Cheltenham-Maths: 18-month tutor answers and marking guide, Booklet 2 (Weeks 34–66)

Covers Booklet 2 (Weeks 34–66). Each practice is about 40 marks (one hour). These are original teaching questions with suggested marks, not an official Pearson or Edexcel mark scheme. Award method marks for correct, clearly shown methods even after an earlier slip; accuracy marks need the correct value.

Basic-skills parts: 2-mark parts are 1 method + 1 accuracy; 3-mark parts are 2 method + 1 accuracy.

Week 34: General binomial expansion

Pearson P2 chapter 4.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). General binomial expansion. Expand each up to and including the term in x^3 and state the validity.

- (a) $(1+x)^{-1}$ – 2 marks. Answer: $1 - x + x^2 - x^3$, $|x| < 1$. Award for: Correct method (1); accurate answer (1).
- (b) $(1-2x)^{-2}$ – 2 marks. Answer: $1 + 4x + 12x^2 + 32x^3$, $|x| < \frac{1}{2}$. Award for: Correct method (1); accurate answer (1).
- (c) $(1+x)^{\frac{1}{2}}$ – 2 marks. Answer: $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$, $|x| < 1$. Award for: Correct method (1); accurate answer (1).
- (d) $(1+3x)^{-\frac{1}{3}}$ – 2 marks. Answer: $1 - x + 2x^2 - \frac{14}{3}x^3$, $|x| < \frac{1}{3}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Using binomial expansions.

- (a) Find the coefficient of x^2 in $(1+4x)^{-\frac{1}{2}}$. – 2 marks. Answer: 6. Award for: Correct method (1); accurate answer (1).
- (b) Use $(1+x)^{\frac{1}{2}}$ with $x = 0.02$ to estimate $\sqrt{1.02}$ to 5 decimal places. – 2 marks. Answer: 1.00995. Award for: Correct method (1); accurate answer (1).
- (c) Expand $\frac{1+x}{1-x}$ up to x^3. – 2 marks. Answer: $1 + 2x + 2x^2 + 2x^3$. Award for: Correct method (1); accurate answer (1).
- (d) Expand $(1+x)^{-3}$ up to x^2. – 2 marks. Answer: $1 - 3x + 6x^2$. Award for: Correct method (1); accurate answer (1).
- (e) Find the first three terms of $(9-x)^{-\frac{1}{2}}$. – 3 marks. Answer: $\frac{1}{3} + \frac{x}{54} + \frac{x^2}{648}$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). Expand $(1+11x)^{-1/2}$ through x^3 and state the interval of validity. Use it to approximate its value at x=0.01 and discuss the approximation.

Answer: $-\frac{6655x^3}{16}+\frac{363x^2}{8}-\frac{11x}{2}+1$; |x|<1/11; approximation 0.94912. Truncation error remains; exact value 0.94916.

- Expand correctly through the cubic term: 3
- State the validity interval: 1
- Calculate the requested approximation: 1
- Discuss truncation and the applicability of the expansion: 1

Q4 (5 marks). Find the first three terms of $(2-11x)^{-1}$. State validity and check by multiplying the truncated expression by 2-11x.

Answer: $1/2+(11/4)x+(121/8)x^2$; |x|<2/11; product=1+terms of order x^3.

- Factor out the constant before expanding: 1
- Find the first three terms: 2
- State validity: 1
- Check by multiplying the truncated series: 1

Q5 (5 marks). Find coefficients of x and x^2 in $(1+2x)(1-11x)^{-2}$, stating the validity condition.

Answer: x coefficient 24, x^2 coefficient 407; |x|<1/11.

- Obtain the needed binomial terms: 2
- Collect the linear and quadratic coefficients of the product: 2
- State the validity interval: 1

Q6 (6 marks). Resolve $\frac{1}{(1-x)(1-11x)}$ into partial fractions and hence expand through x^2. Explain which singularity sets the smaller validity interval.

Answer: $-1/(10)(1-x)^{(-1)}+11/(10)(1-11x)^{(-1)}$; $1+12x+133x^2$; |x|<1/11.

- Find the partial-fraction coefficients: 3
- Expand and combine through the quadratic term: 2
- State validity and explain the controlling singularity: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Binomial expansions with partial fractions.

- (a) Express in partial fractions $\frac{3}{(1+x)(1-2x)}$. – 2 marks. Answer: $-\frac{2}{2x-1}+\frac{1}{x+1}$. Award for: Correct method (1); accurate answer (1).
- (b) Hence expand it up to x^2. – 2 marks. Answer: $3+3x+9x^2$. Award for: Correct method (1); accurate answer (1).

- (c) State the range of validity of that expansion. – 2 marks. Answer: $|x| < \frac{1}{2}$. Award for: Correct method (1); accurate answer (1).
- (d) Express in partial fractions $\frac{5-x}{(2-x)(1+x)}$. – 2 marks. Answer: $\frac{2}{x+1} - \frac{1}{x-2}$. Award for: Correct method (1); accurate answer (1).
- (e) Hence expand it up to x^2. – 3 marks. Answer: $\frac{5}{2} - \frac{7}{4}x + \frac{17}{8}x^2$. Award for: Correct method (2); accurate answer (1).
- (f) State the range of validity. – 3 marks. Answer: $|x| < 1$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks).

- (a) Find the binomial expansion of $(1 - 4x)^{-1/2}$ up to and including the term in x^3. – 4 marks. Award for: Coefficients and powers simplified
- (b) State the values of x for which the expansion is valid. – 1 marks. Award for: |x| < 1/4
- (c) By substituting x = 1/20, use your expansion to estimate √5. – 3 marks. Award for: Show 1/√0.8 = √5/2 and evaluate

Answers: (a) 1 + 2x + 6x² + 20x³. (b) |x| < 1/4. (c) (0.8)^(−1/2) = √5/2 ≈ 1.1175, so √5 ≈ 2.235.

Q3 (exam style) (9 marks).

- (a) Express $\frac{5+3x}{(1+x)(2+x)}$ in partial fractions. – 3 marks. Award for: Identity and constants
- (b) Hence find the expansion in ascending powers of x up to the term in x^2. – 5 marks. Award for: Two expansions, factor 1/2, combine
- (c) State the range of validity. – 1 marks. Award for: Most restrictive condition

Answers: (a) 2/(1 + x) + 1/(2 + x). (b) 5/2 − (9/4)x + (17/8)x². (c) |x| < 1.

Q4 (exam style) (8 marks). In the expansion of $(1 + ax)^n$, where n is not a positive integer, the coefficient of x is −6 and the coefficient of x^2 is 27.

- (a) Find n and a. – 5 marks. Award for: Form two equations and solve
- (b) Find the coefficient of x^3. – 2 marks. Award for: General term
- (c) State the range of validity. – 1 marks. Award for: |ax| < 1

Answers: (a) na = −6, n(n − 1)a²/2 = 27 give n = −2, a = 3. (b) −108. (c) |x| < 1/3.

Week 35: Radians, sectors and approximations

Pearson P2 chapter 5.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Radians, arcs and sectors.

- (a) Convert 150° to radians, in terms of π. – 2 marks. Answer: $\frac{5\pi}{6}$. Award for: Correct method (1); accurate answer (1).

- (b) Convert $\frac{3\pi}{4}$ to degrees. – 2 marks. Answer: 135°. Award for: Correct method (1); accurate answer (1).
- (c) Find the arc length of a sector with radius 6 cm and angle 1.2 radians. – 2 marks. Answer: 7.2 cm. Award for: Correct method (1); accurate answer (1).
- (d) Find the area of that sector. – 2 marks. Answer: 21.6 cm^2. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Exact values and equations in radians, for $0 \le x \le 2\pi$.

- (a) Write down $\sin\frac{\pi}{6}$ and $\cos\frac{\pi}{4}$. – 2 marks. Answer: $\frac{1}{2}$, $\frac{\sqrt{2}}{2}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the exact value of $\tan\frac{2\pi}{3}$. – 2 marks. Answer: $-\sqrt{3}$. Award for: Correct method (1); accurate answer (1).
- (c) Solve $\sin x = \frac{\sqrt{3}}{2}$. – 2 marks. Answer: $\frac{\pi}{3}$, $\frac{2\pi}{3}$. Award for: Correct method (1); accurate answer (1).
- (d) Solve $\cos x = -\frac{1}{2}$. – 2 marks. Answer: $\frac{2\pi}{3}$, $\frac{4\pi}{3}$. Award for: Correct method (1); accurate answer (1).
- (e) Solve $\tan 2x = 1$. – 3 marks. Answer: $\frac{\pi}{8}$, $\frac{5\pi}{8}$, $\frac{9\pi}{8}$, $\frac{13\pi}{8}$. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). A sector has radius 14 cm and angle 1.2 radians. Find arc length, sector area and perimeter; explain why the angle must be in radians in these formulae.

Answer: Arc 16.8 cm; area 117.6 cm^2; perimeter 44.8 cm. Radian definition gives s=rθ.

- Find arc length: 1
- Find sector area: 1
- Find perimeter: 1
- Explain why the formulas require radians: 1

Q4 (5 marks). A circular segment has radius 14 and angle π/3 at its centre. Find its exact area and its chord length. Sketch the region and identify the subtracted triangle.

Answer: Area $-49\sqrt{3} + \frac{98\pi}{3}$; chord 14.

- Form the sector-minus-triangle calculation: 2
- Give the exact segment area: 1
- Find the chord length: 1
- Sketch and identify the subtracted triangle: 1

Q5 (7 marks). A sector has fixed perimeter 48 and angle strictly between0 and2π. Express its area in terms of radius r and maximise it, giving the angle at the maximum and the physical domain.

Answer: Arc=48-2r; area=24r-r^2; max r=12, area=144, angle2 radians. From0<θ<2π, the physical domain is 24/(π+1)<r<24.

- Use the perimeter constraint to form the area function: 2
- Derive the physical radius domain: 2
- Find and establish the maximum: 2
- State the angle at the maximum: 1

Q6 (4 marks). Use small-angle approximations to estimate $\frac{1-\cos(12x)}{x\sin x}$ when x is small. State its limiting value and why using degrees without conversion would invalidate the formulae.

Answer: Numerator≈(144/2)x^2, denominator≈x^2; limit 72. Standard expansions assume radians.

- Use the correct small-angle numerator and denominator approximations: 2
- Simplify to obtain the limiting value: 1
- Explain the radian requirement: 1

Practice 2 (Day 4) – total 37 marks

Q1 (basic skills) (14 marks). Small-angle approximations.

- (a) Write down the small-angle approximations for $\sin\theta$, $\tan\theta$ and $\cos\theta$. – 2 marks. Answer: $\theta,\ \theta,\ 1-\frac{\theta^2}{2}$. Award for: Correct method (1); accurate answer (1).
- (b) Approximate $\sin 3\theta + \tan 2\theta$. – 2 marks. Answer: 5θ. Award for: Correct method (1); accurate answer (1).
- (c) Approximate $\frac{\sin 4\theta}{\theta}$. – 2 marks. Answer: 4. Award for: Correct method (1); accurate answer (1).
- (d) Approximate $1-\cos 2\theta$. – 2 marks. Answer: $2\theta^2$. Award for: Correct method (1); accurate answer (1).
- (e) Approximate $\frac{\theta\tan\theta}{1-\cos\theta}$. – 3 marks. Answer: 2. Award for: Correct method (2); accurate answer (1).
- (f) Find the percentage error in using $\sin\theta\approx\theta$ when $\theta=0.3$. – 3 marks. Answer: $\frac{0.3-0.29552}{0.29552}=1.52\%$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). A sector OAB of a circle with centre O and radius 6 cm has angle θ radians and area 15 cm^2.

- (a) Find θ. – 2 marks. Award for: Sector area
- (b) Find the area of the segment between chord AB and arc AB. – 3 marks. Award for: Sector minus triangle
- (c) Find the perimeter of the segment. – 3 marks. Award for: Arc plus chord

Answers: (a) θ = 5/6. (b) 15 − 18 sin(5/6) ≈ 1.68 cm^2. (c) 5 + 12 sin(5/12) ≈ 9.86 cm.

Q3 (exam style) (7 marks).

- (a) Show that, for small θ, $4\cos 3\theta + 2\theta\sin 2\theta \approx 4-14\theta^2$. – 3 marks. Award for: Small-angle approximations and simplify

- (b) Use this to estimate the value when θ = 0.1, and find the percentage error. – 3 marks. Award for: Approximation, true value, error
- (c) Explain why θ must be measured in radians. – 1 marks. Award for: Approximations derived for radians

Answers: (a) $4(1 - 9\theta^2/2) + 2\theta(2\theta)$. (b) 3.86; true value 3.86108, error ≈ 0.03%. (c) The approximations $\sin\theta \approx \theta$ etc. hold only in radians.

Q4 (exam style) (8 marks). Solve, giving exact answers in terms of π:

- (a) $2\cos^2 x + 3\sin x = 3$, 0 ≤ x < 2π. – 4 marks. Award for: Identity, quadratic and all solutions
- (b) $\tan\left(2x - \frac{\pi}{3}\right) = 1$, 0 ≤ x < π. – 4 marks. Award for: Adjusted interval and solutions

Answers: (a) sin x = 1 or 1/2: x = π/6, π/2, 5π/6. (b) x = 7π/24, 19π/24.

Week 36: Trigonometric functions and inverse trig

Pearson P2 chapter 6.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Reciprocal trigonometric functions.

- (a) Find the exact value of $\sec\frac{\pi}{3}$. – 2 marks. Answer: 2. Award for: Correct method (1); accurate answer (1).
- (b) Find the exact value of $\operatorname{cosec}\frac{\pi}{4}$. – 2 marks. Answer: $\sqrt{2}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the exact value of $\cot\frac{\pi}{6}$. – 2 marks. Answer: $\sqrt{3}$. Award for: Correct method (1); accurate answer (1).
- (d) Simplify $\sin x \sec x$. – 2 marks. Answer: $\tan x$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Pythagorean identities.

- (a) Simplify $1 + \tan^2 x$. – 2 marks. Answer: $\sec^2 x$. Award for: Correct method (1); accurate answer (1).
- (b) Given $\tan x = \frac{3}{4}$, x acute, find $\sec x$. – 2 marks. Answer: $\frac{5}{4}$. Award for: Correct method (1); accurate answer (1).
- (c) Show that $\sec^2 x - \tan^2 x \equiv 1$. – 2 marks. Answer: divide $\sin^2 + \cos^2 = 1$ by $\cos^2 x$. Award for: Correct method (1); accurate answer (1).
- (d) Simplify $\frac{\operatorname{cosec}^2 x - 1}{\cot x}$. – 2 marks. Answer: $\cot x$. Award for: Correct method (1); accurate answer (1).
- (e) Solve $\tan^2 x + \sec x = 1$ for $0 \le x \le 360^\circ$. – 3 marks. Answer: $\sec^2 x + \sec x - 2 = 0$; $x = 0^\circ, 120^\circ, 240^\circ, 360^\circ$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). Solve sec x=2 for 0≤x<2π. Sketch sec x over that interval, showing asymptotes, and explain why the equation is equivalent to cos x=1/2.

Answer: x=π/3,5π/3; vertical asymptotes π/2,3π/2. sec=1/cos on its domain.

- Find both solutions: 2
- Sketch secant with correct branches and asymptotes: 3
- Explain the reciprocal-function equivalence: 1

Q4 (5 marks). Prove 1+tan²x=sec²x and 1+cot²x=cosec²x from sin²x+cos²x=1, stating all domain exclusions.

Answer: Divide by cos²x or sin²x respectively. First excludes cos x=0; second excludes sin x=0.

- Derive the first reciprocal identity: 2
- Derive the second reciprocal identity: 2
- State the domain exclusions for both: 1

Q5 (5 marks). Find principal values arcsin(-1/2), arccos(-1/2) and arctan(1). Then solve sin x=-1/2 on 0≤x<2π, explaining the difference from the inverse-function output.

Answer: Principal values -π/6,2π/3,π/4. Equation roots 7π/6,11π/6; inverse sine selects one value from its restricted range.

- Give the three principal values: 3
- Find all sine-equation solutions: 1
- Explain inverse-function range versus the full solution set: 1

Q6 (6 marks). Sketch $y = 6\csc x$ on 0<x<2π. State range, asymptotes and all intersections with y=12.

Answer: Range y≤-6 or y≥6; asymptotes x=0,π,2π; intersections x=π/6,5π/6.

- Sketch the scaled cosecant branches: 2
- State range and all asymptotes: 2
- Find both intersections: 2

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Inverse trigonometric functions.

- (a) Find $\arcsin \frac{1}{2}$ in radians. – 2 marks. Answer: $\frac{\pi}{6}$. Award for: Correct method (1); accurate answer (1).
- (b) Find $\arccos(-1)$. – 2 marks. Answer: π. Award for: Correct method (1); accurate answer (1).
- (c) Find $\arctan \sqrt{3}$. – 2 marks. Answer: $\frac{\pi}{3}$. Award for: Correct method (1); accurate answer (1).
- (d) State the domain and range of $y = \arcsin x$. – 2 marks. Answer: $-1 \leq x \leq 1$, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$. Award for: Correct method (1); accurate answer (1).

- (e) Find $\sin(\arccos \frac{3}{5})$. – 3 marks. Answer: $\frac{4}{5}$. Award for: Correct method (2); accurate answer (1).
- (f) Solve $\arctan(2x - 1) = \frac{\pi}{4}$. – 3 marks. Answer: $x = 1$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks).

- (a) Prove that $\frac{1+\sec x}{\tan x+\sin x} \equiv \operatorname{cosec} x$. – 4 marks. Award for: Convert to sin and cos, factorise, cancel
- (b) Hence solve $\frac{1+\sec x}{\tan x+\sin x} = 3\sin x$ for 0° < x < 360°, explaining any values that must be excluded. – 4 marks. Award for: $\sin^2 x = 1/3$, four solutions, exclusions

Answers: (a) (1 + c)/c ÷ s(1 + c)/c = 1/s. (b) sin x = ±1/√3: x = 35.3°, 144.7°, 215.3°, 324.7°; values where cos x = 0, cos x = −1 or sin x = 0 are excluded but none arise.

Q3 (exam style) (9 marks).

- (a) Solve $2\cot^2 x - 5\operatorname{cosec} x + 4 = 0$ for 0° ≤ x ≤ 360°. – 5 marks. Award for: Identity, quadratic in cosec, solutions
- (b) Explain why one root of your quadratic gives no solutions. – 1 marks. Award for: |sin x| ≤ 1
- (c) Given sec θ = −3 with 90° < θ < 180°, find the exact values of tan θ and cosec θ. – 3 marks. Award for: Signs in second quadrant, exact values

Answers: (a) cosec x = 2 or 1/2: x = 30°, 150°. (b) cosec x = 1/2 would need sin x = 2. (c) tan θ = −2√2, cosec θ = 3√2/4.

Q4 (exam style) (8 marks).

- (a) Sketch y = arccos x, stating its domain and range. – 3 marks. Award for: Shape, end points, domain and range
- (b) Solve $\arccos(2x - 1) = \frac{\pi}{3}$. – 2 marks. Award for: Take cos of both sides
- (c) Prove that arcsin x + arccos x = π/2 for −1 ≤ x ≤ 1. – 3 marks. Award for: Let y = arcsin x and use cos(π/2 − y) with ranges

Answers: (a) From (−1, π) to (1, 0); domain −1 ≤ x ≤ 1, range 0 ≤ y ≤ π. (b) x = 3/4. (c) If y = arcsin x then x = cos(π/2 − y) with π/2 − y in [0, π], so arccos x = π/2 − y.

Week 37: Compound and double angles

Pearson P2 chapter 7.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Addition formulae.

- (a) Find the exact value of $\sin 75^\circ$. – 2 marks. Answer: $\frac{\sqrt{6}+\sqrt{2}}{4}$. Award for: Correct method (1); accurate answer (1).

- (b) Find the exact value of $\cos 15^\circ$. – 2 marks. Answer: $\frac{\sqrt{6}+\sqrt{2}}{4}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the exact value of $\tan 15^\circ$. – 2 marks. Answer: $2-\sqrt{3}$. Award for: Correct method (1); accurate answer (1).
- (d) Simplify $\sin 3x \cos x + \cos 3x \sin x$. – 2 marks. Answer: $\sin 4x$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Double-angle formulae.

- (a) Given $\cos x = \frac{1}{3}$, find $\cos 2x$. – 2 marks. Answer: $-\frac{7}{9}$. Award for: Correct method (1); accurate answer (1).
- (b) Given $\sin x = \frac{4}{5}$, x acute, find $\sin 2x$. – 2 marks. Answer: $\frac{24}{25}$. Award for: Correct method (1); accurate answer (1).
- (c) Simplify $2\sin 15^\circ \cos 15^\circ$. – 2 marks. Answer: $\frac{1}{2}$. Award for: Correct method (1); accurate answer (1).
- (d) Write $\cos^2 x$ in terms of $\cos 2x$. – 2 marks. Answer: $\frac{1+\cos 2x}{2}$. Award for: Correct method (1); accurate answer (1).
- (e) Solve $\sin 2x = \sin x$ for $0 \le x \le 360^\circ$. – 3 marks. Answer: $0, 60, 180, 300, 360^\circ$. Award for: Correct method (2); accurate answer (1).

Q3 (7 marks). Derive sin(2x) and both forms cos(2x)=1-2sin²x=2cos²x-1 from addition formulae. Use them to obtain exact sin15° and cos15°.

Answer: sin2x=2sin x cos x; cos2x=cos²x-sin²x. sin15°=(sqrt6-sqrt2)/4; cos15°=(sqrt6+sqrt2)/4.

- Derive the double-angle formulas from addition formulas: 3
- Use addition or double-angle identities to obtain both exact values: 4

Q4 (5 marks). Solve sin(2x)=cos x for 0≤x<2π. Show the factorisation and explain which roots would be lost by dividing by cos x.

Answer: cos x(2sin x-1)=0; x=π/6,π/2,5π/6,3π/2. Division loses π/2,3π/2.

- Use the double-angle identity and factorise: 2
- Find all roots in the interval: 2
- Identify the roots lost by division: 1

Q5 (4 marks). Prove $\frac{\sin(2x)}{1+\cos(2x)} = \tan x$ on its domain. State exclusions and explain why equality at a few calculator values is not a proof.

Answer: 2sin x cos x/(2cos²x)=tan x when cos x≠0. Exclude x=π/2+kπ; finite tests cannot establish all real defined inputs.

- Substitute double-angle identities and simplify to prove the result: 2
- State all exclusions from the original expression: 1
- Explain the requested domain comparison or why finite checks are not proof: 1

Q6 (6 marks). Solve cos(2x)+sin x=0 on 0≤x<2π. Use a quadratic in sin x and verify every resulting angle.

Answer: 1-2s²+s=0 ⇒ s=1 or -1/2. x=π/2,7π/6,11π/6.

- Use a double-angle identity to form a quadratic: 1
- Factorise or solve the quadratic: 2
- Find all angles in the interval: 2
- Verify the resulting angles: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Harmonic form $R\sin(x \pm \alpha)$ and $R\cos(x \pm \alpha)$.

- (a) Write $3\sin x + 4\cos x$ as $R\sin(x+\alpha)$. – 2 marks. Answer: $5\sin(x + 53.1^\circ)$. Award for: Correct method (1); accurate answer (1).
- (b) State its maximum value and where it first occurs for $x \geq 0$. – 2 marks. Answer: 5 at $x = 36.9^\circ$. Award for: Correct method (1); accurate answer (1).
- (c) Write $\sin x - \sqrt{3}\cos x$ as $R\sin(x-\alpha)$. – 2 marks. Answer: $2\sin(x - 60^\circ)$. Award for: Correct method (1); accurate answer (1).
- (d) Hence solve $\sin x - \sqrt{3}\cos x = 1$ for $0 \leq x \leq 360^\circ$. – 2 marks. Answer: $90^\circ, 210^\circ$. Award for: Correct method (1); accurate answer (1).
- (e) Write $5\cos x - 12\sin x$ as $R\cos(x+\alpha)$. – 3 marks. Answer: $13\cos(x + 67.4^\circ)$. Award for: Correct method (2); accurate answer (1).
- (f) State the minimum of $\frac{1}{20+5\cos x - 12\sin x}$. – 3 marks. Answer: $\frac{1}{33}$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks).

- (a) Prove that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. – 4 marks. Award for: Compound angle, double angles, simplify
- (b) Hence solve $8\cos^3\theta - 6\cos\theta = 1$ for 0° ≤ θ < 360°. – 4 marks. Award for: cos 3θ = 1/2, adjusted interval, six solutions

Answers: (a) cos(2θ + θ) = (2c² − 1)c − 2s²c = 4c³ − 3c. (b) θ = 20°, 100°, 140°, 220°, 260°, 340°.

Q3 (exam style) (8 marks). Angle A is acute with tan A = 3/4, and angle B is obtuse with cos B = −5/13. Find the exact value of:

- (a) sin(A + B) – 3 marks. Award for: Ratios with signs and formula
- (b) tan(A − B) – 3 marks. Award for: tan B and formula
- (c) cos 2B – 2 marks. Award for: Double angle

Answers: (a) 33/65. (b) −63/16. (c) −119/169.

Q4 (exam style) (8 marks).

- (a) Show that $\frac{\sin 2x}{1-\cos 2x} \equiv \cot x$. – 3 marks. Award for: Double-angle forms and simplify
- (b) Hence solve $\frac{\sin 2x}{1-\cos 2x} = 2\tan x$ for 0° < x < 360°. – 4 marks. Award for: $\tan^2 x = 1/2$, four solutions
- (c) State the values of x in this interval for which the original equation is undefined. – 1 marks. Award for: Where sin x = 0 or cos x = 0

Answers: (a) $2sc/(2s^2) = c/s$. (b) $\tan x = \pm 1/\sqrt{2}$: x = 35.3°, 144.7°, 215.3°, 324.7°. (c) x = 90°, 180°, 270°.

Week 38: Trigonometric modelling

Pearson P2 chapter 7.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Trigonometric models. The depth of water is $D = 5 + 2\sin(30t)^\circ$ metres, t hours after midnight.

- (a) Find the depth at 1 am. – 2 marks. Answer: 6 m. Award for: Correct method (1); accurate answer (1).
- (b) State the maximum depth and the first time it occurs. – 2 marks. Answer: 7 m at 3 am. Award for: Correct method (1); accurate answer (1).
- (c) State the minimum depth. – 2 marks. Answer: 3 m. Award for: Correct method (1); accurate answer (1).
- (d) Find the period of the model. – 2 marks. Answer: 12 hours. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Modelling with $R\sin(x+\alpha)$. The temperature is $T = 12 + 3\sin(15t)^\circ + 4\cos(15t)^\circ$, $0 \le t < 24$.

- (a) Write $3\sin(15t) + 4\cos(15t)$ as $R\sin(15t+\alpha)$. – 2 marks. Answer: $5\sin(15t + 53.1^\circ)$. Award for: Correct method (1); accurate answer (1).
- (b) State the maximum temperature. – 2 marks. Answer: 17. Award for: Correct method (1); accurate answer (1).
- (c) Find the time of the maximum. – 2 marks. Answer: $t = 2.46$. Award for: Correct method (1); accurate answer (1).
- (d) State the minimum temperature. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (e) Find the time of the minimum. – 3 marks. Answer: $t = 14.46$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). Write 24sin x+32cos x in the form Rsin(x+α), with R>0 and 0<α<π/2. Find its maximum/minimum and where the maximum occurs in 0≤x<2π.

