Cheltenham-Maths: 18-month tutor answers and marking guide, Booklet 1 (Weeks 1–33)

Covers Booklet 1 (Weeks 1–33). Each practice is about 40 marks (one hour). These are original teaching questions with suggested marks, not an official Pearson or Edexcel mark scheme. Award method marks for correct, clearly shown methods even after an earlier slip; accuracy marks need the correct value.

Basic-skills parts: 2-mark parts are 1 method + 1 accuracy; 3-mark parts are 2 method + 1 accuracy.

Week 1: Algebra, indices and surds

Pearson P1 chapter 1.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Index laws. Simplify each expression.

- (a) Simplify $x^5 \times x^3$. – 2 marks. Answer: x^8. Award for: Correct method (1); accurate answer (1).
- (b) Simplify $(3x^4)^2$. – 2 marks. Answer: $9x^8$. Award for: Correct method (1); accurate answer (1).
- (c) Simplify $\frac{20x^7}{4x^3}$. – 2 marks. Answer: $5x^4$. Award for: Correct method (1); accurate answer (1).
- (d) Evaluate $16^{\frac{3}{4}}$. – 2 marks. Answer: 8. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Surds. Simplify fully.

- (a) Simplify $\sqrt{50}$. – 2 marks. Answer: $5\sqrt{2}$. Award for: Correct method (1); accurate answer (1).
- (b) Simplify $\sqrt{12} + \sqrt{75}$. – 2 marks. Answer: $7\sqrt{3}$. Award for: Correct method (1); accurate answer (1).
- (c) Expand and simplify $(3 + \sqrt{2})(3 - \sqrt{2})$. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (d) Expand and simplify $(2 + \sqrt{5})^2$. – 2 marks. Answer: $4\sqrt{5} + 9$. Award for: Correct method (1); accurate answer (1).
- (e) Rationalise the denominator of $\frac{6}{\sqrt{3}}$. – 3 marks. Answer: $2\sqrt{3}$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). (a) Simplify $3\sqrt{12} - \sqrt{27} + 2\sqrt{3}$. (b) Rationalise $\frac{3}{2+\sqrt{3}}$. (c) Verify (b) by multiplication.

Answer: (a) $5\sqrt{3}$. (b) $6 - 3\sqrt{3}$. (c) Product with the original denominator is 3.

- Simplify the surds, with correct intermediate simplification: 2
- Rationalise using the conjugate and simplify: 2
- Verify by multiplying by the original denominator: 1

Q4 (5 marks). For positive x, y, simplify $\frac{(3x^{3/2}y^{-1})^2}{x^{1/2}y^{-3}}$. Evaluate it at x=4, y=2. Explain why the positivity condition matters.

Answer: $9x^{5/2}y$; value 576. Fractional powers/denominators require domain care.

- Apply the outer power correctly: 1
- Combine the powers of x: 1
- Combine the powers of y and the numerical coefficient: 1
- Evaluate at the given values: 1
- Explain the real-domain and nonzero-denominator restrictions: 1

Q5 (6 marks). Solve $(3)^{2x-1} = (3)^5$. Then solve $x^{2/3} = 9$, over the real numbers. Explain why the second equation has two solutions.

Answer: x=3; the second gives $\{-27, 27\}$. Cube roots may be positive or negative before squaring.

- Equate exponents and solve the exponential equation: 2
- Use the real cube root to reduce the fractional-power equation: 1
- Find both real roots: 2
- Explain why squaring a cube root permits both signs: 1

Q6 (5 marks). A student claims $\sqrt{x^2 + 9} = x + 3$ for every real x. Disprove this. Solve the equation where it does hold and check the domain of the square root.

Answer: x=1 is a counterexample. Squaring gives 6x=0, so only x=0; both sides then equal 3. The square root is defined for all real x.

- Give a valid counterexample: 1
- Square and solve the resulting equation: 2
- Check the candidate in the original equation: 1
- State and justify the square-root domain: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Expanding and factorising.

- (a) Expand and simplify $(x + 4)(x - 7)$. – 2 marks. Answer: $x^2 - 3x - 28$. Award for: Correct method (1); accurate answer (1).

- (b) Expand and simplify $(2x-3)^2$. – 2 marks. Answer: $4x^2-12x+9$. Award for: Correct method (1); accurate answer (1).
- (c) Expand and simplify $(x+1)(x-2)(x+3)$. – 2 marks. Answer: x^3+2x^2-5x-6. Award for: Correct method (1); accurate answer (1).
- (d) Factorise fully $x^2+8x+15$. – 2 marks. Answer: $(x+3)(x+5)$. Award for: Correct method (1); accurate answer (1).
- (e) Factorise fully $6x^2-x-2$. – 3 marks. Answer: $(2x+1)(3x-2)$. Award for: Correct method (2); accurate answer (1).
- (f) Factorise fully $2x^3-50x$. – 3 marks. Answer: $2x(x-5)(x+5)$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). Do not use a calculator for this question.

- (a) Express $\frac{5+\sqrt{3}}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers. – 3 marks. Award for: Multiply by the conjugate, expand and simplify to 13 + 7√3
- (b) Hence find $\left(\frac{5+\sqrt{3}}{2-\sqrt{3}}\right)^2$ in the form $c+d\sqrt{3}$. – 2 marks. Award for: Square their answer correctly, collecting surd terms
- (c) A rectangle has area $(13+7\sqrt{3})$ cm² and width $(2+\sqrt{3})$ cm. Find its exact length in the form $p+q\sqrt{3}$, explaining how part (a) helps. – 3 marks. Award for: Divide and rationalise, or use (a) in reverse, to obtain 5 + √3 with a reason

Answers: (a) 13 + 7√3. (b) 316 + 182√3. (c) 5 + √3 cm, since (5 + √3)(2 + √3) = 13 + 7√3 by reversing (a).

Q3 (exam style) (8 marks). Let $y=2^x$.

- (a) Express 4^x and 2^{x+1} in terms of y. – 2 marks. Award for: State 4^x = y² and 2^(x+1) = 2y
- (b) Hence solve $4^x-3\times2^{x+1}+8=0$. – 4 marks. Award for: Form y² − 6y + 8 = 0, solve for y, and convert both values to x
- (c) Explain why $4^x+2^{x+1}+3=0$ has no real solutions. – 2 marks. Award for: Use y > 0 or a discriminant/completed square to justify no solutions

Answers: (a) 4^x = y², 2^(x+1) = 2y. (b) y = 2 or 4, so x = 1 or 2. (c) y² + 2y + 3 = (y + 1)² + 2 > 0; also every term is positive because 2^x > 0.

Q4 (exam style) (9 marks). The expression $f(x)=\frac{2x^2-16\sqrt{x}}{x\sqrt{x}}$ is defined for suitable real x.

- (a) Write f(x) in the form Ax^p+Bx^q, where A, B, p and q are constants. – 3 marks. Award for: Rewrite the denominator as x^(3/2) and divide each term correctly
- (b) Hence solve f(x) = 0. – 3 marks. Award for: Form 2x^(1/2) = 16x^(−1), rearrange to x^(3/2) = 8 and solve
- (c) State the set of x for which f(x) is defined, giving a reason. – 2 marks. Award for: Recognise x ≥ 0 for √x and x√x ≠ 0, so x > 0
- (d) Find the exact value of f(16). – 1 marks. Award for: Substitute into the simplified form accurately

Answers: (a) f(x) = 2x^(1/2) − 16x^(−1). (b) x = 4. (c) x > 0: √x needs x ≥ 0 and the denominator cannot be zero. (d) f(16) = 8 − 1 = 7.

Week 2: Quadratics, equations and inequalities

Pearson P1 chapter 2–3.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Solving quadratic equations.

- (a) Solve $x^2 - 7x + 12 = 0$. – 2 marks. Answer: $x = 3, x = 4$. Award for: Correct method (1); accurate answer (1).
- (b) Solve $2x^2 + 5x - 3 = 0$. – 2 marks. Answer: $x = -3, x = \frac{1}{2}$. Award for: Correct method (1); accurate answer (1).
- (c) Solve $x^2 = 6x$. – 2 marks. Answer: $x = 0, x = 6$. Award for: Correct method (1); accurate answer (1).
- (d) Solve $x^2 - 4x - 1 = 0$, giving exact answers. – 2 marks. Answer: $x = 2 - \sqrt{5}, x = 2 + \sqrt{5}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Completing the square and the discriminant.

- (a) Write $x^2 + 6x + 2$ in the form $(x + b)^2 + c$. – 2 marks. Answer: $(x + 3)^2 - 7$. Award for: Correct method (1); accurate answer (1).
- (b) Write $x^2 - 10x + 7$ in the form $(x + b)^2 + c$. – 2 marks. Answer: $(x - 5)^2 - 18$. Award for: Correct method (1); accurate answer (1).
- (c) Write $2x^2 + 8x + 3$ in the form $a(x + b)^2 + c$. – 2 marks. Answer: $2(x + 2)^2 - 5$. Award for: Correct method (1); accurate answer (1).
- (d) Find the discriminant of $x^2 + 5x + 7$ and state the number of real roots. – 2 marks. Answer: -3; no real roots. Award for: Correct method (1); accurate answer (1).

Q3 (6 marks). For $f(x) = x^2 - 8x + 12$, (a) complete the square, (b) find the roots and minimum, (c) sketch, labelling intercepts and vertex.

Answer: $f(x) = (x - 4)^2 - 4$; roots 2,6; minimum -4 at x=4; vertical intercept 12.

- Complete the square: 2
- Find both roots: 1
- State the minimum value and its x-coordinate: 1
- Sketch with correct shape, intercepts and vertex: 2

Q4 (5 marks). Solve $(x - 3)(x - 6) < 0$. Then solve $x^2 - 8x + k = 0$ for the values of k that give (i) two distinct real roots, (ii) a repeated root.

Answer: 3<x<6; k<16 for two distinct roots; k=16 for a repeated root.

- Find the correct interval for the strict inequality: 2

- Form the discriminant condition: 1
- Give the condition for two distinct roots: 1
- Give the condition for a repeated root: 1

Q5 (6 marks). Find all intersections of $y = x^2$ and $y = 4x + 5$. Check each solution in both equations and sketch their relative positions.

Answer: x=-1,5; points (-1,1),(5,25); line above parabola between the roots.

- Equate the expressions and solve for both x-values: 2
- Find both intersection coordinates: 1
- Check in both original equations: 1
- Sketch the relative positions correctly: 2

Q6 (7 marks). A rectangle has perimeter 24 cm. Let one side be x cm. Form its area function, find the maximum area and give the possible x for area greater than 35 square cm.

Answer: A=x(12-x)=$(6)^{2-(x-6)}2$; max 36; 5<x<7. Physical domain 0<x<12.

- Use the perimeter to form an area function: 2
- Find the maximum area: 2
- Solve the strict area inequality: 2
- Respect the physical domain: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (11 marks). Simultaneous equations and inequalities.

