA (3)

6.1.4. $x > 3$ AND $-1 < x < 3$ ✓CA

SOL : $x > -1$; $x \neq 3$ ✓CA (3)

6.1.5. $x > 2$ ✓CA (2)

6.2.

6.2.

$B(3;0)$; $D(0;6)$

$AG = \frac{6-0}{0-3}$ ✓CA ✓CA

$= -2$ ✓CA

(3)

[19]

QUESTION SEVEN :

7.1. $y = \frac{-1}{0-3} + 2$ ✓A

$y = 2\frac{1}{3}$

$(0; 2\frac{1}{3})$ ✓CA (2)

7.2. $x = 3$ ✓A

$y = 2$ ✓A (2)

7.3. $0 = \frac{-1}{x-3} + 2$ ✓A

$\frac{1}{x-3} = 2$

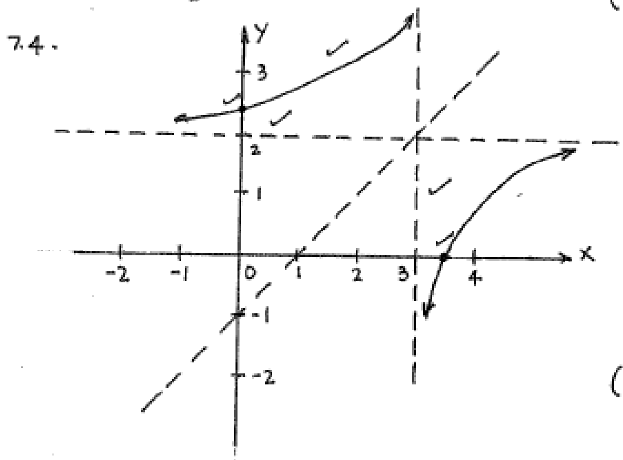
$1 = 2(x-3)$ ✓CA

$1 = 2x - 6$

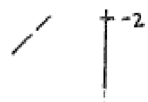
$7 = 2x$

$x = 3\frac{1}{2}$ ✓CA (3)

7.4.



(5)



(5)

7.5. $y = x - 1$ ✓✓ or $y = -x + 5$ (2)

[14]

QUESTION EIGHT :

8.1. $p(x) = t \cdot 3^x + 1$

$0 = t \cdot 3^0 + 1$ ✓A

$0 = t + 1$

$t = -1$ ✓A (2)

8.2. $h(x) = -3^{x+4} + 1$ ✓A ✓A ✓A (3)

[5]

QUESTION/*VRAAG* 4

4.1	$y = 0$	✓ $y = 0$ (1)
4.2	R(0 ; 1)	✓ answer (1)
4.3	$y = a^x$ $9 = a^2$ $\therefore a = 3$	✓ substitution ✓ $a = 3$ (2)
4.4	$DP = 2 - b$ $y = 3^x$ $\frac{1}{81} = 3^b$ $3^{-4} = 3^b$ $b = -4$ $DP = 2 - (-4)$ $= 6$ units	✓ $\frac{1}{81} = 3^b$ ✓ 3^{-4} or use of logs ✓ $b = -4$ ✓ $DP = 6$ units (4)
4.5	$h(x+2) + k = 0$ $h(x+2) = -k$ $0 < -k < \frac{1}{81}$ $-\frac{1}{81} < k < 0$	✓✓ $-k < \frac{1}{81}$ or $k > -\frac{1}{81}$ ✓ $-\frac{1}{81} < k < 0$ (3)
		[11]

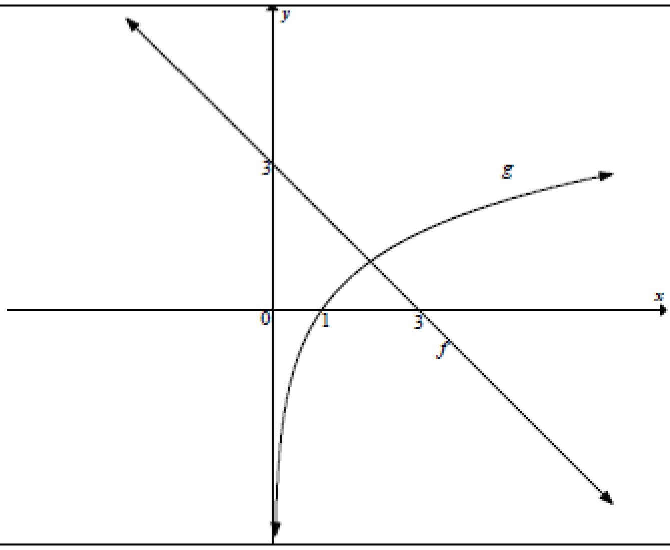
QUESTION/*VRAAG* 5

5.1	$f(x)=-x^2+4x-3$ $f'(x)=0$ or $x=-\frac{4}{2(-1)}$ $-2x+4=0$ $x=2$ $x=2$ $y=-(2)^2+4(2)-3$ $=1$ B$(2\ ;1)$ **OR/OF** $-x^2+4x-3=0$ $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=3$ or $x=1$ $x=\frac{3+1}{2}$ $x=2$ $y=-(2)^2+4(2)-3$ $=1$ B(2 ; 1)	✓ $-2x+4=0$ or $x=-\frac{4}{2(-1)}$ ✓ $y=-(2)^2+4(2)-3$ (2) ✓ $x=\frac{3+1}{2}$ ✓ $y=-(2)^2+4(2)-3$ (2)
5.2	Range/*Waardeversameling* : $y\leq 1$ **OR/OF** Range/*Waardeversameling* : $y\in(-\infty\ ;1]$	✓ $y\leq 1$ (1) ✓ $(-\infty\ ;1]$ (1)
5.3	$x\leq -1$ or $x>2$	✓ critical values ✓ $x\leq -1$ or $x>2$