Answer: R=40; α=atan(4/3); extrema ±40; maximum x=π/2-α.

- Determine R and the correct phase angle: 3
- State both extrema: 1

- Find the maximum location in the interval: 2

Q4 (4 marks). Temperature is modelled by T=28+8sin(πt/12), t hours after midnight. Find period, range and times on 0≤t<24 when T=28. Give one limitation.

Answer: Period24h; range20 to 36; t=0,12. Ideal periodic model ignores day-to-day weather and other influences.

- Find the period: 1
- Find the range: 1
- Find all requested times in the interval: 1
- Give a relevant model limitation: 1

Q5 (6 marks). Solve 24sin x+32cos x=8 on 0≤x<2π. Give numerical roots to 3 decimals and explain how you transformed the interval.

Answer: sin(x+α)=1/5; α=0.9273. Roots x≈2.013, 5.557.

- Rewrite as a single shifted sine: 3
- Find both roots to the required precision: 2
- Explain the transformed-interval selection: 1

Q6 (6 marks). A height model is h=13+3cos(2t)-4sin(2t). Find its range and period; then the earliest t≥0 of maximum height.

Answer: h=13+5cos(2t+α), α=atan(4/3); range [8,18], periodπ; first max t=π-α/2≈2.6779.

- Express the oscillation as a single shifted cosine: 2
- State range and period: 2
- Find the earliest nonnegative time of maximum height: 2

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (11 marks). Interpreting trigonometric models.

- (a) A wheel has height $h = 10 - 8\cos(\frac{\pi t}{15})$. State the height at $t = 0$. – 2 marks. Answer: 2 m. Award for: Correct method (1); accurate answer (1).
- (b) Find the greatest height. – 2 marks. Answer: 18 m. Award for: Correct method (1); accurate answer (1).
- (c) Find the time for one revolution. – 2 marks. Answer: 30 s. Award for: Correct method (1); accurate answer (1).
- (d) Find the first time the height is 10 m. – 2 marks. Answer: 7.5 s. Award for: Correct method (1); accurate answer (1).
- (e) Find the rate of change of height at $t = 5$. – 3 marks. Answer: $\frac{8\pi}{15}\sin\frac{\pi}{3} = 1.45\ \text{m s}^{-1}$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks).

- (a) Express 5 cos x − 12 sin x in the form R cos(x + α), where R > 0 and 0 < α < π/2, giving α to 3 decimal places. – 3 marks. Award for: R and α
- (b) Hence solve 5 cos x − 12 sin x = 6.5 for 0 ≤ x < 2π. – 4 marks. Award for: Adjusted interval and both solutions
- (c) Find the maximum value of $\frac{1}{20+5\cos x-12\sin x}$ and the value of x in 0 ≤ x < 2π at which it occurs. – 3 marks. Award for: Minimum of denominator and x

Answers: (a) 13 cos(x + 1.176). (b) x ≈ 4.060, 6.154. (c) 1/7 when x = π − 1.176 ≈ 1.966.

Q3 (exam style) (10 marks). The depth of water, D m, in a harbour t hours after midnight is modelled by $D = 6 + 2.5\sin\left(\frac{\pi t}{6}\right)$.

- (a) State the maximum depth and the first time it occurs. – 2 marks. Award for: Maximum and time
- (b) A boat needs at least 7 m of water. Find the times in the first 12 hours when it can enter, to the nearest minute. – 4 marks. Award for: Inequality, inverse sine, interval
- (c) Give one criticism of the model. – 1 marks. Award for: Valid criticism
- (d) A refined model is $D = 6 + 2.5\sin\left(\frac{\pi t}{6}\right) + 0.5\cos\left(\frac{\pi t}{6}\right)$. Find its maximum depth. – 3 marks. Award for: R form and maximum

Answers: (a) 8.5 m at 03:00. (b) From 00:47 to 05:13. (c) e.g. tides do not have exactly a 12-hour period; weather ignored. (d) R = √6.5 ≈ 2.55, maximum ≈ 8.55 m.

Q4 (exam style) (8 marks).

- (a) Write √3 sin x + cos x in the form R sin(x + α), where R > 0 and 0 < α < π/2. – 3 marks. Award for: Exact R and α
- (b) Hence find the exact solutions of √3 sin x + cos x = √2 for 0 ≤ x < 2π. – 3 marks. Award for: Both solutions in range
- (c) Describe the transformations mapping y = sin x onto y = √3 sin x + cos x. – 2 marks. Award for: Translation and stretch

Answers: (a) 2 sin(x + π/6). (b) x = π/12, 7π/12. (c) Translate by π/6 to the left, then stretch vertically by factor 2.

Week 39: Conditional probability and correlation tests

Pearson S2 chapter 1–2.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Conditional probability. $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.15$.

- (a) Find $P(A \cup B)$. – 2 marks. Answer: 0.75. Award for: Correct method (1); accurate answer (1).
- (b) Find $P(A \mid B)$. – 2 marks. Answer: 0.375. Award for: Correct method (1); accurate answer (1).

- (c) Find $P(B \mid A)$. – 2 marks. Answer: 0.3. Award for: Correct method (1); accurate answer (1).
- (d) Find $P(A' \cap B)$. – 2 marks. Answer: 0.25. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Two-way tables and tree diagrams.

- (a) In a survey of 200 people, 120 drive and 50 of the drivers also cycle; 30 non-drivers cycle. Find P(cycles). – 2 marks. Answer: 0.4. Award for: Correct method (1); accurate answer (1).
- (b) Find P(drives | cycles). – 2 marks. Answer: $\frac{50}{80} = 0.625$. Award for: Correct method (1); accurate answer (1).
- (c) Find P(cycles | does not drive). – 2 marks. Answer: $\frac{30}{80} = 0.375$. Award for: Correct method (1); accurate answer (1).
- (d) A test is 95% accurate for those with a condition and gives 3% false positives. 2% have the condition. Find P(positive). – 2 marks. Answer: 0.0484. Award for: Correct method (1); accurate answer (1).
- (e) Find P(condition | positive). – 3 marks. Answer: 0.393. Award for: Correct method (2); accurate answer (1).

Q3 (7 marks). A hypothetical screening model has prevalence 0.09, sensitivity 0.95 and false-positive probability 0.08. Draw a probability tree; find P(positive) and P(condition given positive). Explain why the two conditionals differ.

Answer: P(positive)=0.1583; P(condition|positive)=0.54011. Base rate matters; this is a hypothetical mathematics model, not clinical advice.

- Draw and label the full probability tree: 2
- Calculate the total positive probability: 2
- Calculate the posterior event ratio: 2
- Explain why the two conditional probabilities differ: 1

Q4 (5 marks). P(A)=0.6, P(B)=0.4, P(A∩B)=0.18. Find P(A|B), P(B|A), P(A|not B) and decide whether the events are independent.

Answer: $\frac{9}{20}$, $\frac{3}{10}$, $\frac{7}{10}$. Independent iff intersection=0.24; here no.

- Calculate the two direct conditional probabilities: 2
- Calculate the conditional probability given not B: 2
- Test independence using the intersection and product: 1

Q5 (4 marks). A sample of 12 pairs has r=0.53. Test for positive population correlation at 5%, using supplied critical value 0.4973. State hypotheses, decision and a limitation if the sample was self-selected.

Answer: H0:ρ=0, H1:ρ>0. Reject H0, comparing r with0.4973. Self-selection limits generalisation; assume the test's distributional conditions.

- State hypotheses using the population correlation parameter: 1

- Make the correct critical-value comparison: 1
- State the contextual decision: 1
- Explain the self-selection limitation: 1

Q6 (4 marks). A model y=Ab^x gives ln y=9+0.3x. Find A,b, the multiplier for a two-unit increase in x and explain why the model is not an additive linear model in y.

Answer: A=e^9, b=e^0.3; two-unit multiplier=e^0.6≈1.8221. Equal x increments multiply y.

- Recover A and b from the logarithmic model: 2
- Find the two-unit multiplier: 1
- Explain multiplicative rather than additive change in y: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Correlation tests and set notation.

- (a) Write $P(A \mid B)$ in terms of $P(A \cap B)$ and $P(B)$. – 2 marks. Answer: $\frac{P(A \cap B)}{P(B)}$. Award for: Correct method (1); accurate answer (1).
- (b) If A, B are independent with $P(A) = 0.3$, $P(B) = 0.6$, find $P(A \mid B)$. – 2 marks. Answer: 0.3. Award for: Correct method (1); accurate answer (1).
- (c) $n = 15$, $r = 0.48$, 5% one-tailed critical value 0.4409. Test for positive correlation. – 2 marks. Answer: reject H_0: evidence of positive correlation. Award for: Correct method (1); accurate answer (1).
- (d) $n = 20$, $r = -0.35$, 5% two-tailed critical value 0.4438. Test for correlation. – 2 marks. Answer: do not reject H_0. Award for: Correct method (1); accurate answer (1).
- (e) State the hypotheses for a two-tailed test of correlation. – 3 marks. Answer: $H_0 : \rho = 0,\ H_1 : \rho \neq 0$. Award for: Correct method (2); accurate answer (1).
- (f) $P(A) = 0.6$, $P(B \mid A) = 0.5$, $P(B \mid A') = 0.2$. Find $P(B)$. – 3 marks. Answer: 0.38. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). In a sixth form, 60% study Mathematics (M), 35% study Physics (P) and 25% study both.

- (a) Find P(P | M). – 2 marks. Award for: Conditional formula
- (b) Find P(M | P′). – 2 marks. Award for: Correct regions
- (c) Determine whether M and P are independent. – 2 marks. Award for: Compare
- (d) Three students are chosen at random from a large population. Find the probability that exactly one studies both subjects. – 2 marks. Award for: Binomial-type calculation

Answers: (a) 5/12. (b) 0.35/0.65 = 7/13. (c) 0.21 ≠ 0.25, not independent. (d) $3 \times 0.25 \times 0.75^2 \approx 0.422$.

Q3 (exam style) (8 marks). Machine A makes 40% of items and machine B the rest. 3% of A's and 5% of B's items are faulty.

- (a) Find the probability that a random item is faulty. – 2 marks. Award for: Tree diagram

- (b) Given that an item is faulty, find the probability that it came from A. – 2 marks. Award for: Conditional
- (c) Two items are chosen independently. Given that at least one is faulty, find the probability that both are. – 3 marks. Award for: Ratio of probabilities
- (d) Comment on the independence assumption. – 1 marks. Award for: Context

Answers: (a) 0.042. (b) 2/7. (c) $0.042^2/(1 - 0.958^2) \approx 0.0215$. (d) Items from one batch or machine may not be independent.

Q4 (exam style) (8 marks). A researcher records 15 pairs of values and finds r = 0.512.

- (a) Test at the 5% level whether there is positive correlation. The critical value is 0.4409. – 4 marks. Award for: Hypotheses with ρ, comparison, conclusion
- (b) State the conclusion at the 1% level, where the critical value is 0.5923. – 1 marks. Award for: Conclusion
- (c) Explain why the regression line of y on x should not be used to predict x. – 1 marks. Award for: Minimises vertical residuals
- (d) A plot of ln y against ln x has r = 0.94. Explain what this suggests about the relationship between y and x. – 2 marks. Award for: Power law $y = ax^n$

Answers: (a) H_0: ρ = 0, H_1: ρ > 0; 0.512 > 0.4409, reject H_0: evidence of positive correlation. (b) 0.512 < 0.5923: insufficient evidence at 1%. (c) It is designed to predict y from x. (d) A model of the form $y = ax^n$ fits well.

Week 40: The normal distribution

Pearson S2 chapter 3.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (11 marks). Normal probabilities. $X \sim N(50, 4^2)$. Give answers to 4 decimal places.

- (a) Find $P(X < 55)$. – 2 marks. Answer: 0.8944. Award for: Correct method (1); accurate answer (1).
- (b) Find $P(X > 45)$. – 2 marks. Answer: 0.8944. Award for: Correct method (1); accurate answer (1).
- (c) Find $P(46 < X < 58)$. – 2 marks. Answer: 0.8186. Award for: Correct method (1); accurate answer (1).
- (d) Find $P(X > 60)$. – 2 marks. Answer: 0.0062. Award for: Correct method (1); accurate answer (1).
- (e) Find a such that $P(X < a) = 0.9$. – 3 marks. Answer: 55.13. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Standard normal and unknown parameters.

- (a) $Z \sim N(0,1)$. Find $P(Z < 1.2)$. – 2 marks. Answer: 0.8849. Award for: Correct method (1); accurate answer (1).
- (b) Find $P(-1 < Z < 1)$. – 2 marks. Answer: 0.6827. Award for: Correct method (1); accurate answer (1).
- (c) Find z with $P(Z > z) = 0.025$. – 2 marks. Answer: 1.96. Award for: Correct method (1); accurate answer (1).
- (d) $X \sim N(\mu, 3^2)$, $P(X < 20) = 0.8$. Find μ. – 2 marks. Answer: 17.48. Award for: Correct method (1); accurate answer (1).
- (e) $X \sim N(40, \sigma^2)$, $P(X > 46) = 0.1$. Find σ. – 3 marks. Answer: 4.68. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). Let X~N(60,16). Find P(X<56), P(58<X<66) and the 90th percentile. State clearly whether 16 denotes variance or standard deviation.

Answer: 0.15866; 0.62466; percentile 65.126. 16 is variance, sd4.

- Calculate the one-sided probability: 1
- Calculate the interval probability: 1
- Use an inverse normal value to obtain the percentile: 2
- Distinguish variance from standard deviation: 1

Q4 (4 marks). A normal variable has 10th percentile 30 and 90th percentile 50. Find its mean and standard deviation, explaining the symmetry used.

Answer: Mean 40; sd=10/Φ^(-1)(0.9)≈7.803. Symmetric equal-tail quantiles.

- Use symmetry to find the mean: 1
- Use an appropriate standard-normal quantile equation: 2
- Find the standard deviation: 1

Q5 (4 marks). Measurements are N(110,9). Find k so that P(|X-(110)|<k)=0.95. Then find the proportion exceeding 116.

Answer: k=3Φ^(-1)(.975)≈5.8799; proportion 0.02275.

- Identify the two-sided 95% quantile: 1
- Find the interval half-width k: 2
- Calculate the upper-tail proportion: 1

Q6 (5 marks). A normal variable has mean 40 and P(X>46)=0.1. Find its variance and explain why using z=-1.2816 gives an impossible standard deviation in your equation.

Answer: sd=6/Φ^(-1)(.9)≈4.6818; variance≈21.919. Upper 10% cutoff is above the mean, so z>0.

- Use the correct positive quantile equation: 2
- Find the standard deviation and hence variance: 2
- Explain why the negative quantile is inappropriate: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Normal distribution in context. Heights are $N(170, 8^2)$ cm.

- (a) Find the proportion taller than 180 cm. – 2 marks. Answer: 0.1056. Award for: Correct method (1); accurate answer (1).
- (b) Find the proportion between 160 and 175 cm. – 2 marks. Answer: 0.6284. Award for: Correct method (1); accurate answer (1).
- (c) Find the height exceeded by 25% of people. – 2 marks. Answer: 175.4 cm. Award for: Correct method (1); accurate answer (1).
- (d) Three people are chosen at random. Find P(all three are taller than 180 cm). – 2 marks. Answer: 0.00118. Award for: Correct method (1); accurate answer (1).
- (e) State two features of a normal distribution. – 3 marks. Answer: symmetric about the mean; about 95% within 2 standard deviations. Award for: Correct method (2); accurate answer (1).
- (f) Explain why a normal model may be unsuitable for waiting times. – 3 marks. Answer: they are skewed and cannot be negative. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). The masses of bags of flour are X g where X ~ N(500, 12^2).

- (a) Find P(X < 480). – 2 marks. Award for: Standardise
- (b) Find P(490 < X < 520). – 2 marks. Award for: Difference of probabilities
- (c) Find m such that 95% of bags have mass greater than m. – 2 marks. Award for: Inverse normal
- (d) Bags under 480 g are rejected. Find the mean needed, with the same standard deviation, for only 1% to be rejected. – 3 marks. Award for: Inverse normal and rearrange

Answers: (a) 0.0478. (b) 0.7499. (c) 480.3 g. (d) μ = 480 + 2.3263 × 12 ≈ 507.9 g.

Q3 (exam style) (8 marks). X ~ N(μ, σ^2) with P(X > 35) = 0.2 and P(X < 20) = 0.1.

- (a) Find μ and σ. – 5 marks. Award for: Two z-values and simultaneous equations
- (b) Find P(X > 30 | X > 25). – 3 marks. Award for: Conditional probability

Answers: (a) σ ≈ 7.06, μ ≈ 29.05. (b) ≈ 0.623.

Q4 (exam style) (8 marks). The heights of a type of plant are modelled by N(42, 6^2) cm.

- (a) Find the proportion of plants taller than 50 cm. – 2 marks. Award for: Standardise
- (b) In a random sample of 20 plants, find the probability that more than 3 are taller than 50 cm. – 3 marks. Award for: Binomial with p from (a)
- (c) Explain why a normal model cannot be exactly correct for heights. – 1 marks. Award for: Negative values
- (d) Find the interquartile range of heights. – 2 marks. Award for: Quartiles

Answers: (a) 0.0912. (b) ≈ 0.103. (c) It assigns probability to negative heights, though negligible. (d) ≈ 8.09 cm.

Week 41: Normal tests and approximations

Pearson S2 chapter 3.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (11 marks). Sample means. $X \sim N(60, 10^2)$, sample of size 25.

- (a) State the distribution of $\bar{X}$. – 2 marks. Answer: $N(60, 2^2)$. Award for: Correct method (1); accurate answer (1).
- (b) Find $P(\bar{X} > 63)$. – 2 marks. Answer: 0.0668. Award for: Correct method (1); accurate answer (1).
- (c) Find $P(58 < \bar{X} < 62)$. – 2 marks. Answer: 0.6827. Award for: Correct method (1); accurate answer (1).
- (d) Find the 5% upper critical value for $\bar{X}$ when testing $H_0 : \mu = 60$. – 2 marks. Answer: 63.29. Award for: Correct method (1); accurate answer (1).
- (e) A sample of 25 has mean 63.8. Test $H_0 : \mu = 60$ against $H_1 : \mu > 60$ at 5%. – 3 marks. Answer: $z = 1.9 > 1.645$: reject H_0. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Normal approximations to the binomial.

- (a) $X \sim B(100, 0.4)$. State the normal approximation. – 2 marks. Answer: $N(40, 24)$. Award for: Correct method (1); accurate answer (1).
- (b) State the conditions for this approximation. – 2 marks. Answer: n large, p close to 0.5 ($np, nq > 5$). Award for: Correct method (1); accurate answer (1).
- (c) Use a continuity correction to estimate $P(X \leq 45)$. – 2 marks. Answer: $P(Y < 45.5) = 0.8692$. Award for: Correct method (1); accurate answer (1).
- (d) Estimate $P(X = 40)$. – 2 marks. Answer: $P(39.5 < Y < 40.5) = 0.0813$. Award for: Correct method (1); accurate answer (1).
- (e) Estimate $P(35 \leq X < 50)$. – 3 marks. Answer: $P(34.5 < Y < 49.5) = 0.8430$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). A population is normal with known sd=5. A random sample of 25 has mean 113. Test H0:μ=111 against μ>111 at 5%; state assumptions and contextual conclusion.

Answer: SE=1, z=2, p=0.02275; reject H0. Evidence mean exceeds 111; random independent sample and known variance required.

- Use the sample-mean distribution and correct standard error: 2
- Calculate a valid test statistic or tail probability: 1
- Make the correct significance comparison: 1
- State the conclusion in context: 1

- State relevant assumptions: 1

Q4 (4 marks). Find the two-sided 5% critical values for the sample mean when n=25, σ=5 and H0:μ=111. Explain the effect of quadrupling n.

Answer: Reject outside 109.04, 112.96; quadrupling n halves SE and the critical distance from μ0.

- Use the two-sided normal critical quantile: 1
- Find both critical values for the sample mean: 2
- Explain how quadrupling sample size changes the standard error and distances: 1

Q5 (5 marks). For X~B(210,0.4), use a normal approximation to find P(X≤84). State mean, variance, continuity correction and suitability.

Answer: Mean 84, variance 252/5; use boundary 84.5. Approx probability=0.52807. np,n(1-p) both sufficiently large.

- Find the approximating mean and variance: 2
- Apply the correct continuity correction: 1
- Calculate the approximate probability: 1
- Assess suitability of the approximation: 1

Q6 (4 marks). A test returns p=0.07. Explain the decisions at 5% and 10%, and why 'accept H0 as proved true' and '7% probability H0 is true' are both invalid interpretations.

Answer: Do not reject at5%; reject at10%. p is a tail probability assuming H0, not a posterior probability of H0; non-rejection is not proof.

- State the decisions at both significance levels: 2
- Explain why non-rejection is not proof of the null: 1
- Explain why a p-value is not the probability that the null is true: 1

Practice 2 (Day 4) – total 37 marks

Q1 (basic skills) (14 marks). Tests for the mean.

- (a) $\sigma = 4$, $n = 16$, $\bar{x} = 21.8$. Test $H_0 : \mu = 20$, $H_1 : \mu \neq 20$ at 5%. – 2 marks. Answer: $z = 1.8 < 1.96$: do not reject. Award for: Correct method (1); accurate answer (1).
- (b) Find the critical region for that test. – 2 marks. Answer: $\bar{X} < 18.04$ or $\bar{X} > 21.96$. Award for: Correct method (1); accurate answer (1).
- (c) $\sigma = 12$, $n = 36$, $\bar{x} = 96.1$. Test $H_0 : \mu = 100$, $H_1 : \mu < 100$ at 1%. – 2 marks. Answer: $z = -1.95 > -2.326$: do not reject. Award for: Correct method (1); accurate answer (1).
- (d) State what a Type I error would mean here. – 2 marks. Answer: concluding the mean has fallen when it has not. Award for: Correct method (1); accurate answer (1).
- (e) Explain why a larger sample makes the test more sensitive. – 3 marks. Answer: standard error $\frac{\sigma}{\sqrt{n}}$ is smaller. Award for: Correct method (2); accurate answer (1).

- (f) State the distribution of $\bar{X}$ for $n = 50$ from $N(\mu, 9^2)$. – 3 marks. Answer: $N(\mu, 1.62)$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). Commuting times are N(μ, 4^2) minutes and historically μ = 32. After roadworks begin, a random sample of 16 journeys has mean 34.1 minutes.

- (a) Test at the 5% level whether the mean time has increased. – 5 marks. Award for: Hypotheses, distribution of mean, z, comparison, conclusion
- (b) Find the critical region for the sample mean. – 2 marks. Award for: Boundary
- (c) State one assumption needed. – 1 marks. Award for: Valid assumption

Answers: (a) z = 2.1 > 1.6449: reject H_0, evidence of an increase. (b) $\bar{x} > 33.64$. (c) e.g. σ unchanged; journeys independent.

Q3 (exam style) (7 marks). X ~ B(120, 0.35).

- (a) Explain why a normal approximation is suitable and state its parameters. – 2 marks. Award for: np and n(1 − p) large; parameters
- (b) Use the approximation to estimate P(36 ≤ X < 45). – 3 marks. Award for: Continuity correction and standardise
- (c) The exact probability is 0.5807. Comment on the approximation. – 2 marks. Award for: Percentage error comment

Answers: (a) n large, p near 0.5: N(42, 27.3). (b) P(35.5 < Y < 44.5) ≈ 0.577. (c) Good, within about 0.6%.

Q4 (exam style) (8 marks). A machine's bottle volumes are normally distributed with known standard deviation 6 ml. The historical mean is 750 ml. A random sample of 25 bottles has mean 747.4 ml.

- (a) Test at the 5% level whether the mean has changed. – 5 marks. Award for: Two-tailed test
- (b) Find the p-value. – 1 marks. Award for: Two tails
- (c) State the conclusion at the 1% level. – 1 marks. Award for: Compare
- (d) Explain what a Type I error would mean in this context. – 1 marks. Award for: Context

Answers: (a) z = −2.17, |z| > 1.96: reject H_0, mean has changed. (b) 0.0303. (c) Do not reject. (d) Concluding the mean changed when it is still 750 ml.

Week 42: Differentiation rules

Pearson P2 chapter 9.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Differentiating standard functions.

- (a) Given $y = e^{3x}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $3e^{3x}$. Award for: Correct method (1); accurate answer (1).
- (b) Given $y = \ln 5x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $\frac{1}{x}$. Award for: Correct method (1); accurate answer (1).
- (c) Given $y = \sin 4x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $4\cos 4x$. Award for: Correct method (1); accurate answer (1).
- (d) Given $y = 2\cos\frac{x}{2}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $-\sin\frac{x}{2}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Chain, product and quotient rules.

- (a) Given $y = (2x+1)^5$, find $\frac{dy}{dx}$. – 2 marks. Answer: $10(2x+1)^4$. Award for: Correct method (1); accurate answer (1).
- (b) Given $y = \sqrt{x^2+4}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $\frac{x}{\sqrt{x^2+4}}$. Award for: Correct method (1); accurate answer (1).
- (c) Given $y = x^2e^x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $e^x(x^2+2x)$. Award for: Correct method (1); accurate answer (1).
- (d) Given $y = x\sin x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $\sin x + x\cos x$. Award for: Correct method (1); accurate answer (1).

Q3 (6 marks). Differentiate $y = \left(x^2+1\right)e^{12x}$. Factorise the result and find the tangent at x=0, explaining where the product and chain rules enter.

Answer: y'=$2\left(6x^2+x+6\right)e^{12x}$; tangent y=1+12x.

- Apply the product and chain rules correctly: 2
- Factorise the derivative: 1
- Find the tangent: 2
- Explain where both rules enter: 1

Q4 (6 marks). Differentiate $y = \frac{\sin(12x)}{x+1}$. State its domain and evaluate y'(0), checking its plausibility using the local sine approximation.

Answer: Derivative $\frac{12\cos(12x)}{x+1} - \frac{\sin(12x)}{(x+1)^2}$; x≠-1; y'(0)=12. sin(ax)≈ax near0.

- Use the quotient and chain rules to obtain the derivative: 3
- State the domain: 1
- Evaluate the derivative at zero: 1
- Check plausibility using the local sine approximation: 1

Q5 (6 marks). For $y = \ln\left(12x^2+1\right)$, find first and second derivatives and classify x=0. State the real domain.

Answer: y'=$\frac{24x}{12x^2+1}$; y"=$-\frac{24(12x^2-1)}{(12x^2+1)^2}$; local/global minimum at0; all real x.

- Find the first derivative: 2
- Find the second derivative: 2
- Classify the stationary point: 1
- State the real domain: 1

Q6 (5 marks). Find d/dx of $\sin^2(12x)$ in two ways: chain rule and a double-angle identity. Show agreement.

Answer: 24sin(12x)cos(12x)=12sin(24x). Identity $\sin^2$u=(1-cos2u)/2.

- Differentiate using the chain rule: 2
- Differentiate independently using a double-angle identity: 2
- Show the resulting expressions agree: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Applications of the rules.