- (a) Solve $2x + y = 7$ and $x - y = 2$. – 2 marks. Answer: $x = 3,\ y = 1$. Award for: Correct method (1); accurate answer (1).
- (b) Solve $y = x + 1$ and $x^2 + y^2 = 13$. – 2 marks. Answer: $(2, 3)$ and $(-3, -2)$. Award for: Correct method (1); accurate answer (1).
- (c) Solve $3x - 5 > 2x + 4$. – 2 marks. Answer: $x > 9$. Award for: Correct method (1); accurate answer (1).
- (d) Solve $x^2 - x - 12 < 0$. – 2 marks. Answer: $-3 < x < 4$. Award for: Correct method (1); accurate answer (1).
- (e) Solve $2x^2 + x - 6 \geq 0$. – 3 marks. Answer: $x \leq -2$ or $x \geq \frac{3}{2}$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). $f(x) = 2x^2 - 12x + 23$.

- (a) Write f(x) in the form $a(x + b)^2 + c$. – 3 marks. Award for: Factor out 2, complete the square and find c correctly
- (b) Hence state the coordinates of the minimum point of y = f(x) and explain why f(x) = 0 has no real roots. – 2 marks. Award for: Minimum point and a reason based on the minimum value

- (c) The line y = k meets y = f(x) at exactly one point. State the value of k. – 1 marks. Award for: k equals the minimum value
- (d) Solve the inequality f(x) < 13. – 3 marks. Award for: Use the completed square or a quadratic, find critical values and select the correct interval

Answers: (a) $2(x - 3)^2 + 5$. (b) (3, 5); the minimum value 5 is positive. (c) k = 5. (d) $2(x - 3)^2 < 8$, so $1 < x < 5$.

Q3 (exam style) (9 marks). The equation $kx^2 + (k + 3)x + 1 = 0$ involves a real constant k.

- (a) Show that the discriminant is $k^2 + 2k + 9$. – 2 marks. Award for: Substitute into $b^2 - 4ac$ and simplify
- (b) By completing the square, prove that the equation has two distinct real roots for every non-zero real k. – 3 marks. Award for: Write $(k + 1)^2 + 8$ and argue it is always positive, with $k \neq 0$ for a quadratic
- (c) Explain what happens when k = 0. – 1 marks. Award for: Linear equation with one root −1/3
- (d) Find the value of k for which x = 1 is a root, and find the other root. – 3 marks. Award for: Substitute x = 1, solve for k and factorise the resulting quadratic

Answers: (a) $(k + 3)^2 - 4k = k^2 + 2k + 9$. (b) $(k + 1)^2 + 8 > 0$, so two distinct roots whenever the equation is quadratic. (c) It becomes $3x + 1 = 0$, one root $x = -1/3$. (d) $k = -2$; $-2x^2 + x + 1 = 0$ gives other root $x = -1/2$.

Q4 (exam style) (11 marks). A ball is thrown so that its height, h m, t seconds after release is modelled by $h = 1.5 + 14t - 4.9t^2$.

- (a) Find, to 3 significant figures, the time at which the ball reaches the ground. – 3 marks. Award for: Set h = 0, use the formula and reject the negative root
- (b) By completing the square, find the maximum height and the time at which it occurs. – 4 marks. Award for: Complete the square on the t terms and interpret the vertex
- (c) Find the length of time for which the ball is more than 9.5 m above the ground. – 3 marks. Award for: Form and solve the inequality, then find the difference between the times
- (d) Give one limitation of the model. – 1 marks. Award for: A sensible limitation, e.g. air resistance or validity after landing

Answers: (a) 2.96 s. (b) $h = 11.5 - 4.9(t - 10/7)^2$, maximum 11.5 m at $t = 10/7 \approx 1.43$ s. (c) $|t - 10/7| < \sqrt{(2/4.9)}$, duration ≈ 1.28 s. (d) e.g. ignores air resistance; invalid after the ball lands.

Week 3: Graphs and transformations

Pearson P1 chapter 4.

Practice 1 (Day 2) – total 42 marks

Q1 (basic skills) (8 marks). Sketching curves. For each, state where the graph meets the coordinate axes.

- (a) $y = (x-2)(x+3)$ – 2 marks. Answer: $x = 2,\ x = -3$; $y = -6$. Award for: Correct method (1); accurate answer (1).
- (b) $y = x^2 - 6x + 5$ – 2 marks. Answer: $x = 1,\ x = 5$; $y = 5$. Award for: Correct method (1); accurate answer (1).
- (c) $y = x(x-1)(x+4)$ – 2 marks. Answer: $x = 0, 1, -4$; $y = 0$. Award for: Correct method (1); accurate answer (1).
- (d) $y = (x-3)^2(x+1)$ – 2 marks. Answer: $x = -1$, touches at $x = 3$; $y = 9$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Transformations. The curve $y = f(x)$ has a maximum point at $(2, 5)$. State the coordinates of the maximum on each curve.

- (a) $y = f(x) + 3$ – 2 marks. Answer: $(2, 8)$. Award for: Correct method (1); accurate answer (1).
- (b) $y = f(x - 4)$ – 2 marks. Answer: $(6, 5)$. Award for: Correct method (1); accurate answer (1).
- (c) $y = 2f(x)$ – 2 marks. Answer: $(2, 10)$. Award for: Correct method (1); accurate answer (1).
- (d) $y = f(3x)$ – 2 marks. Answer: $\left(\frac{2}{3}, 5\right)$. Award for: Correct method (1); accurate answer (1).

Q3 (7 marks). Starting with $y = x^2$, describe a sequence of transformations to $y = -2(x-5)^2 + 6$. State vertex, range and intercept with the vertical axis; sketch.

Answer: Translate right 5, stretch vertically by 2, reflect in x-axis, translate up 6; vertex (5,6); y≤6; y-intercept -44.

- Describe the four transformations, with a valid order: 3
- State vertex and range: 2
- Find the vertical intercept: 1
- Sketch the transformed parabola consistently: 1

Q4 (6 marks). Sketch $y = (x+5)(x-1)^2$, identifying roots, repeated roots and the y-intercept. State the sign on each interval between roots and end behaviour.

Answer: Roots -5,1 (double); y-intercept 5; negative for x<-5, positive for x>-5 except zero at 1. Falls to left, rises to right; touches at 1.

- Identify the simple and repeated roots: 2
- Find the y-intercept: 1
- Give signs between roots: 1

- Show both end behaviours and correct crossing/touching in the sketch: 2

Q5 (8 marks). For $y = \frac{5}{x-2} + 1$, give asymptotes, domain and range. Find intercepts and solve y>1, explaining graphically.

Answer: Asymptotes x=2,y=1; domain x≠2, range y≠1; x-intercept -3, y-intercept -3/2; y>1 for x>2.

- State both asymptotes: 2
- State domain and range: 2
- Find both intercepts: 2
- Solve the inequality and explain it using the graph: 2

Q6 (5 marks). A point (2,5) lies on y=f(x). Find its corresponding point on (i) y=3f(x-1), (ii) y=f(2x)+4. Explain why the horizontal coordinate is not multiplied by 2 in (ii).

Answer: (i) (3,15); (ii) (1,9). To obtain the old input 2, the new x must satisfy 2x=2.

- Find the first transformed point: 2
- Find the second transformed point: 2
- Explain the inverse horizontal-coordinate change: 1

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (11 marks). Intersections of graphs.

- (a) Find the points where $y = x^2$ meets $y = x + 6$. – 2 marks. Answer: $(3, 9)$ and $(-2, 4)$. Award for: Correct method (1); accurate answer (1).
- (b) Show that $y = x^2 + 4$ and $y = 2x$ do not meet. – 2 marks. Answer: $x^2 - 2x + 4 = 0$ has discriminant $-12 < 0$. Award for: Correct method (1); accurate answer (1).
- (c) Find the points where $y = x^3$ meets $y = 4x$. – 2 marks. Answer: $(0, 0), (2, 8), (-2, -8)$. Award for: Correct method (1); accurate answer (1).
- (d) How many solutions has $\frac{1}{x} = x^2$? Use a sketch. – 2 marks. Answer: one ($x = 1$). Award for: Correct method (1); accurate answer (1).
- (e) Find the coordinates where $y = \frac{6}{x}$ meets $y = x + 1$. – 3 marks. Answer: $(2, 3)$ and $(-3, -2)$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). $f(x) = (x + 2)(x - 1)^2$. The curve y = f(x) has a local maximum at (−1, 4).

- (a) Sketch y = f(x), showing the coordinates of all points where the curve meets the axes. – 3 marks. Award for: Cubic shape, root at −2, touching at 1, y-intercept 2
- (b) Sketch y = f(x − 2) and state the coordinates where it meets the axes. – 2 marks. Award for: Translation 2 right; roots 0 and 3 (touching), through the origin
- (c) Verify that f(−1) = 4 and hence find the values of k for which f(x) = k has exactly three real roots. – 3 marks. Award for: Verify the maximum value and use the sketch to give 0 < k < 4

- (d) State the number of real roots of f(x) + 4 = 0, giving a reason. – 2 marks. Award for: Interpret as f(x) = −4 below the local minimum, so one root

Answers: (a) Roots x = −2 and x = 1 (touch), y-intercept (0, 2). (b) Meets axes at (0, 0) and touches at (3, 0). (c) f(−1) = 1 × 4 = 4; 0 < k < 4. (d) One root, since y = −4 lies below the local minimum (1, 0) and meets only the left branch.

Q3 (exam style) (11 marks). The curve C has equation $y = \frac{3}{x-2} + 1$.

- (a) Sketch C, stating the equations of its asymptotes and the coordinates of any points where it meets the axes. – 4 marks. Award for: Correct shape, asymptotes x = 2 and y = 1, intercepts (−1, 0) and (0, −1/2)
- (b) Find the exact coordinates of the points where C meets the line y = x − 1. – 4 marks. Award for: Form $(x - 2)^2 = 3$, solve exactly and find both y-values
- (c) Hence solve $\frac{3}{x-2} + 1 > x - 1$. – 3 marks. Award for: Use the sketch or sign analysis, including the asymptote, to give both intervals

Answers: (a) Asymptotes x = 2, y = 1; intercepts (−1, 0), (0, −1/2). (b) (2 + √3, 1 + √3) and (2 − √3, 1 − √3). (c) x < 2 − √3 or 2 < x < 2 + √3.

Q4 (exam style) (9 marks). The point P(3, −2) is a maximum point on the curve y = f(x).

