Physics for Machine Learning

Physics formulas sometimes find their way into finance, machine learning, data analysis, or other seemingly unrelated fields, and we sometimes collaborate with physicists. I therefore find it important to know some of this physics stuff even though I am not a physicist.

The Forces: A force is any interaction that, if unopposed, will alter the motion of an object. (Remember vector calculus)?

Forces can be based on contact or noncontact.

There are four fundamental forces, and all other forces are a variation of one of these four forces:

- Strong nuclear
- Weak nuclear
- Electromagnetic
- Gravity

Force is measured in newtons

$$1\ Newton = 1kg \cdot \frac{m}{s^2}$$

Work: An action done on an object that displaces the object. (In other words, work is a scalar).

$$Work = (Magnitude\ of\ a\ force)(Displacement)$$
$$W = Fd$$

Work will be a positive number when the applied force is in the same direction as displacement and negative otherwise.

Energy: The capacity of something to do work. Measured in Joules.

Work-Energy Theorem

$$W_{net} = \Delta KE \quad where\ \Delta KE\ is\ a\ change\ in\ kinetic\ energy$$

Simple Harmonic Motion: A vibration back and forth between more and less compressed than its resting state. Example: A spring.

Frictional forces will eventually return the spring back to its equilibrium state **(Hooke's Law)**.

$$F_{spring} = -kx \quad where\ k\ is\ the\ spring\ constant$$

Law of Conservation of Momentum:

Momentum is a vector, and mass is a scalar.

$\vec{p} = m\vec{v}$

With any collision, there must exist a conservation of linear momentum.

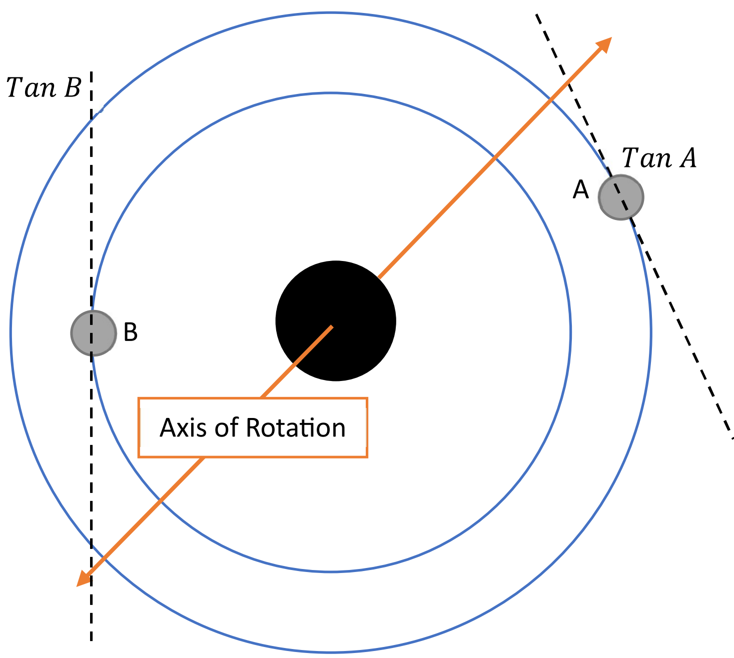
Elastic collisions: the objects involved remain separate.

Nearly elastic collisions: some of the kinetic energy is lost to heat and/or sound

Perfectly inelastic collisions: the objects collide and become one mass. Kinetic energy is lost and become heat and sound.

Circular Motion

Axis of Rotation: An axis that is always perpendicular or orthogonal to the plane containing the revolving object.



Tangential Speed: Instantaneous speed along the tangent line (orthogonal to the radius).

- If the tangential speed is constant, it is uniform circular motion.
- Tangential speed depends on the distance from the axis of rotation (Object A travels farther than Object B.)

Velocity is tangent to an orbit. If velocity changes, there must be acceleration.

Velocity is the derivative of location

Acceleration is the derivative of velocity, or the second derivative of location.

Recall from Calculus 2: Integration in Polar Coordinates:

$$Area\ of\ the\ circle = \int_a^b \frac{1}{2} r^2 d\theta \quad = \quad \int_a^b \frac{1}{2} [f(\theta)]^2 d\theta$$

If you end up with an ellipse, you can still use the circular motion properties to solve it after transforming the ellipse into a circle with a Jacobian Matrix.