- (a) Find the gradient of $y = xe^{2x}$ at $x = 0$. – 2 marks. Answer: 1. Award for: Correct method (1); accurate answer (1).
- (b) Find the stationary point of $y = xe^{-x}$. – 2 marks. Answer: $(1, e^{-1})$. Award for: Correct method (1); accurate answer (1).
- (c) Find the equation of the tangent to $y = \ln x$ at $x = e$. – 2 marks. Answer: $y = \frac{x}{e}$. Award for: Correct method (1); accurate answer (1).
- (d) Given $y = \sec x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $\sec x \tan x$. Award for: Correct method (1); accurate answer (1).
- (e) Given $y = \operatorname{cosec} x$, find $\frac{dy}{dx}$. – 3 marks. Answer: $-\operatorname{cosec} x \cot x$. Award for: Correct method (2); accurate answer (1).
- (f) Find the x-coordinates of the stationary points of $y = \sin x + \cos x$, $0 \leq x \leq 2\pi$. – 3 marks. Answer: $\frac{\pi}{4}$, $\frac{5\pi}{4}$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). The curve C has equation $y = x^2e^{-2x}$.

- (a) Find dy/dx, factorising your answer. – 3 marks. Award for: Product and chain rules
- (b) Find the exact coordinates of the stationary points and determine their nature. – 5 marks. Award for: Solve, coordinates, nature
- (c) Find the set of values of k for which $x^2e^{-2x} = k$ has three solutions. – 1 marks. Award for: Interval

Answers: (a) 2x(1 − x)e^(−2x). (b) (0, 0) minimum, (1, e^(−2)) maximum. (c) 0 < k < e^(−2).

Q3 (exam style) (8 marks). $f(x) = \frac{3x-1}{x^2+1}$.

- (a) Show that $f'(x) = \frac{-3x^2+2x+3}{(x^2+1)^2}$. – 3 marks. Award for: Quotient rule

- (b) Find the exact x-coordinates of the stationary points. – 3 marks. Award for: Solve quadratic
- (c) Find the equation of the tangent where x = 0. – 2 marks. Award for: Point and gradient

Answers: (a) Quotient rule and simplify. (b) x = (1 ± √10)/3. (c) y = 3x − 1.

Q4 (exam style) (8 marks).

- (a) Given y = ln(cos x), show that dy/dx = −tan x. – 2 marks. Award for: Chain rule
- (b) Differentiate $e^{3x}\sin 4x$ and find the exact gradient at x = π/4. – 3 marks. Award for: Product rule and evaluate
- (c) Find the equation of the normal to $y = (2x+1)^5$ at x = 0. – 3 marks. Award for: Chain rule and normal

Answers: (a) −sin x/cos x. (b) e^(3x)(3 sin 4x + 4 cos 4x); −4e^(3π/4). (c) y = 1 − x/10.

Week 43: Implicit differentiation and rates of change

Pearson P2 chapter 9.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Implicit differentiation. Find $\frac{dy}{dx}$ in terms of x and y.

- (a) $x^2 + y^2 = 25$ – 2 marks. Answer: $-\frac{x}{y}$. Award for: Correct method (1); accurate answer (1).
- (b) $y^3 = x^2 + 1$ – 2 marks. Answer: $\frac{2x}{3y^2}$. Award for: Correct method (1); accurate answer (1).
- (c) $xy = 6$ – 2 marks. Answer: $-\frac{y}{x}$. Award for: Correct method (1); accurate answer (1).
- (d) $x^2 + xy + y^2 = 7$ – 2 marks. Answer: $-\frac{2x+y}{x+2y}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Tangents and normals to implicit curves.

- (a) Find the gradient of $x^2 + y^2 = 25$ at $(3, 4)$. – 2 marks. Answer: $-\frac{3}{4}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the tangent to $x^2 + y^2 = 25$ at $(3, 4)$. – 2 marks. Answer: $3x + 4y = 25$. Award for: Correct method (1); accurate answer (1).
- (c) Find the gradient of $x^2 + xy + y^2 = 7$ at $(1, 2)$. – 2 marks. Answer: $-\frac{4}{5}$. Award for: Correct method (1); accurate answer (1).
- (d) Find the normal to $x^2 + xy + y^2 = 7$ at $(1, 2)$. – 2 marks. Answer: $5x - 4y + 3 = 0$. Award for: Correct method (1); accurate answer (1).
- (e) Find the points on $x^2 + y^2 - 4x = 0$ where the tangent is parallel to the x-axis. – 3 marks. Answer: $(2, \pm 2)$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). For $x^2+y^2=37$, find dy/dx implicitly and the tangent/normal at (1,6). Explain where dy/dx is undefined.

Answer: dy/dx=-x/y. Tangent y-6=-(x-1)/6; normal y-6=6(x-1). Undefined at y=0 on the circle.

- Differentiate implicitly and isolate the derivative: 2
- Find the tangent equation: 1
- Find the normal equation: 1
- Identify where the derivative is undefined on the circle: 1

Q4 (5 marks). For $x^2 + xy + y^2 = 108$, find dy/dx and evaluate it at (6,6). Find the tangent there.

Answer: y'=-(2x+y)/(x+2y); at point -1; tangent x+y=12.

- Differentiate the mixed xy term correctly: 1
- Collect terms and isolate the derivative: 2
- Evaluate at the given point: 1
- Find the tangent: 1

Q5 (5 marks). A sphere has radius 6 cm increasing at 0.2 cm/s at an instant. Find rates of change of volume and surface area at that instant; state units.

Answer: dV/dt=4πr²dr/dt=$\frac{144\pi}{5}$ cm³/s; dA/dt=8πrdr/dt=$\frac{48\pi}{5}$ cm²/s.

- Apply the volume chain rule and evaluate its rate: 2
- Apply the surface-area chain rule and evaluate its rate: 2
- Give both correct units: 1

Q6 (6 marks). Differentiate $y = x^6 e^{-x}$ and find/classify its positive stationary point. Explain behaviour as x becomes large.

Answer: y'=x^{5e}(-x)(6-x); positive maximum at x=6, y=$\frac{46656}{e^6}$. y→0 as x→∞.

- Differentiate and factorise: 2
- Find the positive stationary point and its value: 2
- Establish its classification: 1
- Explain the large-x behaviour: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Connected rates of change.

- (a) $A = \pi r^2$ and $\frac{dr}{dt} = 0.5$. Find $\frac{dA}{dt}$ when $r = 4$. – 2 marks. Answer: 4π. Award for: Correct method (1); accurate answer (1).
- (b) $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = 10$. Find $\frac{dr}{dt}$ when $r = 2$. – 2 marks. Answer: $\frac{10}{16\pi} = 0.199$. Award for: Correct method (1); accurate answer (1).
- (c) A cube's edge grows at 0.2 cm s^{-1}. Find the rate of increase of volume when the edge is 5 cm. – 2 marks. Answer: 15 cm^3 s^{-1}. Award for: Correct method (1); accurate answer (1).

- (d) Write a differential equation: the rate of decrease of mass m is proportional to m. – 2 marks. Answer: $\frac{dm}{dt} = -km$. Award for: Correct method (1); accurate answer (1).
- (e) Write a differential equation: the rate of increase of N is proportional to $\sqrt{N}$. – 3 marks. Answer: $\frac{dN}{dt} = k\sqrt{N}$. Award for: Correct method (2); accurate answer (1).
- (f) $y = x^2$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at $x = 2$. – 3 marks. Answer: 12. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). The curve C has equation $x^2 + 3xy + y^2 = 11$.

- (a) Find dy/dx in terms of x and y. – 4 marks. Award for: Implicit differentiation including product rule
- (b) Find the equation of the tangent at (1, 2). – 2 marks. Award for: Gradient and line
- (c) Show that C has no points where the tangent is horizontal. – 3 marks. Award for: Substitute 2x + 3y = 0 into C, no real solutions

Answers: (a) −(2x + 3y)/(3x + 2y). (b) 8x + 7y − 22 = 0. (c) x = −3y/2 gives $-5y^2/4 = 11$: no real solutions.

Q3 (exam style) (8 marks). Water is poured at 20 cm³/s into an inverted cone of height 30 cm and top radius 10 cm.

- (a) Show that the volume of water at depth h cm is $V = \frac{\pi h^3}{27}$. – 2 marks. Award for: Similar triangles
- (b) Find the rate at which the depth increases when h = 12. – 4 marks. Award for: Chain rule
- (c) Find the rate of increase of the water surface area at this instant. – 2 marks. Award for: Chain rule

Answers: (a) r = h/3. (b) dh/dt = 20/(16π) ≈ 0.398 cm/s. (c) 10/3 cm²/s.

Q4 (exam style) (8 marks).

- (a) Show that if $y = 2^x$ then $\frac{dy}{dx} = 2^x \ln 2$. – 3 marks. Award for: Rewrite with e and differentiate
- (b) Find the exact gradient of $y = 3^{2x}$ at x = 1. – 2 marks. Award for: Correct derivative
- (c) Show that (0, 1) lies on $e^y + xy = e$ and find dy/dx there. – 3 marks. Award for: Implicit differentiation

Answers: (a) y = e^(x ln 2). (b) 18 ln 3. (c) e + 0 = e; dy/dx = −y/(e^y + x) = −1/e.

Week 44: Parametric equations and differentiation

Pearson P2 chapter 8–9.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Parametric equations to Cartesian form.

- (a) $x = t + 1$, $y = t^2$. Find the Cartesian equation. – 2 marks. Answer: $y = (x-1)^2$. Award for: Correct method (1); accurate answer (1).
- (b) $x = 2t$, $y = \frac{3}{t}$. – 2 marks. Answer: $xy = 6$. Award for: Correct method (1); accurate answer (1).
- (c) $x = 3\cos t$, $y = 3\sin t$. – 2 marks. Answer: $x^2 + y^2 = 9$. Award for: Correct method (1); accurate answer (1).
- (d) $x = 2 + \cos t$, $y = \sin t - 1$. – 2 marks. Answer: $(x-2)^2 + (y+1)^2 = 1$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Parametric differentiation.

- (a) $x = t^2$, $y = 2t$. Find $\frac{dy}{dx}$ in terms of t. – 2 marks. Answer: $\frac{1}{t}$. Award for: Correct method (1); accurate answer (1).
- (b) $x = t^3$, $y = t^2 - 1$. Find $\frac{dy}{dx}$. – 2 marks. Answer: $\frac{2}{3t}$. Award for: Correct method (1); accurate answer (1).
- (c) $x = \cos t$, $y = \sin t$. Find $\frac{dy}{dx}$. – 2 marks. Answer: $-\cot t$. Award for: Correct method (1); accurate answer (1).
- (d) For $x = t^2$, $y = 2t$, find the tangent at $t = 2$. – 2 marks. Answer: $2y = x + 4$. Award for: Correct method (1); accurate answer (1).
- (e) $x = e^t$, $y = e^{2t}$. Find $\frac{dy}{dx}$ and simplify. – 3 marks. Answer: $2e^t = 2x$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). $x=t^2+1$, $y=7t$, $t\geq 0$. Eliminate t, state the Cartesian domain/range, and find dy/dx and the tangent at t=1.

Answer: $y=7\sqrt{x-1}$, $x\geq 1, y\geq 0$; $\frac{dy}{dx}=\frac{7}{2t}$; tangent $y-7=\frac{7}{2}(x-2)$.

- Eliminate the parameter on the correct branch: 2
- State Cartesian domain and range: 2
- Find dy/dx: 1
- Find the tangent: 1

Q4 (5 marks). $x=7\cos t$, $y=7\sin t$, $0\leq t\leq \pi$. Eliminate t with the correct restriction, find dy/dx and the tangent at $t=\pi/4$.

Answer: $x^2+y^2=49$, $y\geq 0$; $\frac{dy}{dx}=-\cot t$; tangent $x+y=7\sqrt{2}$.

- Eliminate the parameter and retain the semicircle restriction: 2
- Find dy/dx: 2
- Find the tangent: 1

Q5 (4 marks). $x=t^3$, $y=t^2+7$. Find dy/dx and $\frac{d^2y}{dx^2}$ for $t\neq 0$. Explain why differentiating dy/dx with respect to t alone does not give $\frac{d^2y}{dx^2}$.

Answer: dy/dx=2/(3t); second derivative=-2/(9t^4). Must divide d/dt(dy/dx) by dx/dt.

- Find dy/dx: 1
- Differentiate it with respect to t: 1
- Divide by dx/dt to obtain the second derivative: 1
- Explain the extra chain-rule division: 1

Q6 (6 marks). x=t+1/t, y=7(t-1/t), t>0. Eliminate t and state the restriction on x. Find points where y=0 and the tangent gradient there.

Answer: x^2-(y/7)^2=4; x≥2. y=0 at t=1 gives (2,0); dx/dt=0,dy/dt≠0, so vertical tangent, undefined finite gradient.

- Eliminate t to obtain the Cartesian relation: 2
- State the restriction on x: 1
- Find the point where y is zero: 1
- Identify and justify the vertical tangent: 2

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Using parametric curves.

- (a) $x = t^2 - 1$, $y = t^3 - 3t$. Find where the curve crosses the y-axis. – 2 marks. Answer: $t = \pm 1$: $(0, \mp 2)$. Award for: Correct method (1); accurate answer (1).
- (b) Find the values of t at stationary points. – 2 marks. Answer: $t = \pm 1$. Award for: Correct method (1); accurate answer (1).
- (c) $x = 4t$, $y = \frac{4}{t}$. Find the normal at $t = 2$. – 2 marks. Answer: $y = 4x - 30$. Award for: Correct method (1); accurate answer (1).
- (d) Find where $x = t + 2$, $y = t^2$ meets $y = 4$. – 2 marks. Answer: $(4, 4)$, $(0, 4)$. Award for: Correct method (1); accurate answer (1).
- (e) State the domain of x for $x = \cos t$, $0 \le t \le \pi$. – 3 marks. Answer: $-1 \le x \le 1$. Award for: Correct method (2); accurate answer (1).
- (f) A ball has position $x = 10t$, $y = 15t - 4.9t^2$. Find when it lands. – 3 marks. Answer: 3.06 s. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). The curve C has parametric equations $x = 2t^2$, y = 4t.

- (a) Find a Cartesian equation of C. – 2 marks. Award for: Eliminate t
- (b) Find dy/dx in terms of t. – 2 marks. Award for: Chain rule
- (c) The normal at the point where t = 2 meets C again at Q. Find Q. – 5 marks. Award for: Normal, substitute parametric form, solve

Answers: (a) y^2 = 8x. (b) 1/t. (c) Normal y = 24 − 2x; t^2 + t − 6 = 0, t = −3, Q(18, −12).

Q3 (exam style) (8 marks). The curve C has parametric equations x = 4 sin θ, y = 3 cos 2θ, −π/2 ≤ θ ≤ π/2.

- (a) Show that a Cartesian equation is $y = 3 - \frac{3x^2}{8}$. – 3 marks. Award for: Double angle and eliminate
- (b) Find dy/dx when θ = π/6. – 3 marks. Award for: Parametric differentiation
- (c) State the domain and range of the Cartesian equation for C. – 2 marks. Award for: Both ranges

Answers: (a) cos 2θ = 1 − 2(x/4)². (b) −3/2. (c) −4 ≤ x ≤ 4, −3 ≤ y ≤ 3.

Q4 (exam style) (8 marks). A curve has parametric equations $x = t^2 - 4t$, $y = t^3 - 3t$, t ≥ 0.

- (a) Find the points where the curve meets the y-axis. – 2 marks. Award for: Solve x = 0
- (b) Find the point where the tangent is parallel to the x-axis. – 3 marks. Award for: dy/dt = 0
- (c) Find the point where the tangent is parallel to the y-axis. – 2 marks. Award for: dx/dt = 0
- (d) Find the gradient when t = 3. – 1 marks. Award for: Ratio

Answers: (a) (0, 0) and (0, 52). (b) t = 1: (−3, −2). (c) t = 2: (−4, 2). (d) 12.

Week 45: Integration and substitution

Pearson P2 chapter 11.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (11 marks). Integrating standard functions.

- (a) Find $\int e^{2x}\,dx$. – 2 marks. Answer: $\frac{e^{2x}}{2} + c$. Award for: Correct method (1); accurate answer (1).
- (b) Find $\int \frac{3}{x}\,dx$. – 2 marks. Answer: $3\ln(x) + c$. Award for: Correct method (1); accurate answer (1).
- (c) Find $\int \cos 3x\,dx$. – 2 marks. Answer: $\frac{\sin(3x)}{3} + c$. Award for: Correct method (1); accurate answer (1).
- (d) Find $\int \sec^2 4x\,dx$. – 2 marks. Answer: $\frac{1}{4}\tan 4x + c$. Award for: Correct method (1); accurate answer (1).
- (e) Find $\int \frac{1}{2x+1}\,dx$. – 3 marks. Answer: $\frac{1}{2}\ln|2x+1| + c$. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Reverse chain rule and identities.

- (a) Find $\int 2x(x^2+1)^4\,dx$. – 2 marks. Answer: $\frac{(x^2+1)^5}{5} + c$. Award for: Correct method (1); accurate answer (1).
- (b) Find $\int \frac{2x}{x^2+3}\,dx$. – 2 marks. Answer: $\ln(x^2+3) + c$. Award for: Correct method (1); accurate answer (1).
- (c) Find $\int \tan x\,dx$. – 2 marks. Answer: $\ln|\sec x| + c$. Award for: Correct method (1); accurate answer (1).
- (d) Find $\int \sin^2 x\,dx$. – 2 marks. Answer: $\frac{x}{2} - \frac{\sin 2x}{4} + c$. Award for: Correct method (1); accurate answer (1).

- (e) Find $\int \cos x \sin^3 x \, dx$. – 3 marks. Answer: $\frac{\sin^4 x}{4} + c$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). Find $\int 2x(x^2+8)^3 \, dx$ by substitution. State the substitution, integrate and differentiate your answer to check.

Answer: u=x^2+8; integral $(x^2+8)^4/4$+C.

- State the substitution and transformed differential: 1
- Transform and integrate: 2
- Back-substitute and include the constant: 1
- Check by differentiation: 1

Q4 (4 marks). Find $\int \frac{16x}{8x^2+1} \, dx$. Explain why a logarithm appears and state any domain issue.

Answer: ln($8x^2$+1)+C; denominator derivative is numerator; argument positive for all real x.

- Recognise the logarithmic derivative or make a valid substitution: 1
- Integrate correctly, including the constant: 1
- Explain why a logarithm appears: 1
- Resolve the real-domain issue: 1

Q5 (5 marks). Evaluate $\int_0^{\pi/16} \sin(8x)\cos(8x) \, dx$ by substitution and independently using a double-angle identity.

Answer: Antiderivative $\sin^2$(8x)/16; between limits=$\frac{1}{16}$. sin2u=2sin u cos u gives same.

- Use substitution to evaluate the definite integral: 3
- Evaluate independently using a double-angle identity: 2

Q6 (4 marks). Find $\int \frac{x}{\sqrt{x^2+8}} \, dx$, then use the result to evaluate it from 0 to 2. Explain why no absolute value is needed in the square root.

Answer: sqrt(x^2+8)+C; definite=$-2\sqrt{2} + 2\sqrt{3}$. Radicand positive.

- Use a valid substitution or reverse chain rule: 1
- Find the antiderivative with the constant: 1
- Evaluate at both limits: 1
- Explain positivity of the radicand: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Integration by substitution.

- (a) Use $u = 2x + 1$ to find $\int x(2x+1)^3 \, dx$. – 2 marks. Answer: $\frac{(2x+1)^5}{20} - \frac{(2x+1)^4}{16} + c$. Award for: Correct method (1); accurate answer (1).
- (b) Use $u = x^2 + 1$ to find $\int \frac{x}{\sqrt{x^2+1}} \, dx$. – 2 marks. Answer: $\sqrt{x^2+1} + c$. Award for: Correct method (1); accurate answer (1).

- (c) Use $u = \sin x$ to find $\int \cos x\, e^{\sin x}\, dx$. – 2 marks. Answer: $e^{\sin x} + c$. Award for: Correct method (1); accurate answer (1).
- (d) Use $u = x - 1$ to evaluate $\int_1^2 x(x-1)^4\, dx$. – 2 marks. Answer: $\frac{1}{6} + \frac{1}{5} = \frac{11}{30}$. Award for: Correct method (1); accurate answer (1).
- (e) Use $u = \ln x$ to find $\int \frac{\ln x}{x}\, dx$. – 3 marks. Answer: $\frac{(\ln x)^2}{2} + c$. Award for: Correct method (2); accurate answer (1).
- (f) Use $x = \sin\theta$ to evaluate $\int_0^1 \sqrt{1-x^2}\, dx$. – 3 marks. Answer: $\frac{\pi}{4}$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks).

- (a) Use the substitution $u = 1 + \sqrt{x}$ to find $\int \frac{1}{1+\sqrt{x}}\, dx$. – 5 marks. Award for: dx in terms of du, simplify, integrate, back-substitute
- (b) Hence find the exact value of $\int_0^4 \frac{1}{1+\sqrt{x}}\, dx$. – 2 marks. Award for: Limits
- (c) Explain why the limits must be changed, or the answer rewritten in x, before evaluating. – 1 marks. Award for: Limits refer to the variable

Answers: (a) 2(1 + √x) − 2 ln(1 + √x) + c. (b) 4 − 2 ln 3. (c) The original limits are values of x, not u.

Q3 (exam style) (8 marks).

- (a) Use the substitution u = 1 + cos x to show that $\int_0^{\pi/3} \frac{\sin x}{(1+\cos x)^2}\, dx = \frac{1}{6}$. – 4 marks. Award for: Substitution, limits, evaluate
- (b) Find $\int \frac{6x}{(x^2+4)^3}\, dx$. – 2 marks. Award for: Reverse chain rule
- (c) Show that $\int \tan x\, dx = \ln|\sec x| + c$. – 2 marks. Award for: f′/f form

Answers: (a) ∫ u^(−2) du from 3/2 to 2 = 1/6. (b) −3/(2(x² + 4)²) + c. (c) −ln|cos x| = ln|sec x|.

Q4 (exam style) (8 marks).

- (a) Use the substitution u = x + 1 to find the exact value of $\int_0^3 x\sqrt{x+1}\, dx$. – 5 marks. Award for: Substitute, expand, integrate, evaluate
- (b) Find the area of the region between $y = x\sqrt{x+1}$ and the x-axis for −1 ≤ x ≤ 0. – 3 marks. Award for: Integral is negative; area positive

Answers: (a) 116/15. (b) 4/15.

Week 46: Integration by parts and partial fractions

Pearson P2 chapter 11.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Integration by parts.

- (a) Find $\int xe^x\, dx$. – 2 marks. Answer: $e^x(x-1) + c$. Award for: Correct method (1); accurate answer (1).

- (b) Find $\int x\cos x\,dx$. – 2 marks. Answer: $x\sin x+\cos x+c$. Award for: Correct method (1); accurate answer (1).
- (c) Find $\int x\ln x\,dx$. – 2 marks. Answer: $\frac{x^2}{2}\ln x-\frac{x^2}{4}+c$. Award for: Correct method (1); accurate answer (1).
- (d) Find $\int \ln x\,dx$. – 2 marks. Answer: $x\ln x-x+c$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Integration using partial fractions.

- (a) Find $\int \frac{1}{x(x+1)}\,dx$. – 2 marks. Answer: $\ln\left|\frac{x}{x+1}\right|+c$. Award for: Correct method (1); accurate answer (1).
- (b) Find $\int \frac{5}{(x-2)(x+3)}\,dx$. – 2 marks. Answer: $\ln\left|\frac{x-2}{x+3}\right|+c$. Award for: Correct method (1); accurate answer (1).
- (c) Find $\int \frac{3x+5}{(x+1)(x+2)}\,dx$. – 2 marks. Answer: $2\ln|x+1|+\ln|x+2|+c$. Award for: Correct method (1); accurate answer (1).
- (d) Find $\int \frac{1}{(x+1)^2}\,dx$. – 2 marks. Answer: $-\frac{1}{x+1}+c$. Award for: Correct method (1); accurate answer (1).
- (e) Find $\int \frac{x+3}{(x+1)^2}\,dx$. – 3 marks. Answer: $\ln|x+1|-\frac{2}{x+1}+c$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). Find $\int xe^{9x}\,dx$ by parts. Evaluate it from 0 to1 and verify the antiderivative.

Answer: e^(9x)(x/9-1/81)+C; definite=$\frac{1}{81}+\frac{8e^9}{81}$.

- Choose u and dv and apply integration by parts: 2
- Obtain the antiderivative with its constant: 1
- Evaluate the definite integral: 2
- Check the antiderivative: 1

Q4 (5 marks). Find $\int x\cos(9x)\,dx$. Then differentiate your answer and show the cancellation explicitly.

Answer: x sin(9x)/9+cos(9x)/81+C. Product-rule cross terms cancel.

- Choose and apply integration by parts: 2
- Complete the antiderivative: 1
- Differentiate the answer: 1
- Show the cross-term cancellation: 1

Q5 (6 marks). Resolve $\frac{19x+1}{x(x+1)}$ into partial fractions and integrate, stating excluded x-values.

Answer: 1/x+18/(x+1); ln|x|+18ln|x+1|+C; x≠0,-1.

- Set up and determine the partial fractions: 3
- Integrate with correct logarithms and constant: 2

- State the excluded values: 1

Q6 (5 marks). Evaluate $\int_1^9 \ln x \, dx$. Explain the choice of u and dv, and why no elementary power-rule shortcut applies.

Answer: u=lnx,dv=dx; xlnx-x gives 9ln(9)-9+1.

- Choose u and dv and explain the choice: 1
- Apply integration by parts: 2
- Evaluate the definite integral: 1
- Explain why the elementary power rule is inapplicable: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Definite integration by parts.

- (a) Evaluate $\int_0^1 xe^x \, dx$. – 2 marks. Answer: 1. Award for: Correct method (1); accurate answer (1).
- (b) Evaluate $\int_0^{\pi/2} x \sin x \, dx$. – 2 marks. Answer: 1. Award for: Correct method (1); accurate answer (1).
- (c) Evaluate $\int_1^e \ln x \, dx$. – 2 marks. Answer: 1. Award for: Correct method (1); accurate answer (1).
- (d) Evaluate $\int_1^e x \ln x \, dx$. – 2 marks. Answer: $\frac{e^2+1}{4}$. Award for: Correct method (1); accurate answer (1).
- (e) Evaluate $\int_0^{\ln 2} xe^x \, dx$. – 3 marks. Answer: $2 \ln 2 - 1$. Award for: Correct method (2); accurate answer (1).
- (f) Explain how to choose u and $\frac{dv}{dx}$ for $\int x^3 \ln x \, dx$. – 3 marks. Answer: $u = \ln x$, since $\ln x$ is hard to integrate. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks).

- (a) Find $\int x^2 e^x \, dx$. – 4 marks. Award for: Parts twice
- (b) Hence find the exact area under $y = x^2 e^x$ between x = 0 and x = 1. – 2 marks. Award for: Evaluate
- (c) Find $\int x^2 \ln x \, dx$. – 3 marks. Award for: Parts with ln x differentiated

Answers: (a) e^x(x² − 2x + 2) + c. (b) e − 2. (c) (x³ ln x)/3 − x³/9 + c.

Q3 (exam style) (8 marks). $f(x) = \frac{5x+3}{(2x-3)(x+2)}$.