- (a) Find the image of P on y = −f(x) and state whether it is a maximum or minimum. – 2 marks. Award for: Correct image and nature
- (b) Find the image of P on y = f(2x) + 1 and state its nature. – 2 marks. Award for: Correct image and nature
- (c) Find the image of P on y = 3f(x + 1) and state its nature. – 2 marks. Award for: Correct image and nature
- (d) Find the image of P on y = f(−x) − 2 and state its nature. – 2 marks. Award for: Correct image and nature
- (e) The curve y = f(x) is translated by the vector $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$. Write down the equation of the new curve. – 1 marks. Award for: y = f(x + 2) + 5

Answers: (a) (3, 2), minimum. (b) (1.5, −1), maximum. (c) (2, −6), maximum. (d) (−3, −4), maximum. (e) y = f(x + 2) + 5.

Week 4: Straight lines and circles

Pearson P1 chapter 5–6.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Straight lines.

- (a) Find the gradient of the line through $(1, 4)$ and $(5, 12)$. – 2 marks. Answer: 2. Award for: Correct method (1); accurate answer (1).

- (b) Find the equation of the line with gradient 3 through $(2, -1)$, in the form $y = mx + c$. – 2 marks. Answer: $y = 3x - 7$. Award for: Correct method (1); accurate answer (1).
- (c) Find the gradient of $3x + 4y = 12$. – 2 marks. Answer: $-\frac{3}{4}$. Award for: Correct method (1); accurate answer (1).
- (d) Find the midpoint and length of the segment joining $(-2, 3)$ and $(4, 11)$. – 2 marks. Answer: $(1, 7)$; 10. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Circles.

- (a) State the centre and radius of $(x - 3)^2 + (y + 1)^2 = 16$. – 2 marks. Answer: $(3, -1)$, 4. Award for: Correct method (1); accurate answer (1).
- (b) Write the equation of the circle centre $(-2, 5)$, radius 3. – 2 marks. Answer: $(x + 2)^2 + (y - 5)^2 = 9$. Award for: Correct method (1); accurate answer (1).
- (c) Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$. – 2 marks. Answer: $(3, -2)$, 5. Award for: Correct method (1); accurate answer (1).
- (d) Show that $(6, 2)$ lies on $(x - 2)^2 + (y + 1)^2 = 25$. – 2 marks. Answer: $16 + 9 = 25$. Award for: Correct method (1); accurate answer (1).
- (e) Find the equation of the circle with diameter from $(1, 2)$ to $(7, 10)$. – 3 marks. Answer: $(x - 4)^2 + (y - 6)^2 = 25$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). A circle has centre (6,2) and radius 5. Find its equation, its intersections with y=2, and the tangent at (9,6).

Answer: $(x - 6)^2 + (y - 2)^2 = 25$; (1,2),(11,2). Tangent $3(x - 9) + 4(y - 6) = 0$.

- Write the circle equation: 1
- Find the two intersections: 2
- Use the radius to obtain the tangent gradient: 1
- Find the tangent equation: 2

Q4 (6 marks). A=(6,1), B=(10,7). Find AB's length and midpoint, then its perpendicular bisector. Decide whether C=(11,2) is equidistant from A and B.

Answer: Length $2\sqrt{13}$; midpoint (8,4); bisector y-4=-(2/3)(x-8). CA^2=26, CB^2=26, so yes.

- Find length and midpoint: 2
- Find the perpendicular-bisector gradient and equation: 2
- Compare the distances from C and conclude: 2

Q5 (5 marks). Find intersections of $(x - 6)^2 + y^2 = 25$ with y=3. Find the chord length. Explain why y=6 gives no intersection.

Answer: (2,3),(10,3); length 8. Distance 6 from centre exceeds radius 5.

- Substitute the line and find both intersection points: 3
- Find the chord length: 1
- Explain non-intersection for the other line: 1

Q6 (4 marks). The line $y = mx + 6$ passes through (0,6). Find its intersection with x=2 and its perpendicular through (0,0). Discuss separately m=0.

Answer: (2,2m+6); perpendicular y=-x/m for m≠0. If m=0 the perpendicular is x=0.

- Find the intersection in terms of m: 1
- Find the perpendicular for nonzero m: 2
- Handle m=0 separately: 1

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (14 marks). Lines meeting circles.

- (a) Find where the line $y = x$ meets $x^2 + y^2 = 18$. – 2 marks. Answer: $(3, 3), (-3, -3)$. Award for: Correct method (1); accurate answer (1).
- (b) Find where $y = 2$ meets $(x - 1)^2 + (y - 2)^2 = 9$. – 2 marks. Answer: $(4, 2), (-2, 2)$. Award for: Correct method (1); accurate answer (1).
- (c) Show that $y = x + 10$ does not meet $x^2 + y^2 = 25$. – 2 marks. Answer: $2x^2 + 20x + 75 = 0$, discriminant $-200 < 0$. Award for: Correct method (1); accurate answer (1).
- (d) Find where $x + y = 7$ meets $x^2 + y^2 = 25$. – 2 marks. Answer: $(3, 4), (4, 3)$. Award for: Correct method (1); accurate answer (1).
- (e) Find the perpendicular bisector of $A(2, 1)$, $B(6, 5)$. – 3 marks. Answer: $y = -x + 7$. Award for: Correct method (2); accurate answer (1).
- (f) Find the values of c for which $y = x + c$ is a tangent to $x^2 + y^2 = 8$. – 3 marks. Answer: $c = \pm 4$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). The circle C has equation $x^2 + y^2 - 6x + 4y - 12 = 0$.

- (a) Find the centre and radius of C. – 3 marks. Award for: Complete the square in x and y; centre and radius
- (b) Show that A(6, 2) lies on C and find the equation of the tangent to C at A, in the form ax + by + c = 0. – 4 marks. Award for: Verify, find radius gradient, perpendicular gradient and the equation
- (c) The tangent meets the x-axis at T. Find the exact length AT. – 3 marks. Award for: Find T and use the distance formula

Answers: (a) Centre (3, −2), radius 5. (b) 9 + 16 = 25; radius gradient 4/3, tangent 3x + 4y − 26 = 0. (c) T(26/3, 0), AT = 10/3.

Q3 (exam style) (8 marks). The line l has equation y = 2x + k and the circle C has equation $x^2 + y^2 = 20$.

- (a) Show that the x-coordinates of any intersection points satisfy $5x^2 + 4kx + k^2 - 20 = 0$. – 2 marks. Award for: Substitute and expand correctly

- (b) Find the values of k for which l is a tangent to C. – 3 marks. Award for: Set the discriminant to zero and solve
- (c) For the positive value of k, find the point of contact. – 2 marks. Award for: Solve the repeated-root quadratic and find y
- (d) State the set of k for which l does not meet C. – 1 marks. Award for: |k| > 10

Answers: (a) $x^2 + (2x + k)^2 = 20$ gives $5x^2 + 4kx + k^2 - 20 = 0$. (b) $400 - 4k^2 = 0$, $k = \pm 10$. (c) $(-4, 2)$. (d) $k < -10$ or $k > 10$.

Q4 (exam style) (9 marks). The points A(−1, 4), B(5, 2) and C(3, −4) are given.

- (a) Show that triangle ABC has a right angle at B. – 3 marks. Award for: Find gradients of AB and BC and show their product is −1
- (b) Hence find the equation of the circle passing through A, B and C. – 4 marks. Award for: Use AC as diameter: midpoint and radius, then equation
- (c) Find the area of triangle ABC. – 2 marks. Award for: Use the perpendicular sides AB and BC

Answers: (a) Gradients −1/3 and 3, product −1. (b) Centre (1, 0), radius $\sqrt{20}$: $(x - 1)^2 + y^2 = 20$. (c) $AB = BC = \sqrt{40}$, area 20.

Week 5: Circles and polynomial methods

Pearson P1 chapter 6–7.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Algebraic division and the factor theorem.

- (a) Find the remainder when $x^3 + 2x^2 - 5x + 1$ is divided by $(x - 2)$. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (b) Show that $(x + 3)$ is a factor of $x^3 + x^2 - 5x + 3$. – 2 marks. Answer: $f(-3) = -27 + 9 + 15 + 3 = 0$. Award for: Correct method (1); accurate answer (1).
- (c) Divide $x^3 - 6x^2 + 11x - 6$ by $(x - 1)$. – 2 marks. Answer: $x^2 - 5x + 6$. Award for: Correct method (1); accurate answer (1).
- (d) Factorise fully $x^3 - 6x^2 + 11x - 6$. – 2 marks. Answer: $(x - 1)(x - 2)(x - 3)$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Circle geometry.

- (a) Find the gradient of the radius of $(x - 1)^2 + (y - 2)^2 = 25$ to the point $(4, 6)$. – 2 marks. Answer: $\frac{4}{3}$. Award for: Correct method (1); accurate answer (1).
- (b) Hence find the tangent at $(4, 6)$. – 2 marks. Answer: $3x + 4y = 36$. Award for: Correct method (1); accurate answer (1).
- (c) Find the equation of the circle centre $(2, -3)$ passing through $(5, 1)$. – 2 marks. Answer: $(x - 2)^2 + (y + 3)^2 = 25$. Award for: Correct method (1); accurate answer (1).

- (d) Find the length of the tangent from $(10, 0)$ to $x^2 + y^2 = 36$. – 2 marks. Answer: 8. Award for: Correct method (1); accurate answer (1).
- (e) A circle has centre $(3, 4)$ and touches the x-axis. Find its equation. – 3 marks. Answer: $(x - 3)^2 + (y - 4)^2 = 16$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A circle has centre (2, −1) and passes through (5, 3). Find its equation, then find the exact y-coordinates of the points where it meets the y-axis.

Answer: $(x - 2)^2 + (y + 1)^2 = 25$; $y = -1 \pm \sqrt{21}$.

- Find the radius from the two points: 1
- Write the circle equation: 1
- Substitute x = 0: 1
- Solve exactly for both y-values: 2

Q4 (6 marks). Use the factor theorem to show that (x − 2) is a factor of $p(x) = x^3 - 3x^2 - 4x + 12$. Factorise p(x) fully and sketch y = p(x), showing where it meets the axes.

Answer: p(2) = 0; (x − 2)(x − 3)(x + 2); roots −2, 2, 3 and y-intercept 12.

- Show p(2) = 0 and conclude: 1
- Divide or compare coefficients: 2
- Factorise fully: 1
- Sketch with roots and intercept: 2

Q5 (5 marks). Use the remainder theorem to find the remainder when $2x^3 + x^2 - 5x + 3$ is divided by (x + 2) and when it is divided by (2x − 1). Explain why neither is a factor.

Answer: Both remainders are 1, so neither is a factor (a factor gives remainder 0).

- Evaluate f(−2): 2
- Evaluate f(1/2): 2
- Explain the factor condition: 1

Q6 (6 marks). For $p(x) = x^3 + 4x^2 - 19x + 14$, verify that x=1 is a root, divide by x-1 and factorise fully. Solve p(x)≥0 using a sign diagram.

Answer: $p(x) = (x - 1)(x + 7)(x - 2)$; quotient $x^2 + 5x - 14$; -7≤x≤1 or x≥2.

- Verify the stated root: 1
- Divide by the stated factor: 2
- Factorise fully: 1
- Use a sign diagram to obtain all intervals, including the endpoints: 2

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (8 marks). Polynomial graphs.

