

QUESTION FOUR:

4.1. Consider the sequence:

8 ; 11 ; 14 ;

4.1.1. Extend the sequence by two terms. (2)

4.1.2. Is this sequence linear or quadratic?
Give a reason for your answer. (2)

4.1.3. Find the general term. (2)

4.1.4. Find the tenth term. (2)

4.2. Consider the sequence:

-1 ; 2 ; 7 ; 14 ;

4.2.1. Extend the sequence by two terms. (2)

4.2.2. Is this sequence linear or quadratic?
Give a reason for your answer. (2)

4.2.3. Find the general term. (2)

4.2.4. Which term of the sequence is equal to 623? (2)

4.3. If $T_n = (-2)^{n-1}$

4.3.1. Write down the first 4 terms of the sequence. (4)

4.3.2. What pattern do you notice? (2)

4.3.3. What is the sum of the first 4 terms? (2)

4.3.4. Which term will be equal to 64? (2) [26]

QUESTION 2

Given the finite arithmetic sequence: 5 ; 1 ; -3 ; ... ; -83 ; -87

2.1 Write down the fourth term (T_4) of the sequence. (1)

2.2 Calculate the number of terms in the sequence. (3)

2.3 Calculate the sum of all the negative numbers in the sequence. (3)

2.4 Consider the sequence: 5 ; 1 ; -3 ; ... ; -83 ; -87 ; ... ; -4 187
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)
[11]

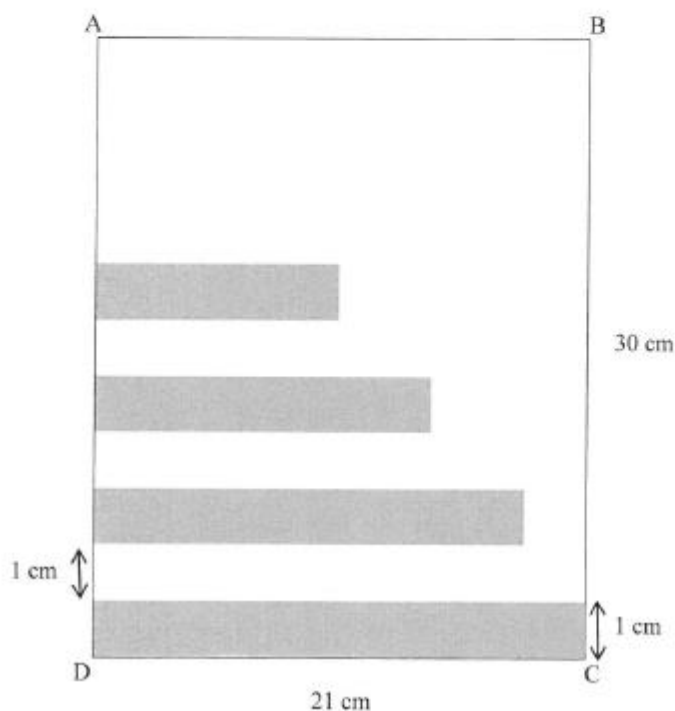
QUESTION 3

3.1 The first four terms of a quadratic number pattern are -1 ; x ; 3 ; $x + 8$

3.1.1 Calculate the value(s) of x . (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first n first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.



3.2.1 Calculate the length of the 10th rectangle. (3)

3.2.2 Calculate the percentage of the paper that is coloured grey. (4)
[15]

QUESTION 3

- 3.1 Given the finite linear pattern: 12 ; 17 ; 22 ; ... ; 172
- 3.1.1 Determine a formula for the n^{th} term of the pattern. (2)
- 3.1.2 Calculate the value of T_{12} . (2)
- 3.1.3 Determine the number of terms in the pattern. (2)
- 3.2 Given the first four terms of a linear pattern: 3; x; y; 30
- Calculate the values of x and y. (4)
- [10]**

QUESTION 4

Given the quadratic pattern: 244 ; 193 ; 148 ; 109 ...

- 4.1 Write down the next term of the pattern. (2)
- 4.2 Determine a formula for the n^{th} term of the pattern. (4)
- 4.3 Which term of the pattern will have a value of 508? (4)
- 4.4 Between which TWO consecutive terms of the quadratic pattern will the first difference be 453? (3)
- 4.5 Show that all the terms of the quadratic pattern are positive. (4)
- [17]**

ANSWERS

QUESTION FOUR :	
4.1.1. 17; 20	(2)
4.1.2. LINEAR; $d = 3$	(2)
4.1.3. $T_n = a + (n-1)d$ $T_n = 8 + (n-1)(3)$ $T_n = 8 + 3n - 3$ $T_n = 3n + 5$	(2)
4.1.4. $T_{10} = 3(10) + 5$ $= 35$	(2)
4.2.1. 23; 34	(2)
4.2.2. QUADRATIC. SECOND DIFF = 2	(2)
4.2.3. $a = 1, b = 0, c = -2$ $T_n = n^2 - 2$	(2)
4.2.4. $n^2 - 2 = 625$ $n^2 = 625$ $n = 25$ $\therefore 25^{\text{th}} \text{ term is } 623.$	(2)