- (a) Express f(x) in partial fractions. – 3 marks. Award for: Constants
- (b) Find $\int_2^3 f(x) \, dx$ in the form ln k. – 4 marks. Award for: Integrate including factor 1/2, combine logs
- (c) Explain why $\int_1^3 f(x) \, dx$ cannot be found in this way. – 1 marks. Award for: Discontinuity at x = 1.5

Answers: (a) 3/(2x − 3) + 1/(x + 2). (b) (3/2) ln 3 + ln(5/4) = ln(15√3/4). (c) f is undefined at x = 3/2.

Q4 (exam style) (8 marks).

- (a) Use integration by parts twice to find $\int e^x \sin x \, dx$. – 5 marks. Award for: Two applications and solve for the integral
- (b) Hence find the exact value of $\int_0^{\pi} e^x \sin x \, dx$. – 2 marks. Award for: Evaluate
- (c) Explain why the answer to (b) is positive. – 1 marks. Award for: Integrand positive on the interval

Answers: (a) ½e^x(sin x − cos x) + c. (b) ½(e^π + 1). (c) e^x sin x > 0 for 0 < x < π.

Week 47: Definite and parametric integration

Pearson P2 chapter 8, 11.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Definite integrals with standard functions. Give exact answers.

- (a) Evaluate $\int_0^{\ln 3} e^x \, dx$. – 2 marks. Answer: 2. Award for: Correct method (1); accurate answer (1).
- (b) Evaluate $\int_1^{e^2} \frac{1}{x} \, dx$. – 2 marks. Answer: 2. Award for: Correct method (1); accurate answer (1).
- (c) Evaluate $\int_0^{\pi/2} \cos x \, dx$. – 2 marks. Answer: 1. Award for: Correct method (1); accurate answer (1).
- (d) Evaluate $\int_0^{\pi/4} \sec^2 x \, dx$. – 2 marks. Answer: 1. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Areas involving curves and parametric integration.

- (a) Find the area under $y = e^x$ from $x = 0$ to $x = 2$. – 2 marks. Answer: $e^2 - 1$. Award for: Correct method (1); accurate answer (1).
- (b) Find the area under $y = \frac{2}{x}$ from $x = 1$ to $x = 4$. – 2 marks. Answer: $2 \ln 4 = 4 \ln 2$. Award for: Correct method (1); accurate answer (1).
- (c) Find the area between $y = \sin x$ and the x-axis for $0 \le x \le \pi$. – 2 marks. Answer: 2. Award for: Correct method (1); accurate answer (1).
- (d) Write $\int y \, dx$ as an integral in t for $x = t^2, y = t^3$. – 2 marks. Answer: $\int 2t^4 \, dt$. Award for: Correct method (1); accurate answer (1).
- (e) Hence find the area under that curve from $t = 0$ to $t = 1$. – 3 marks. Answer: $\frac{2}{5}$. Award for: Correct method (2); accurate answer (1).

Q3 (8 marks). Find the area between y=10sin x and y=10sin(2x) for 0≤x≤π. Locate all intersections first and explain the necessary split.

Answer: Intersections 0,π/3,π. First area $\frac{5}{2}$, second $\frac{45}{2}$; total 25.

- Find all intersections: 2
- Identify the upper curve on each subinterval: 1
- Set up the split area integral: 1
- Integrate and combine the exact areas: 3
- Explain the need to split: 1

Q4 (5 marks). A curve is $x=t^2$, $y=10t$, $0\le t\le 2$. Find the area under it against the x-axis using parametric integration. Check by eliminating t.

Answer: $\int y(dx/dt)dt=\int_0^2 20t^2\,dt=\frac{160}{3}$; Cartesian $y=10\sqrt{x}$, $x\in[0,4]$.

- Set up the parametric area integral with dx/dt: 2
- Integrate and evaluate: 1
- Eliminate t and confirm using a Cartesian integral: 2

Q5 (5 marks). Evaluate $\int_0^{\pi/2}\sin^2 x\,dx$ in two ways, using a double-angle identity and a symmetry relation with $\cos^2 x$. Explain why both agree.

Answer: $\pi/4$. $\sin^2 x=(1-\cos 2x)/2$; equal integrals of $\sin^2$ and $\cos^2$ sum to $\pi/2$.

- Evaluate using a double-angle identity: 2
- Evaluate independently using symmetry and the Pythagorean identity: 2
- Explain agreement between the methods: 1

Q6 (4 marks). Find the exact area between $y=10e^x$ and the x-axis from x=-1 to1. Compare it with the signed integral from1 to-1 and explain the sign.

Answer: Area $-\frac{10}{e}+10e$; reversed-limits integral is its negative.

- Integrate and evaluate the positive area: 2
- Give the reversed-limits integral: 1
- Explain the sign reversal: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Areas between curves and trapezium checks.

- (a) Find where $y=\sin x$ and $y=\cos x$ meet for $0\le x\le\pi$. – 2 marks. Answer: $\frac{\pi}{4}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the area between $y=\cos x$, $y=\sin x$ and $x=0$. – 2 marks. Answer: $\sqrt{2}-1$. Award for: Correct method (1); accurate answer (1).
- (c) Find the area between $y=e^x$ and $y=x+1$ from $x=0$ to $x=1$. – 2 marks. Answer: $e-\frac{5}{2}$. Award for: Correct method (1); accurate answer (1).
- (d) Find the area between $y=x^2$ and $y=\sqrt{x}$. – 2 marks. Answer: $\frac{1}{3}$. Award for: Correct method (1); accurate answer (1).

- (e) Estimate $\int_0^2 e^x\,dx$ with the trapezium rule and 2 strips. – 3 marks. Answer: 6.91. Award for: Correct method (2); accurate answer (1).
- (f) State whether this is an over- or under-estimate and why. – 3 marks. Answer: over-estimate; the curve is convex. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). The curve C has parametric equations $x = 3t^2$, $y = t^3 - 3t$.

- (a) Show that C crosses itself at (9, 0). – 2 marks. Award for: Two values of t give the same point
- (b) Find the exact area enclosed by the loop. – 6 marks. Award for: Area integral with dx/dt, limits, sign

Answers: (a) t = ±√3 both give (9, 0). (b) $|\int (t^3 - 3t)(6t)\,dt|$ from $-\sqrt{3}$ to $\sqrt{3} = 72\sqrt{3}/5$.

Q3 (exam style) (8 marks).

- (a) Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{4x+1}\,dx$. – 3 marks. Award for: Ordinates and rule
- (b) Find the exact value of the integral. – 3 marks. Award for: Reverse chain rule
- (c) Find the percentage error and explain why the trapezium rule underestimates. – 2 marks. Award for: Error and concavity

Answers: (a) ≈ 4.307. (b) 13/3. (c) ≈ 0.61%; the curve is concave, so the chords lie below it.

Q4 (exam style) (8 marks). The ellipse E has parametric equations x = 2 cos t, y = 3 sin t.

- (a) Show that the area of the quarter of E in the first quadrant is $\int_0^{\pi/2} 6\sin^2 t\,dt$, and evaluate it. – 6 marks. Award for: Parametric area, limits, double angle
- (b) Hence state the area enclosed by E. – 1 marks. Award for: Multiply
- (c) Find a Cartesian equation of E. – 1 marks. Award for: Eliminate

Answers: (a) 3π/2. (b) 6π. (c) $x^2/4 + y^2/9 = 1$.

Week 48: Differential equations and numerical integration

Pearson P2 chapter 11.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Separating variables.

- (a) Solve $\frac{dy}{dx} = 2xy$, $y = 1$ when $x = 0$. – 2 marks. Answer: $y = e^{x^2}$. Award for: Correct method (1); accurate answer (1).
- (b) Solve $\frac{dy}{dx} = \frac{x}{y}$, $y = 2$ when $x = 0$. – 2 marks. Answer: $y^2 = x^2 + 4$. Award for: Correct method (1); accurate answer (1).
- (c) Find the general solution of $\frac{dy}{dx} = 3y$. – 2 marks. Answer: $y = Ae^{3x}$. Award for: Correct method (1); accurate answer (1).

- (d) Solve $\frac{dy}{dx} = y^2$, $y = 1$ when $x = 0$. – 2 marks. Answer: $y = \frac{1}{1-x}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Exponential growth and decay.

- (a) $\frac{dN}{dt} = 0.2N$, $N = 50$ at $t = 0$. Find N in terms of t. – 2 marks. Answer: $N = 50e^{0.2t}$. Award for: Correct method (1); accurate answer (1).
- (b) Find N when $t = 5$. – 2 marks. Answer: 135.9. Award for: Correct method (1); accurate answer (1).
- (c) Find when N doubles. – 2 marks. Answer: $5 \ln 2 = 3.47$. Award for: Correct method (1); accurate answer (1).
- (d) $\frac{dm}{dt} = -km$, m halves in 10 years. Find k. – 2 marks. Answer: $\frac{\ln 2}{10} = 0.0693$. Award for: Correct method (1); accurate answer (1).

Q3 (6 marks). Solve dy/dx=11y with y(0)=2 by separation. Find when y=10, and explain why dividing by y would require checking y=0 in the general equation.

Answer: y=2e^(11x); x=ln5/11. y=0 is another general solution but does not satisfy this initial condition.

- Separate variables and integrate: 2
- Apply the initial condition: 1
- Find when the target value is reached: 2
- Explain the possible zero solution in the general equation: 1

Q4 (7 marks). A model has dP/dt=0.2(100-P), P(0)=21. Solve and find when P first reaches90. Interpret its limiting value.

Answer: P=100-(79)e^(-0.2t); t=5ln((79)/10)≈10.334. Limit100.

- Separate variables and integrate: 2
- Apply the initial condition to obtain the model: 2
- Solve for the target time: 2
- Interpret the limiting value: 1

Q5 (5 marks). Solve $\frac{dy}{dx} = \frac{11x}{y}$ with y(0)=2. State the appropriate branch and check the result by implicit differentiation.

Answer: $y^2=11x^2+4$; positive branch y=sqrt($11x^2$+4); y never zero.

- Separate variables and integrate both sides: 2
- Apply the initial condition and choose the branch: 2
- Check by implicit differentiation: 1

Q6 (5 marks). Use the trapezium rule with four equal strips to estimate $\int_0^2 (x^2 + 11)\,dx$. Find the exact value and explain the direction of error using a sketch.

Answer: h=0.5; estimate $\frac{99}{4}$; exact $\frac{74}{3}$; overestimate by1/12 because curve convex.

- Use the correct strip width and function values: 1
- Apply the trapezium formula and calculate the estimate: 2
- Calculate the exact integral: 1
- Explain the error direction using a sketch: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Forming and solving differential equations.

- (a) The rate of cooling of a liquid is proportional to $\theta - 20$. Write a differential equation. – 2 marks. Answer: $\frac{d\theta}{dt} = -k(\theta - 20)$. Award for: Correct method (1); accurate answer (1).
- (b) Solve it with $\theta = 80$ at $t = 0$. – 2 marks. Answer: $\theta = 20 + 60e^{-kt}$. Award for: Correct method (1); accurate answer (1).
- (c) Given $\theta = 50$ at $t = 5$, find k. – 2 marks. Answer: $\frac{\ln 2}{5} = 0.139$. Award for: Correct method (1); accurate answer (1).
- (d) State the long-term temperature. – 2 marks. Answer: 20. Award for: Correct method (1); accurate answer (1).
- (e) Solve $\frac{dy}{dx} = \frac{1}{x(x+1)}$ for $x > 0$, given $y = 0$ at $x = 1$. – 3 marks. Answer: $y = \ln \frac{2x}{x+1}$. Award for: Correct method (2); accurate answer (1).
- (f) Verify that $y = e^{-x}$ satisfies $\frac{dy}{dx} + y = 0$. – 3 marks. Answer: $-e^{-x} + e^{-x} = 0$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). A cup of tea cools according to $\frac{d\theta}{dt} = -k(\theta - 20)$, where θ °C is the temperature after t minutes. Initially θ = 90, and θ = 55 when t = 10.

- (a) Solve the differential equation to show that $\theta = 20 + 70e^{-kt}$. – 4 marks. Award for: Separate, integrate, initial condition
- (b) Find k. – 2 marks. Award for: Use θ = 55
- (c) Find the time to cool to 30 °C. – 2 marks. Award for: Solve

Answers: (a) ln(θ − 20) = −kt + c. (b) k = ln 2/10 ≈ 0.0693. (c) t = 10 ln 7/ln 2 ≈ 28.1 min.

Q3 (exam style) (8 marks). $\frac{dy}{dx} = \frac{y(1+x)}{x}$, x > 0, y > 0.

- (a) Find the general solution in the form $y = Axe^x$. – 4 marks. Award for: Separate and integrate
- (b) Find the particular solution with y = 2e when x = 1. – 2 marks. Award for: A
- (c) Find y when x = 2. – 1 marks. Award for: Substitute
- (d) If the restriction y > 0 is removed, explain why y = 0 also satisfies the differential equation. – 1 marks. Award for: Both sides zero

Answers: (a) ln y = ln x + x + c. (b) y = 2xe^x. (c) $4e^2$. (d) dy/dx = 0 and the right-hand side is 0.

Q4 (exam style) (8 marks). A population, x thousands, satisfies $\frac{dx}{dt} = \frac{x(4-x)}{8}$, with x = 1 when t = 0.

- (a) Write $\frac{1}{x(4-x)}$ in partial fractions. – 1 marks. Award for: Constants
- (b) Solve the differential equation to show that $x = \frac{4}{1+3e^{-t/2}}$. – 6 marks. Award for: Separate, integrate, constant, rearrange
- (c) State the long-term population. – 1 marks. Award for: Limit

Answers: (a) ¼(1/x + 1/(4 − x)). (b) 2 ln(x/(4 − x)) = t + 2 ln(1/3), then rearrange. (c) 4 thousand.

Week 49: Numerical methods

Pearson P2 chapter 10.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (11 marks). Locating roots.

- (a) Show that $x^3 - 2x - 5 = 0$ has a root between 2 and 3. – 2 marks. Answer: $f(2) = -1 < 0$, $f(3) = 16 > 0$, continuous. Award for: Correct method (1); accurate answer (1).
- (b) Show that $e^x = 3x$ has a root between 1.5 and 1.6. – 2 marks. Answer: sign change of $e^x - 3x$: $-0.018, 0.153$. Award for: Correct method (1); accurate answer (1).
- (c) Explain why $\frac{1}{x} = 0$ shows no root despite $f(-1) < 0 < f(1)$. – 2 marks. Answer: discontinuity at $x = 0$. Award for: Correct method (1); accurate answer (1).
- (d) Show that the root of $x^3 - 2x - 5 = 0$ is 2.09 to 2 d.p. – 2 marks. Answer: $f(2.085) < 0 < f(2.095)$. Award for: Correct method (1); accurate answer (1).
- (e) Show that $x^3 - 2x - 5 = 0$ can be written $x = \sqrt[3]{2x + 5}$. – 3 marks. Answer: rearrange. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). The Newton–Raphson method.

- (a) Write down the Newton–Raphson formula. – 2 marks. Answer: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Award for: Correct method (1); accurate answer (1).
- (b) Use it once on $f(x) = x^2 - 5$ with $x_0 = 2$. – 2 marks. Answer: 2.25. Award for: Correct method (1); accurate answer (1).
- (c) Apply it again. – 2 marks. Answer: 2.2361. Award for: Correct method (1); accurate answer (1).
- (d) Use it once on $f(x) = x^3 - 2x - 5$ with $x_0 = 2$. – 2 marks. Answer: 2.1. Award for: Correct method (1); accurate answer (1).
- (e) Explain why Newton–Raphson fails if $f'(x_0) = 0$. – 3 marks. Answer: division by zero; tangent is horizontal. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). For f(x)=x^3-x-12, find consecutive nonnegative integers between which a root lies, using a sign change. Explain the role of continuity and why the argument is insufficient for 1/x across zero.

Answer: f(2)=-6, f(3)=12; root lies in (2,3). Continuity is required; 1/x is discontinuous at0, so its sign change across0 does not imply a root.

- Evaluate at suitable consecutive integers: 1
- Identify a valid bracket or an exact endpoint root: 1
- Explain the continuity requirement: 1
- Explain why the argument fails across the discontinuity of 1/x: 1

Q4 (5 marks). Apply Newton–Raphson to x^3-x-12=0 starting at x0=2. Calculate x1,x2 and state the iteration formula. Give one possible cause of failure in general.

Answer: xnext=x-(x^3-x-12)/(3x^2-1); x1=2.5455, x2=2.4398. Near-zero derivative can cause divergence/undefined step.

- State the correct Newton-Raphson iteration: 2
- Calculate both iterates: 2
- Give a valid possible cause of failure: 1

Q5 (4 marks). For x^3-x-12=0, use $x_{n+1} = (x_n + 12)^{1/3}$ with x0=2. Find three iterates and explain a local convergence condition.

Answer: Iterates: 2.4101, 2.4335, 2.4348. Locally |g'(root)|<1 supports convergence; g'=1/[3(x+12)^(2/3)].

- Calculate the three iterates: 2
- State an appropriate local convergence condition: 1
- Relate it to the derivative of this iteration: 1

Q6 (4 marks). A root is reported as 1.42 to two decimal places. Explain how a sign change on [1.415,1.425] can certify the rounded value for a continuous function with a unique root there. Explain why small successive iteration changes alone may be misleading.

Answer: The true root lies within the rounding interval (with endpoint/tie care). Must establish continuity, signs and the relevant uniqueness. Stagnation/rounding or a faulty iteration can produce small changes without root accuracy.

- Explain the rounding interval and endpoint care: 1
- Explain the need for continuity and a sign change: 1
- Respect the specified uniqueness condition: 1
- Explain why small iteration changes need not imply accuracy: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). The trapezium rule.

- (a) Estimate $\int_0^2 x^2\,dx$ using 4 strips. – 2 marks. Answer: 2.75. Award for: Correct method (1); accurate answer (1).
- (b) Find the percentage error compared with the exact value. – 2 marks. Answer: $\frac{0.0833}{2.6667} = 3.1\%$. Award for: Correct method (1); accurate answer (1).

- (c) Estimate $\int_1^3 \ln x \, dx$ using 4 strips, to 3 s.f. – 2 marks. Answer: 1.28. Award for: Correct method (1); accurate answer (1).
- (d) State whether this is an over- or under-estimate. – 2 marks. Answer: under-estimate; the curve is concave. Award for: Correct method (1); accurate answer (1).
- (e) Estimate $\int_0^1 \sqrt{1+x^2}\, dx$ using 2 strips. – 3 marks. Answer: 1.16. Award for: Correct method (2); accurate answer (1).
- (f) Explain how to improve the accuracy. – 3 marks. Answer: use more, narrower strips. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). $f(x) = e^x - 3x - 2$.

- (a) Show that f(x) = 0 has a root α between 2 and 3. – 2 marks. Award for: Sign change and continuity
- (b) Using $x_0 = 2$, apply the Newton–Raphson method twice, giving x_2 to 4 decimal places. – 3 marks. Award for: f′ and iterations
- (c) Show that α = 2.125 to 3 decimal places. – 2 marks. Award for: Sign change on [2.1245, 2.1255]
- (d) Explain why $x_0 = 1.1$ would be a poor starting value. – 1 marks. Award for: f′ near zero

Answers: (a) f(2) = −0.611, f(3) = 9.09. (b) $x_1 = 2.1392$, $x_2 = 2.1255$. (c) f(2.1245) < 0 < f(2.1255). (d) f′(x) = 0 at x = ln 3 ≈ 1.099, so the tangent is nearly horizontal.

Q3 (exam style) (9 marks).

- (a) Show that $x^3 + 2x^2 - 6 = 0$ can be written as $x = \sqrt{\frac{6}{x+2}}$. – 2 marks. Award for: Rearrange
- (b) Using $x_{n+1} = \sqrt{\frac{6}{x_n+2}}$ with $x_0 = 1.5$, find x_1, x_2 and x_3 to 4 decimal places. – 3 marks. Award for: Iterations
- (c) Show that the root is 1.340 to 3 decimal places. – 2 marks. Award for: Sign change
- (d) Describe the convergence shown on a cobweb diagram. – 2 marks. Award for: Oscillating convergence

Answers: (a) $x^2(x + 2) = 6$. (b) 1.3093, 1.3465, 1.3390. (c) f(1.3395) < 0 < f(1.3405). (d) A cobweb converging to the root, alternating sides.

Q4 (exam style) (8 marks). $h(t) = 10te^{-0.5t} - 2$.

- (a) Show that h(t) = 0 has a root between 0.2 and 0.3. – 2 marks. Award for: Sign change
- (b) Apply the Newton–Raphson method once with $t_0 = 0.2$. – 3 marks. Award for: Derivative and iteration
- (c) Explain why $t_0 = 2$ cannot be used. – 1 marks. Award for: h′(2) = 0
- (d) Find the maximum value of h. – 2 marks. Award for: Stationary point

Answers: (a) h(0.2) = −0.190, h(0.3) = 0.582. (b) h′(t) = 10e^(−0.5t)(1 − 0.5t); $t_1 \approx 0.2234$. (c) Division by zero since h′(2) = 0. (d) 20e^(−1) − 2 ≈ 5.36.

Week 50: Three-dimensional vectors

Pearson P2 chapter 12.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (11 marks). Vectors in three dimensions. $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

- (a) Find $\mathbf{a} + 2\mathbf{b}$. – 2 marks. Answer: $4\mathbf{i} + 7\mathbf{j} - \mathbf{k}$. Award for: Correct method (1); accurate answer (1).
- (b) Find $|\mathbf{a}|$. – 2 marks. Answer: $\sqrt{14}$. Award for: Correct method (1); accurate answer (1).
- (c) Find a unit vector parallel to $\mathbf{b}$. – 2 marks. Answer: $\frac{1}{\sqrt{21}}(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$. Award for: Correct method (1); accurate answer (1).
- (d) Find $|\mathbf{a} - \mathbf{b}|$. – 2 marks. Answer: $\sqrt{51}$. Award for: Correct method (1); accurate answer (1).
- (e) Find the angle between $\mathbf{a}$ and the positive x-axis. – 3 marks. Answer: $\cos^{-1}\frac{2}{\sqrt{14}} = 57.7^\circ$. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Points and geometry in 3D. $A(1, 2, -1)$, $B(3, -2, 4)$, $C(5, 0, 1)$.

- (a) Find $\overrightarrow{AB}$. – 2 marks. Answer: $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the distance AB. – 2 marks. Answer: $\sqrt{45} = 3\sqrt{5}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the midpoint of AC. – 2 marks. Answer: $(3, 1, 0)$. Award for: Correct method (1); accurate answer (1).
- (d) Find D so that $ABCD$ is a parallelogram. – 2 marks. Answer: $(3, 4, -4)$. Award for: Correct method (1); accurate answer (1).
- (e) Find $|\overrightarrow{BC}|$. – 3 marks. Answer: $\sqrt{4 + 4 + 9} = \sqrt{17}$. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). A=(1,2,6), B=(3,5,12). Find AB, its length and a unit vector. Find C on AB extended such that AC=3AB.

Answer: AB=(2,3,6), length7, unit=(2/7,3/7,6/7); C=(7,11,24).

- Find AB: 1
- Find its length and unit vector: 2
- Find the extended point C: 1

Q4 (4 marks). A=(0,1,6), B=(2,3,8), C=(5,6,11). Prove collinearity. Find ratio AB:BC and the midpoint of AC.

Answer: AB=(2,2,2), BC=(3,3,3); ratio2:3; midpoint=(5/2,7/2,$\frac{17}{2}$).

- Find appropriate difference vectors and prove collinearity: 2

- Give the segment ratio: 1
- Find the midpoint: 1

Q5 (4 marks). A point P has vector $(1+2t,\ 3-t,\ 6+2t)$. Find its distance from Q=(1,3,6) in terms of t, and all t for which the distance is 9.

Answer: PQ=3|t|; t=±3. Both directions along the line are possible.

- Obtain the displacement vector: 1
- Express distance using an absolute value: 1
- Find both parameter values: 1
- Explain why both directions are possible: 1

Q6 (5 marks). Let O=(0,0,0), A=(2,0,0), B=(0,3,6). Find position vectors of the midpoints of OA and OB. Show their joining vector is half of AB, and explain the resulting parallelism.

Answer: Midpoints (1,0,0),(0,3/2,3); joining=(-1,3/2,3)=AB/2.

- Find both midpoint position vectors: 2
- Show that their difference is half of AB: 2
- Conclude parallelism: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Using vectors to solve problems.

- (a) Find the distance of $(2, -3, 6)$ from the origin. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (b) Find the vector of magnitude 14 in the direction of $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. – 2 marks. Answer: $4\mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$. Award for: Correct method (1); accurate answer (1).
- (c) P divides AB in the ratio $1:2$, $A(0,3,6)$, $B(6,0,3)$. Find P. – 2 marks. Answer: $(2,2,5)$. Award for: Correct method (1); accurate answer (1).
- (d) A force $(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N acts on a 3 kg mass. Find the acceleration magnitude. – 2 marks. Answer: 1 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (e) Find k if $|k\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}| = 3$. – 3 marks. Answer: $k = \pm 1$. Award for: Correct method (2); accurate answer (1).
- (f) Find the angle between $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the z-axis. – 3 marks. Answer: $\cos^{-1}\frac{1}{3} = 70.5°$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). The points A(2, −1, 4), B(5, 3, −8) and C(−1, 2, 6) are given.

- (a) Find $|\overrightarrow{AB}|$. – 2 marks. Award for: Vector and magnitude
- (b) Find the angle between $\overrightarrow{AB}$ and the positive x-axis. – 2 marks. Award for: Direction cosine
- (c) Find angle BAC. – 4 marks. Award for: Three lengths and cosine rule

- (d) Find the area of triangle ABC. – 2 marks. Award for: ½ab sin C

Answers: (a) 13. (b) 76.7°. (c) ≈ 110.1°. (d) ≈ 28.6.

Q3 (exam style) (8 marks). The points A(1, 2, 3), B(4, −2, 3) and C(4, 2, −1) are given.

- (a) Show that triangle ABC is isosceles. – 3 marks. Award for: Three lengths
- (b) Find D such that ABCD is a parallelogram. – 2 marks. Award for: D = A + C − B
- (c) Find the area of ABCD. – 3 marks. Award for: Height and area

Answers: (a) AB = AC = 5, BC = √32. (b) D(1, 6, −1). (c) Triangle area √136, parallelogram 2√136 ≈ 23.3.

Q4 (exam style) (8 marks). A cube has vertices O(0, 0, 0), A(4, 0, 0), C(0, 4, 0), D(0, 0, 4) and F(4, 4, 4).

- (a) Find |OF| and the angle between OF and the x-axis. – 3 marks. Award for: Magnitude and angle
- (b) P, Q and R are the midpoints of OA, OC and OD. Show that triangle PQR is equilateral and find its exact area. – 3 marks. Award for: Three lengths and area
- (c) Show that PQ is parallel to AC and half its length. – 2 marks. Award for: Scalar multiple

Answers: (a) 4√3, 54.7°. (b) Sides 2√2, area 2√3. (c) PQ = −2i + 2j = ½(−4i + 4j).

Week 51: Further kinematics

Pearson S2 chapter 8.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Vectors in kinematics. A particle has velocity $\mathbf{v} = (3t\mathbf{i} + (4 - t^2)\mathbf{j})$ m s^{-1}.