- (a) Factorise $x^3 - 4x$ and sketch $y = x^3 - 4x$. – 2 marks. Answer: $x(x-2)(x+2)$; roots $0, \pm 2$. Award for: Correct method (1); accurate answer (1).
- (b) Show $x = 1$ is a root of $x^3 - 3x + 2 = 0$ and find the other roots. – 2 marks. Answer: $(x-1)^2(x+2)$; $x = -2$. Award for: Correct method (1); accurate answer (1).
- (c) Find the remainder when $2x^3 - x + 4$ is divided by $(2x + 1)$. – 2 marks. Answer: $f(-\frac{1}{2}) = \frac{17}{4}$. Award for: Correct method (1); accurate answer (1).
- (d) Find the quotient when $2x^3 + 3x^2 - 8x + 3$ is divided by $(x - 1)$. – 2 marks. Answer: $2x^2 + 5x - 3$. Award for: Correct method (1); accurate answer (1).

Q2 (exam style) (11 marks). $f(x) = 2x^3 - 5x^2 - 4x + 3$.

- (a) Use the factor theorem to show that (x − 3) is a factor of f(x). – 2 marks. Award for: Evaluate f(3) = 0 and conclude
- (b) Hence factorise f(x) completely. – 3 marks. Award for: Divide by (x − 3) and factorise the quadratic
- (c) Hence solve f(x) ≤ 0. – 3 marks. Award for: Use roots and the cubic shape to state both intervals
- (d) Hence find all real solutions of $f(x^2) = 0$. – 3 marks. Award for: Set x^2 equal to each valid root and reject $x^2 = -1$

Answers: (a) f(3) = 54 − 45 − 12 + 3 = 0. (b) (x − 3)(2x − 1)(x + 1). (c) x ≤ −1 or 1/2 ≤ x ≤ 3. (d) x = ±√3 or x = ±√2/2.

Q3 (exam style) (11 marks). The circle C has centre (4, 3) and radius 5. The line L has equation y = mx.

- (a) Show that the x-coordinates of the intersections of L and C satisfy $(1 + m^2)x^2 - (8 + 6m)x = 0$. – 3 marks. Award for: Substitute into $(x - 4)^2 + (y - 3)^2 = 25$ and simplify
- (b) Explain why L meets C at the origin for every value of m. – 2 marks. Award for: Show the origin lies on C or that x = 0 is always a root
- (c) Find the value of m for which L is a tangent to C. – 3 marks. Award for: Require the second root to be zero
- (d) For m = 1, find the exact length of the chord cut off by C on L. – 3 marks. Award for: Find the second intersection and its distance from the origin

Answers: (a) $(x - 4)^2 + (mx - 3)^2 = 25$ gives the result. (b) $4^2 + 3^2 = 25$, so O lies on C. (c) 8 + 6m = 0, m = −4/3. (d) Other point (7, 7), chord 7√2.

Q4 (exam style) (10 marks). $p(x) = x^3 + ax^2 + bx - 6$, where a and b are constants. (x + 1) is a factor of p(x), and the remainder when p(x) is divided by (x − 2) is −12.

- (a) Find the values of a and b. – 4 marks. Award for: Form two equations from the factor and remainder theorems and solve
- (b) Factorise p(x) completely. – 2 marks. Award for: Correct full factorisation
- (c) Sketch y = p(x), showing where it meets the axes. – 2 marks. Award for: Correct cubic shape with roots −2, −1, 3 and intercept −6
- (d) Find the number of real solutions of p(x) = x − 6, giving the solutions. – 2 marks. Award for: Reduce to $x^3 - 8x = 0$ and solve

Answers: (a) a − b = 7 and 2a + b = −7, so a = 0, b = −7. (b) (x + 1)(x + 2)(x − 3). (c) Roots −2, −1, 3; y-intercept −6. (d) Three: x = 0, ±2√2.

Week 6: Data collection, spread and representation

Pearson S1 chapter 1–3.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Measures of location and spread. The data are 3, 5, 6, 6, 8, 9, 12, 15.

- (a) Find the mean. – 2 marks. Answer: 8. Award for: Correct method (1); accurate answer (1).
- (b) Find the median. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (c) Find the mode and the range. – 2 marks. Answer: mode 6, range 12. Award for: Correct method (1); accurate answer (1).
- (d) Find the lower and upper quartiles. – 2 marks. Answer: $Q_1 = 5.5,\ Q_3 = 10.5$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Summary statistics.

- (a) Given $n = 10$, $\sum x = 85$, find the mean. – 2 marks. Answer: 8.5. Award for: Correct method (1); accurate answer (1).
- (b) Given also $\sum x^2 = 810$, find the variance. – 2 marks. Answer: $81 - 72.25 = 8.75$. Award for: Correct method (1); accurate answer (1).
- (c) Hence find the standard deviation. – 2 marks. Answer: 2.96. Award for: Correct method (1); accurate answer (1).
- (d) Each value is multiplied by 2 and then 3 is added. State the new mean and standard deviation. – 2 marks. Answer: 20, 5.92. Award for: Correct method (1); accurate answer (1).
- (e) Find S_{xx} for $n = 5$, $\sum x = 30$, $\sum x^2 = 200$. – 3 marks. Answer: $200 - 180 = 20$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A school wants the average journey time of all sixth-form students. Compare a random sample from the full register with asking the first 30 students at the gate. Define population, sampling frame and bias; suggest one practical improvement.

Answer: Population: all sixth-form students. Frame: full register. Gate sample is opportunity sampling and can exclude late/non-gate arrivals. Random selection from the register with follow-up reduces selection bias; nonresponse can remain.

- Define population and sampling frame: 2
- Compare the random and gate samples, identifying selection bias: 2
- Suggest a practical improvement: 1

Q4 (6 marks). For the five values 8, 9, 10, 11, 17, find mean, median and population variance. Explain the effect of adding 10 to every observation on mean and variance.

Answer: Mean 11, median 10, variance 10. Adding 10 increases mean by 10 and leaves variance unchanged.

- Find mean and median: 2
- Calculate population variance with a valid method: 2
- Explain the translation effect on mean and variance: 2

Q5 (5 marks). A grouped table has intervals $0 \le x < 10$, $10 \le x < 30$, $30 \le x < 50$ and frequencies 8, 16, 8. Find frequency densities and a midpoint estimate of the mean. State one limitation.

Answer: Densities $\frac{4}{5}, \frac{4}{5}, \frac{2}{5}$; estimated mean $\frac{85}{4}$=21.25. Individual values within classes are unknown.

- Find frequency densities for all three intervals: 2
- Use class midpoints and frequencies to estimate the mean: 2
- State a valid limitation: 1

Q6 (5 marks). A distribution has lower quartile 12, median 18 and upper quartile 24. Use the 1.5 IQR rule to find outlier fences. Classify -8 and 53. Explain why a box plot cannot establish a causal relationship.

Answer: IQR=12, fences -6 and 42. -8 is an outlier; 53 is an outlier. A box plot shows a distribution, not causation.

- Calculate the IQR and both fences: 2
- Classify the two observations correctly: 2
- Explain why the display does not establish causation: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Grouped data and interpolation. Times (minutes): 0–10: 6, 10–20: 14, 20–30: 12, 30–50: 8.

- (a) Write down the modal class. – 2 marks. Answer: 10–20. Award for: Correct method (1); accurate answer (1).
- (b) Estimate the mean. – 2 marks. Answer: $(30 + 210 + 300 + 320)/40 = 21.5$. Award for: Correct method (1); accurate answer (1).

- (c) Estimate the median by interpolation. – 2 marks. Answer: $10 + \frac{14}{14} \times 10 = 20$. Award for: Correct method (1); accurate answer (1).
- (d) Estimate the lower quartile. – 2 marks. Answer: $10 + \frac{4}{14} \times 10 = 12.9$. Award for: Correct method (1); accurate answer (1).
- (e) A histogram is drawn with the 10–20 bar of height 2.8 cm. Find the height of the 30–50 bar. – 3 marks. Answer: 0.8 cm. Award for: Correct method (2); accurate answer (1).
- (f) Explain why the mean is only an estimate. – 3 marks. Answer: actual values within classes are unknown; midpoints are used. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). The lengths, x minutes, of 40 phone calls are summarised by $\sum x = 520$ and $\sum x^2 = 7480$.

- (a) Find the mean and standard deviation of the lengths. – 3 marks. Award for: Mean and variance formula, correct sd
- (b) A further call of 13 minutes is added. Without calculating, state the effect on the mean and the standard deviation, giving reasons. – 2 marks. Award for: Mean unchanged; sd decreases with reason
- (c) The original data are coded using y = (x − 10)/2. Find the mean and standard deviation of y. – 2 marks. Award for: Apply coding to mean and sd
- (d) Explain why the median and interquartile range might be preferred if the data contained an extreme value. – 1 marks. Award for: Resistant to outliers

Answers: (a) Mean 13, sd $\sqrt{18} \approx 4.24$ min. (b) Mean unchanged; sd decreases slightly because the squared deviations are unchanged but n increases. (c) Mean 1.5, sd ≈ 2.12. (d) They are not distorted by extreme values.

Q3 (exam style) (9 marks). The times, t minutes, taken by 60 students to complete a task are grouped: 0 ≤ t < 10: 8; 10 ≤ t < 15: 14; 15 ≤ t < 20: 20; 20 ≤ t < 30: 12; 30 ≤ t < 50: 6.

- (a) On a histogram, the bar for 10 ≤ t < 15 has height 5.6 cm. Find the height of the bar for 30 ≤ t < 50. – 3 marks. Award for: Frequency densities and consistent scale
- (b) Use linear interpolation to estimate the median. – 3 marks. Award for: Cumulative frequencies, correct class and interpolation
- (c) Estimate the number of students who took between 25 and 40 minutes. – 2 marks. Award for: Proportional parts of two classes
- (d) Explain why your answer to (c) is only an estimate. – 1 marks. Award for: Assumes uniform spread within classes

Answers: (a) 2 cm per unit density; 30–50 bar is 0.6 cm. (b) 15 + (8/20) × 5 = 17 min. (c) 6 + 3 = 9 students. (d) Assumes values are evenly distributed within each class.

Q4 (exam style) (8 marks). For one set of exam scores, $Q_1 = 22$, median = 28 and $Q_3 = 34$. The lowest score is 9 and the highest is 58. An outlier is a value more than 1.5 × IQR beyond the nearer quartile.

- (a) Determine whether the lowest and highest scores are outliers. – 3 marks. Award for: IQR, both fences and conclusions
- (b) The next highest score after 58 is 49. Draw a box plot for the data. – 2 marks. Award for: Correct box, whiskers and outlier marked
- (c) A second group has median 31 and IQR 8. Compare the two groups. – 2 marks. Award for: Compare location and spread in context
- (d) Explain whether the outlier should be removed. – 1 marks. Award for: Only if it is an error; otherwise genuine data

Answers: (a) IQR 12, fences 4 and 52: 58 is an outlier, 9 is not. (b) Whiskers 9 to 49, box 22–28–34, 58 marked. (c) The second group scored higher on average and was more consistent. (d) Remove only if it is an error; a genuine score should be kept.