	$-3; -7; \dots; -87$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{22} = \frac{22}{2}[2(-3) + (22-1)(-4)]$ $= -990$ OR/OF All negative terms can be written down and added to get the answer of -990 . / <i>Alle negatiewe terme kan neergeskryf word en dan bymekaar getel word om -990 te kry.</i> OR/OF $\text{Sum} = S_{24} - (5+1)$ $= \frac{24}{2}[5 - 87] - 6$ $= -990$	$\checkmark n = 22$ $\checkmark a = -3$ $\checkmark$ answer (3) $\checkmark a = -3$ $\checkmark \checkmark$ answer (3) $\checkmark \frac{24}{2}[5 - 87]$ $\checkmark -6$ $\checkmark$ answer (3)
2.4	$5; -15; -35; \dots$ $d = -20$ $T_n = -20n + 25$ Last term in the sequence divisible by 5 is: / <i>Laaste term in die ry deelbaar deur 5 is:</i> $-4187 + 4(3)$ $= -4175$ $T_n = -20n + 25$ $-4175 = -20n + 25$ $20n = 4200$ $n = 210$ There will be 210 terms in the sequence that is divisible by 5. / <i>Daar is 210 terme in die ry deelbaar deur 5.</i>	$\checkmark d = -20$ $\checkmark T_n = -20n + 25$ $\checkmark -4175 = -20n + 25$ $\checkmark n = 210$ (4)

QUESTION/VRAG 3

3.1.1	$-1 ; x ; 3 ; x+8 ; \dots$ $-2x+2 = 2x+2$ $4x=0$ $x=0$	✓ $x+1; 3-x$ and $x+5$ ✓ calculating second differences ✓ $-2x+2 = 2x+2$ ✓ $x=0$ (4)
3.1.2	First differences/<i>Eerste verskille</i>: $1 ; 3 ; 5 ; \dots$ $S_n = \frac{n}{2}[2(1) + (n-1)(2)]$ $= n^2$ $250 < n^2$ $n > \sqrt{250}$ $\therefore n > 15,8$ The sum of the 16 first differences will be greater than 250. Therefore the 17th term of the quadratic number pattern is the first satisfying this condition. / <i>Die som van 16 eerste verskille sal groter as 250 wees. Gevolglik sal die 17^{de} term van die kwadrasiese getalpatroon die eerste wees wat aan die voorwaarde voldoen.</i>	✓ $S_n = n^2$ ✓ $S_n > 250$ ✓ $n > 15,8$ ✓ $n = 17$ (4)
3.2.1	$21 + 21(0,85) + 21(0,85)^2 + \dots$ $T_n = ar^{n-1}$ $T_{10} = (21)(0,85)^9$ $= 4,86 \text{ cm}$	✓ $n = 10 ; r = 0,85$ or $\frac{17}{20}$ ✓ substitution into correct formula ✓ answer (3)

3.2.2	$S_n = \frac{a(1-r^n)}{1-r}$ $S_{15} = \frac{21(1-(0,85)^{15})}{1-0,85}$ $= 127,77$ Area of the page = $30 \times 21 = 630$ Percentage of paper covered in grey ink: $= \frac{127,77}{630} \times 100\%$ $= 20,28\%$	✓ $n = 15$ ✓ 127,77 ✓ 630 ✓ 20,28 (4) [15]
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QUESTION/VRAAG 4

4.1	$y = 0$	✓ $y = 0$ (1)
4.2	$R(0; 1)$	✓ answer (1)
4.3	$y = a^x$ $9 = a^2$ $\therefore a = 3$	✓ substitution ✓ $a = 3$ (2)
4.4	$DP = 2 - b$ $y = 3^x$ $\frac{1}{81} = 3^b$ $3^{-4} = 3^b$ $b = -4$ $DP = 2 - (-4)$ $= 6$ units	✓ $\frac{1}{81} = 3^b$ ✓ 3^{-4} or use of logs ✓ $b = -4$ ✓ $DP = 6$ units (4)
4.5	$h(x+2) + k = 0$ $h(x+2) = -k$ $0 < -k < \frac{1}{81}$ $-\frac{1}{81} < k < 0$	✓ ✓ $-k < \frac{1}{81}$ or $k > -\frac{1}{81}$ ✓ $-\frac{1}{81} < k < 0$ (3) [11]