- (a) Find the acceleration in terms of t. – 2 marks. Answer: $3\mathbf{i} - 2t\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the speed when $t = 1$. – 2 marks. Answer: $\sqrt{9 + 9} = 4.24$ m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (c) Find when the particle moves parallel to $\mathbf{i}$. – 2 marks. Answer: $t = 2$. Award for: Correct method (1); accurate answer (1).
- (d) Find the position at $t = 3$ if it starts at the origin. – 2 marks. Answer: $13.5\mathbf{i} + 3\mathbf{j}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Constant acceleration vectors.

- (a) $\mathbf{u} = (2\mathbf{i} + 3\mathbf{j})$, $\mathbf{a} = (\mathbf{i} - \mathbf{j})$. Find $\mathbf{v}$ at $t = 4$. – 2 marks. Answer: $6\mathbf{i} - \mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the displacement after 4 s. – 2 marks. Answer: $16\mathbf{i} + 4\mathbf{j}$. Award for: Correct method (1); accurate answer (1).

- (c) Find when the particle is moving parallel to $\mathbf{i}$. – 2 marks. Answer: 3 s. Award for: Correct method (1); accurate answer (1).
- (d) A particle starts at $(1, 2)$ with $\mathbf{v} = (4\mathbf{i} - 2\mathbf{j})$ m s^{-1}. Find its position after 3 s. – 2 marks. Answer: $13\mathbf{i} - 4\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (e) Find the velocity required to move from $(0, 0)$ to $(12, -9)$ in 3 s. – 3 marks. Answer: $4\mathbf{i} - 3\mathbf{j}$. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). A particle has position $\mathbf{r} = (t^2 + 1)\mathbf{i} + (7t - t^2)\mathbf{j}$. Find velocity, acceleration, speed at t=1 and when motion is parallel to the i-axis.

Answer: v=(2t,7-2t), acc=(2,-2); speed at1=sqrt(4+(7-2)^2); j velocity zero at t=$\frac{7}{2}$.

- Differentiate to obtain velocity: 1
- Differentiate to obtain acceleration: 1
- Find speed at the specified time: 1
- Find the time the velocity is parallel to the i-axis: 1

Q4 (6 marks). Acceleration is $\mathbf{a} = 2\mathbf{i} + 7t\mathbf{j}$. At t=0, velocity=(1,-2), position=(3,4). Find velocity and position functions, applying both vector constants.

Answer: v=(1+2t,-2+7t^2/2); r=(3+t+t^2,4-2t+7t^3/6).

- Integrate both acceleration components: 2
- Apply the initial velocity: 1
- Integrate both velocity components: 2
- Apply the initial position: 1

Q5 (5 marks). A particle has velocity $\mathbf{v} = (7 - t)\mathbf{i} + 2\mathbf{j}$. Find the time of minimum speed for t≥0 and the minimum speed, explaining why minimising speed squared is convenient.

Answer: Speed^2=(7-t)^2+4, minimum at t=7, speed2. Square root is increasing on nonnegative inputs.

- Form speed squared: 1
- Find and justify the minimum time: 2
- Find the minimum speed: 1
- Explain why speed squared has the same minimiser: 1

Q6 (5 marks). A particle moves with r=(2t+1,t^2-7). Find all t≥0 when |r|=sqrt(50), or show how to reduce this to an equation and isolate the physical roots.

Answer: (2t+1)^2+(t^2-7)^2=50 ⇒ t[t^3+(4-14)t+4]=0. t=0 and any nonnegative real roots of the cubic; numerical values 0, 0.40673, 2.9392.

- Form the squared-distance equation: 1
- Expand and reduce it correctly: 2

- Retain t=0 and isolate the nonnegative real cubic roots or specify a valid isolation procedure: 2

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Variable acceleration in one dimension.

- (a) $a = 2t - 3$, $v = 4$ at $t = 0$. Find v. – 2 marks. Answer: $t^2 - 3t + 4$. Award for: Correct method (1); accurate answer (1).
- (b) Show the particle is never at rest. – 2 marks. Answer: discriminant $9 - 16 < 0$. Award for: Correct method (1); accurate answer (1).
- (c) $v = e^{-t}$. Find the displacement from $t = 0$ to $t = 2$. – 2 marks. Answer: $1 - e^{-2} = 0.865$ m. Award for: Correct method (1); accurate answer (1).
- (d) $s = \sin 2t$. Find the maximum speed. – 2 marks. Answer: 2 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (e) $v = \frac{1}{t+1}$. Find the displacement from 0 to 3 s. – 3 marks. Answer: $\ln 4 = 1.39$ m. Award for: Correct method (2); accurate answer (1).
- (f) State the terminal velocity for $v = 10(1 - e^{-0.5t})$. – 3 marks. Answer: 10 m s^{-1}. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). A particle P moves in a plane with velocity $\mathbf{v} = (6 - 2t)\mathbf{i} + (t^2 - 4)\mathbf{j}$ m/s, t ≥ 0. P starts at the origin.

- (a) Find when P is moving parallel to j. – 2 marks. Award for: i-component zero
- (b) Find the magnitude of the acceleration when t = 1. – 2 marks. Award for: Differentiate and magnitude
- (c) Find the position vector of P when t = 3. – 3 marks. Award for: Integrate with initial condition
- (d) Find the speed when t = 3. – 1 marks. Award for: Magnitude

Answers: (a) t = 3. (b) a = −2i + 2tj, 2√2 m/s^2. (c) 9i − 3j. (d) 5 m/s.

Q3 (exam style) (8 marks). Unit vectors $\mathbf{i}$ and $\mathbf{j}$ point east and north respectively. Ship A starts at the origin with constant velocity $(4\mathbf{i} + 3\mathbf{j})$ km/h. Ship B starts at $(10\mathbf{i} + 2\mathbf{j})$ km with constant velocity $(2\mathbf{i} + 4\mathbf{j})$ km/h.

- (a) Find the position vectors of A and B after t hours. – 2 marks. Award for: Both
- (b) Find the time of closest approach and the least distance. – 5 marks. Award for: Separation, square, minimise
- (c) Find the bearing of B from A at that time. – 1 marks. Award for: Angle

Answers: (a) A: 4ti + 3tj; B: (10 + 2t)i + (2 + 4t)j. (b) |AB|^2 = 5t^2 − 36t + 104: t = 3.6 h, distance √39.2 ≈ 6.26 km. (c) 026.6°.

Q4 (exam style) (8 marks). A particle has acceleration (6ti − 4j) m/s^2. At t = 0, its velocity is (−3i + 8j) m/s and its position vector is 2i m.

- (a) Find the velocity at time t. – 2 marks. Award for: Integrate
- (b) Find the speed when t = 2. – 2 marks. Award for: Magnitude
- (c) Find the position vector at time t. – 2 marks. Award for: Integrate
- (d) Find when the particle is due east of the origin for t > 0, and its distance from O. – 2 marks. Award for: y = 0

Answers: (a) $(3t^2 - 3)\mathbf{i} + (8 - 4t)\mathbf{j}$. (b) 9 m/s. (c) $(t^3 - 3t + 2)\mathbf{i} + (8t - 2t^2)\mathbf{j}$. (d) t = 4, 54 m.

Week 52: Moments and rigid-body equilibrium

Pearson S2 chapter 4.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (11 marks). Moments. Take $g = 9.8$ m s^{-2}.

- (a) A force of 20 N has a perpendicular distance of 0.5 m from a pivot. Find its moment. – 2 marks. Answer: 10 N m. Award for: Correct method (1); accurate answer (1).
- (b) Downward forces of 30 N and 40 N act on a horizontal light rod at distances 2 m and 1.5 m on the same side of a pivot. Find the total moment. – 2 marks. Answer: 120 N m. Award for: Correct method (1); accurate answer (1).
- (c) A uniform 4 m plank of mass 10 kg rests on a support at its centre with a 5 kg mass 1.5 m from the centre. Find the distance from the centre for a 6 kg mass to balance it. – 2 marks. Answer: 1.25 m. Award for: Correct method (1); accurate answer (1).
- (d) A uniform 6 m beam of weight 200 N rests on supports at its ends. A 300 N load is 2 m from the left end. Find the reaction at the left support. – 2 marks. Answer: 300 N. Award for: Correct method (1); accurate answer (1).
- (e) Find the reaction at the right support. – 3 marks. Answer: 200 N. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Tilting and non-uniform rods.

- (a) A uniform 5 m rod of weight 80 N rests on supports at 1 m and 4 m from end A. Find the reactions. – 2 marks. Answer: 40 N each. Award for: Correct method (1); accurate answer (1).
- (b) A 60 N load is placed at A. Find the new reactions. – 2 marks. Answer: $R_1 = 120$ N, $R_2 = 20$ N. Award for: Correct method (1); accurate answer (1).
- (c) Find the largest load at A before the rod tilts. – 2 marks. Answer: 120 N. Award for: Correct method (1); accurate answer (1).
- (d) A non-uniform rod of length 3 m, weight 50 N, balances at 1.2 m from one end. Where is its centre of mass? – 2 marks. Answer: 1.2 m from that end. Award for: Correct method (1); accurate answer (1).
- (e) A uniform ladder of length 4 m and weight 120 N is held horizontally at its ends. Find the force at each end. – 3 marks. Answer: 60 N. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). A uniform horizontal rod length6 m and mass 8 kg is supported at both ends. A 16 kg load is placed2 m from the left. Find both reactions using moments and vertical equilibrium.

Answer: Right R=91.467 N; left R=143.73 N. Total weight=235.2 N.

- Place the rod weight and load correctly: 1
- Form a valid moment equation: 1
- Find one reaction: 1
- Use vertical equilibrium to find the other reaction: 1

Q4 (5 marks). A light beam length4 m is supported at x=0 and x=3. A 8 kg load acts at x=4 and an unknown mass m at x=1. Find the minimum m for both reactions to be nonnegative; interpret the boundary.

Answer: Moments about x=3 give2mg≥8g, so m≥4. At equality left reaction0, on point of tipping.

- Form a suitable moment or reaction equation: 2
- Apply the nonnegative-reaction condition: 1
- Find the minimum mass: 1
- Interpret the zero-reaction tipping boundary: 1

Q5 (3 marks). A horizontal force 80 N acts perpendicular to a spanner at distance0.25 m from its pivot. Find its moment. Find the force needed for the same moment at0.4 m, then discuss an oblique force.

Answer: Moment 20 N m; new perpendicular force 50 N. Oblique moment Fd $\sin\theta$, using perpendicular distance.

- Find the first moment: 1
- Find the new perpendicular force: 1
- Explain the oblique-force adjustment using perpendicular distance: 1

Q6 (5 marks). A uniform rod length5 m and weight 160 N rests horizontally on supports1 m and4 m from its left end. An additional 80 N force acts down at the left end. Find reactions and explain why the rod's weight acts at2.5 m.

Answer: Moments about x=1: right R×3=(160)(1.5)-80; right=$\frac{160}{3}$ N; left=$\frac{560}{3}$ N. Uniform mass distribution gives midpoint centre of mass.

- Place the rod weight at its midpoint: 1
- Form and solve a suitable moment equation: 2
- Use vertical equilibrium for the other reaction: 1
- Explain the midpoint centre of mass: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Equilibrium of rigid bodies.

- (a) A uniform 2 m rod of weight 40 N is hinged at one end and held horizontal by a vertical string at the other. Find the tension. – 2 marks. Answer: 20 N. Award for: Correct method (1); accurate answer (1).
- (b) Find the hinge reaction. – 2 marks. Answer: 20 N. Award for: Correct method (1); accurate answer (1).
- (c) The string is moved to 1.5 m from the hinge. Find the new tension. – 2 marks. Answer: 26.7 N. Award for: Correct method (1); accurate answer (1).
- (d) A rod is in limiting equilibrium. State the two conditions for equilibrium. – 2 marks. Answer: resultant force zero; resultant moment zero. Award for: Correct method (1); accurate answer (1).
- (e) A see-saw has children of 30 kg and 40 kg. The 40 kg child sits 1.5 m from the pivot. Find where the 30 kg child sits. – 3 marks. Answer: 2 m from the pivot. Award for: Correct method (2); accurate answer (1).
- (f) State a modelling assumption about the children. – 3 marks. Answer: particles. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). A uniform plank AB, 6 m long and of mass 40 kg, rests horizontally on supports at C and D, where AC = 1 m and DB = 1.5 m. A child of mass 30 kg stands on the plank x m from A.

- (a) For x = 2, find the reactions at C and D. – 4 marks. Award for: Moments and resolving
- (b) Determine whether the plank can tilt for any position of the child on the plank. – 4 marks. Award for: Tilting conditions at both supports
- (c) State one modelling assumption. – 1 marks. Award for: Valid assumption

Answers: (a) $R_D = 110g/3.5 \approx 308$ N, $R_C \approx 378$ N. (b) Tilting about C needs $x < -5/3$; about D needs $x > 6.5$: neither is on the plank, so it never tilts. (c) e.g. child as a particle; plank rigid.

Q3 (exam style) (8 marks). A non-uniform rod AB, 4 m long and of mass 12 kg, hangs horizontally from vertical strings at A and B. The tension at A is twice the tension at B.

- (a) Find the distance of the centre of mass from A. – 4 marks. Award for: Tensions and moments
- (b) A particle of mass m kg is attached at B so that the tensions become equal. Find m. – 3 marks. Award for: New equations
- (c) Explain what 'non-uniform' means for the model. – 1 marks. Award for: Centre of mass not at midpoint

Answers: (a) $T_B = 4g$, $T_A = 8g$; 4/3 m from A. (b) m = 4. (c) The mass is not evenly distributed, so the centre of mass need not be at the midpoint.

Q4 (exam style) (9 marks). A uniform rod AB of length 2a and weight W is hinged at A to a vertical wall and held horizontal by a light string from B to a point C on the wall above A. Angle

ABC = 30°.

- (a) Find the tension in the string. – 3 marks. Award for: Moments about A
- (b) Find the magnitude and direction of the force at the hinge. – 4 marks. Award for: Resolve both directions
- (c) The string breaks if its tension exceeds 2W. Find the greatest weight that can be hung at B. – 2 marks. Award for: Moments with extra load

Answers: (a) T = W. (b) Horizontal W√3/2, vertical W/2: magnitude W at 30° above the horizontal. (c) W/2.

Week 53: Resolving forces and friction

Pearson S2 chapter 5.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Resolving forces. Take $g = 9.8$ m s^{-2}.

- (a) Resolve a 20 N force at 30° above the horizontal into components. – 2 marks. Answer: $10\sqrt{3} = 17.3$ N, 10 N. Award for: Correct method (1); accurate answer (1).
- (b) Find the resultant of 5 N east and 12 N north. – 2 marks. Answer: 13 N at 67.4° north of east. Award for: Correct method (1); accurate answer (1).
- (c) A 2 kg particle is placed on a smooth plane inclined at 30°. Find the component of weight down the plane. – 2 marks. Answer: 9.8 N. Award for: Correct method (1); accurate answer (1).
- (d) Find the normal reaction for that particle. – 2 marks. Answer: 17.0 N. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Friction on horizontal surfaces ($g = 9.8$).

- (a) State the friction law $F \leq \mu R$ and when equality holds. – 2 marks. Answer: when sliding or in limiting equilibrium. Award for: Correct method (1); accurate answer (1).
- (b) A 5 kg box on rough ground, $\mu = 0.3$. Find the maximum friction. – 2 marks. Answer: 14.7 N. Award for: Correct method (1); accurate answer (1).
- (c) A horizontal force of 20 N is applied. Find the acceleration. – 2 marks. Answer: 1.06 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) A 10 kg box is pulled by 50 N at 30° above horizontal, $\mu = 0.4$. Find the normal reaction. – 2 marks. Answer: 73 N. Award for: Correct method (1); accurate answer (1).
- (e) Find the acceleration of that box. – 3 marks. Answer: 1.41 m s^{-2}. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). A 9 kg block lies on a rough horizontal surface, coefficient μ=0.3. Find R and limiting friction. A horizontal force 9 N is applied: decide if it moves and find actual friction.

Answer: R=88.2 N; limiting friction=26.46 N. Applied force 9 N is below that limit, so actual friction=9 N, not μR.

- Find the normal reaction: 1
- Find the limiting friction: 1
- Compare the applied force with that limit: 1
- State whether it moves and give the actual friction: 1

Q4 (6 marks). A 9 kg particle is on a rough slope at30°, μ=0.4. Determine whether it slides from rest, and if so find acceleration. Draw all forces and justify friction direction.

Answer: Downhill component=44.1 N; maximum friction=30.553 N. It slides down, with acceleration g(1/2-.4cos30°)≈1.5052 m/s^2.

- Draw the forces and justify the friction direction: 2
- Resolve perpendicular to the slope: 1
- Compare downhill force with limiting friction: 1
- Use Newton's second law to find acceleration: 2

Q5 (6 marks). A 9 kg block is pulled along a horizontal surface by tension 54 N at30° above horizontal. With μ=0.2, find reaction, friction during sliding and acceleration; check contact is maintained.

Answer: R=9(g-3)=61.2 N>0; friction=.2R; acceleration=6cos30°-.2(g-3)≈3.8362 m/s^2.

- Resolve vertically to find reaction: 2
- Find the sliding friction: 1
- Resolve horizontally and calculate acceleration: 2
- Check continued contact: 1

Q6 (4 marks). A block is in limiting equilibrium on a rough slope at angle θ with no other applied force. Derive μ=tanθ. Explain why this equality does not describe every block that is merely at rest on the slope.

Answer: R=mgcosθ, mg sinθ=μR ⇒ μ=tanθ. For non-limiting static equilibrium only tanθ≤μ is required.

- Resolve perpendicular and parallel to the slope: 2
- Use limiting friction to derive the equality: 1
- Explain why ordinary static equilibrium gives only an inequality: 1

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (14 marks). Friction on inclined planes ($g = 9.8$).

- (a) A 3 kg block is on the point of slipping on a plane at 25°. Find μ. – 2 marks. Answer: $\tan 25^\circ = 0.466$. Award for: Correct method (1); accurate answer (1).
- (b) A 4 kg block on a plane at 30°, $\mu = 0.2$, slides down. Find the acceleration. – 2 marks. Answer: 3.20 m s^{-2}. Award for: Correct method (1); accurate answer (1).

- (c) Find its speed after sliding 5 m from rest. – 2 marks. Answer: 5.66 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (d) Find the force parallel to the plane needed to push it up at constant speed. – 2 marks. Answer: 26.4 N. Award for: Correct method (1); accurate answer (1).
- (e) A block is projected up a plane at 20°, $\mu = 0.3$, at 8 m s^{-1}. Find the deceleration. – 3 marks. Answer: 6.11 m s^{-2}. Award for: Correct method (2); accurate answer (1).
- (f) Find how far it travels up the plane. – 3 marks. Answer: 5.24 m. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). A box of mass 5 kg is placed on a rough plane inclined at α, where tan α = 3/4. The coefficient of friction is 0.5.

- (a) Show that the box slides down the plane when released. – 3 marks. Award for: Compare component of weight with maximum friction
- (b) Find its acceleration. – 2 marks. Award for: Newton's second law
- (c) A horizontal force P N holds the box in limiting equilibrium, about to slide up the plane. Find P. – 5 marks. Award for: Resolve both directions with friction down the slope

Answers: (a) 3g > 2g. (b) g/5 = 1.96 m/s^2. (c) R = 4g + 0.6P; 0.8P = 3g + 0.5R gives P = 10g = 98 N.

Q3 (exam style) (9 marks). A particle of mass 2 kg is pulled up a rough plane inclined at 30° by a rope parallel to the plane with tension 20 N. The coefficient of friction is 0.25.

- (a) Find the acceleration. – 4 marks. Award for: Normal reaction, friction, equation
- (b) Starting from rest, the rope breaks after 3 s. Find the further distance travelled up the plane. – 4 marks. Award for: Speed, deceleration, distance
- (c) Determine whether the particle then slides back down. – 1 marks. Award for: Compare tan 30° with μ

Answers: (a) ≈ 2.98 m/s^2. (b) v ≈ 8.93 m/s; deceleration ≈ 7.02 m/s^2; ≈ 5.68 m. (c) tan 30° = 0.577 > 0.25, so it slides back.

Q4 (exam style) (8 marks). Block A of mass 3 kg lies on a rough horizontal table (μ = 0.4). It is connected by a light string over a smooth pulley to particle B of mass 2 kg hanging freely.

- (a) Show that the system moves when released. – 2 marks. Award for: Compare 2g with maximum friction
- (b) Find the acceleration and the tension. – 4 marks. Award for: Equations for A and B
- (c) Find the least extra mass placed on A for the system to remain at rest. – 2 marks. Award for: Inequality

Answers: (a) 2g > 1.2g. (b) a = 0.16g ≈ 1.57 m/s^2, T ≈ 16.5 N. (c) 2 kg.

Week 54: Projectiles

Pearson S2 chapter 6.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Projectiles from level ground. Speed 20 m s^{-1} at 30° above horizontal; $g = 9.8$.

- (a) Find the horizontal and vertical components of the initial velocity. – 2 marks. Answer: 17.3, 10 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (b) Find the time to the highest point. – 2 marks. Answer: 1.02 s. Award for: Correct method (1); accurate answer (1).
- (c) Find the greatest height. – 2 marks. Answer: 5.10 m. Award for: Correct method (1); accurate answer (1).
- (d) Find the time of flight. – 2 marks. Answer: 2.04 s. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Horizontal projection ($g = 9.8$).

- (a) A ball is thrown horizontally at 12 m s^{-1} from a 20 m cliff. Find the time to land. – 2 marks. Answer: 2.02 s. Award for: Correct method (1); accurate answer (1).
- (b) Find the horizontal distance travelled. – 2 marks. Answer: 24.2 m. Award for: Correct method (1); accurate answer (1).
- (c) Find the vertical speed on landing. – 2 marks. Answer: 19.8 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (d) Find the speed and direction on landing. – 2 marks. Answer: 23.2 m s^{-1}, 58.8° below horizontal. Award for: Correct method (1); accurate answer (1).
- (e) Find the initial horizontal speed needed to land 30 m from the base of the cliff. – 3 marks. Answer: 14.8 m s^{-1}. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). A projectile is launched from level ground at speed 20 m/s and angle30°. Neglecting air resistance, find flight time, maximum height and range; state units and assumptions.

Answer: Time 2.0408 s; height 5.102 m; range 35.348 m. Constant g, returns to launch height.

- Resolve the launch velocity: 1
- Find flight time: 2
- Find maximum height: 1
- Find range: 1
- State correct units and relevant assumptions: 1

Q4 (5 marks). A particle is launched horizontally at 20 m/s from height20 m. Find impact time, horizontal distance and impact speed.

Answer: Time=2.0203 s; range=40.406 m; speed=28.142 m/s.

- Use vertical motion to find impact time: 2
- Find horizontal distance: 1
- Find vertical impact velocity: 1
- Combine components to find impact speed: 1

Q5 (5 marks). A projectile has initial velocity (12,18) m/s from the origin. Eliminate t to find its trajectory, and find its height when x=12 m. State the flight-time restriction.

Answer: $y=((18)/12)x-(g/288)x^2$; at12 y=13.1 m; $0\le t\le 3.6735$ for return to level ground.

- Form horizontal and vertical equations: 2
- Eliminate time for the trajectory: 1
- Find the requested height: 1
- State the flight-time restriction: 1

Q6 (5 marks). For fixed launch speed 20 from level ground, derive range $R=u^2\sin(2\theta)/g$ and show its maximum. Explain why this conclusion may fail for an elevated launch or air resistance.

Answer: tflight=2u sinθ/g; multiply by u cosθ. Max range=40.816 m at45° for this model only.

- Derive flight time from vertical motion: 2
- Combine with horizontal speed to derive the range formula: 1
- Find and justify the maximum: 1
- Explain the limitations for elevated launch or air resistance: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Projectile problems ($g = 9.8$).

- (a) Show that the range on level ground is $\frac{u^2 \sin 2\alpha}{g}$. – 2 marks. Answer: $T = \frac{2u \sin \alpha}{g}$, $R = u \cos \alpha \, T$. Award for: Correct method (1); accurate answer (1).
- (b) Find the angle giving maximum range. – 2 marks. Answer: 45°. Award for: Correct method (1); accurate answer (1).
- (c) Find the maximum range for $u = 14$. – 2 marks. Answer: 20 m. Award for: Correct method (1); accurate answer (1).
- (d) A ball is kicked at 15 m s^{-1} at 40°. Find whether it clears a 2 m wall 18 m away. – 2 marks. Answer: height 3.08 m at the wall: yes. Award for: Correct method (1); accurate answer (1).
- (e) Find the two angles for range 20 m with $u = 16$. – 3 marks. Answer: 25.0° and 65.0°. Award for: Correct method (2); accurate answer (1).
- (f) Write the trajectory equation for $u = 10$, $\alpha = 45^\circ$. – 3 marks. Answer: $y = x - 0.098x^2$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). A ball is projected from O on horizontal ground at 24.5 m/s at an angle θ above the horizontal, where tan θ = 3/4.

- (a) Find the greatest height. – 2 marks. Award for: Vertical component
- (b) Find the time of flight and the range. – 3 marks. Award for: Both
- (c) A wall 4 m high stands 50 m from O. Determine whether the ball clears the wall. – 3 marks. Award for: Time to wall and height

Answers: (a) 11.0 m. (b) 3 s, 58.8 m. (c) At t ≈ 2.55 s, height ≈ 5.6 m > 4 m: clears.

Q3 (exam style) (9 marks). A stone is thrown from the top of a cliff 40 m above the sea at 15 m/s at 30° above the horizontal.

- (a) Find the time taken to reach the sea. – 4 marks. Award for: Vertical equation and quadratic
- (b) Find the horizontal distance travelled. – 2 marks. Award for: Horizontal motion
- (c) Find the speed and direction of motion at impact. – 3 marks. Award for: Both components

Answers: (a) ≈ 3.72 s. (b) ≈ 48.4 m. (c) ≈ 31.8 m/s at 65.9° below the horizontal.

Q4 (exam style) (8 marks). A particle is projected from O with speed u at angle θ above the horizontal.

- (a) Show that its trajectory is $y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$. – 4 marks. Award for: Eliminate t and use sec²
- (b) With u = 14 m/s, the particle passes through (10, 5). Find the possible values of θ. – 4 marks. Award for: Quadratic in tan θ

Answers: (a) x = ut cos θ, eliminate t. (b) $\tan^2\theta - 4\tan\theta + 3 = 0$: θ = 45° or 71.6°.

Week 55: Applications of forces

Pearson S2 chapter 7.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Statics problems ($g = 9.8$).

- (a) A 5 kg mass hangs from two strings at 30° and 60° to the horizontal. Find the tension in each. – 2 marks. Answer: 24.5 N and 42.4 N. Award for: Correct method (1); accurate answer (1).
- (b) A 2 kg particle on a smooth slope at 20° is held by a horizontal force P. Find P. – 2 marks. Answer: 7.13 N. Award for: Correct method (1); accurate answer (1).
- (c) Find the normal reaction for that particle. – 2 marks. Answer: 20.9 N. Award for: Correct method (1); accurate answer (1).
- (d) Forces $(3\mathbf{i}+4\mathbf{j})$, $(-5\mathbf{i}+2\mathbf{j})$ and $\mathbf{F}$ N are in equilibrium. Find $\mathbf{F}$. – 2 marks. Answer: $2\mathbf{i}-6\mathbf{j}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Dynamics with forces at angles ($g = 9.8$).