Week 7: Proof and positive-integer binomial expansion

Pearson P1 chapter 7–8.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Binomial expansions.

- (a) Expand $(1+x)^4$. – 2 marks. Answer: $1+4x+6x^2+4x^3+x^4$. Award for: Correct method (1); accurate answer (1).
- (b) Expand $(2+x)^3$. – 2 marks. Answer: $8+12x+6x^2+x^3$. Award for: Correct method (1); accurate answer (1).
- (c) Find the first three terms of $(1-2x)^6$. – 2 marks. Answer: $1-12x+60x^2$. Award for: Correct method (1); accurate answer (1).
- (d) Find the coefficient of x^3 in $(3+x)^5$. – 2 marks. Answer: $\binom{5}{3}3^2=90$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Choose and nCr.

- (a) Evaluate $\binom{7}{2}$. – 2 marks. Answer: 21. Award for: Correct method (1); accurate answer (1).
- (b) Evaluate $\binom{10}{3}$. – 2 marks. Answer: 120. Award for: Correct method (1); accurate answer (1).
- (c) Use Pascal's triangle to write down row 6. – 2 marks. Answer: 1, 6, 15, 20, 15, 6, 1. Award for: Correct method (1); accurate answer (1).
- (d) Use $(1+x)^8$ with a suitable x to estimate 1.01^8 to 4 decimal places. – 2 marks. Answer: $1+0.08+0.0028+0.000056\approx 1.0829$. Award for: Correct method (1); accurate answer (1).
- (e) Find n if the coefficient of x in $(1+2x)^n$ is 24. – 3 marks. Answer: $n=12$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). Expand $(1+9x)^5$ through the x^3 term. Find the coefficient of x^2 in $(1+9x)^5(1-x)$. Explain why terms beyond x^2 cannot affect that coefficient.

Answer: $7290x^3 + 810x^2 + 45x + 1$; coefficient 765; multiplication by 1 or -x cannot lower degree.

- Expand through the cubic term: 3
- Collect the requested product coefficient: 2
- Explain why higher powers cannot contribute: 1

Q4 (5 marks). Find the coefficient of x^3 in $(2 + x)^{12}$. State the general term used, and check a nearby coefficient independently using Pascal's relation.

Answer: Coefficient 112640; term $\binom{12}{r}2^{12-r}x^r$. Adjacent coefficients follow binomial coefficients.

- State an appropriate general term: 1
- Obtain the cubic coefficient: 2
- Check a nearby coefficient using Pascal's relation: 2

Q5 (6 marks). Prove that the sum of two odd integers is even. Then prove that if an integer has an odd square, the integer is odd. State whether each proof is direct or uses a contrapositive.

Answer: Write 2r+1,2s+1; sum=2(r+s+1), direct. If n is even, n=2k so $n^2=4k^2$ is even; contrapositive proves odd square implies odd n.

- Give a complete direct proof for the odd-integer sum: 2
- Give a complete contrapositive proof for the square statement: 3
- Identify both proof types: 1

Q6 (4 marks). Disprove the claim that $n^2 + n + 9$ is prime for every positive integer n. Give a constructive choice of n and explain why one counterexample is sufficient.

Answer: Take n=9; expression=9×11, composite because both factors exceed1. A universal claim fails at one counterexample.

- Construct a suitable integer counterexample: 1
- Show the resulting number is composite: 2
- Explain why one counterexample suffices: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Proof basics.

- (a) Prove that the sum of two consecutive integers is odd. – 2 marks. Answer: $n + (n + 1) = 2n + 1$. Award for: Correct method (1); accurate answer (1).
- (b) Prove that the square of an odd number is odd. – 2 marks. Answer: $(2n + 1)^2 = 2(2n^2 + 2n) + 1$. Award for: Correct method (1); accurate answer (1).
- (c) Disprove: $n^2 + n + 1$ is prime for all positive integers n. – 2 marks. Answer: $n = 4$: 21. Award for: Correct method (1); accurate answer (1).

- (d) Prove that $x^2 - 6x + 10 > 0$ for all real x. – 2 marks. Answer: $(x-3)^2 + 1 > 0$. Award for: Correct method (1); accurate answer (1).
- (e) Prove by exhaustion that $n^2 + 2$ is not divisible by 4 for $n = 1, 2, 3, 4$. – 3 marks. Answer: 3, 6, 11, 18. Award for: Correct method (2); accurate answer (1).
- (f) Prove that the product of three consecutive integers is even. – 3 marks. Answer: at least one factor is even. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks).

- (a) Find the first four terms, in ascending powers of x, of $\left(2 - \frac{x}{4}\right)^8$, giving each term in its simplest form. – 4 marks. Award for: Binomial coefficients and powers, simplified
- (b) Use your expansion, with a suitable value of x, to estimate 1.975^8. State the value of x used. – 3 marks. Award for: x = 0.1 and accurate substitution
- (c) Find the coefficient of x^3 in the expansion of $(1 + 2x)\left(2 - \frac{x}{4}\right)^8$. – 2 marks. Award for: Combine the two contributing products

Answers: (a) 256 − 256x + 112x² − 28x³. (b) x = 0.1: 231.492 (true value 231.4924...). (c) −28 + 224 = 196.

Q3 (exam style) (9 marks).

- (a) Prove that $n^3 - n$ is a multiple of 6 for every integer n. – 4 marks. Award for: Factorise to consecutive integers and argue divisibility by 2 and 3
- (b) Disprove the statement: 'n² + n + 41 is prime for every positive integer n'. – 2 marks. Award for: Counter-example with explicit factorisation
- (c) Prove by exhaustion that the square of any integer leaves remainder 0 or 1 on division by 3. – 3 marks. Award for: Consider 3k, 3k + 1 and 3k + 2 and simplify each

Answers: (a) $n^3 - n = (n-1)n(n+1)$, three consecutive integers, containing a multiple of 2 and of 3. (b) n = 40: 1681 = 41². (c) $(3k)^2 = 3(3k^2)$; $(3k \pm 1)^2 = 3(3k^2 \pm 2k) + 1$; $(3k+2)^2 = 3(3k^2 + 4k + 1) + 1$.

Q4 (exam style) (8 marks). In the expansion of $(1 + kx)^n$, where n is a positive integer and k is a constant, the coefficient of x is 15 and the coefficient of x^2 is 90.

- (a) Show that nk = 15 and $n(n-1)k^2 = 180$. – 2 marks. Award for: Form both equations correctly
- (b) Find the values of n and k. – 4 marks. Award for: Eliminate k and solve for n, then k
- (c) Find the coefficient of x^3. – 2 marks. Award for: Correct coefficient

Answers: (a) nk = 15; $C(n, 2)k^2 = 90$ gives $n(n-1)k^2 = 180$. (b) $225(n-1)/n = 180$, n = 5, k = 3. (c) $C(5, 3) \times 27 = 270$.

Week 8: Trigonometric ratios and graphs

Pearson P1 chapter 9.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Sine and cosine rules. Give answers to 3 significant figures.

- (a) In triangle ABC, $a = 7$, $b = 9$, $C = 50^\circ$. Find c. – 2 marks. Answer: $c = 7.00$. Award for: Correct method (1); accurate answer (1).
- (b) In triangle ABC, $A = 40^\circ$, $B = 65^\circ$, $a = 8$. Find b. – 2 marks. Answer: $b = 11.3$. Award for: Correct method (1); accurate answer (1).
- (c) A triangle has sides 5, 6, 8. Find the largest angle. – 2 marks. Answer: 92.9°. Award for: Correct method (1); accurate answer (1).
- (d) Find the area of a triangle with sides 6 and 10 enclosing 30°. – 2 marks. Answer: 15. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Exact values and graphs.

- (a) Write down the exact value of $\sin 60^\circ$. – 2 marks. Answer: $\frac{\sqrt{3}}{2}$. Award for: Correct method (1); accurate answer (1).
- (b) Write down the exact value of $\tan 45^\circ + \cos 60^\circ$. – 2 marks. Answer: $\frac{3}{2}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the exact value of $\cos 150^\circ$. – 2 marks. Answer: $-\frac{\sqrt{3}}{2}$. Award for: Correct method (1); accurate answer (1).
- (d) State the period of $y = \sin 2x$. – 2 marks. Answer: 180°. Award for: Correct method (1); accurate answer (1).

Q3 (5 marks). A triangle has sides 6 and 8 with included angle 60°. Find its area exactly and its third side. Explain why these data determine the side uniquely.

Answer: Area $12\sqrt{3}$; side $2\sqrt{13}$; SAS fixes it.

- Use the area formula to obtain an exact area: 2
- Use the cosine rule to find the third side: 2
- Explain uniqueness from the given SAS data: 1

Q4 (6 marks). In a triangle A=30°, side a=5 and side b=8. Find both possible values of B and the corresponding C. Explain the ambiguous case with a sketch.

Answer: sin B=0.8; B≈53.13° or 126.87°; C≈96.870° or 23.130°.

- Apply the sine rule: 1
- Find both possible values of B: 2
- Find both corresponding values of C: 1
- Explain the ambiguity with an appropriate sketch: 2

Q5 (8 marks). Sketch $y = 3\sin(2x) - 1$ for 0°≤x≤360°. State amplitude, period, range and all x for which y=-1.

Answer: Amplitude 3, period 180°, range [-4,2]; x=0°,90°,180°,270°,360°.

- Sketch the correct curve over the full interval: 3
- State amplitude, period and range: 3
- Find every requested x-value including endpoints: 2

Q6 (4 marks). A right triangle has opposite side 3 and adjacent side 4 relative to angle θ. Find sin θ, cos θ and tan θ exactly, then explain why changing both lengths by the same factor preserves θ.

Answer: Hypotenuse 5; sin=$\frac{3}{\sqrt{25}}$, cos=$\frac{4}{\sqrt{25}}$, tan=$\frac{3}{4}$. Ratios unchanged.

- Find the hypotenuse: 1
- Give all three exact trigonometric ratios: 2
- Explain why a common scaling preserves the angle: 1

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (14 marks). Using the unit circle.

