Hypothesis Testing

I have said for a long time that statistics should be considered a science and not a math, and that is because of its reliance on hypotheses, which are science.

Hypothesis: A statement or claim that an experiment tests to determine validity. If you want to support a claim, you must state it as an Alternative Hypothesis.

Null Hypothesis H_0: The opposite of your hypothesis, which you will be attempting to prove wrong. States that the population parameter is equal to some value. Statistics cannot prove anything correct. It can just fail to prove something correct. You always start by assuming the null hypothesis is true and then use evidence to reach a conclusion.

Two types of null hypotheses:

- Proportions: Most people get their jobs through networking: $P > 0.50$
- Averages: The average payload of trucks on Highway 550 is 18,000 pounds: $\mu = 18{,}000$

Results: either reject or fail to reject null hypothesis

Alternative Hypothesis H_1: States that the parameter (μ, P) has a value different than H_0.

RARE EVENT RULE: If the probability of an assumption occurring is "very small," then the assumption is probably wrong. And the quantitative value of "very small" will change on a test-by-test basis.

How to Identify the Hypotheses

State the original claim symbolically. State the OPPOSITE of the original claim also. The original claim could be H_0 or H_1. It depends on where the equality is.

Example: The mean volume of fluid in a can of Pepsi is at least 12 ounces.

Claim: $\mu \geq 12\ ounces$ ← has equality, so this is H_0

Opposite: $\mu < 12\ ounces$

RESTATE AS: $\boldsymbol{H_0: \mu = 12;}$ $\boldsymbol{H_1: \mu < 12}$

Testing the Thing

T-TEST: Developing and Using Test Statistics (The traditional way)

For Proportions (P): **just use a standard Z-Score**

$$z = \frac{\hat{P} - P}{\sqrt{\frac{PQ}{n}}}$$

For Mean (μ): **two options: Z or T.**

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \quad or \quad T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \; if\ \sigma\ is\ unknown$$

Example: A sample of 706 companies found that 85% of CEOs were male. Claim: most CEOs are male.

Claim: $P > 50\%$; Opposite: $P \leq 50\%$ ← has equality, so this is H_0

∴ $H_o: P = 0.50$, and $H_1: P > 0.50$

Choose test statistic. Since this is a proportion, we will be using Z-Score.

$$z = \frac{\hat{P} - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.85 - 0.50}{\sqrt{\frac{(0.50)(0.50)}{706}}} = \frac{0.35}{The\ mess} \approx 18.60$$

Whoa… that's a very big z-score. That means H_0 (female CEO) is very rare, and we proved our claim.

T-Test for Two Samples: Tests whether the means of two separate populations are significantly different from one another.

Paired: Each value of one group corresponds directly to a value in the other group. If it's paired, subtract the values for each pair to get one set of values.

Unpaired: The two populations are independent.

T Statistic:

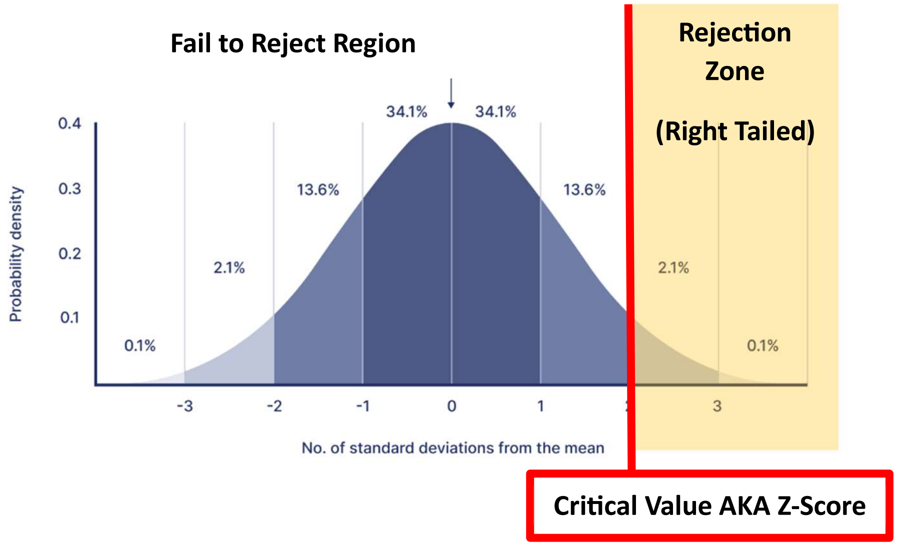
For samples with equal variances:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

For samples with unequal variances:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}}$$

Reject the test statistic if it falls in the rejection zone.



Determining the Tailedness of the Thing

IF:	THEN:
$H_1: \ P < 0.5$	Left Tailed Test and $\alpha = 0.05$ and $z = -1.645$
$H_1: \ P > 0.5$	Right Tailed Test and $\alpha = 0.05$ and $z = 1.645$
$H_1: \ P \neq 0.5$	Two-Tailed Test and $\alpha = 0.025$ and $z = \pm 1.96$

Note: The < or > could be seen as an arrow pointing to the direction of the tailedness

There is another way to do testing:

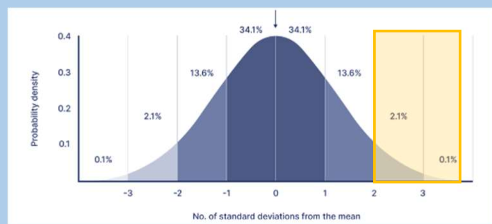
P-Value: Probability associated with your Test Statistic, known as a P Value. There are only two options:

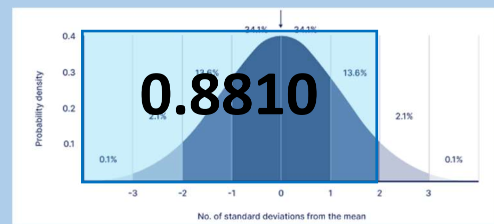
1) **Reject H_0, and therefore accept H_1 iff $P - Value \ \leq \alpha$**
2) **Fail to reject H_0 and therefore YOU KNOW NOTHING iff $P - Value > \alpha$**

Example: Find P-Values: $\alpha = 0.05 \quad and \ H_1: P > 0.25$

You should get a test statistic of $z = 1.18$

Okay let's do this both ways so we can get a feel for how to do these problems.

<table>
<tr><th></th><th>Traditional Way</th><th>P-Value Way</th></tr>
<tr><td>Step 1: Determine Tailedness</td><td colspan="2">> pointing right, so it's right tailed</td></tr>
<tr><td>Step 2: Draw the thing</td><td>

Critical value: $z = 1.645$ because $\alpha = 0.05$</td><td>

Test statistic: $z = 1.18$
P-Value = Area associated with that z-score (0.8810)
Normal cdf function or look up on table = 0.1190</td></tr>
<tr><td>Make decision</td><td>Fail to reject H_0</td><td>Compare P-Value to Alpha:
$0.1190 > 0.05$
Since P-Value is greater:
Fail to Reject H_0</td></tr>
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