- (a) A 4 kg box is pulled by 30 N at 25° above horizontal on smooth ground. Find the acceleration. – 2 marks. Answer: 6.80 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (b) Find the normal reaction. – 2 marks. Answer: 26.5 N. Award for: Correct method (1); accurate answer (1).
- (c) A 1500 kg car climbs a slope at 5° with resistance 600 N and driving force 3000 N. Find the acceleration. – 2 marks. Answer: 0.746 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) Find the driving force needed for constant speed. – 2 marks. Answer: 1880 N. Award for: Correct method (1); accurate answer (1).
- (e) A 50 kg skier descends a 15° slope with $\mu = 0.1$. Find the acceleration. – 3 marks. Answer: 1.59 m s^{-2}. Award for: Correct method (2); accurate answer (1).

Q3 (7 marks). A uniform ladder length5 m, weight 220 N, rests at60° to rough horizontal ground against a smooth vertical wall. Find reactions and the minimum coefficient of ground friction.

Answer: Ground normal=W; wall reaction=W cos60°/(2sin60°)=W/(2sqrt3); friction equals wall reaction; μmin=1/(2sqrt3)≈0.28868.

- Identify the reaction and friction forces: 2
- Use moments to find the wall reaction: 2
- Use horizontal and vertical equilibrium for the ground forces: 2
- Find the minimum friction coefficient: 1

Q4 (5 marks). A 11 kg mass lies on a rough table (μ=0.2), connected to a hanging 13 kg mass over a smooth pulley. Find acceleration and tension when the hanging mass descends.

Answer: a=g[(13)-.2(11)]/(24)=4.41; T=11(a+.2g)=70.07 N.

- Find reaction and friction for the table mass: 1
- Form the two equations of motion: 2
- Solve for acceleration: 1
- Obtain the tension: 1

Q5 (4 marks). A uniform horizontal rod length4 m and mass 11 kg is hinged at its left end. A vertical string supports its right end; a load of mass 12 kg hangs at x=3. Find string tension and vertical hinge reaction.

Answer: 4T=22g+36g; T=142.1 N; hinge=83.3 N upward.

- Form and solve the moment equation for tension: 2
- Use vertical equilibrium to find the hinge reaction: 2

Q6 (5 marks). A dynamics solution assumes a light inextensible string, a smooth pulley and no air resistance. Explain one mathematical consequence of each, and describe how one assumption could be evaluated against observed motion.

Answer: Uniform/equal tension under ideal string/pulley assumptions; equal linked acceleration magnitudes while taut; no drag term. Compare measured acceleration/trajectory with prediction and discuss systematic deviations; discrepancies need not identify a unique cause.

- Explain the consequences of the ideal string assumptions: 2
- Explain the smooth-pulley and no-drag consequences: 2
- Describe a meaningful observation-model comparison: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Ladders and rough contact.

- (a) A uniform ladder of length 6 m and weight 150 N rests against a smooth wall with its foot on rough ground at 60° to the ground. Find the normal reaction at the ground. – 2 marks. Answer: 150 N. Award for: Correct method (1); accurate answer (1).
- (b) Take moments about the foot to find the wall reaction. – 2 marks. Answer: 43.3 N. Award for: Correct method (1); accurate answer (1).
- (c) Find the friction at the foot. – 2 marks. Answer: 43.3 N. Award for: Correct method (1); accurate answer (1).
- (d) Find the least μ needed. – 2 marks. Answer: 0.289. Award for: Correct method (1); accurate answer (1).
- (e) State why the wall reaction is horizontal. – 3 marks. Answer: the wall is smooth. Award for: Correct method (2); accurate answer (1).
- (f) Explain why a person climbing makes slipping more likely. – 3 marks. Answer: the moment of their weight about the foot increases. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). A uniform ladder of length 6 m and mass 20 kg rests with its foot on rough horizontal ground and its top against a smooth vertical wall, at 60° to the ground.

- (a) Find the reaction at the wall. – 3 marks. Award for: Moments about the foot
- (b) Find the least coefficient of friction for equilibrium. – 3 marks. Award for: Resolve and $F \leq \mu R$
- (c) Given that $\mu = 0.4$, a man of mass 80 kg climbs the ladder. Find how far up he can climb before it slips. – 4 marks. Award for: Limiting friction and moments

Answers: (a) ≈ 56.6 N. (b) $1/(2\sqrt{3}) \approx 0.289$. (c) ≈ 4.45 m.

Q3 (exam style) (8 marks). Particle A (4 kg) is on a smooth plane inclined at 30°, connected by a light string over a smooth pulley at the top to particle B (3 kg) hanging freely.

- (a) Find the acceleration. – 3 marks. Award for: Equations of motion
- (b) Find the tension. – 1 marks. Award for: Substitute
- (c) The system starts from rest and B hits the ground after 1.5 s. Find the further distance A moves up the plane. – 4 marks. Award for: Speed, deceleration, distance

Answers: (a) g/7 = 1.4 m/s^2. (b) 25.2 N. (c) v = 2.1 m/s, deceleration 4.9 m/s^2, distance 0.45 m.

Q4 (exam style) (8 marks). A particle of mass 0.5 kg is acted on by F = (2ti + (3 − t)j) N. At t = 0 its velocity is (i − 2j) m/s.

- (a) Find the velocity at time t. – 2 marks. Award for: a = F/m, integrate
- (b) Find the times when the particle moves parallel to i. – 2 marks. Award for: j-component zero
- (c) Find the speed when t = 3. – 2 marks. Award for: Magnitude
- (d) State the direction of the force when t = 3. – 1 marks. Award for: Vector
- (e) Given that it starts at the origin, find its position vector at t = 3. – 1 marks. Award for: Integrate

Answers: (a) (2t^2 + 1)i + (6t − t^2 − 2)j. (b) t = 3 ± √7. (c) v = 19i + 7j, speed √410 ≈ 20.2 m/s. (d) 6i: parallel to i. (e) 21i + 12j.

Week 56: Mixed Pure: algebra, graphs and calculus

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 31] Simplify $\frac{x^2+5x+6}{x^2-4}$. – 2 marks. Answer: $\frac{x+3}{x-2}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 32] $f(x) = 3x - 1$. Find $f^{-1}(x)$. – 2 marks. Answer: $\frac{x+1}{3}$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 33] Find the 15th term of $2, 7, 12, \ldots$ – 2 marks. Answer: 72. Award for: Correct method (1); accurate answer (1).
- (d) [Week 42] Given $y = e^{-2x}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $-2e^{-2x}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $\frac{x+3}{x-2}$; (b) $\frac{x+1}{3}$; (c) 72; (d) $-2e^{-2x}$

Q2 (retrieval from Week 31) (6 marks). Resolve $\frac{31x+14}{(x-1)(x+2)}$ into partial fractions. State excluded x-values, recombine to check, and explain why denominators must retain their original restrictions.

Answer: $\frac{16}{x+2} + \frac{15}{x-1}$; x≠1,-2. Recombination verifies numerator; simplification does not restore undefined original inputs.

- Set up the decomposition: 1
- Find both coefficients: 2
- State exclusions: 1
- Recombine to check: 1

- Explain why original restrictions remain: 1

Q3 (retrieval from Week 32) (6 marks). Let f(x)=sqrt(x-15), g(x)=2x+1. Find fg(x), gf(x), with full domains, and solve fg(x)=3. State your convention for composition.

Answer: fg=f∘g=sqrt(2x+1-15), x≥7; gf=2sqrt(x-15)+1, x≥15; solution x=$\frac{23}{2}$.

- State the composition convention: 1
- Find both compositions: 2
- Give the full domain of each: 2
- Solve the composition equation within its domain: 1

Q4 (retrieval from Week 33) (4 marks). An arithmetic sequence starts at 15 with common difference 3. Find the 20th term and sum of the first 20 terms. Find the least n for which the nth term exceeds 100.

Answer: u20=72; S20=870; n>88/3, least integer 30.

- Find the requested term: 1
- Find the finite sum: 1
- Set up the threshold inequality: 1
- Select the least admissible integer n: 1

Q5 (retrieval from Week 42) (6 marks). Differentiate $y=\left(x^2+1\right)e^{15x}$. Factorise the result and find the tangent at x=0, explaining where the product and chain rules enter.

Answer: y'=$\left(15x^2+2x+15\right)e^{15x}$; tangent y=1+15x.

- Apply the product and chain rules correctly: 2
- Factorise the derivative: 1
- Find the tangent: 2
- Explain where both rules enter: 1

Q6 (retrieval from Week 43) (5 marks). For $x^2+y^2=226$, find dy/dx implicitly and the tangent/normal at (1,15). Explain where dy/dx is undefined.

Answer: dy/dx=-x/y. Tangent y-15=-(x-1)/15; normal y-15=15(x-1). Undefined at y=0 on the circle.

- Differentiate implicitly and isolate the derivative: 2
- Find the tangent equation: 1
- Find the normal equation: 1
- Identify where the derivative is undefined on the circle: 1

Q7 (retrieval from Week 45) (5 marks). Find $\int 2x(x^2+15)^3\,dx$ by substitution. State the substitution, integrate and differentiate your answer to check.

Answer: u=x^2+15; integral $(x^2+15)^4/4$+C.

- State the substitution and transformed differential: 1

- Transform and integrate: 2
- Back-substitute and include the constant: 1
- Check by differentiation: 1

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (10 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 31] Express in partial fractions $\frac{4}{(x-1)(x+3)}$. – 2 marks. Answer: $-\frac{1}{x+3}+\frac{1}{x-1}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 32] $f(x)=x+4$, $g(x)=2x^2$. Find $gf(1)$. – 2 marks. Answer: 50. Award for: Correct method (1); accurate answer (1).
- (c) [Week 33] Find the sum to infinity of $8+4+2+\ldots$ – 2 marks. Answer: 16. Award for: Correct method (1); accurate answer (1).
- (d) [Week 42] Given $y=\ln(3x+1)$, find $\frac{dy}{dx}$. – 2 marks. Answer: $\frac{3}{3x+1}$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 43] Find $\frac{dy}{dx}$ for $x^2-y^2=9$. – 2 marks. Answer: $\frac{x}{y}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $-\frac{1}{x+3}+\frac{1}{x-1}$; (b) 50; (c) 16; (d) $\frac{3}{3x+1}$; (e) $\frac{x}{y}$

Q2 (exam style) (9 marks). f(x) = 3 − 2e^(−x), x ∈ ℝ, and g(x) = x².

- (a) State the range of f. – 1 marks. Award for: f < 3
- (b) Find $f^{-1}(x)$ and its domain. – 3 marks. Award for: Inverse
- (c) Solve fg(x) = 1. – 3 marks. Award for: Composition
- (d) Find the tangent to y = f(x) where it crosses the y-axis. – 2 marks. Award for: Gradient

Answers: (a) f(x) < 3. (b) ln(2/(3 − x)), x < 3. (c) x = 0. (d) y = 2x + 1.

Q3 (exam style) (9 marks). The curve C has equation $y=(x^2-3)e^x$.

- (a) Find dy/dx. – 2 marks. Award for: Product rule
- (b) Find the exact stationary points. – 3 marks. Award for: Solve
- (c) Determine their nature. – 2 marks. Award for: Second derivative
- (d) Find the values of k for which $(x^2-3)e^x=k$ has exactly two solutions. – 2 marks. Award for: Use graph features

Answers: (a) (x² + 2x − 3)e^x. (b) (1, −2e), (−3, 6e^(−3)). (c) Minimum, maximum. (d) k = 6e^(−3) or −2e < k ≤ 0.

Q4 (exam style) (11 marks). $f(x)=\frac{3x+5}{(1+x)(1+3x)}$.

- (a) Express f(x) in partial fractions. – 3 marks. Award for: Constants
- (b) Find the expansion up to the term in x^2. – 4 marks. Award for: Two expansions

- (c) State the range of validity. – 1 marks. Award for: |x| < 1/3
- (d) Find $\int_0^1 f(x)\,dx$ exactly. – 3 marks. Award for: Logs

Answers: (a) −1/(1 + x) + 6/(1 + 3x). (b) 5 − 17x + 53x^2. (c) |x| < 1/3. (d) 3 ln 2.

Week 57: Mixed Pure: trig, models and methods

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 44 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 34] Expand $(1-x)^{-1}$ up to x^3. – 2 marks. Answer: $1+x+x^2+x^3$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 35] Convert 240° to radians. – 2 marks. Answer: $\frac{4\pi}{3}$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 36] Find the exact value of sec $\frac{\pi}{6}$. – 2 marks. Answer: $\frac{2}{\sqrt{3}}$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 37] Find the exact value of cos 75°. – 2 marks. Answer: $\frac{\sqrt{6}-\sqrt{2}}{4}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $1+x+x^2+x^3$; (b) $\frac{4\pi}{3}$; (c) $\frac{2}{\sqrt{3}}$; (d) $\frac{\sqrt{6}-\sqrt{2}}{4}$

Q2 (retrieval from Week 34) (6 marks). Expand $(1+9x)^{-1/2}$ through x^3 and state the interval of validity. Use it to approximate its value at x=0.01 and discuss the approximation.

Answer: $-\frac{3645x^3}{16}+\frac{243x^2}{8}-\frac{9x}{2}+1$; |x|<1/9; approximation 0.95781. Truncation error remains; exact value 0.95783.

- Expand correctly through the cubic term: 3
- State the validity interval: 1
- Calculate the requested approximation: 1
- Discuss truncation and the applicability of the expansion: 1

Q3 (retrieval from Week 35) (4 marks). A sector has radius 11 cm and angle 1.2 radians. Find arc length, sector area and perimeter; explain why the angle must be in radians in these formulae.

Answer: Arc 13.2 cm; area 72.6 cm^2; perimeter 35.2 cm. Radian definition gives s=rθ.

- Find arc length: 1
- Find sector area: 1
- Find perimeter: 1
- Explain why the formulas require radians: 1

Q4 (retrieval from Week 36) (6 marks). Solve sec x=-2 for 0≤x<2π. Sketch sec x over that interval, showing asymptotes, and explain the equivalent cosine equation.

Answer: cos x=-1/2 gives x=2π/3,4π/3. Asymptotes π/2,3π/2; sec=1/cos where defined.

- Find both solutions: 2
- Sketch secant with correct branches and asymptotes: 3
- Explain the reciprocal-function equivalence: 1

Q5 (retrieval from Week 37) (8 marks). Derive the double-angle formulae from addition formulae. Use addition formulae to find exact sin75° and cos75°, explaining why each sign is positive.

Answer: sin75°=(sqrt6+sqrt2)/4; cos75°=(sqrt6-sqrt2)/4. Both positive in first quadrant.

- Derive the double-angle formulas from addition formulas: 3
- Use addition or double-angle identities to obtain both exact values: 4
- Explain why both exact trigonometric values are positive: 1

Q6 (retrieval from Week 38) (6 marks). Write 27sin x+36cos x in the form Rsin(x+α), with R>0 and 0<α<π/2. Find its maximum/minimum and where the maximum occurs in 0≤x<2π.

Answer: R=45; α=atan(4/3); extrema ±45; maximum x=π/2-α.

- Determine R and the correct phase angle: 3
- State both extrema: 1
- Find the maximum location in the interval: 2

Q7 (retrieval from Week 44) (6 marks). $x=t^2+1$, $y=9t$, $t\geq 0$. Eliminate t, state the Cartesian domain/range, and find dy/dx and the tangent at t=1.

Answer: y=9sqrt(x-1), x≥1,y≥0; dy/dx=9/(2t); tangent y-9=(9/2)(x-2).

- Eliminate the parameter on the correct branch: 2
- State Cartesian domain and range: 2
- Find dy/dx: 1
- Find the tangent: 1

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (14 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 34] Expand $(1+2x)^{\frac{1}{2}}$ up to x^2. – 2 marks. Answer: $1+x-\frac{x^2}{2}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 35] Find the arc length for radius 10 cm, angle 0.8 rad. – 2 marks. Answer: 8 cm. Award for: Correct method (1); accurate answer (1).
- (c) [Week 36] Simplify $\cos x \operatorname{cosec} x$. – 2 marks. Answer: $\cot x$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 37] Simplify $\cos^2 x - \sin^2 x$. – 2 marks. Answer: $\cos 2x$. Award for: Correct method (1); accurate answer (1).

- (e) [Week 38] $y = 3 + \cos(20t)^\circ$. State the maximum. – 2 marks. Answer: 4. Award for: Correct method (1); accurate answer (1).
- (f) [Week 44] $x = t - 2$, $y = 3t$. Find the Cartesian equation. – 2 marks. Answer: $y = 3x + 6$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 48] Find the general solution of $\frac{dy}{dx} = -2y$. – 2 marks. Answer: $y = Ae^{-2x}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $1 + x - \frac{x^2}{2}$; (b) 8 cm; (c) $\cot x$; (d) $\cos 2x$; (e) 4; (f) $y = 3x + 6$; (g) $y = Ae^{-2x}$

Q2 (exam style) (9 marks).

- (a) Express 8 sin θ − 6 cos θ in the form R sin(θ − α). – 3 marks. Award for: R and α
- (b) The height of a seat on a water wheel is $H = 12 + 8\sin(0.5t) - 6\cos(0.5t)$. Find the maximum height and the first time it occurs. – 3 marks. Award for: Maximum and time
- (c) Find the first two times when H = 17. – 3 marks. Award for: Solve

Answers: (a) 10 sin(θ − 0.6435). (b) 22 at t ≈ 4.43. (c) t ≈ 2.33, 6.52.

Q3 (exam style) (8 marks).

- (a) Show that $\cos^4 x - \sin^4 x \equiv \cos 2x$. – 2 marks. Award for: Factorise
- (b) Hence find $\int_0^{\pi/4} (\cos^4 x - \sin^4 x)\,dx$. – 2 marks. Award for: Integrate
- (c) Find $\int_0^{\pi/2} \sin^2 x \cos^2 x\,dx$. – 4 marks. Award for: Double angles twice

Answers: (a) $(c^2 - s^2)(c^2 + s^2)$. (b) 1/2. (c) π/16.

Q4 (exam style) (8 marks). $y = \frac{3\sin x}{2 + \cos x}$.

- (a) Show that $\frac{dy}{dx} = \frac{3(2\cos x + 1)}{(2 + \cos x)^2}$. – 3 marks. Award for: Quotient rule
- (b) Find the stationary points for 0 ≤ x < 2π. – 3 marks. Award for: Solve
- (c) State the range of y. – 1 marks. Award for: Range
- (d) Find the tangent at x = 0. – 1 marks. Award for: Line

Answers: (a) Simplify using $\sin^2 + \cos^2 = 1$. (b) (2π/3, √3), (4π/3, −√3). (c) −√3 ≤ y ≤ √3. (d) y = x.

Week 58: Mixed Statistics and Mechanics

Pearson S2 chapter 1–8.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (10 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 39] $P(A \cap B) = 0.12$, $P(B) = 0.4$. Find $P(A \mid B)$. – 2 marks. Answer: 0.3. Award for: Correct method (1); accurate answer (1).
- (b) [Week 40] $X \sim N(20, 5^2)$. Find $P(X < 25)$. – 2 marks. Answer: 0.8413. Award for: Correct method (1); accurate answer (1).

- (c) [Week 41] $X \sim N(30, 6^2)$, $n = 9$. State the distribution of $\bar{X}$. – 2 marks. Answer: $N(30, 2^2)$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 51] $\mathbf{r} = (t^2\mathbf{i} + 3t\mathbf{j})$. Find $\mathbf{v}$. – 2 marks. Answer: $2t\mathbf{i} + 3\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 52] A force of 15 N has a perpendicular distance of 0.4 m from a pivot. Find its moment. – 2 marks. Answer: 6 N m. Award for: Correct method (1); accurate answer (1).

Answers: (a) 0.3; (b) 0.8413; (c) $N(30, 2^2)$; (d) $2t\mathbf{i} + 3\mathbf{j}$; (e) 6 N m

Q2 (retrieval from Week 39) (7 marks). A hypothetical screening model has prevalence 0.10, sensitivity 0.95 and false-positive probability 0.08. Draw a probability tree; find P(positive) and P(condition given positive). Explain why the two conditionals differ.

Answer: P(positive)=0.167; P(condition|positive)=0.56886. Base rate matters; this is a hypothetical mathematics model, not clinical advice.

- Draw and label the full probability tree: 2
- Calculate the total positive probability: 2
- Calculate the posterior event ratio: 2
- Explain why the two conditional probabilities differ: 1

Q3 (retrieval from Week 40) (5 marks). Let X~N(61,16). Find P(X<57), P(59<X<67) and the 90th percentile. State clearly whether 16 denotes variance or standard deviation.

Answer: 0.15866; 0.62466; percentile 66.126. 16 is variance, sd4.

- Calculate the one-sided probability: 1
- Calculate the interval probability: 1
- Use an inverse normal value to obtain the percentile: 2
- Distinguish variance from standard deviation: 1

Q4 (retrieval from Week 41) (6 marks). A population is normal with known sd=5. A random sample of 25 has mean 112. Test H0:μ=110 against μ>110 at 5%; state assumptions and contextual conclusion.

Answer: SE=1, z=2, p=0.02275; reject H0. Evidence mean exceeds 110; random independent sample and known variance required.

- Use the sample-mean distribution and correct standard error: 2
- Calculate a valid test statistic or tail probability: 1
- Make the correct significance comparison: 1
- State the conclusion in context: 1
- State relevant assumptions: 1

Q5 (retrieval from Week 51) (4 marks). A particle has position $\mathbf{r} = (t^2 + 1)\mathbf{i} + (10t - t^2)\mathbf{j}$. Find velocity, acceleration, speed at t=1 and when motion is parallel to the i-axis.

Answer: v=(2t,10-2t), acc=(2,-2); speed at1=sqrt(4+(10-2)^2); j velocity zero at t=5.

- Differentiate to obtain velocity: 1
- Differentiate to obtain acceleration: 1
- Find speed at the specified time: 1
- Find the time the velocity is parallel to the i-axis: 1

Q6 (retrieval from Week 52) (4 marks). A uniform horizontal rod length6 m and mass 10 kg is supported at both ends. A 20 kg load is placed2 m from the left. Find both reactions using moments and vertical equilibrium.

Answer: Right R=114.33 N; left R=179.67 N. Total weight=294 N.

- Place the rod weight and load correctly: 1
- Form a valid moment equation: 1
- Find one reaction: 1
- Use vertical equilibrium to find the other reaction: 1

Q7 (retrieval from Week 53) (4 marks). A 10 kg block lies on a rough horizontal surface, coefficient μ=0.3. Find R and limiting friction. A horizontal force 10 N is applied: decide if it moves and find actual friction.

Answer: R=98 N; limiting friction=29.4 N. Applied force 10 N is below that limit, so actual friction=10 N, not μR.

- Find the normal reaction: 1
- Find the limiting friction: 1
- Compare the applied force with that limit: 1
- State whether it moves and give the actual friction: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (16 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 39] $P(A) = 0.7$, $P(B \mid A) = 0.2$. Find $P(A \cap B)$. – 2 marks. Answer: 0.14. Award for: Correct method (1); accurate answer (1).
- (b) [Week 40] $X \sim N(20, 5^2)$. Find $P(15 < X < 30)$. – 2 marks. Answer: 0.8186. Award for: Correct method (1); accurate answer (1).
- (c) [Week 41] A random sample of 9 independent observations is taken from $N(30, 6^2)$. Find $P(\bar{X} > 33)$. – 2 marks. Answer: 0.0668. Award for: Correct method (1); accurate answer (1).
- (d) [Week 51] A particle has position vector $\mathbf{r} = t^2\mathbf{i} + 3t\mathbf{j}$ metres, with t in seconds. Find its speed at $t = 2$. – 2 marks. Answer: 5 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (e) [Week 52] A uniform 2 m rod of weight 30 N is supported at its ends. Find each reaction. – 2 marks. Answer: 15 N. Award for: Correct method (1); accurate answer (1).

- (f) [Week 53] Find the maximum friction for a 2 kg block on horizontal ground, $\mu = 0.5$. – 2 marks. Answer: 9.8 N. Award for: Correct method (1); accurate answer (1).
- (g) [Week 54] A ball is projected at 14 m s^{-1} vertically upwards component and 10 m s^{-1} horizontally. Find the time of flight. – 2 marks. Answer: 2.86 s. Award for: Correct method (1); accurate answer (1).
- (h) [Week 55] Find the tension when a 3 kg mass hangs from one vertical string. – 2 marks. Answer: 29.4 N. Award for: Correct method (1); accurate answer (1).

Answers: (a) 0.14; (b) 0.8186; (c) 0.0668; (d) 5 m s^{-1}; (e) 15 N; (f) 9.8 N; (g) 2.86 s; (h) 29.4 N

Q2 (exam style) (8 marks). Lengths of rods are N(80, 2.5^2) mm. Rods outside 76–84 mm are rejected.

- (a) Find the proportion rejected. – 3 marks. Award for: Two tails
- (b) Find the probability that at most 3 of a box of 50 rods are rejected. – 3 marks. Award for: Binomial
- (c) The standard deviation is adjusted so that 95% are within the limits. Find it. – 2 marks. Award for: Inverse normal

Answers: (a) 0.1096. (b) ≈ 0.188. (c) 4/1.96 ≈ 2.04 mm.

Q3 (exam style) (8 marks). A ball is kicked from the ground with velocity (12i + 14.7j) m/s.

- (a) Find the greatest height and when it occurs. – 2 marks. Award for: Vertical
- (b) The ball lands on a ledge 4.9 m high on its way down. Find the horizontal distance. – 4 marks. Award for: Quadratic
- (c) Find the speed on landing. – 2 marks. Award for: Components

Answers: (a) 11.0 m at 1.5 s. (b) t = (3 + √5)/2 ≈ 2.62 s, ≈ 31.4 m. (c) ≈ 16.3 m/s.

Q4 (exam style) (8 marks). Of 120 students, 50 play football, 40 play tennis and 30 swim. 15 play football and tennis, 10 play tennis and swim, 8 play football and swim, and 4 do all three.

- (a) Draw a Venn diagram. – 3 marks. Award for: All regions
- (b) Find P(none). – 1 marks. Award for: Complement
- (c) Find P(exactly one activity). – 2 marks. Award for: Sum
- (d) Find P(F | T ∪ S). – 2 marks. Award for: Conditional

Answers: (a) F only 31, T only 19, S only 16. (b) 29/120. (c) 11/20. (d) 19/60.

Week 59: Exam practice: Pure selection and accuracy

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 42 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 31] Express in partial fractions $\frac{2x+3}{x(x+3)}$. – 2 marks. Answer: $\frac{1}{x+3}+\frac{1}{x}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 32] Solve $|x+2|=6$. – 2 marks. Answer: $x=4,\ -8$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 33] Find $\sum_{r=1}^{12}(3r-1)$. – 2 marks. Answer: 222. Award for: Correct method (1); accurate answer (1).
- (d) [Week 42] Given $y=x^3\cos x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $3x^2\cos x - x^3\sin x$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $\frac{1}{x+3}+\frac{1}{x}$; (b) $x=4,\ -8$; (c) 222; (d) $3x^2\cos x - x^3\sin x$

Q2 (retrieval from Week 46) (6 marks). Find $\int xe^{11x}\,dx$ by parts. Evaluate it from 0 to1 and verify the antiderivative.