Thermodynamics

0th Law of Thermodynamics: If two systems are in thermal equilibrium with a third system, then those two systems are in thermal equilibrium with each other.

1st Law of Thermodynamics – Law of Conservation of Energy: The total energy of a closed system is conserved.

$$Potential\ energy < - - - - - > Kinetic\ energy$$

$$Mechanical\ energy = Potential\ energy + Kinetic\ energy$$

$$ME = \frac{1}{2}mv^2 + mgh \quad (assuming\ negligible\ or\ no\ friction)$$

Non-mechanical energies (chemical, thermal, nuclear, electrical, acoustic, etc.) will also all be conserved.

$$Total\ energy = Mechanical\ energy + Nonmechanical\ energy$$

$$\Delta U = Q - W; \quad where:$$
$$\Delta U = the\ internal\ energy, Q = heat, and\ W = work$$

So basically, in plain English:

All changes in energy must be the result of heat transfer either into or out of the process, and it assumes multiple things including constant temperature, in order to eliminate noise in the data.

- “Q” will be positive when heat is absorbed and negative when heat is released.
- “W” will be positive when work is done by the system and negative when work is done to the system.
- “Q” and “W” can both be 0

Example: 100 Joules of work is done on a system and internal energy increases by 74 Joules. How much energy is transferred as heat?

$$Q = 74J + W$$

$$\therefore Q = -26J$$

So, 26 Joules of heat is lost or released.

2nd Law of Thermodynamics – Entropy: The inevitable partial loss of a system's ability to convert energy into work. And the dispersal of matter and energy always involves an increase in entropy and will tend to be spontaneous.

In plain english: Things like to be messy and tend to move toward disorder.

Microstates: Ways that stuff can be arranged.

$$S = K \ln W$$

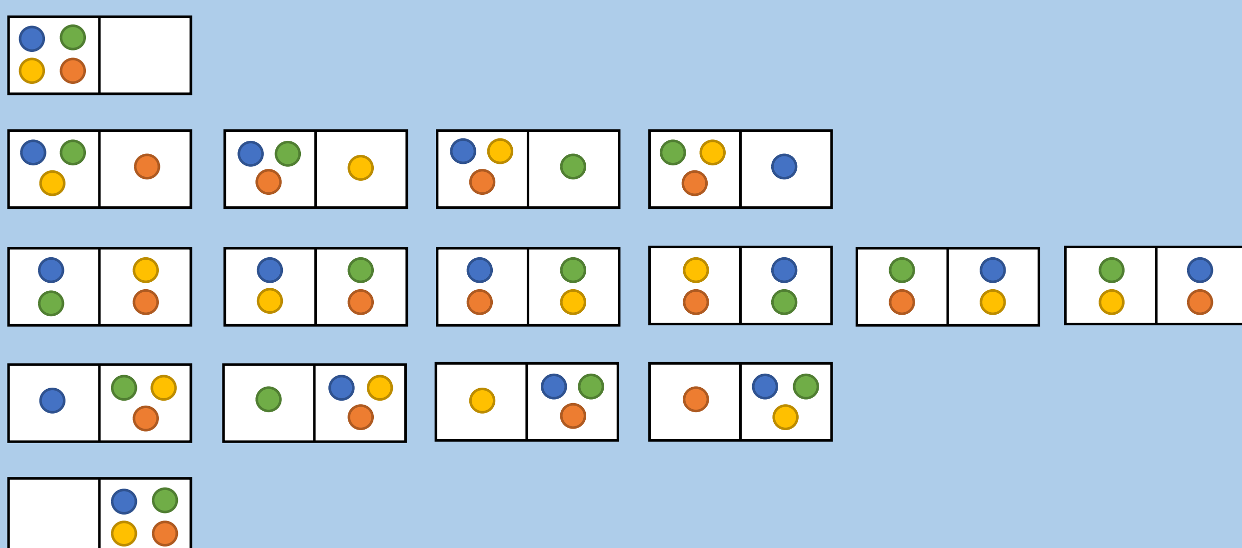
Physicists do this annoying thing where they reuse the same letters for different things in equations. In this case…

$$K = the\ Boltzman\ constant, and$$
$$W = the\ number\ of\ microstates\ possible.$$

Statistically, the distribution with the greatest number of microstates will be preferred. To get us on the same page, let's go ahead and call a microstate a "combination" and recognize this as a combinatorics problem.

