Limits and Derivatives

You should also print out my Trig review, as you might need some of that information.

Limit = Moving something really really close to another thing without having them touch

$$\lim_{x \to a} f(x) = L$$

This limit gets infinitely close to L without touching it.

Note that $x \neq a$ or else the slope of the line will be undefined.

A function must approach the same value from the left and right in order for the limit to exist.

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$$

Right hand limit = When a is going *LEFT* on a number line

$$\lim_{x \to a^+} f(x) = L$$

Left hand limit = When a is going *RIGHT* on a number line

$$\lim_{x \to a^-} f(x) = L$$

You can remember this because it makes no sense

PROPERTIES OF LIMITS

$\lim_{x \to a}[cf(x)]$	=	$c \lim_{x \to a} f(x)$
$\lim_{x \to a}[f(x) \pm g(x)]$	=	$\lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
$\lim_{x \to a} f(x)g(x)$	=	$\lim_{x \to a} f(x) \lim_{x \to a} g(x)$
$\lim_{x \to a} \frac{f(x)}{g(x)}$	=	$\frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ *Provided that $\lim_{x \to a} g(x) \neq 0$
$\lim_{x \to a}[f(x)]^n$	=	$[\lim_{x \to a} f(x)]^n$
$\lim_{x \to a} \sqrt[n]{f(x)}$	=	$\sqrt[n]{\lim_{x \to a} f(x)}$

Continuity = A function is continuous if it doesn't have any holes, breaks or asymptotes. (Otherwise, it is *discontinuous*, not *incontinent*.)

If a function is continuous at point c, then:

1) *f(c)* is defined
2) $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$

Removable Discontinuity = you have a hole. If you fill in the hole with a point, it is continuous.

Indeterminate Limits and L'HOPITAL'S RULE

When limits are sick, we send them to the L'Hospital.

Sick limits are those that have an undefined denominator. This happens if the numerator and denominator both approach a value at different rates.

$$IFF: \quad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \ or \ \frac{\infty}{\infty} \quad \Rightarrow \quad L'Hopital \quad \Rightarrow \quad \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Derivatives

Derivative = slope of a line tangent to a curve

A function is called differentiable if a derivative exists.

$$D = \lim_{n \to 0} \frac{f(a+h) - f(a)}{h}$$

Where: f(a) and f(a+h) are points on a curve

h is the distance on the x axis between the two points

COMMON DERIVATIVES

Derivative of a constant	$\frac{d}{dx} C = 0$
Derivative times a constant	$\frac{d}{dx} cf(x) = c \frac{d}{dx}$
Derivative raised to a power **(POWER RULE)**	$\frac{d}{dx} (x^n) = nx^{n-1}$
Adding derivatives	$(f + g)' = f' + g'$
Difference of derivatives	$(f - g)' = f' - g'$

DERIVATIVES OF TRIG and LOG FUNCTIONS

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = (\tan x)(\sec x)$$

$$\frac{d}{dx}\csc x = -(\cot x)(\csc x)$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}\ log_b\ (x) = \frac{1}{\ln(b)(x)}$$

$$\frac{d}{dx}\ e^x = e^x$$

PRODUCT RULE FOR MULTIPLICATION OF DERIVATIVES

$$\frac{d}{dx}[\,f(x)\cdot g(x)] = \frac{d}{dx}(x)\cdot g(x) + f(x)\cdot\frac{d}{dx}\ (g)$$

QUOTIENT RULE FOR DIVIDING DERIVATIVES

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot\frac{d}{dx}\ [f(x)] - f(x)\cdot\frac{d}{dx}\ [g(x)]}{[g(x)]^2}$$

A song to remember this :

Low D high minus high D low: Divide by squared low and away you go!

CHAIN RULE FOR COMPOSITION OF FUNCTIONS OF DERIVATIVES

$$F(x) = f\big(g(x)\big)$$

$$F'(x) = \big[f'\big(g(x)\big)\big][g'(x)]$$

This is going to resemble solving an equation by substitution from college algebra class.

I always recommend that people come back and review this section after learning integration by parts because it might make more sense at that point.

Using the chain rule:

An example:

$$\frac{d}{dx}[(3x^2-4)^{100}]$$

Break the function up into two separate functions and compose them.

$$\frac{dy}{du} = u^{100}, \quad \frac{du}{dx} = (3x^2-4), \quad \textit{And now you can multiply and substitute}$$

$$\frac{d}{dx}[(3x^2-4)^{100}] = \frac{d}{du}(u^{100})\frac{d}{dx}(3x^2-4)$$

$$= (100u^{99})6x$$

$$= 600x(3x^2-4)^{99}$$

COMMON CHAIN RULE VARIANTS

$$\frac{d}{dx}([f(x)]^n) = n[f(x)^{n-1}f-(x)$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(sin\,[f(x)]) = f'(x)cos[f(x)]$$

$$\frac{d}{dx}(cos[f(x)]) = -f'(x)sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x)sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

Concave Up: When you draw the function, it could hold water (like a bowl).

Concave Down: When you draw the function, water would fall off it (like an umbrella).

Inflection Point: Where the function changes concavity.

Second Derivative = Derivative of the derivative of a function

$$\frac{d^2f}{dx^2} = f''(x) = f^{(2)}(x) = (f'(x))'$$

nth Derivative =

$$\frac{d^nf}{dx^n} = f^{(n)}(x) = (f^{(n-1)}(x))'$$