Answer: e^(11x)(x/11-1/121)+C; definite=$\frac{1}{121}+\frac{10e^{11}}{121}$.

- Choose u and dv and apply integration by parts: 2
- Obtain the antiderivative with its constant: 1
- Evaluate the definite integral: 2
- Check the antiderivative: 1

Q3 (retrieval from Week 47) (8 marks). Find the area between y=11sin x and y=11sin(2x) for 0≤x≤π. Locate all intersections first and explain the necessary split.

Answer: Intersections 0,π/3,π. First area $\frac{11}{4}$, second $\frac{99}{4}$; total $\frac{55}{2}$.

- Find all intersections: 2
- Identify the upper curve on each subinterval: 1
- Set up the split area integral: 1
- Integrate and combine the exact areas: 3
- Explain the need to split: 1

Q4 (retrieval from Week 1) (5 marks). A student claims $\sqrt{x^2+121}=x+11$ for every real x. Disprove this. Solve the equation where it does hold and check the domain of the square root.

Answer: x=1 is a counterexample. Squaring gives 22x=0, so only x=0; both sides then equal 11. The square root is defined for all real x.

- Give a valid counterexample: 1
- Square and solve the resulting equation: 2
- Check the candidate in the original equation: 1
- State and justify the square-root domain: 1

Q5 (retrieval from Week 2) (7 marks). A rectangle has perimeter 52 cm. Let one side be x cm. Form its area function, find the maximum area and give the possible x for area greater than 168 square cm.

Answer: A=x(26-x)=(13)$^{2-(x-13)}$2; max 169; 12<x<14. Physical domain 0<x<26.

- Use the perimeter to form an area function: 2
- Find the maximum area: 2
- Solve the strict area inequality: 2
- Respect the physical domain: 1

Q6 (retrieval from Week 4) (4 marks). The line $y = mx + 11$ passes through (0,11). Find its intersection with x=2 and its perpendicular through (0,0). Discuss separately m=0.

Answer: (2,2m+11); perpendicular y=-x/m for m≠0. If m=0 the perpendicular is x=0.

- Find the intersection in terms of m: 1
- Find the perpendicular for nonzero m: 2
- Handle m=0 separately: 1

Q7 (retrieval from Week 7) (4 marks). Disprove the claim that $n^2 + n + 11$ is prime for every positive integer n. Give a constructive choice of n and explain why one counterexample is sufficient.

Answer: Take n=11; expression=11×13, composite because both factors exceed1. A universal claim fails at one counterexample.

- Construct a suitable integer counterexample: 1
- Show the resulting number is composite: 2
- Explain why one counterexample suffices: 1

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (14 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 31] Express in partial fractions $\frac{x+3}{(x-1)^2}$. – 2 marks. Answer: $\frac{1}{x-1} + \frac{4}{(x-1)^2}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 32] State the range of $f(x) = (x-1)^2 - 5$. – 2 marks. Answer: $f(x) \geq -5$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 33] Find the 6th term of the geometric sequence with $a = 2$, $r = -3$. – 2 marks. Answer: -486. Award for: Correct method (1); accurate answer (1).
- (d) [Week 42] Given $y = (x^2+1)^3$, find $\frac{dy}{dx}$. – 2 marks. Answer: $6x(x^2+1)^2$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 43] Find $\frac{dy}{dx}$ for $y^2 = 4x$. – 2 marks. Answer: $\frac{2}{y}$. Award for: Correct method (1); accurate answer (1).

- (f) [Week 45] Find $\int e^{-x}\,dx$. – 2 marks. Answer: $-e^{-x}+c$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 46] Find $\int xe^{2x}\,dx$. – 2 marks. Answer: $\frac{x}{2}e^{2x}-\frac{1}{4}e^{2x}+c$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $\frac{1}{x-1}+\frac{4}{(x-1)^2}$; (b) $f(x)\geq -5$; (c) -486; (d) $6x(x^2+1)^2$; (e) $\frac{2}{y}$; (f) $-e^{-x}+c$; (g) $\frac{x}{2}e^{2x}-\frac{1}{4}e^{2x}+c$

Q2 (exam style) (8 marks). The curve C has equation $y = xe^{-x/2}$, x ≥ 0.

- (a) Find the maximum point. – 3 marks. Award for: Differentiate
- (b) Find the exact area bounded by C, the x-axis and x = 4. – 5 marks. Award for: Integration by parts

Answers: (a) (2, 2/e). (b) 4 − 12e^(−2).

Q3 (exam style) (9 marks). The curve C has equation $x^3 + y^3 = 6xy$.

- (a) Find dy/dx. – 4 marks. Award for: Implicit differentiation
- (b) Find the tangent at (3, 3). – 2 marks. Award for: Line
- (c) Find the point with x > 0 where the tangent is horizontal. – 3 marks. Award for: Substitute y = $x^2/2$

Answers: (a) $(2y - x^2)/(y^2 - 2x)$. (b) x + y = 6. (c) (16^(1/3), 16^(2/3)/2) ≈ (2.52, 3.17).

Q4 (exam style) (8 marks). f(x) = 2|x + 1| − 3 and g(x) = 5 − x.

- (a) Sketch both graphs. – 3 marks. Award for: V-shape and line
- (b) Solve f(x) = g(x). – 3 marks. Award for: Both branches
- (c) Solve f(x) < g(x). – 2 marks. Award for: Interval

Answers: (a) Vertex (−1, −3). (b) x = 2 or x = −10. (c) −10 < x < 2.

Week 60: Exam practice: applied models and interpretation

Pearson S2 chapter 1–8.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 39] $P(A)=0.2$, $P(B)=0.5$, independent. Find $P(A\cup B)$. – 2 marks. Answer: 0.6. Award for: Correct method (1); accurate answer (1).
- (b) [Week 40] $X\sim N(20,5^2)$. Find a with $P(X<a)=0.95$. – 2 marks. Answer: 28.22. Award for: Correct method (1); accurate answer (1).
- (c) [Week 41] $X\sim B(50,0.5)$. State a normal approximation. – 2 marks. Answer: $N(25,12.5)$. Award for: Correct method (1); accurate answer (1).

- (d) [Week 51] $\mathbf{u} = \mathbf{i}$, $\mathbf{a} = 2\mathbf{j}$. Find $\mathbf{v}$ at $t = 3$. – 2 marks. Answer: $\mathbf{i} + 6\mathbf{j}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 0.6; (b) 28.22; (c) $N(25, 12.5)$; (d) $\mathbf{i} + 6\mathbf{j}$

Q2 (retrieval from Week 54) (6 marks). A projectile is launched from level ground at speed 22 m/s and angle30°. Neglecting air resistance, find flight time, maximum height and range; state units and assumptions.

Answer: Time 2.2449 s; height 6.1735 m; range 42.771 m. Constant g, returns to launch height.

- Resolve the launch velocity: 1
- Find flight time: 2
- Find maximum height: 1
- Find range: 1
- State correct units and relevant assumptions: 1

Q3 (retrieval from Week 55) (7 marks). A uniform ladder length5 m, weight 240 N, rests at60° to rough horizontal ground against a smooth vertical wall. Find reactions and the minimum coefficient of ground friction.

Answer: Ground normal=W; wall reaction=W cos60°/(2sin60°)=W/(2sqrt3); friction equals wall reaction; μmin=1/(2sqrt3)≈0.28868.

- Identify the reaction and friction forces: 2
- Use moments to find the wall reaction: 2
- Use horizontal and vertical equilibrium for the ground forces: 2
- Find the minimum friction coefficient: 1

Q4 (retrieval from Week 10) (6 marks). Let X~B(20,0.3). Calculate P(X=3), P(X≤2) and P(X≥4). State two modelling assumptions in the context of repeated independent attempts.

Answer: 0.071604, 0.035483, 0.89291; fixed n, independent attempts, constant success probability, two outcomes.

- Calculate the point probability: 1
- Calculate the lower-tail probability: 1
- Use the correct complementary tail: 2
- State two valid binomial assumptions: 2

Q5 (retrieval from Week 11) (3 marks). A student observes 25 successes in 32 trials and tests H0:p=0.5 against H1:p>0.5. Calculate the p-value. Explain why the probability of the single observed count is not enough.

Answer: P(X≥25)=0.0010512; includes every outcome at least as extreme in the alternative direction.

- Identify the upper-tail event including the observed count: 1

- Calculate its probability: 1
- Explain why the single-count probability is insufficient: 1

Q6 (retrieval from Week 12) (4 marks). A displacement-time graph is described by s=12t for 0≤t≤2, then s=24 for 2≤t≤5. Sketch it, give velocities in each stage and discuss instantaneous velocity at t=2.

Answer: Velocities 12,0 m/s. The idealised corner has no single derivative at t=2; model contains an instantaneous velocity change.

- Sketch both displacement-time segments: 2
- Give the velocities in both stages: 1
- Explain the undefined instantaneous velocity at the idealised corner: 1

Q7 (retrieval from Week 13) (5 marks). A car starts at 12 m/s and accelerates at 2 m/s^2 over 40 m. Find its final speed and travel time; explain why a negative quadratic time root is rejected.

Answer: v=17.436 m/s; t=2.7178 s; elapsed time must be nonnegative.

- Use an appropriate constant-acceleration equation for final speed: 2
- Find the travel time: 2
- Explain rejection of a negative elapsed time: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (16 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 39] Two cards are drawn from a standard pack without replacement. Find P(second is a heart | first is a heart). – 2 marks. Answer: $\frac{12}{51}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 40] State the points of inflection of the $N(20, 5^2)$ curve. – 2 marks. Answer: $x = 15,\ 25$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 41] $X \sim B(50, 0.5)$. Use a normal approximation with a continuity correction to estimate $P(X \geq 30)$. – 2 marks. Answer: $P(Y > 29.5) = 0.1015$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 51] $v = 4t^3$. Find s given $s = 2$ at $t = 0$. – 2 marks. Answer: $t^4 + 2$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 52] A uniform horizontal rod is 2 m long, weighs 30 N and is supported at both ends. A 10 N load hangs 0.5 m from the left end. Find the left reaction. – 2 marks. Answer: 22.5 N. Award for: Correct method (1); accurate answer (1).
- (f) [Week 53] Resolve 10 N at 60° to the horizontal. – 2 marks. Answer: 5 N, 8.66 N. Award for: Correct method (1); accurate answer (1).
- (g) [Week 54] A ball is projected from horizontal ground with horizontal velocity 10 m/s and upward vertical velocity 14 m/s. Find its range when it returns to the ground. – 2 marks. Answer: 28.6 m. Award for: Correct method (1); accurate answer (1).

- (h) [Week 55] Resolve 50 N at 37° to the horizontal. – 2 marks. Answer: 39.9 N, 30.1 N. Award for: Correct method (1); accurate answer (1).

Answers: (a) $\frac{12}{51}$; (b) $x = 15,\ 25$; (c) $P(Y > 29.5) = 0.1015$; (d) $t^4 + 2$; (e) 22.5 N; (f) 5 N, 8.66 N; (g) 28.6 m; (h) 39.9 N, 30.1 N

Q2 (exam style) (8 marks). A uniform rod AB, 3 m long and weighing 40 N, rests with A on rough horizontal ground. It is held at 40° to the ground by a string attached at B and perpendicular to the rod.

- (a) Find the tension. – 3 marks. Award for: Moments
- (b) Find the normal reaction and the friction at A. – 4 marks. Award for: Resolve
- (c) Find the least coefficient of friction. – 1 marks. Award for: F/R

Answers: (a) 20 cos 40° ≈ 15.3 N. (b) R ≈ 28.3 N, F ≈ 9.85 N. (c) ≈ 0.348.

Q3 (exam style) (8 marks). Battery life is N(μ, 3.2^2) hours. The manufacturer claims μ = 20. A sample of 40 batteries has mean 19.1 hours.

- (a) Test at the 1% level whether the mean is less than 20. – 5 marks. Award for: Test
- (b) Find the largest sample mean that would lead to rejection. – 2 marks. Award for: Boundary
- (c) Explain what the significance level represents. – 1 marks. Award for: Type I error probability

Answers: (a) z = −1.78 > −2.326: insufficient evidence. (b) 18.82 h. (c) The probability of rejecting H_0 when it is true.

Q4 (exam style) (8 marks). A particle of mass 2 kg has position vector $\mathbf{r} = (t^3 - 3t)\mathbf{i} + (2t^2 - 8t)\mathbf{j}$ m.

- (a) Find v and a. – 2 marks. Award for: Differentiate
- (b) Find the magnitude of the force at t = 2. – 2 marks. Award for: F = ma
- (c) Show that the particle moves parallel to j when t = 1. – 2 marks. Award for: i-component
- (d) Find when it moves parallel to i. – 1 marks. Award for: j-component
- (e) Find the speed at that time. – 1 marks. Award for: Magnitude

Answers: (a) v = $(3t^2 - 3)$i + (4t − 8)j, a = 6ti + 4j. (b) $2\sqrt{160} \approx 25.3$ N. (c) v = −4j. (d) t = 2. (e) 9 m/s.

Week 61: Exam practice: Pure reasoning and transfer

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (10 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 34] State the validity of $(1-3x)^{-2}$. – 2 marks. Answer: $|x| < \frac{1}{3}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 35] Find the sector area for radius 3 cm, angle 2 rad. – 2 marks. Answer: 9 cm^2. Award for: Correct method (1); accurate answer (1).
- (c) [Week 36] Given $\sec x = 3$, find $\tan^2 x$. – 2 marks. Answer: 8. Award for: Correct method (1); accurate answer (1).
- (d) [Week 37] Given $\tan x = \frac{1}{2}$, find $\tan 2x$. – 2 marks. Answer: $\frac{4}{3}$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 38] For $y = 3 + \cos((20t)^\circ)$, state the period in units of t. – 2 marks. Answer: 18. Award for: Correct method (1); accurate answer (1).

Answers: (a) $|x| < \frac{1}{3}$; (b) 9 cm^2; (c) 8; (d) $\frac{4}{3}$; (e) 18

Q2 (retrieval from Week 48) (6 marks). Solve dy/dx=16y with y(0)=2 by separation. Find when y=10, and explain why dividing by y would require checking y=0 in the general equation.

Answer: y=2e^(16x); x=ln5/16. y=0 is another general solution but does not satisfy this initial condition.

- Separate variables and integrate: 2
- Apply the initial condition: 1
- Find when the target value is reached: 2
- Explain the possible zero solution in the general equation: 1

Q3 (retrieval from Week 49) (4 marks). For f(x)=x^3-x-16, find consecutive nonnegative integers between which a root lies, using a sign change. Explain the role of continuity and why the argument is insufficient for 1/x across zero.

Answer: f(2)=-10, f(3)=8; root lies in (2,3). Continuity is required; 1/x is discontinuous at0, so its sign change across0 does not imply a root.

- Evaluate at suitable consecutive integers: 1
- Identify a valid bracket or an exact endpoint root: 1
- Explain the continuity requirement: 1
- Explain why the argument fails across the discontinuity of 1/x: 1

Q4 (retrieval from Week 50) (4 marks). A=(1,2,16), B=(3,5,22). Find AB, its length and a unit vector. Find C on AB extended such that AC=3AB.

Answer: AB=(2,3,6), length7, unit=(2/7,3/7,6/7); C=(7,11,34).

- Find AB: 1
- Find its length and unit vector: 2

- Find the extended point C: 1

Q5 (retrieval from Week 34) (5 marks). Find the first three terms of $(2-16x)^{-1}$. State validity and check by multiplying the truncated expression by 2-16x.

Answer: 1/2+(16/4)x+(256/8)x^2; |x|<2/16; product=1+terms of order x^3.

- Factor out the constant before expanding: 1
- Find the first three terms: 2
- State validity: 1
- Check by multiplying the truncated series: 1

Q6 (retrieval from Week 35) (5 marks). A circular segment has radius 18 and angle π/3 at its centre. Find its exact area and its chord length. Sketch the region and identify the subtracted triangle.

Answer: Area $-81\sqrt{3}+54\pi$; chord 18.

- Form the sector-minus-triangle calculation: 2
- Give the exact segment area: 1
- Find the chord length: 1
- Sketch and identify the subtracted triangle: 1

Q7 (retrieval from Week 37) (5 marks). Solve sin(2x)=-cos x for 0≤x<2π. Show the factorisation and identify roots that dividing by cos x would lose.

Answer: cos x(2sin x+1)=0; x=π/2,7π/6,3π/2,11π/6. Division loses π/2,3π/2.

- Use the double-angle identity and factorise: 2
- Find all roots in the interval: 2
- Identify the roots lost by division: 1

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (12 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 34] Find the coefficient of x^2 in $(1-x)^{-2}$. – 2 marks. Answer: 3. Award for: Correct method (1); accurate answer (1).
- (b) [Week 35] Solve $\cos x = \frac{\sqrt{2}}{2}$ for $0 \le x \le 2\pi$. – 2 marks. Answer: $\frac{\pi}{4}, \frac{7\pi}{4}$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 36] Find $\arccos \frac{\sqrt{3}}{2}$. – 2 marks. Answer: $\frac{\pi}{6}$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 37] Write $\sin x + \cos x$ as $R\sin(x+\alpha)$. – 2 marks. Answer: $\sqrt{2}\sin(x+45^\circ)$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 38] For $y = 10 + 4\sin x$, state the range. – 2 marks. Answer: $6 \le y \le 14$. Award for: Correct method (1); accurate answer (1).

- (f) [Week 44] $x = 5\cos t$, $y = 5\sin t$. Find the Cartesian equation. – 2 marks. Answer: $x^2 + y^2 = 25$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 3; (b) $\frac{\pi}{4}$, $\frac{7\pi}{4}$; (c) $\frac{\pi}{6}$; (d) $\sqrt{2}\sin(x + 45^\circ)$; (e) $6 \le y \le 14$; (f) $x^2 + y^2 = 25$

Q2 (exam style) (9 marks). A drug dose of 200 mg decays according to dx/dt = −kx. After 4 hours, 140 mg remain.

- (a) Show that $x = 200e^{-kt}$ and find k. – 4 marks. Award for: Separate and solve
- (b) The drug is effective above 50 mg. Find how long it remains effective. – 2 marks. Award for: Logs
- (c) A second 200 mg dose is given at t = 8. Find the amount at t = 12. – 3 marks. Award for: Superpose doses

Answers: (a) k = ln(10/7)/4 ≈ 0.0892. (b) ≈ 15.5 h. (c) 68.6 + 140 = 208.6 mg.

Q3 (exam style) (8 marks). $f(x) = x - 2\cos x$.

- (a) Show that f(x) = 0 has a root between 1 and 1.1. – 2 marks. Award for: Sign change
- (b) Apply the Newton–Raphson method once from $x_0 = 1$. – 3 marks. Award for: Iteration
- (c) Explain why the iteration $x_{n+1} = 2\cos x_n$ fails to converge to this root. – 3 marks. Award for: Derivative magnitude

Answers: (a) f(1) = −0.081, f(1.1) = 0.193. (b) 1.0300. (c) $|g'(x)| = 2|\sin x| \approx 1.71 > 1$ near the root.

Q4 (exam style) (10 marks). A curve has parametric equations x = 2 cos t, y = sin 2t, 0 ≤ t ≤ π/2.

- (a) Find dy/dx in terms of t. – 2 marks. Award for: Parametric differentiation
- (b) Find the point where the tangent is horizontal. – 2 marks. Award for: cos 2t = 0
- (c) Find a Cartesian equation. – 3 marks. Award for: Eliminate t
- (d) Find the area under the curve. – 3 marks. Award for: Parametric integral

Answers: (a) −cos 2t/sin t. (b) (√2, 1). (c) $y = (x/2)\sqrt{(4 - x^2)}$. (d) 4/3.

Week 62: Exam practice: Pure selection and accuracy

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 43 marks

Q1 (basic skills) (12 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 31] Express in partial fractions $\frac{x^2}{x^2-4}$. – 3 marks. Answer: $1 - \frac{1}{x+2} + \frac{1}{x-2}$. Award for: Correct method (2); accurate answer (1).
- (b) [Week 32] Solve $|3x - 2| \ge 4$. – 3 marks. Answer: $x \le -\frac{2}{3}$ or $x \ge 2$. Award for: Correct method (2); accurate answer (1).
- (c) [Week 33] $u_{n+1} = u_n + 4$, $u_1 = 3$. Find u_{10}. – 3 marks. Answer: 39. Award for: Correct method (2); accurate answer (1).

- (d) [Week 42] Given $y = \frac{x}{x+2}$, find $\frac{dy}{dx}$. – 3 marks. Answer: $\frac{2}{(x+2)^2}$. Award for: Correct method (2); accurate answer (1).

Answers: (a) $1 - \frac{1}{x+2} + \frac{1}{x-2}$; (b) $x \leq -\frac{2}{3}$ or $x \geq 2$; (c) 39; (d) $\frac{2}{(x+2)^2}$

Q2 (retrieval from Week 8) (4 marks). A right triangle has opposite side 17 and adjacent side 18 relative to angle θ. Find sin θ, cos θ and tan θ exactly, then explain why changing both lengths by the same factor preserves θ.

Answer: Hypotenuse $\sqrt{613}$; sin=$\frac{17}{\sqrt{613}}$, cos=$\frac{18}{\sqrt{613}}$, tan=$\frac{17}{18}$. Ratios unchanged.

- Find the hypotenuse: 1
- Give all three exact trigonometric ratios: 2
- Explain why a common scaling preserves the angle: 1

Q3 (retrieval from Week 9) (4 marks). Prove (1-cos^{2}x)/sin x=sin x wherever the left side is defined. State the exclusions. Explain why cancelling sin x can lose solutions when solving an equation.

Answer: 1-cos^{2}x=sin^{2}x; division gives sin x when sin x≠0. Exclude multiples of 180°. Dividing by a potentially zero expression may discard its zero cases.

- Use the Pythagorean identity: 1
- Complete the algebraic proof on the valid domain: 1
- State all exclusions: 1
- Explain the possible loss of roots when dividing by a zero factor: 1

Q4 (retrieval from Week 14) (5 marks). Differentiate $y = 17x^{3/2} + \frac{4}{x} - 2\sqrt{x}$, for x>0. Find its gradient at x=4 and explain why the stated domain is needed.

Answer: Derivative $\frac{51\sqrt{x}}{2} - \frac{4}{x^2} - \frac{1}{\sqrt{x}}$; at 4=$\frac{201}{4}$. Denominator and fractional powers constrain x.

- Differentiate the three terms correctly: 3
- Evaluate the gradient: 1
- Explain the domain restriction: 1

Q5 (retrieval from Week 15) (7 marks). For $y = x^3 - 51x$, find and classify every stationary point using the second derivative. Sketch the curve with stationary points and intercepts.

Answer: x=±sqrt(17); points (-sqrt(17),34sqrt(17)) maximum and (sqrt(17),-34sqrt(17)) minimum; roots 0,±sqrt(51).

- Differentiate and solve for both stationary x-values: 2
- Find both y-coordinates: 1
- Use the second derivative to classify both: 2
- Sketch with stationary points and all intercepts: 2

Q6 (retrieval from Week 16) (4 marks). Integrate $17\sqrt{x} + \frac{3}{x^2}$, for x>0. Explain why the same power rule does not integrate 1/x.

Answer: $\frac{34}{3}x^{3/2} - \frac{3}{x} + C$; power rule denominator n+1 vanishes at n=-1.

- Rewrite powers and integrate both terms, including the constant: 3
- Explain the failure of the power-rule expression when the exponent is minus one: 1

Q7 (retrieval from Week 17) (7 marks). Evaluate $\int_{-1}^{2}(17x - 1)\,dx$. Then find the total area between the line and x-axis over that interval, explaining why the answers differ.

Answer: Signed $\frac{45}{2}$; total area $\frac{1413}{34}$. Split at x=1/17; negative contributions must be reversed.

- Evaluate the signed integral: 2
- Locate the sign-change point: 1
- Set up the split total-area calculation: 1
- Evaluate the total area: 2
- Explain the difference between signed integral and area: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (15 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 43] Find the gradient of $xy = 12$ at $(3, 4)$. – 2 marks. Answer: $-\frac{4}{3}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 45] Find $\int \sin 2x\,dx$. – 2 marks. Answer: $-\frac{1}{2}\cos 2x + c$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 46] Find $\int \frac{2}{x(x+2)}\,dx$. – 2 marks. Answer: $\ln\left|\frac{x}{x+2}\right| + c$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 47] Evaluate $\int_0^{\pi} \sin x\,dx$. – 2 marks. Answer: 2. Award for: Correct method (1); accurate answer (1).
- (e) [Week 8] State the period of $y = \tan x$. – 2 marks. Answer: $180°$. Award for: Correct method (1); accurate answer (1).
- (f) [Week 9] Solve $3\sin x = \cos x$ for $0 \le x \le 360°$. – 2 marks. Answer: $18.4°, 198.4°$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 14] Find $\frac{d^2y}{dx^2}$ for $y = x^5$. – 3 marks. Answer: $20x^3$. Award for: Correct method (2); accurate answer (1).

Answers: (a) $-\frac{4}{3}$; (b) $-\frac{1}{2}\cos 2x + c$; (c) $\ln\left|\frac{x}{x+2}\right| + c$; (d) 2; (e) $180°$; (f) $18.4°, 198.4°$; (g) $20x^3$

Q2 (exam style) (8 marks). A ball is dropped from 8 m. Each bounce reaches 3/4 of the previous height.

- (a) Find the height after the 5th bounce. – 2 marks. Award for: Geometric term
- (b) Show that the total distance travelled is 56 m. – 3 marks. Award for: Sum to infinity

- (c) Find the first bounce whose height is below 0.5 m. – 3 marks. Award for: Logs

Answers: (a) ≈ 1.90 m. (b) 8 + 2 × 6/(1 − 0.75) = 56. (c) The 10th.

Q3 (exam style) (8 marks). The depth h m of water in a tank satisfies $\frac{dh}{dt} = -0.2\sqrt{h}$, with h = 9 at t = 0.

- (a) Solve to find h in terms of t. – 4 marks. Award for: Separate and integrate
- (b) Find when the tank empties. – 1 marks. Award for: h = 0
- (c) Find dh/dt at t = 10. – 2 marks. Award for: Substitute
- (d) Explain why the model is valid only for 0 ≤ t ≤ 30. – 1 marks. Award for: After emptying

Answers: (a) $h = (3 - 0.1t)^2$. (b) t = 30. (c) −0.4 m per minute. (d) After t = 30 the formula would wrongly show refilling.

Q4 (exam style) (9 marks). $f(x) = \ln x + x^2 - 4$.

- (a) Show that f(x) = 0 has a root α between 1.8 and 1.9. – 2 marks. Award for: Sign change
- (b) Apply the Newton–Raphson method once from 1.8. – 3 marks. Award for: Iteration
- (c) Show that α = 1.84 to 2 decimal places. – 2 marks. Award for: Interval check
- (d) Explain how a staircase or cobweb diagram shows whether an iteration converges. – 2 marks. Award for: Gradient condition

Answers: (a) f(1.8) = −0.172, f(1.9) = 0.252. (b) 1.8414. (c) f(1.835) < 0 < f(1.845). (d) Convergence when |g′(x)| < 1 near the root.

