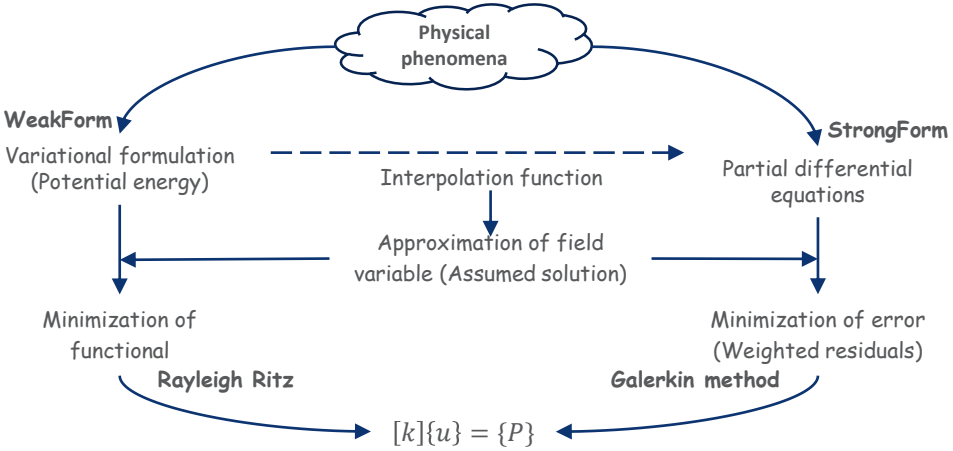
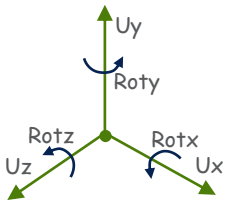
Structural FEA (Cheat Sheet)



Nodal degrees of freedom



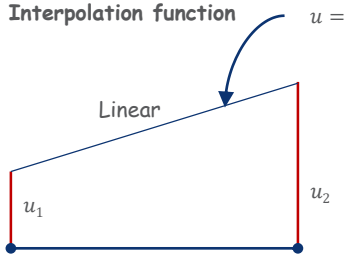
Translation Ux, Uy, Uz
Rotational Rotx, Roty, Rotz

Finite element equations

- Load vs displacement $[k]\{u\} = \{P\}$
- Strain vs displacement $\{\varepsilon\} = [\partial]\{u\}$
- Stress vs strain $\{\sigma\} = [E]\{\varepsilon\}$
- Boundary conditions $\{u\} = known\ displacements$

Interpolation function

$u = N_1 u_1 + N_2 u_2$



$N_1 = \frac{u_2 - u}{u_2 - u_1}$

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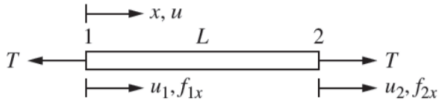
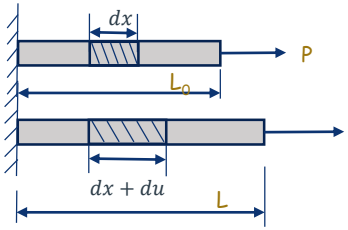
Bar element FE equation



$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Strain-Displacement (Linear, nonlinear measures)



Engineering strain $\varepsilon_{eng} = \frac{L - L_0}{L_0}$

Logarithmic strain $\varepsilon = \ln\left(\frac{L}{L_0}\right)$

Cauchy-Green strain $\varepsilon_{xx} = \frac{L^2 - L_0^2}{2L_0^2}$

Beam element stiffness matrix

$$\mathbf{k} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Plane stress-strain equation

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & 0.5-\nu \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Nonlinearity

Geometric nonlinearity (types)

- Nonlinear strain-displacement
- Loads varying with displacement
- Stress stiffening or softening
- Spin softening

Contact nonlinearity (formulations)

- Normal Lagrange
- Pure Penalty
- Augmented Penalty

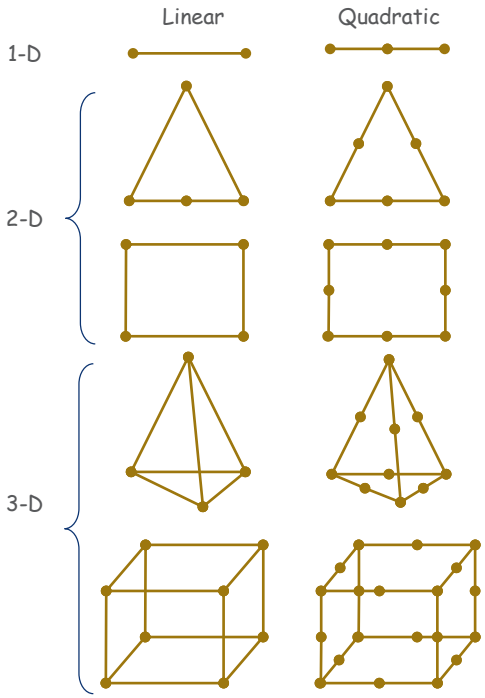
Material nonlinearity (types)

- Elastic-plastic
- Hyper-elasticity
- Visco-elasticity (Creep)

Solution methods

- Gauss elimination
- Jacobi Conjugate Gradient
- Direct Iteration
- Newton Raphson
- Modified Newton Raphson

Types of elements



Theories of failure

Static loading → Brittle material → Modified Mohr's method
→ Ductile material → Distortion Energy method
→ Maximum Shear Stress

Cyclic loading

- Constant amplitude → Brittle material → Not recommended
  → Ductile material → Fluctuating stress
  - Zero mean → Combined reversed stress
  - Non zero mean → Modified Goodman method
- Varying amplitude → Brittle material → Not recommended
  → Ductile material → Miner's rule (Damage accumulation)