5.4	$(x-p)(y+t)=3$ Vertical asymptote of h(x) /*vertikale asimptoot* at $x = 2$ Translation 4 units to the left / *Translasie 4 eenhede links* $x = 2 - 4 = -2$ is the equation of the vertical asymptote of h(x + 4) $x = 2 - 4 = -2$ *is die vergelyking van die vertikale asimptoot* **OR/OF**	✓ $x=-2$ (1)

	OR/OF $h(x)=\frac{3}{x-2+4}+1$ $=\frac{3}{x+2}+1$ $x=-2$ is the equation of the vertical asymptote / *is die vergelyking van die vertikale asimptoot*	✓ $x=-2$ (1)
5.5	$(x-p)(y+t)=3$ $(y+t)=\frac{3}{(x-p)}$ $y=\frac{3}{x-p}-t$ B(2;1) Point of intersection of the asymptotes *Snypunt van die asimptote* $p=2$ $-t=1$ $t=-1$	✓ $\frac{3}{x-p}$ ✓ $-t$ ✓ $p=2$ ✓ $t=-1$ (4)
5.6	x-intercepts of f / *x-afsnitte van f*: $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or $x=3$ $g'(x)<0$ for $x \in R;\ x \neq 2$ Hence $f(x)<0$ $x \leq 1$ or $x \geq 3$ **OR/OF** $(-\infty;1]\cup[3;\infty)$	✓ both critical values ✓ $x\leq 1$ ✓ or ✓ $x \geq 3$ (4) **[14]**

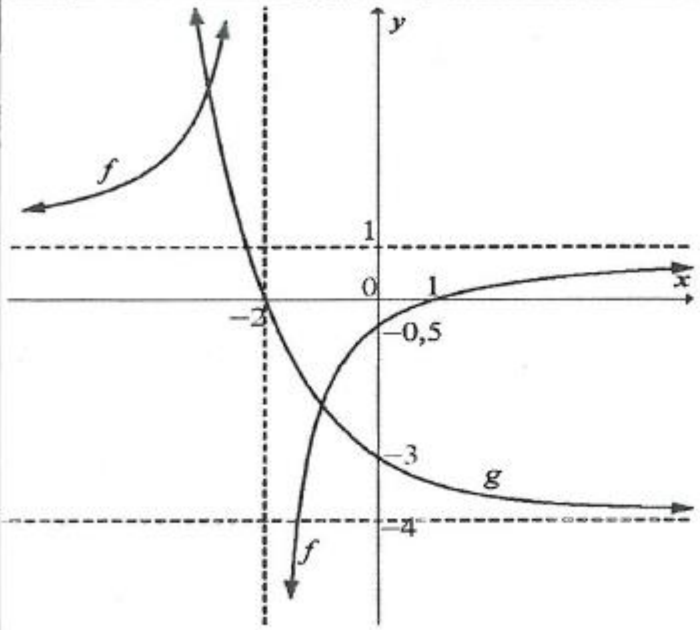
QUESTION/*VRAAG* 6

6.1		g: ✓shape: increasing curve ✓(1 ; 0): only on log graph f: ✓(3 ; 0) ✓(0 ; 3) (4)
6.2	$y=\log_2 x$ $g^{-1}:\ x=\log_2 y$ $y=2^x$	✓ interchange x and y ✓ $y=2^x$ (2)
6.3	$\log_2(3-x)=x$ $2^x=3-x$ $2^x=-x+3$ Reflect the graph of g about the line $y=x$ to obtain g^{-1} and determine the point of intersection of f and g^{-1}./ *Reflekteer die grafiek van g om die lyn* $y=x$ *en bepaal die snypunt van* f and g^{-1}	✓✓ $2^x=-x+3$ ✓ point of intersection of f and g^{-1} (3)
6.4	$x=1$	✓ answer (1) **[10]**

QUESTION/*VRAAG* 5

5.1	$f(-3)=\frac{-3}{-3+2}+1$ $=4$	✓answer/*antw.* (1)
5.2	$4=2^{-x}-4$ $8=2^{-x}$ $2^3=2^{-x}$ $x=-3$	✓ $4=2^{-x}-4$ ✓answer /*antw.* (2)
5.3	$x=-2$ $y=1$	✓ $x=-2$ ✓ $y=1$ (2)
5.4	$y>-4$ **OR/*OF*** $y\in(-4\ ;\ \infty)$	✓answer/*antw.* (1) ✓answer/*antw.* (1)