Week 63: Exam practice: applied models and interpretation

Pearson S2 chapter 1–8.

Practice 1 (Day 2) – total 42 marks

Q1 (basic skills) (12 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 39] State the meaning of ρ. – 3 marks. Answer: population correlation coefficient. Award for: Correct method (2); accurate answer (1).
- (b) [Week 40] $X \sim N(\mu, 2^2)$, $P(X > 12) = 0.3$. Find μ. – 3 marks. Answer: 10.95. Award for: Correct method (2); accurate answer (1).
- (c) [Week 41] State the 5% two-tailed critical z-values. – 3 marks. Answer: ± 1.96. Award for: Correct method (2); accurate answer (1).
- (d) [Week 51] $v = \cos t$. Find the maximum acceleration. – 3 marks. Answer: 1 m s^{-2}. Award for: Correct method (2); accurate answer (1).

Answers: (a) population correlation coefficient; (b) 10.95; (c) ± 1.96; (d) 1 m s^{-2}

Q2 (retrieval from Week 20) (5 marks). A lift carries a particle of mass 20 kg. Find the support force when accelerating (i) upward at 2 m/s², (ii) downward at 2 m/s². Interpret the difference.

Answer: Up 236 N; down 156 N. Apparent weight changes with acceleration.

- Form and solve the upward-acceleration force equation: 2
- Form and solve the downward-acceleration force equation: 2
- Interpret the change in support force: 1

Q3 (retrieval from Week 21) (7 marks). Masses 18 kg and 20 kg hang over a smooth pulley on a light inextensible string. Find acceleration and tension, drawing separate force diagrams. State the common-acceleration assumption.

Answer: a=0.51579 m/s^2, heavier mass down; T=185.68 N. Equations 20g-T=20a and T-18g=18a; string taut/inextensible.

- Draw separate force diagrams: 2
- Form both equations of motion: 2
- Solve for acceleration and tension: 2
- State the linked-acceleration assumption: 1

Q4 (retrieval from Week 22) (5 marks). A particle has velocity $v(t) = t^2 - 19t + 18$ for 0≤t≤19. Find acceleration, stationary times and intervals of forward/backward motion.

Answer: Acceleration $2t - 19$; v=0 at 1,18; forward [0,1) and (18,19], backward (1,18).

- Differentiate to find acceleration: 1
- Find stationary times: 2
- Use velocity signs to state the motion intervals: 2

Q5 (retrieval from Week 39) (5 marks). P(A)=0.6, P(B)=0.4, P(A∩B)=0.36. Find P(A|B), P(B|A), P(A|not B) and decide whether the events are independent.

Answer: $\frac{9}{10}$, $\frac{3}{5}$, $\frac{2}{5}$. Independent iff intersection=0.24; here no.

- Calculate the two direct conditional probabilities: 2
- Calculate the conditional probability given not B: 2
- Test independence using the intersection and product: 1

Q6 (retrieval from Week 40) (4 marks). A normal variable has 10th percentile 38 and 90th percentile 58. Find its mean and standard deviation, explaining the symmetry used.

Answer: Mean 48; sd=10/Φ^(-1)(0.9)≈7.803. Symmetric equal-tail quantiles.

- Use symmetry to find the mean: 1
- Use an appropriate standard-normal quantile equation: 2
- Find the standard deviation: 1

Q7 (retrieval from Week 41) (4 marks). Find the two-sided 5% critical values for the sample mean when n=25, σ=5 and H0:μ=118. Explain the effect of quadrupling n.

Answer: Reject outside 116.04, 119.96; quadrupling n halves SE and the critical distance from μ0.

- Use the two-sided normal critical quantile: 1
- Find both critical values for the sample mean: 2
- Explain how quadrupling sample size changes the standard error and distances: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (16 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 52] State the principle of moments. – 2 marks. Answer: clockwise moments equal anticlockwise moments in equilibrium. Award for: Correct method (1); accurate answer (1).
- (b) [Week 53] Find the normal reaction for 5 kg on a smooth plane at 40°. – 2 marks. Answer: 37.5 N. Award for: Correct method (1); accurate answer (1).
- (c) [Week 54] A ball is projected from horizontal ground with horizontal velocity 10 m/s and upward vertical velocity 14 m/s. Find its greatest height above the ground. – 2 marks. Answer: 10 m. Award for: Correct method (1); accurate answer (1).
- (d) [Week 55] Find the acceleration of 2 kg down a smooth 45° slope. – 2 marks. Answer: 6.93 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (e) [Week 6] State one advantage of the median over the mean. – 2 marks. Answer: not affected by extreme values. Award for: Correct method (1); accurate answer (1).
- (f) [Week 10] Two fair dice: find P(double). – 2 marks. Answer: $\frac{1}{6}$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 11] $X \sim B(30, 0.1)$. Given $P(X = 0) = 0.0424$, state the 5% lower critical region. – 2 marks. Answer: $X = 0$. Award for: Correct method (1); accurate answer (1).
- (h) [Week 24] State what interpolation means. – 2 marks. Answer: estimating within the data range. Award for: Correct method (1); accurate answer (1).

Answers: (a) clockwise moments equal anticlockwise moments in equilibrium; (b) 37.5 N; (c) 10 m; (d) 6.93 m s^{-2}; (e) not affected by extreme values; (f) $\frac{1}{6}$; (g) $X = 0$; (h) estimating within the data range

Q2 (exam style) (8 marks). For 10 pairs of data, r = −0.62.

- (a) Test at the 5% level for negative correlation. The critical value is 0.5494. – 3 marks. Award for: Test
- (b) The regression line is y = 45.2 − 1.3x. Interpret the gradient. – 1 marks. Award for: Context
- (c) Explain why this line should not be used to predict x. – 1 marks. Award for: Direction
- (d) A plot of ln y against x gives ln y = 4.1 − 0.06x. Find the model in the form $y = ae^{bx}$. – 3 marks. Award for: Exponentiate

Answers: (a) |−0.62| > 0.5494: reject H_0, evidence of negative correlation. (b) y falls 1.3 per unit x. (c) It predicts y from x. (d) y = 60.3e^(−0.06x).

Q3 (exam style) (8 marks). A 5 kg block on a rough plane inclined at 25° (μ = 0.3) is attached by a string over a smooth pulley at the top to a 4 kg mass hanging freely.

- (a) Find the acceleration. – 4 marks. Award for: Equations of motion
- (b) Find the tension. – 2 marks. Award for: Substitute
- (c) Find the magnitude of the force on the pulley. – 2 marks. Award for: Angle 65° between strings

Answers: (a) ≈ 0.574 m/s^2. (b) ≈ 36.9 N. (c) 2T cos 32.5° ≈ 62.2 N.

Q4 (exam style) (8 marks). Times X minutes are N(μ, σ^2). 15% exceed 50 minutes and 30% are under 40 minutes.

- (a) Find μ and σ. – 5 marks. Award for: Simultaneous equations
- (b) Given a time exceeds 45 minutes, find the probability it exceeds 50. – 3 marks. Award for: Conditional

Answers: (a) μ ≈ 43.4, σ ≈ 6.41. (b) ≈ 0.376.

Week 64: Exam practice: Pure reasoning and transfer

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 42 marks

Q1 (basic skills) (12 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 34] Expand $(8+x)^{\frac{1}{3}}$ up to x. – 3 marks. Answer: $2+\frac{x}{12}$. Award for: Correct method (2); accurate answer (1).
- (b) [Week 35] Approximate $\cos 3\theta$ for small θ. – 3 marks. Answer: $1-\frac{9\theta^2}{2}$. Award for: Correct method (2); accurate answer (1).
- (c) [Week 36] Solve $\cot x = 1$ for $0 \le x \le 360^\circ$. – 3 marks. Answer: $45^\circ, 225^\circ$. Award for: Correct method (2); accurate answer (1).
- (d) [Week 37] Simplify $\frac{2\tan x}{1-\tan^2 x}$. – 3 marks. Answer: $\tan 2x$. Award for: Correct method (2); accurate answer (1).

Answers: (a) $2+\frac{x}{12}$; (b) $1-\frac{9\theta^2}{2}$; (c) $45^\circ, 225^\circ$; (d) $\tan 2x$

Q2 (retrieval from Week 38) (4 marks). Temperature is modelled by T=32+12sin(πt/12), t hours after midnight. Find period, range and times on 0≤t<24 when T=32. Give one limitation.

Answer: Period24h; range20 to 44; t=0,12. Ideal periodic model ignores day-to-day weather and other influences.

- Find the period: 1
- Find the range: 1
- Find all requested times in the interval: 1
- Give a relevant model limitation: 1

Q3 (retrieval from Week 44) (5 marks). x=12cos t, y=12sin t, 0≤t≤π. Eliminate t with the correct restriction, find dy/dx and the tangent at t=π/4.

Answer: $x^2+y^2=144$, y≥0; dy/dx=-cot t; tangent x+y=12sqrt2.

- Eliminate the parameter and retain the semicircle restriction: 2
- Find dy/dx: 2
- Find the tangent: 1

Q4 (retrieval from Week 48) (7 marks). A model has dP/dt=0.2(100-P), P(0)=22. Solve and find when P first reaches90. Interpret its limiting value.

Answer: P=100-(78)e^(-0.2t); t=5ln((78)/10)≈10.271. Limit100.

- Separate variables and integrate: 2
- Apply the initial condition to obtain the model: 2
- Solve for the target time: 2
- Interpret the limiting value: 1

Q5 (retrieval from Week 49) (5 marks). Apply Newton–Raphson to x^3-x-13=0 starting at x0=2. Calculate x1,x2 and state the iteration formula. Give one possible cause of failure in general.

Answer: xnext=x-(x^3-x-13)/(3x^2-1); x1=2.6364, x2=2.501. Near-zero derivative can cause divergence/undefined step.

- State the correct Newton-Raphson iteration: 2
- Calculate both iterates: 2
- Give a valid possible cause of failure: 1

Q6 (retrieval from Week 50) (4 marks). A=(0,1,12), B=(2,3,14), C=(5,6,17). Prove collinearity. Find ratio AB:BC and the midpoint of AC.

Answer: AB=(2,2,2), BC=(3,3,3); ratio2:3; midpoint=(5/2,7/2, $\frac{29}{2}$).

- Find appropriate difference vectors and prove collinearity: 2
- Give the segment ratio: 1
- Find the midpoint: 1

Q7 (retrieval from Week 34) (5 marks). Find coefficients of x and x^2 in $(1+2x)(1-12x)^{-2}$, stating the validity condition.

Answer: x coefficient 26, x^2 coefficient 480; |x|<1/12.

- Obtain the needed binomial terms: 2
- Collect the linear and quadratic coefficients of the product: 2
- State the validity interval: 1

Practice 2 (Day 4) – total 41 marks

Q1 (exam style) (16 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 38] Write $\sqrt{3}\sin x + \cos x$ as $R\sin(x+\alpha)$. – 2 marks. Answer: $2\sin(x+30^\circ)$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 44] $x = 2t$, $y = t^2$. Find $\frac{dy}{dx}$. – 2 marks. Answer: t. Award for: Correct method (1); accurate answer (1).
- (c) [Week 48] Solve $\frac{dy}{dx} = 6x^2y$, $y = 2$ at $x = 0$. – 2 marks. Answer: $y = 2e^{2x^3}$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 49] Show $x^2 - 3 = 0$ has a root in $[1.7, 1.8]$. – 2 marks. Answer: $-0.11, 0.24$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 50] Find $|\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}|$. – 2 marks. Answer: 3. Award for: Correct method (1); accurate answer (1).
- (f) [Week 8] Find all x in $0 \le x \le 360^\circ$ with $\sin x = -\frac{\sqrt{2}}{2}$. – 3 marks. Answer: $225^\circ, 315^\circ$. Award for: Correct method (2); accurate answer (1).
- (g) [Week 9] Solve $\cos 2x = 0$ for $0 \le x \le 360^\circ$. – 3 marks. Answer: $45^\circ, 135^\circ, 225^\circ, 315^\circ$. Award for: Correct method (2); accurate answer (1).

Answers: (a) $2\sin(x+30^\circ)$; (b) t; (c) $y = 2e^{2x^3}$; (d) $-0.11, 0.24$; (e) 3; (f) $225^\circ, 315^\circ$; (g) $45^\circ, 135^\circ, 225^\circ, 315^\circ$

Q2 (exam style) (8 marks).

- (a) Expand $\sqrt{4+x}$ up to the term in x^2. – 4 marks. Award for: Factor and expand
- (b) State the range of validity. – 1 marks. Award for: |x| < 4
- (c) Use your expansion to estimate √3.96. – 2 marks. Award for: x = −0.04
- (d) Explain why x = 5 would not be appropriate. – 1 marks. Award for: Outside range

Answers: (a) 2 + x/4 − x²/64. (b) |x| < 4. (c) ≈ 1.98998. (d) The expansion is not valid there.

Q3 (exam style) (8 marks).

- (a) Show that for small θ, $\frac{1-\cos 4\theta}{\theta \tan 2\theta} \approx 4$. – 3 marks. Award for: Approximations
- (b) Prove from first principles that the derivative of sin x is cos x. – 5 marks. Award for: Compound angle and limits

Answers: (a) $8\theta^2/(2\theta^2) = 4$. (b) Use sin(x + h) = sin x cos h + cos x sin h with sin h/h → 1 and (cos h − 1)/h → 0.

Q4 (exam style) (9 marks).

- (a) Solve $3\sec^2\theta + 5\tan\theta = 5$ for 0 < θ < 2π. – 4 marks. Award for: Identity and four solutions
- (b) Prove that tan θ + cot θ ≡ 2 cosec 2θ. – 3 marks. Award for: Identity
- (c) Hence solve tan θ + cot θ = 4 for 0 < θ < 2π. – 2 marks. Award for: sin 2θ = 1/2

Answers: (a) θ ≈ 0.322, 2.034, 3.463, 5.176. (b) 1/(sin θ cos θ). (c) π/12, 5π/12, 13π/12, 17π/12.

Week 65: Exam practice: Pure and final repairs

Pearson P2 chapter 1–12.

Practice 1 (Day 2) – total 42 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 43] $A = x^2$, $\frac{dx}{dt} = 2$. Find $\frac{dA}{dt}$ at $x = 3$. – 2 marks. Answer: 12. Award for: Correct method (1); accurate answer (1).
- (b) [Week 45] Find $\int \frac{5}{x}\,dx$. – 2 marks. Answer: $5\ln|x| + c$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 46] Find $\int x\cos 2x\,dx$. – 2 marks. Answer: $\frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 47] Evaluate $\int_1^3 \frac{1}{x}\,dx$. – 2 marks. Answer: $\ln 3$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 12; (b) $5\ln|x| + c$; (c) $\frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$; (d) $\ln 3$

Q2 (retrieval from Week 18) (6 marks). A model is N=1300e^(-0.2t), t in hours. Find N after 5 hours, its half-life and when N first falls below 260. Explain why the model never reaches zero.

Answer: N(5)=478.24; half-life=3.4657 h; threshold t>8.0472 h; exponential always positive.

- Calculate the amount after the stated time: 1
- Find the half-life: 2
- Solve the threshold inequality: 2
- Explain why zero is never reached: 1

Q3 (retrieval from Week 19) (5 marks). A=(1,13), B=(5,16). Find AB, its magnitude and a unit vector from A to B. Find C such that B is the midpoint of AC.

Answer: AB=(4,3), magnitude5, unit=(4/5,3/5); C=(9,19).

- Find the displacement vector: 1
- Find its magnitude and unit vector: 2
- Use the midpoint condition to find C: 2

Q4 (retrieval from Week 31) (4 marks). Divide $x^3 + 13x^2 + 2x + 1$ by x+1, giving quotient and remainder. Use the remainder theorem to check.

Answer: Quotient $x^2 + 12x - 10$; remainder 11=p(-1).

- Carry out polynomial division and give the quotient: 2
- Give the remainder: 1
- Check using the remainder theorem: 1

Q5 (retrieval from Week 32) (8 marks). Let f(x)=(x-13)^2+1 on x≥13. Find its inverse, its domain/range and the fixed point(s) of f if any. Explain why the restriction is needed.

Answer: Inverse 13+sqrt(x-1), x≥1; inverse range x≥13. Fixed points solve x^2-(27)x+170=0 subject to x≥13; roots 10, 17; only the upper root satisfies the restricted domain. Restriction makes f one-to-one.

- Rearrange and choose the correct inverse branch: 2
- Give inverse domain and range: 2
- Solve the fixed-point quadratic: 2
- Filter using the original domain: 1
- Explain the need for a one-to-one restriction: 1

Q6 (retrieval from Week 33) (5 marks). A geometric series starts at 13 with ratio 0.8. Find S10 and its sum to infinity. Find the least n with the remainder after n terms less than 0.1.

Answer: S10=58.021; S∞=65; need 65(0.8)^n<0.1, least n=30.

- Find the finite geometric sum: 1
- Find the infinite sum: 1
- Form the remainder inequality: 1
- Use logarithms correctly: 1
- Choose the least integer satisfying the strict inequality: 1

Q7 (retrieval from Week 42) (6 marks). Differentiate $y = \frac{\sin(13x)}{x+1}$. State its domain and evaluate y'(0), checking its plausibility using the local sine approximation.

Answer: Derivative $\frac{13\cos(13x)}{x+1} - \frac{\sin(13x)}{(x+1)^2}$; x≠-1; y'(0)=13. sin(ax)≈ax near0.

- Use the quotient and chain rules to obtain the derivative: 3
- State the domain: 1
- Evaluate the derivative at zero: 1
- Check plausibility using the local sine approximation: 1

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (11 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 43] Find $\frac{dy}{dx}$ for $y = 3^x$. – 3 marks. Answer: $3^x \ln 3$. Award for: Correct method (2); accurate answer (1).
- (b) [Week 45] Find $\int (4x+1)^3\,dx$. – 2 marks. Answer: $\frac{(4x+1)^4}{16} + c$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 46] Evaluate $\int_1^2 \frac{1}{x}\,dx$. – 2 marks. Answer: $\ln 2$. Award for: Correct method (1); accurate answer (1).

- (d) [Week 47] Evaluate $\int_0^2 e^x\,dx$. – 2 marks. Answer: $e^2 - 1$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 15] The curve $y = x^3 - 12x + 1$ has stationary points $(2, -15)$ and $(-2, 17)$. Use $\frac{d^2y}{dx^2}$ to classify them. – 2 marks. Answer: $(2, -15)$ min, $(-2, 17)$ max. Award for: Correct method (1); accurate answer (1).

Answers: (a) $3^x \ln 3$; (b) $\frac{(4x+1)^4}{16} + c$; (c) $\ln 2$; (d) $e^2 - 1$; (e) $(2, -15)$ min, $(-2, 17)$ max

Q2 (exam style) (9 marks). A solid consists of a cylinder of radius r cm and height h cm with a hemisphere on top. Its volume is 288π cm³.

- (a) Show that its surface area is $S = \frac{576\pi}{r} + \frac{5}{3}\pi r^2$. – 4 marks. Award for: Volume, h and area
- (b) Find r for which S is minimised. – 3 marks. Award for: Differentiate
- (c) Justify that this is a minimum. – 1 marks. Award for: Second derivative
- (d) Find the minimum surface area. – 1 marks. Award for: Substitute

Answers: (a) h = 288/r² − 2r/3. (b) r = 172.8^(1/3) ≈ 5.57. (c) S″ > 0. (d) ≈ 487 cm².

Q3 (exam style) (10 marks).

- (a) Prove that $x^2 - 6x + 10 > 0$ for all real x. – 2 marks. Award for: Completed square
- (b) Prove that there is no greatest even integer. – 2 marks. Award for: Contradiction
- (c) Prove that the sum of the squares of two consecutive integers is odd. – 2 marks. Award for: Algebra
- (d) Disprove: '$2^n + 1$ is prime for every positive integer n'. – 1 marks. Award for: Counter-example
- (e) Prove that if n is not divisible by 3, then $n^2 - 1$ is divisible by 3. – 3 marks. Award for: Cases

Answers: (a) $(x - 3)^2 + 1 > 0$. (b) If N is greatest, N + 2 is larger and even. (c) $2n^2 + 2n + 1$. (d) n = 3: 9. (e) n = 3k ± 1 gives $3(3k^2 \pm 2k)$.

Q4 (exam style) (9 marks). $S_n = \sum_{r=1}^{n} \ln(2^r)$.

- (a) Show that the series is arithmetic and find S_n. – 3 marks. Award for: Common difference and sum
- (b) Find the least n for which $S_n > 100$. – 3 marks. Award for: Quadratic inequality
- (c) Explain why the series does not converge. – 1 marks. Award for: Terms increase
- (d) Find $\sum_{r=1}^{\infty} (\ln 2)^r$. – 2 marks. Award for: Geometric

Answers: (a) S_n = n(n + 1)ln 2/2. (b) n = 17. (c) Terms grow without bound. (d) ln 2/(1 − ln 2) ≈ 2.26.

Week 66: Exam practice: applied and final repairs

Pearson S2 chapter 1–8.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (9 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 52] A uniform horizontal rod is 2 m long, weighs 30 N and is supported at both ends. A 10 N load hangs 0.5 m from the left end. Find the right reaction. – 3 marks. Answer: 17.5 N. Award for: Correct method (2); accurate answer (1).
- (b) [Week 53] A block is on the point of sliding at 35°. Find μ. – 2 marks. Answer: 0.700. Award for: Correct method (1); accurate answer (1).
- (c) [Week 54] A stone is thrown horizontally at 8 m s^{-1} from 10 m. Find the time to land. – 2 marks. Answer: 1.43 s. Award for: Correct method (1); accurate answer (1).
- (d) [Week 55] State the condition for limiting friction. – 2 marks. Answer: $F = \mu R$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 17.5 N; (b) 0.700; (c) 1.43 s; (d) $F = \mu R$

Q2 (retrieval from Week 51) (6 marks). Acceleration is $\mathbf{a} = 2\mathbf{i} + 14t\mathbf{j}$. At t=0, velocity=(1,-2), position=(3,4). Find velocity and position functions, applying both vector constants.

Answer: v=(1+2t,-2+14t^2/2); r=(3+t+t^2,4-2t+14t^3/6).

- Integrate both acceleration components: 2
- Apply the initial velocity: 1
- Integrate both velocity components: 2
- Apply the initial position: 1

Q3 (retrieval from Week 52) (5 marks). A light beam length4 m is supported at x=0 and x=3. A 14 kg load acts at x=4 and an unknown mass m at x=1. Find the minimum m for both reactions to be nonnegative; interpret the boundary.

Answer: Moments about x=3 give2mg≥14g, so m≥7. At equality left reaction0, on point of tipping.

- Form a suitable moment or reaction equation: 2
- Apply the nonnegative-reaction condition: 1
- Find the minimum mass: 1
- Interpret the zero-reaction tipping boundary: 1

Q4 (retrieval from Week 53) (6 marks). A 14 kg particle is on a rough slope at30°, μ=0.4. Determine whether it slides from rest, and if so find acceleration. Draw all forces and justify friction direction.

Answer: Downhill component=68.6 N; maximum friction=47.527 N. It slides down, with acceleration g(1/2-.4cos30°)≈1.5052 m/s^2.

- Draw the forces and justify the friction direction: 2
- Resolve perpendicular to the slope: 1
- Compare downhill force with limiting friction: 1
- Use Newton's second law to find acceleration: 2

Q5 (retrieval from Week 54) (5 marks). A particle is launched horizontally at 24 m/s from height20 m. Find impact time, horizontal distance and impact speed.

Answer: Time=2.0203 s; range=48.487 m; speed=31.113 m/s.

- Use vertical motion to find impact time: 2
- Find horizontal distance: 1
- Find vertical impact velocity: 1
- Combine components to find impact speed: 1

Q6 (retrieval from Week 55) (5 marks). A 14 kg mass lies on a rough table (μ=0.2), connected to a hanging 16 kg mass over a smooth pulley. Find acceleration and tension when the hanging mass descends.

Answer: a=g[(16)-.2(14)]/(30)=4.312; T=14(a+.2g)=87.808 N.

- Find reaction and friction for the table mass: 1
- Form the two equations of motion: 2
- Solve for acceleration: 1
- Obtain the tension: 1

Q7 (retrieval from Week 10) (4 marks). A discrete variable takes values 0,1,2 with probabilities k,2k,14k. Find k, P(X≥1) and P(X=2 given X≥1), using an event ratio rather than a memorised rule.

Answer: k=$\frac{1}{17}$; P≥1=$\frac{16}{17}$; conditional=$\frac{7}{8}$.

- Normalise the probabilities to find k: 1
- Find the probability of X at least one: 1
- Set up and evaluate the conditional event ratio: 2

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (15 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 53] State the direction of friction on a block about to slide down a plane. – 3 marks. Answer: up the plane. Award for: Correct method (2); accurate answer (1).
- (b) [Week 54] State the acceleration of a projectile at its highest point. – 3 marks. Answer: 9.8 m s^{-2} downwards. Award for: Correct method (2); accurate answer (1).
- (c) [Week 55] Find the weight of a 70 kg person. – 3 marks. Answer: 686 N. Award for: Correct method (2); accurate answer (1).

- (d) [Week 6] Quartiles are 20 and 32. Find the upper outlier limit using $1.5\times$IQR. – 3 marks. Answer: 50. Award for: Correct method (2); accurate answer (1).
- (e) [Week 10] Explain what mutually exclusive means. – 3 marks. Answer: the events cannot both happen. Award for: Correct method (2); accurate answer (1).

Answers: (a) up the plane; (b) 9.8 m s^{-2} downwards; (c) 686 N; (d) 50; (e) the events cannot both happen

Q2 (exam style) (8 marks). A ball is projected at 20 m/s and passes through a point 20 m horizontally and 5 m vertically from the point of projection.

- (a) Find the possible angles of projection. – 5 marks. Award for: Trajectory quadratic
- (b) For the smaller angle, find the time to reach the point. – 2 marks. Award for: Horizontal motion
- (c) Suggest one improvement to the model. – 1 marks. Award for: Air resistance

Answers: (a) ≈ 74.1° or 30.0°. (b) ≈ 1.15 s. (c) Include air resistance.

Q3 (exam style) (8 marks). A biased die shows six with probability p. Rolls are independent. In 10 rolls, P(no sixes) = 0.1074 to 4 decimal places.

- (a) Show that $p = 0.2$ to 1 decimal place. Use $p = 0.2$ in the remaining parts. – 2 marks. Award for: Solve
- (b) Find P(at least 3 sixes in 10 rolls). – 2 marks. Award for: Binomial
- (c) In 50 rolls, there are 16 sixes. Test at the 5% level whether p > 0.2. – 4 marks. Award for: Test

Answers: (a) $(1 - p)^{10} = 0.1074$. (b) 0.3222. (c) $P(X \geq 16) = 0.0308 < 0.05$: reject H_0.

Q4 (exam style) (8 marks). A particle of weight 30 N hangs from two strings inclined at 30° and 60° to the horizontal.

- (a) Find both tensions. – 4 marks. Award for: Resolve
- (b) The particle is then placed on a rough plane at 20° (μ = 0.4). Find the least horizontal force needed to keep it in equilibrium. – 4 marks. Award for: Limiting friction

Answers: (a) 15 N and $15\sqrt{3} \approx 26.0$ N. (b) $\tan 20° = 0.364 < 0.4$: friction alone holds it, so the least force is 0.