- (a) Given $\sin\theta = \frac{3}{5}$ and θ acute, find $\cos\theta$ and $\tan\theta$. – 2 marks. Answer: $\frac{4}{5}$, $\frac{3}{4}$. Award for: Correct method (1); accurate answer (1).
- (b) Find all x in $0 \le x \le 360^\circ$ with $\sin x = 0.5$. – 2 marks. Answer: $30^\circ, 150^\circ$. Award for: Correct method (1); accurate answer (1).
- (c) Find all x in $0 \le x \le 360^\circ$ with $\cos x = -0.5$. – 2 marks. Answer: $120^\circ, 240^\circ$. Award for: Correct method (1); accurate answer (1).
- (d) State the sign of $\tan 200^\circ$ and explain. – 2 marks. Answer: positive, third quadrant. Award for: Correct method (1); accurate answer (1).
- (e) Given $\cos\theta = -\frac{5}{13}$, $90^\circ < \theta < 180^\circ$, find $\sin\theta$. – 3 marks. Answer: $\frac{12}{13}$. Award for: Correct method (2); accurate answer (1).
- (f) Find all x in $0 \le x \le 360^\circ$ with $\tan x = -1$. – 3 marks. Answer: $135^\circ, 315^\circ$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). In triangle ABC, AB = 8 cm, AC = 11 cm and angle BAC = 50°.

- (a) Find BC. – 2 marks. Award for: Cosine rule
- (b) Find angle ABC, explaining why only one value is possible. – 3 marks. Award for: Sine or cosine rule with ambiguity checked
- (c) Find the area of the triangle. – 2 marks. Award for: ½ab sin C
- (d) Find the shortest distance from A to the line BC. – 2 marks. Award for: Use area to find the perpendicular height

Answers: (a) BC ≈ 8.48 cm. (b) ≈ 83.7° (the cosine rule gives a positive cosine, so B is acute). (c) ≈ 33.7 cm^2. (d) ≈ 7.95 cm.

Q3 (exam style) (9 marks). The curve C has equation y = 2 sin(x + 30°), 0° ≤ x ≤ 360°.

- (a) Sketch C, giving the coordinates of the maximum and minimum points and the points where C meets the axes. – 4 marks. Award for: Shape, translation and all key coordinates
- (b) State the number of solutions of 2 sin(x + 30°) = 1.5 in this interval, explaining your answer. – 2 marks. Award for: Two, with reference to the line crossing the graph
- (c) Solve 2 sin(x + 30°) = 1 for 0° ≤ x ≤ 360°. – 3 marks. Award for: Adjusted interval and all solutions

Answers: (a) Max (60°, 2), min (240°, −2), x-intercepts 150° and 330°, y-intercept (0, 1). (b) Two, since 0 < 1.5 < 2 and the line crosses the curve twice. (c) x = 0°, 120°, 360°.

Q4 (exam style) (9 marks). A ship sails from port P on a bearing of 060° for 12 km to Q, then on a bearing of 150° for 5 km to R.

- (a) Show that angle PQR = 90°. – 2 marks. Award for: Use back bearing 240° and 150°
- (b) Find the distance PR and the bearing of R from P. – 4 marks. Award for: Pythagoras and angle QPR then bearing
- (c) A lighthouse L is 7 km from P on a bearing of 100°. Find the distance QL. – 3 marks. Award for: Angle QPL = 40° and cosine rule

Answers: (a) Back bearing 240° − 150° = 90°. (b) PR = 13 km, bearing 060° + 22.6° = 082.6°. (c) QL ≈ 8.02 km.

Week 9: Trigonometric identities and equations

Pearson P1 chapter 10.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (11 marks). Solve for $0 \leq x \leq 360^\circ$, to 1 decimal place where necessary.

- (a) $\sin x = 0.4$ – 2 marks. Answer: $23.6^\circ, 156.4^\circ$. Award for: Correct method (1); accurate answer (1).
- (b) $\tan x = 3$ – 2 marks. Answer: $71.6^\circ, 251.6^\circ$. Award for: Correct method (1); accurate answer (1).
- (c) $2\cos x + 1 = 0$ – 2 marks. Answer: $120^\circ, 240^\circ$. Award for: Correct method (1); accurate answer (1).
- (d) $\sin 2x = 0.5$ – 2 marks. Answer: $15^\circ, 75^\circ, 195^\circ, 255^\circ$. Award for: Correct method (1); accurate answer (1).
- (e) $\cos(x - 20^\circ) = 0.6$ – 3 marks. Answer: $73.1^\circ, 326.9^\circ$. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Identities.

- (a) Simplify $\frac{\sin x}{\cos x} \times \cos x$. – 2 marks. Answer: $\sin x$. Award for: Correct method (1); accurate answer (1).
- (b) Simplify $1 - \cos^2 x$. – 2 marks. Answer: $\sin^2 x$. Award for: Correct method (1); accurate answer (1).
- (c) Show that $\frac{1-\sin^2 x}{\cos x} = \cos x$. – 2 marks. Answer: $\cos^2 x / \cos x$. Award for: Correct method (1); accurate answer (1).
- (d) Show that $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$. – 2 marks. Answer: expand, use $\sin^2 + \cos^2 = 1$. Award for: Correct method (1); accurate answer (1).
- (e) Show that $\tan x + \frac{1}{\tan x} = \frac{1}{\sin x \cos x}$. – 3 marks. Answer: common denominator, $\sin^2 + \cos^2 = 1$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). Solve $2\cos^2 x - 3\cos x + 1 = 0$ for $0° \le x < 360°$. Show the factorisation, list every solution and verify by substitution.

Answer: $(2\cos x - 1)(\cos x - 1) = 0$; $x = 0°, 60°, 300°$.

- Factorise the quadratic in the trigonometric function: 2
- Find every solution in the interval: 2
- Verify the solutions: 1

Q4 (5 marks). Solve $\cos(2x) = 1/2$ for $0° \le x < 720°$. State the transformed interval first and explain your method for ensuring no roots are missed.

Answer: $0° \le 2x < 1440°$. Solutions $x = 30° + 180°k$ or $150° + 180°k$, for $k = 0, \ldots, 3$.

- State the transformed interval: 1
- Find the two solution families: 2
- Restrict them to the given interval without missing roots: 1
- Explain the completeness method: 1

Q5 (4 marks). Prove $(1 - \sin^2 x)/\cos x = \cos x$ wherever the left side is defined. State exclusions and explain why dividing by a potentially zero factor when solving an equation can lose roots.

Answer: Use $1 - \sin^2 x = \cos^2 x$; cancel $\cos x$ only for $\cos x \neq 0$. Exclude $90° + 180°k$. Check zero-factor cases separately in equations.

- Use the Pythagorean identity: 1
- Complete the algebraic proof on the valid domain: 1
- State all exclusions: 1
- Explain the possible loss of roots when dividing by a zero factor: 1

Q6 (5 marks). Find all x in $0° \le x \le 360°$ satisfying $4\tan x = 4\sqrt{3}$. Then solve $\sin x = \cos x$ on the same interval. Identify undefined points before dividing.

Answer: First: $x = 60°, 240°$. Second: $x = 45°, 225°$. tan undefined at $90°, 270°$; sin=cos has neither cos=0 nor sin=0 solutions.

- Solve the tangent equation over the interval: 2
- Solve the sine-cosine equation over the interval: 2
- Identify undefined or zero-denominator cases before division: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Equations needing identities, for $0 \leq x \leq 360^\circ$.

- (a) $\sin x = 2\cos x$ – 2 marks. Answer: $63.4^\circ, 243.4^\circ$. Award for: Correct method (1); accurate answer (1).
- (b) $2\sin^2 x = 1$ – 2 marks. Answer: $45^\circ, 135^\circ, 225^\circ, 315^\circ$. Award for: Correct method (1); accurate answer (1).
- (c) $\sin x(2\cos x - 1) = 0$ – 2 marks. Answer: $0, 60, 180, 300, 360^\circ$. Award for: Correct method (1); accurate answer (1).
- (d) $2\cos^2 x - \cos x - 1 = 0$ – 2 marks. Answer: $0^\circ, 120^\circ, 240^\circ, 360^\circ$. Award for: Correct method (1); accurate answer (1).
- (e) $2\sin^2 x + 3\cos x = 3$ – 3 marks. Answer: $0^\circ, 60^\circ, 300^\circ, 360^\circ$. Award for: Correct method (2); accurate answer (1).
- (f) $\tan^2 x = 3$ – 3 marks. Answer: $60^\circ, 120^\circ, 240^\circ, 300^\circ$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks).

- (a) Show that the equation $6\cos^2 x + \sin x = 5$ can be written as $6\sin^2 x - \sin x - 1 = 0$. – 2 marks. Award for: Use $\cos^2x = 1 - \sin^2x$ and rearrange
- (b) Hence solve $6\cos^2 x + \sin x = 5$ for $0° \leq x < 360°$, giving answers to 1 decimal place where appropriate. – 4 marks. Award for: Factorise, solve both sine equations, all four solutions
- (c) Hence find the solutions of $6\cos^2 2\theta + \sin 2\theta = 5$ for $0° \leq \theta < 90°$. – 2 marks. Award for: Use 2θ in $0°$ to $180°$ and halve

Answers: (a) $6(1 - s^2) + s = 5$. (b) $(3s + 1)(2s - 1) = 0$: $x = 30°, 150°, 199.5°, 340.5°$. (c) $2\theta = 30°, 150°$: $\theta = 15°, 75°$.

Q3 (exam style) (8 marks).

- (a) Show that $\frac{\sin^2 x}{1 - \cos x} \equiv 1 + \cos x$, where $\cos x \neq 1$. – 3 marks. Award for: Use $\sin^2x = 1 - \cos^2x$ and factorise the difference of squares
- (b) Hence solve $\frac{\sin^2 x}{1 - \cos x} = 4\cos^2 x$ for $0° \leq x < 360°$, giving answers to 1 decimal place. – 4 marks. Award for: Form $4c^2 - c - 1 = 0$, solve and find all angles
- (c) Explain why $x = 0°$ cannot be a solution, even though the right-hand side of (a) is defined there. – 1 marks. Award for: Original denominator is zero

Answers: (a) $(1 - c)(1 + c)/(1 - c) = 1 + c$. (b) $\cos x = (1 \pm \sqrt{17})/8$: $x = 50.2°, 113.0°, 247.0°, 309.8°$. (c) At $x = 0°$, $1 - \cos x = 0$, so the original expression is undefined.

Q4 (exam style) (8 marks). Solve each equation for −180° ≤ θ ≤ 180°.

- (a) $\tan(\theta - 40^\circ) = \sqrt{3}$ – 3 marks. Award for: Principal value, period of tan, correct interval
- (b) $3\sin\theta = 2\tan\theta$ – 5 marks. Award for: Rewrite with cos θ, factorise without dividing by sin θ, solve both factors

Answers: (a) θ − 40° = 60° or −120°: θ = 100°, −80°. (b) sin θ(3cos θ − 2) = 0: θ = −180°, 0°, 180°, ±48.2°.

Week 10: Probability and binomial models

Pearson S1 chapter 5–6.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Probability basics.