5.5	y-intercept/*afsnit*: $y=\frac{-3}{0+2}+1$ $=\frac{-1}{2}$ y-intercept/*afsnit* is $\left(0\ ;-\frac{1}{2}\right)$ x-intercept/*afsnit*: $0=\frac{-3}{x+2}+1$ $-1=\frac{-3}{x+2}$ $-x-2=-3$ $-x=-1$ $x=1$ x-intercept/*afsnit* is $(1\ ;\ 0)$	✓subst/*verv* $x=0$ ✓ $y=\frac{-1}{2}$ ✓subst/*verv* $y=0$ ✓simplification/*vereenv.* ✓ $x=1$ (5)
5.6	$y=-x+c$ $1=-(-2)+c$ $-1=c$ $y=-x-1$ **OR/*OF*** $y-1=-(x-(-2))$ $y=-x-2+1$ $y=-x-1$	✓subst/*verv* ✓answer/*antw.* (2) ✓subst/*verv* ✓answer/*antw.* (2)

5.7		*f* ✓asympt/*asimpt* ✓Shape / *vorm* ✓*x* and/*en y* intercepts / *afsnitte* *g* ✓asymptote/*asimpt* ✓*x*-intercept/*afsnit* $(-2\ ;\ 0)$ ✓*y*-intercept/*afsnit* $(0\ ;\ -3)$ (6)
5.8	$x \leq -3$ or $-2 < x \leq -1$ **OR/*OF*** $x \in (-\infty\ ;\ -3) \cup (-2\ ;\ -1]$	✓$x \leq -3$ ✓$-2 < x \leq -1$ (2) ✓$(-\infty\ ;\ -3)$ ✓$(-2\ ;\ -1]$ (2)
		[21]

QUESTION/*VRAAG* 6

6.1	$0=-x^2-x+6$ $x^2+x-6=0$ $(x+3)(x-2)=0$ $x=-3$ *or / of* $x=2$ B(–3 ; 0) and C(2 ; 0)	✓$y=0$ ✓standard form/*vorm* ✓factors/*faktore* ✓both answers/*beide antw* (4)
6.2	$x=\frac{-b}{2a}$ $x=\frac{-(-1)}{2(-1)}$ $=-\frac{1}{2}$ **NOTE/ *LET WEL:*** If answer only: award 2/2 marks *Slegs antwoord : gee 2/2 punte*	✓method/*metode* ✓answer/*antw.* (2)
	OR/ *OF* $x=\frac{x_1+x_2}{2}$ $=\frac{(-3)+(2)}{2}$ $=-\frac{1}{2}$	✓method/*metode* ✓answer/*antw* (2)
6.3	$f\left(-\frac{1}{2}\right)$ $=-\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)+6$ $=6\frac{1}{4}$ TP / *DP* $\left(-\frac{1}{2};6\frac{1}{4}\right)$ Range/*waardeversameling* $y\in\left(-\infty\ ;6\frac{1}{4}\right]$	✓Subst ✓$6\frac{1}{4}$ ✓Answer/*antw.*

6.4	D(0 ; 6) $m_{AD} = \frac{6-4}{0-(-2)}$ $=1$ Equation of/*vergelyking van g*: $g(x) = x+6$	✓coordinates/*koördinate* D ✓gradient. ✓answer/*antw* (3)

 Please turn over/*Blaai om asseblief*

6.5	Average/*Gemid.* gradient = gradient of/*van g* $=1$	✓answer/*antw.* (1)
6.6	$f(x) = -\left(x+\frac{1}{2}\right)^2 + \frac{25}{4}$ $h(x) = \left(x+\frac{1}{2}-3\right)^2 - \frac{25}{4}$ $h(x) = \left(x-\frac{5}{2}\right)^2 - \frac{25}{4}$ OR/OF	✓ in the form/ *in die vorm* $f(x) = -\left(x+\frac{1}{2}\right)^2 + \frac{25}{4}$ ✓ $\left(x-\frac{5}{2}\right)^2$ ✓ $-\frac{25}{4}$

	OR/*OF*	(3)
	$f(x)=-x^2-x+6$ $h(x)=(x-3)^2+(x-3)-6$ $h(x)=x^2-5x$ $h(x)=\left(x-\frac{5}{2}\right)^2-\frac{25}{4}$	✓ $h(x)=(x-3)^2+(x-3)-6$ ✓ $\left(x-\frac{5}{2}\right)^2$ ✓ $-\frac{25}{4}$ (3)
6.7	$-3<x<2$ **OR/*OF*** $x\in(-3\,;2)$	✓✓answer/*antw.* (2) ✓✓answer/*antw.* (2)
6.8	$r=-2$ By symmetry/*deur simmetrie* $p=1$ $p-r=3$ **OR/*OF*** $-x^2-x+6=4$ $-x^2-x+2=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ *or/of* $x=1$ $r=-2$ $p=1$ $p-r=3$	✓$r=-2$ ✓✓$p=1$ ✓answer/*antw.* (4) ✓$r=-2$ ✓✓$p=1$ ✓answer/*antw.* (4)