QUESTION/VRAAG 3

3.1.1	$ \begin{array}{ccccc} 12 & & 17 & & 22 \\ & \searrow & \nearrow & \searrow & \nearrow \\ & 5 & & 5 & \\ T_n = 5n + 7 \end{array} $	$\checkmark 5n$ $\checkmark +7$ (2)
3.1.2	$ \begin{aligned} T_{12} &= 5(12) + 7 \\ &= 67 \end{aligned} $	$\checkmark$ subst/verv $\checkmark$ answer/antw (2)
3.1.3	$ \begin{aligned} 5n + 7 &= 172 \\ 5n &= 165 \\ n &= 33 \end{aligned} $	$\checkmark 5n + 7 = 172$ $\checkmark$ answer/antw (2)
3.2	$ \begin{array}{ccccccc} 3 & & x & & y & & 30 \\ & \searrow & \nearrow & \searrow & \nearrow & \searrow & \nearrow \\ & x-3 & & y-x & & 30-y & \end{array} $ $ \begin{aligned} x-3 &= y-x \\ y &= 2x-3 \end{aligned} $ $ \begin{aligned} x-3 &= 30-y \\ x-3 &= 30-2x+3 \\ 3x &= 36 & \text{OR/OF} & 30-y &= y-x \\ x &= 12 & & 30+x &= 2y \\ y &= 21 & & 30+x &= 2(2x-3) \\ & & & 36 &= 3x \\ & & & 12 &= x \\ & & & y &= 21 \end{aligned} $ OR/OF $ \begin{aligned} 30-3 &= 3d \\ 3d &= 27 \\ d &= 9 \end{aligned} $ $ \begin{aligned} x &= 3+9=12 \\ y &= 12+9=21 \end{aligned} $	$\checkmark x-3 = y-x$ $\checkmark 30-y = y-x$ $\checkmark$ equating/verg. $\checkmark$ both /beide $x = 12$ and/en $y = 21$ (4) $\checkmark \checkmark 30-3 = 3d$ $\checkmark d = 9$ $\checkmark$ both /beide $x = 12$ and/en $y = 21$ (4) 110

QUESTION/VRAAG 4

4.1	$ \begin{array}{ccccccc} 244 & & 193 & & 148 & & 109 \\ & \searrow & \nearrow & \searrow & \nearrow & \searrow & \nearrow \\ & -51 & & -45 & & -39 & \\ & \nearrow & \searrow & \nearrow & \searrow & \nearrow & \\ & 6 & & 6 & & & \end{array} $ $T_5 = 76$ NOTE/LET WEL: Calc. differences 1/2 marks bereken verskille: 1/2 punte	✓✓ answer/antw. (2)
4.2	$ \begin{aligned} 2a &= 6 \\ a &= 3 \\ 3a + b &= -51 \\ 3(3) + b &= -51 \\ b &= -60 \\ a + b + c &= 244 \\ 3 + -60 + c &= 244 \\ c &= 301 \\ T_n &= 3n^2 - 60n + 301 \end{aligned} $	✓ $a = 3$ ✓ $b = -60$ ✓ $c = 301$ ✓ $T_n = 3n^2 - 60n + 301$ (4)
4.3	$ \begin{aligned} 3n^2 - 60n + 301 &= 508 \\ 3n^2 - 60n - 207 &= 0 \\ n^2 - 20n - 69 &= 0 \\ (n+3)(n-23) &= 0 \\ n &= 23 \text{ or } n = -3 \end{aligned} $	✓ equating/verg. ✓ std form/stand vorm ✓ factors/fakore ✓ select/ kies $n = 23$ (4)
4.4	using first diff./ gebruik eerste versk $ \begin{aligned} T_n &= 6n - 57 \\ 453 &= 6n - 57 \\ 510 &= 6n \\ n &= 85 \end{aligned} $ between T_{85} and T_{86} in the quadratic pattern tussen T_{85} en T_{86} in die kwadratiese patroon	✓ $6n - 57$ ✓ $453 = 6n - 57$ ✓ between T_{85} and T_{86} tussen T_{85} en T_{86} (3)

4.5	$T_n = 3n^2 - 60n + 300 + 1$ $= 3(n-10)^2 + 1$ $(n-10)^2 \geq 0$ for $\forall n \in \mathbb{N}$ $3(n-10)^2 \geq 0$ $3(n-10)^2 + 1 > 0$ All terms in the pattern are positive/ <i>al die terme in die patroon is positief</i> OR/OF $Tn = 3n^2 - 60n + 301$ $= 3(n-10)^2 + 1$ T is a minimum valued function with minimum value 1 Range of T: $y \geq 1$ All terms in the pattern are positive. <i>T is funksie met minimum waarde van 1</i> <i>Waardeversameling van T: $y \geq 1$</i> <i>al die terme in die patroon is positief</i> OR/OF $p = \frac{-b}{2a}$ $= \frac{-(-60)}{6}$ $= 10$ $q = 3(10)^2 - 60(10) + 301$ $= 1$ T is a minimum valued function with minimum value 1 Range of T: $y \geq 1$ All terms in the pattern are positive.	$\checkmark\checkmark T_n = 3(n-10)^2 + 1$ $\checkmark\checkmark$ argument (4) $\checkmark\checkmark T_n = 3(n-10)^2 + 1$ $\checkmark\checkmark$ argument $\checkmark p = 10$ $\checkmark q = 1$ $\checkmark\checkmark$ argument (4)
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