- (a) A fair dice is rolled. Find P(a prime number). – 2 marks. Answer: $\frac{1}{2}$. Award for: Correct method (1); accurate answer (1).
- (b) Two fair coins are tossed. Find P(at least one head). – 2 marks. Answer: $\frac{3}{4}$. Award for: Correct method (1); accurate answer (1).
- (c) $P(A) = 0.4$, $P(B) = 0.3$, A and B are mutually exclusive. Find $P(A \cup B)$. – 2 marks. Answer: 0.7. Award for: Correct method (1); accurate answer (1).
- (d) $P(A) = 0.5$, $P(B) = 0.4$, A, B independent. Find $P(A \cap B)$ and $P(A \cup B)$. – 2 marks. Answer: 0.2, 0.7. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Binomial probabilities. $X \sim B(10, 0.3)$.

- (a) Find $P(X = 3)$. – 2 marks. Answer: 0.267. Award for: Correct method (1); accurate answer (1).
- (b) Find $P(X \leq 2)$. – 2 marks. Answer: 0.383. Award for: Correct method (1); accurate answer (1).
- (c) Find $P(X \geq 4)$. – 2 marks. Answer: 0.350. Award for: Correct method (1); accurate answer (1).
- (d) Find $P(2 \leq X \leq 5)$. – 2 marks. Answer: 0.803. Award for: Correct method (1); accurate answer (1).
- (e) State two conditions for a binomial model. – 3 marks. Answer: fixed number of independent trials, constant probability of success. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). A bag contains 8 red and 5 blue counters. Two are drawn without replacement. Find P(both red), P(exactly one red) and P(second red). Explain why the draws are not independent.

Answer: $\frac{14}{39}$, $\frac{20}{39}$, $\frac{8}{13}$. Composition changes after the first draw.

- Find the probability of two reds without replacement: 2
- Include both orders for exactly one red: 2

- Find the probability that the second counter is red: 1
- Explain the dependence of the draws: 1

Q4 (4 marks). For events A and B, P(A)=0.4, P(B)=0.5 and P(A∩B)=0.2. Find P(A∪B), P(neither), and decide whether A,B are independent or mutually exclusive.

Answer: Union 0.7; neither 0.3. Independent since 0.2=0.4×0.5; not mutually exclusive since intersection is nonzero.

- Find the union probability: 1
- Find the probability of neither event: 1
- Test independence using the product rule: 1
- Decide mutual exclusivity with a reason: 1

Q5 (6 marks). Let X~B(13,0.3). Calculate P(X=3), P(X≤2) and P(X≥4). State two modelling assumptions in the context of repeated independent attempts.

Answer: 0.21813, 0.20248, 0.57939; fixed n, independent attempts, constant success probability, two outcomes.

- Calculate the point probability: 1
- Calculate the lower-tail probability: 1
- Use the correct complementary tail: 2
- State two valid binomial assumptions: 2

Q6 (4 marks). A discrete variable takes values 0,1,2 with probabilities k,2k,5k. Find k, P(X≥1) and P(X=2 given X≥1), using an event ratio rather than a memorised rule.

Answer: k=$\frac{1}{8}$; P≥1=$\frac{7}{8}$; conditional=$\frac{5}{7}$.

- Normalise the probabilities to find k: 1
- Find the probability of X at least one: 1
- Set up and evaluate the conditional event ratio: 2

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (14 marks). Tree diagrams and Venn diagrams.

- (a) A bag has 4 red and 6 blue counters. Two are taken without replacement. Find P(both red). – 2 marks. Answer: $\frac{2}{15}$. Award for: Correct method (1); accurate answer (1).
- (b) For the same bag, find P(one of each colour). – 2 marks. Answer: $\frac{8}{15}$. Award for: Correct method (1); accurate answer (1).
- (c) With replacement, find P(both blue). – 2 marks. Answer: 0.36. Award for: Correct method (1); accurate answer (1).
- (d) In a class of 30, 18 study French, 12 Spanish and 5 both. Find P(neither). – 2 marks. Answer: $\frac{5}{30} = \frac{1}{6}$. Award for: Correct method (1); accurate answer (1).

- (e) For that class, find P(French only). – 3 marks. Answer: $\frac{13}{30}$. Award for: Correct method (2); accurate answer (1).
- (f) A biased coin has P(head)=0.6. Find P(first head on the third toss). – 3 marks. Answer: $0.4^2 \times 0.6 = 0.096$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). A spinner lands on red with probability 0.35. It is spun 20 times and X is the number of reds.

- (a) State two conditions needed for X to be modelled by a binomial distribution. – 2 marks. Award for: Fixed number of independent trials with constant probability
- (b) Find P(X = 7). – 2 marks. Award for: Binomial probability
- (c) Find P(4 < X ≤ 9). – 2 marks. Award for: Correct cumulative probabilities
- (d) The spinner is spun in 5 separate sets of 20. Find the probability that exactly 2 sets contain more than 9 reds. – 3 marks. Award for: p = P(X > 9) then a second binomial

Answers: (a) e.g. independent spins; constant probability of red. (b) 0.1844. (c) 0.7600. (d) p = 0.1218; $10p^2(1 - p)^3 \approx 0.100$.

Q3 (exam style) (9 marks). For events A and B, P(A) = 0.45, P(B) = 0.3 and P(A ∪ B) = 0.6.

- (a) Find P(A ∩ B). – 1 marks. Award for: Addition rule
- (b) Draw a Venn diagram showing all probabilities. – 2 marks. Award for: Four correct regions
- (c) Determine whether A and B are independent. – 2 marks. Award for: Compare P(A)P(B) with P(A ∩ B)
- (d) Event C is mutually exclusive of A, independent of B, and P(C) = 0.2. Find P(B ∩ C) and P(A′ ∩ B′ ∩ C′). – 4 marks. Award for: Use independence and place C outside A

Answers: (a) 0.15. (b) A only 0.30, both 0.15, B only 0.15, neither 0.40. (c) 0.135 ≠ 0.15, not independent. (d) P(B ∩ C) = 0.06; P(A′ ∩ B′ ∩ C′) = 0.40 − 0.14 = 0.26.

Q4 (exam style) (9 marks). The discrete random variable X has $P(X = x) = k(x^2 + 1)$ for x = 0, 1, 2, 3.

- (a) Show that k = 1/18. – 2 marks. Award for: Sum of probabilities equals 1
- (b) Find P(X ≥ 2). – 1 marks. Award for: Correct probability
- (c) Two independent observations X_1 and X_2 are taken. Find $P(X_1 + X_2 = 3)$. – 3 marks. Award for: List all pairs and add products
- (d) Find $P(X_1 > X_2)$. – 3 marks. Award for: Use symmetry or list cases

Answers: (a) 18k = 1. (b) 15/18 = 5/6. (c) (20 + 20)/324 = 10/81. (d) P(equal) = 130/324, so (1 − 130/324)/2 = 97/324.

Week 11: Binomial hypothesis testing

Pearson S1 chapter 7.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (11 marks). Hypothesis test language.

- (a) State the null and alternative hypotheses for testing whether a coin is biased towards heads. – 2 marks. Answer: $H_0 : p = 0.5,\ H_1 : p > 0.5$. Award for: Correct method (1); accurate answer (1).
- (b) Explain what is meant by a critical region. – 2 marks. Answer: values of the test statistic that lead to rejecting H_0. Award for: Correct method (1); accurate answer (1).
- (c) Explain what the significance level represents. – 2 marks. Answer: probability of rejecting H_0 when it is true. Award for: Correct method (1); accurate answer (1).
- (d) State when a two-tailed test should be used. – 2 marks. Answer: when testing for a change in either direction. Award for: Correct method (1); accurate answer (1).
- (e) $X \sim B(20, 0.5)$. Find $P(X \geq 15)$. – 3 marks. Answer: 0.0207. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Critical regions. $X \sim B(20, 0.3)$.

- (a) Find $P(X \leq 2)$. – 2 marks. Answer: 0.0355. Award for: Correct method (1); accurate answer (1).
- (b) Find $P(X \leq 3)$. – 2 marks. Answer: 0.107. Award for: Correct method (1); accurate answer (1).
- (c) Hence find the 5% lower-tail critical region. – 2 marks. Answer: $X \leq 2$. Award for: Correct method (1); accurate answer (1).
- (d) Find $P(X \geq 10)$. – 2 marks. Answer: 0.0480. Award for: Correct method (1); accurate answer (1).
- (e) Find the 5% upper-tail critical region. – 3 marks. Answer: $X \geq 10$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). A device is claimed to work with probability 0.5 independently on each trial. In 26 trials it works 8 times. Test at 5% whether the success probability has fallen. State hypotheses, probability and conclusion.

Answer: H0:p=0.5; H1:p<0.5. X~B(26,0.5), P(X≤8)=0.037759≤0.05; reject H0, evidence of a fall, assuming independent representative trials.

- State both hypotheses using the population parameter: 1
- State the null binomial model: 1
- Calculate the correct cumulative tail probability: 2
- Compare with the significance level: 1
- Give the conclusion in context: 1

Q4 (5 marks). For X~B(26,0.5), find the lower-tail critical region at 5% and the actual significance level. Explain why it need not equal exactly 5%.

Answer: X≤8; actual level 0.037759. Next cumulative probability 0.084319>.05; discreteness prevents exact equality.

- Find the boundary count: 1
- Check the next count to establish the boundary: 1
- State the critical region: 1
- Find its actual significance level: 1
- Explain the effect of discreteness: 1

Q5 (3 marks). A student observes 20 successes in 26 trials and tests H0:p=0.5 against H1:p>0.5. Calculate the p-value. Explain why the probability of the single observed count is not enough.

Answer: P(X≥20)=0.0046777; includes every outcome at least as extreme in the alternative direction.

- Identify the upper-tail event including the observed count: 1
- Calculate its probability: 1
- Explain why the single-count probability is insufficient: 1

Q6 (5 marks). For X~B(20,0.5), construct a two-tailed critical region using no more than 2.5% in each tail. State its actual significance level. Explain a Type I error in a coin-fairness context.

Answer: X≤5 or X≥15; each tail 0.020695; total 0.041389. Type I: rejecting fairness when the coin is fair.

- Establish and state both critical tails: 3
- Calculate the actual total significance level: 1
- Explain a Type I error in the coin context: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Carrying out tests.

- (a) $H_0 : p = 0.2$, $H_1 : p < 0.2$, $n = 25$, observed 1. Find $P(X \leq 1)$. – 2 marks. Answer: 0.0274. Award for: Correct method (1); accurate answer (1).
- (b) Hence state the conclusion at 5%. – 2 marks. Answer: reject H_0: evidence $p < 0.2$. Award for: Correct method (1); accurate answer (1).
- (c) $H_0 : p = 0.4$, $H_1 : p \neq 0.4$, $n = 15$, observed 10. Find $P(X \geq 10)$. – 2 marks. Answer: 0.0338. Award for: Correct method (1); accurate answer (1).
- (d) State the conclusion at 5% (two-tailed). – 2 marks. Answer: 0.0338 > 0.025: do not reject H_0. Award for: Correct method (1); accurate answer (1).
- (e) $n = 10$, $p = 0.5$. Find the smallest c with $P(X \geq c) < 0.05$. – 3 marks. Answer: $c = 9$. Award for: Correct method (2); accurate answer (1).