Example: Consider dominos with 4 dots of different colors that can be located into either the left or right side of the domino. Which distribution has the greatest number of microstates?

Step 1: Draw out every combination.



The distribution with the greatest number of microstates is the one with two dots on each side of the domino. It contains $\frac{6}{16}$ of the possible combinations, which is 37.5%.

Why is this so magic???!

Well, the number 37.5% comes up quite often in statistics. You can apply this same problem to multiple different examples. Like… what is the probability of hiring the perfect candidate for a perfect job? Approximately 37.5%.

3rd Law of Thermodynamics – Absolute Zero: The temperature where all matter is a solid. Essentially, the law states that the entropy of any crystalline substance at absolute zero will equal zero. In addition, it is impossible to lower the temperature of any system to absolute zero in a finite number of steps.

Electromagnetic Fields

I'm just going to summarize these and give formulas really quick. If you need them, you probably know what to use them for.

Gauss's law for electric fields: usually involves simple shapes: spheres, infinite planes, or infinite cylinders, and an electric field is either parallel or orthogonal to the surface and constant over the surface itself.

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad or \quad \int \vec{E} \cdot \hat{n} da = \frac{q_{enc}}{\varepsilon_0} \quad where\ \varepsilon_0 = free\ space$$

Gauss' law for magnetic fields: formed by a moving electric charge. The electric field propagates on the y-axis and the magnetic field on the z-axis. And they're orthogonal.

$$\vec{\nabla} \cdot \vec{B} = 0$$

Faraday's Law: A circulating electric field is produced by a magnetic field that changes with time. The curl of the electric field (cross product) quantifies the circulation of the field.

$$\vec{\nabla} \times \vec{E} = \frac{-d\vec{B}}{dt} \quad or \quad \int\int_C \vec{E} \cdot d\vec{l} = \frac{-d}{dt} \int\int_S \vec{B} \cdot \hat{n} da$$

Ampere-Maxwell Equation: Simply, this bugger is used when the other ones won't work.

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \right) \quad or \quad \int_C \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{enc} + \varepsilon_0 \frac{d}{dt} \int\int_S \vec{E} \cdot \hat{n} da \right)$$

Blackbodies and Quantization

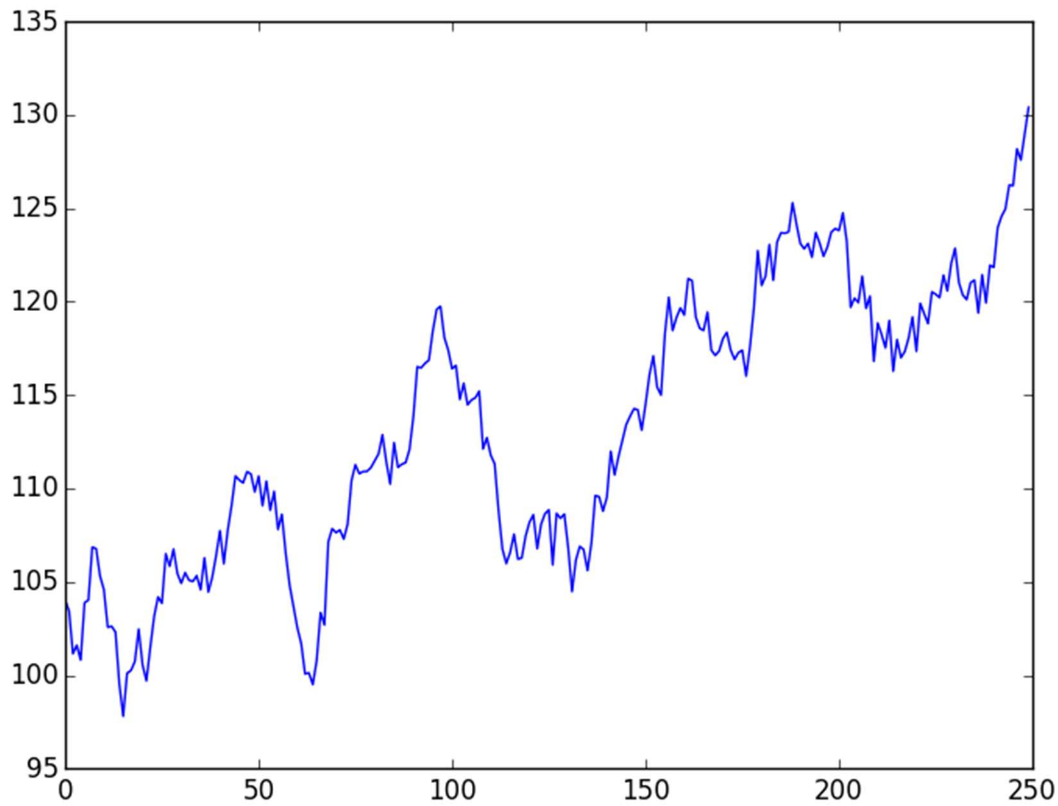
So, again in plain English, classical electromagnetism cannot account for blackbody radiation, which includes ultraviolet and other types of natural radiation we are exposed to. And in short, vibrational energy of atoms are quantized.

Quantization returns discrete values, and this field of research ultimately led to the revelation that light is both a particle and a wave. Later, a dude named DeBroglie determined that matter also exists as a wave.

Brownian Motion is a process developed by Albert Einstein which we see emerge into the field of finance due to its relevancy of stochastic processes. It is used to help model the potential movement of stocks or other securities. Here's the formula:

$$\frac{\partial f}{\partial t} = \left(\frac{D}{2}\right) \frac{\partial^2 f}{\partial x^2}$$

Note: gradient must be positive because particles diffuse from higher to lower concentrations.



For more on this: See my notes on Partial Derivatives and Gradient, Stochastic and Ito Calculus, and the Derivation of The Black Scholes Options Pricing Model with Commentary.

Waves are also applicable to financial machine learning. Lots of things were discovered to be a thing and a wave, and there is lots of research into whether or not this is also true of stocks and other securities.

For more on this, see my paper: Testing Medicare Sales Data and Predictability of Stock Moves: A Wavelet Transform Experiment.

Properties of Waves:

- Any standing wave must have $n \in \mathbb{Z}$ wavelengths in order to exist, and $\lambda \in \mathbb{Z}$
- Quantization must make sense
- More wavelengths mean more energy
- Photon absorption promotes the electron to a higher energy level and increases the number of wavelengths in the standing wave

Schrodinger's Equation

$$H(t)\left|\Psi(t) >= ih\frac{\partial}{\partial t}\right| \Psi(t) >$$

Max Born added that: Ψ^2 tells us the probability that an electron can be found in a certain spot, which is illustrated by diffraction. Thus:

- Electrons are just clouds of probability densities.
- Measuring an electron's position yields a value at random from a probability distribution.

Heisenberg's Uncertainty Principle: The more we know about one parameter, the less we know about the other, and the parameters are complementary.

$$(\Delta x)(\Delta p) \geq \frac{h}{\pi^4}$$

Special Relativity

Okay so this is a big subject, but one thing that might be relevant to machine learning is time dilation. Basically, the speed of light is postulated to be the same in each inertial reference frame. An inertial reference frame is a moving thing that we are presuming to be stationary.

Most of the time, we consider this to be Planet Earth, which is careening through space at 67,000 miles per hour.

The speed of light has been observed to be the same regardless of which moving object one is on, which then shows that time moves at a different rate on the two objects.

Time Dilation: The faster you move, the slower time passes.

$$Time = \frac{distance}{speed}$$

Or more specifically:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\begin{aligned}&Where: \Delta t \text{ is time dilation}: \text{in motion with repect to other events}\\ &\quad \text{and } \Delta t_0 \text{ is the proper time interval}: \text{at rest with respect to other event}\\ &\qquad v = velocity\\ &\qquad c = \text{the speed of light}\end{aligned}$$

Example: A ship travelling to Alpha Centauri at 0.95c takes 4.5 years to get there as measured from Earth. How long does it seem to the passengers?

$$4.5\ years = \frac{\Delta t_0}{\sqrt{1 - \frac{0.95c^2}{c^2}}} = \frac{\Delta t_0}{0.312}$$

You should get 1.4 years.

Example: A ship travelling from Earth to another star system at 90% the speed of light has a person on board. This person measures the trip to be 8.2 light years in length. How far away is the destination according to someone on Earth?

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad 8.2\ LY = L_0\sqrt{1 - \frac{0.9c^2}{c^2}}$$

You should get 18.8 Light Years!