- (f) Explain why the test conclusion should be written in context. – 3 marks. Answer: it answers the original question about the population. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). Historically, 30% of customers choose a premium option. After a website redesign, 12 of a random sample of 25 customers choose it.

- (a) Test, at the 5% significance level, whether the proportion has increased. State your hypotheses clearly. – 5 marks. Award for: Hypotheses, distribution, probability, comparison and contextual conclusion
- (b) Find the critical region for this test. – 2 marks. Award for: Both boundary probabilities checked
- (c) State the actual significance level of the test. – 1 marks. Award for: Correct probability

Answers: (a) H_0: $p = 0.3$, H_1: $p > 0.3$; $P(X \geq 12) = 0.0442 < 0.05$, reject H_0: evidence of an increase. (b) $X \geq 12$ ($P(X \geq 11) = 0.0978$). (c) 0.0442.

Q3 (exam style) (8 marks). A coin is tossed 40 times to test whether it is biased.

- (a) Using a 5% two-tailed test, with each tail as close as possible to but not exceeding 2.5%, find the critical region. – 4 marks. Award for: Hypotheses, both tails and probabilities
- (b) State the actual significance level. – 1 marks. Award for: Sum of both tails
- (c) The coin shows 26 heads. State the conclusion. – 2 marks. Award for: Not in the critical region, conclusion in context
- (d) Explain why this conclusion does not prove the coin is fair. – 1 marks. Award for: Insufficient evidence is not proof

Answers: (a) $X \leq 13$ or $X \geq 27$. (b) 0.0385. (c) 26 is not in the critical region: insufficient evidence of bias. (d) Failing to reject H_0 only means the data are consistent with fairness.

Q4 (exam style) (8 marks). A website claims that 1 in 5 visitors click on an advert. A manager believes the proportion is lower and samples 30 visitors.

- (a) Write down suitable hypotheses. – 1 marks. Award for: Correct one-tailed hypotheses
- (b) Find the critical region at the 10% level. – 3 marks. Award for: Cumulative probabilities and boundary
- (c) In the sample, 3 visitors clicked. State the conclusion. – 2 marks. Award for: Compare with critical region and conclude in context
- (d) The manager says, 'there is a 12% chance that the claim is true'. Criticise this statement. – 2 marks. Award for: Probability refers to data under H_0, not the truth of H_0

Answers: (a) H_0: $p = 0.2$, H_1: $p < 0.2$. (b) $X \leq 2$ ($P(X \leq 2) = 0.0442$, $P(X \leq 3) = 0.1227$). (c) 3 is not in the region: insufficient evidence. (d) 0.1227 is $P(X \leq 3)$ if the claim is true; it is not the probability that the claim is true.

Week 12: Mechanics models and motion graphs

Pearson S1 chapter 8–9.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (11 marks). Constant acceleration formulae. Take each motion in a straight line.

- (a) $u = 3$, $a = 2$, $t = 5$. Find v. – 2 marks. Answer: 13 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (b) $u = 0$, $a = 4$, $t = 6$. Find s. – 2 marks. Answer: 72 m. Award for: Correct method (1); accurate answer (1).
- (c) $u = 12$, $v = 0$, $s = 36$. Find a. – 2 marks. Answer: -2 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) $u = 5$, $v = 15$, $t = 4$. Find s. – 2 marks. Answer: 40 m. Award for: Correct method (1); accurate answer (1).
- (e) A car accelerates from 8 m s^{-1} at 1.5 m s^{-2} over 60 m. Find its final speed. – 3 marks. Answer: $\sqrt{244} = 15.6$ m s^{-1}. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Velocity–time graphs. A cyclist accelerates uniformly from rest to 10 m s^{-1} in 4 s, travels at 10 m s^{-1} for 20 s, then decelerates uniformly to rest in 6 s.

- (a) Find the initial acceleration. – 2 marks. Answer: 2.5 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (b) Find the deceleration. – 2 marks. Answer: $\frac{5}{3}$ m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (c) Find the total distance travelled. – 2 marks. Answer: 20 + 200 + 30 = 250 m. Award for: Correct method (1); accurate answer (1).
- (d) Find the average speed for the journey. – 2 marks. Answer: $\frac{250}{30} = 8.33$ m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (e) Find the distance travelled in the first 10 s. – 3 marks. Answer: 20 + 60 = 80 m. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A particle accelerates uniformly from 7 to 15 m/s in 4 s, then continues at 15 m/s for 3 s. Sketch its velocity-time graph, find acceleration and total displacement.

Answer: Acceleration 2 m/s^2; displacement 89 m, by trapezium plus rectangle. Graph continuous with a straight rising segment then horizontal.

- Sketch and label both stages of the velocity-time graph: 2
- Find the acceleration: 1
- Calculate both displacement contributions and their total: 2

Q4 (4 marks). A displacement-time graph is described by s=7t for 0≤t≤2, then s=14 for 2≤t≤5. Sketch it, give velocities in each stage and discuss instantaneous velocity at t=2.

Answer: Velocities 7,0 m/s. The idealised corner has no single derivative at t=2; model contains an instantaneous velocity change.

- Sketch both displacement-time segments: 2
- Give the velocities in both stages: 1
- Explain the undefined instantaneous velocity at the idealised corner: 1

Q5 (5 marks). A cyclist travels 9 km east and then 6 km west in 30 minutes. Find distance, displacement, average speed and average velocity; state directions and units.

Answer: Distance 15 km, displacement 3 km east; average speed 30 km/h, velocity 6 km/h east.

- Distinguish and calculate distance and displacement: 2
- Calculate average speed and average velocity: 2
- Give correct directions and consistent units: 1

Q6 (4 marks). A model treats a car as a particle moving on a horizontal straight road. Explain what the particle assumption removes and what it does not imply. Convert 72 km/h to m/s and state SI units of acceleration and force.

Answer: Dimensions/rotation ignored; it does not imply zero mass or zero resistance. 20 m/s; acceleration m/s², force N=kg m/s².

- Explain what the particle assumption removes and does not imply: 2
- Convert the speed to SI units: 1
- State the SI units of acceleration and force: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Modelling and units.

- (a) Convert 72 km h^{-1} to m s^{-1}. – 2 marks. Answer: 20 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (b) State what is meant by modelling an object as a particle. – 2 marks. Answer: its size is ignored; mass at a point. Award for: Correct method (1); accurate answer (1).
- (c) A displacement–time graph is a straight line from (0, 0) to (8, 40). Find the velocity. – 2 marks. Answer: 5 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (d) Explain the difference between distance and displacement. – 2 marks. Answer: displacement has direction; distance is total length travelled. Award for: Correct method (1); accurate answer (1).
- (e) A runner travels 400 m north in 80 s, then 200 m south in 50 s. Find the average velocity. – 3 marks. Answer: $\frac{200}{130} = 1.54$ m s^{-1} north. Award for: Correct method (2); accurate answer (1).
- (f) For the runner, find the average speed. – 3 marks. Answer: $\frac{600}{130} = 4.62$ m s^{-1}. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). A car passes point A at 8 m/s and accelerates uniformly at 1.5 m/s^2 for 10 s. It then travels at constant speed for T s and finally decelerates uniformly to rest at B in 6 s. AB = 546 m.

- (a) Sketch a velocity–time graph for the motion. – 2 marks. Award for: Three stages with correct shape
- (b) Find the maximum speed. – 1 marks. Award for: suvat
- (c) Find T. – 4 marks. Award for: Areas of each stage and solve
- (d) Find the deceleration. – 1 marks. Award for: Gradient of final stage

Answers: (a) Trapezium shape starting at 8. (b) 23 m/s. (c) 155 + 23T + 69 = 546, T = 14. (d) 23/6 ≈ 3.83 m/s^2.

Q3 (exam style) (8 marks). A train travels between two stations 2.7 km apart. It accelerates uniformly from rest for 40 s to V m/s, travels at V m/s for 60 s, then decelerates uniformly to rest in 20 s.

- (a) Sketch a velocity–time graph. – 2 marks. Award for: Correct trapezium
- (b) Show that V = 30. – 3 marks. Award for: Area equation 90V = 2700
- (c) Find the total time for which the speed exceeds 15 m/s. – 3 marks. Award for: Times in both sloping stages plus constant stage

Answers: (a) Trapezium. (b) V(20 + 60 + 10) = 2700. (c) 20 + 60 + 10 = 90 s.

Q4 (exam style) (9 marks). A cyclist leaves home H and rides along a straight road. Displacement east of H: 6 km after 20 min; stationary until 30 min; then 4 km east of H at 50 min, each stage at constant velocity.

- (a) Find the velocity, in km/h, for each stage. – 3 marks. Award for: Three gradients with direction
- (b) Find the average speed and the average velocity over the 50 minutes. – 3 marks. Award for: Total distance and displacement over time
- (c) Sketch the velocity–time graph. – 2 marks. Award for: Three horizontal segments
- (d) Explain why the model is unrealistic at the instants the velocity changes. – 1 marks. Award for: Instantaneous change requires infinite acceleration

Answers: (a) 18 km/h, 0, −6 km/h (west). (b) Speed 8 km/(5/6 h) = 9.6 km/h; velocity 4.8 km/h east. (c) Steps at 18, 0, −6. (d) Velocity cannot change instantly.

Week 13: Constant acceleration and vertical motion

Pearson S1 chapter 9.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Vertical motion under gravity. Take $g = 9.8$ m s^{-2}.

- (a) A stone is dropped from rest. Find its speed after 2 s. – 2 marks. Answer: 19.6 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (b) Find how far it falls in 3 s. – 2 marks. Answer: 44.1 m. Award for: Correct method (1); accurate answer (1).
- (c) A ball is thrown up at 14.7 m s^{-1}. Find the time to reach the top. – 2 marks. Answer: 1.5 s. Award for: Correct method (1); accurate answer (1).
- (d) Find the greatest height of that ball. – 2 marks. Answer: 11.025 = 11.0 m. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Vertical motion problems. Take $g = 9.8$ m s^{-2}.

- (a) A ball is projected up at 20 m s^{-1}. Find its velocity after 3 s. – 2 marks. Answer: -9.4 m s^{-1} (downwards). Award for: Correct method (1); accurate answer (1).
- (b) Find the times at which it is 15 m above the point of projection. – 2 marks. Answer: 0.99 s and 3.09 s. Award for: Correct method (1); accurate answer (1).
- (c) Find how long it is above 15 m. – 2 marks. Answer: 2.10 s. Award for: Correct method (1); accurate answer (1).
- (d) A stone falls from a 45 m cliff. Find the time to reach the sea. – 2 marks. Answer: 3.03 s. Award for: Correct method (1); accurate answer (1).
- (e) A ball is thrown up at 8 m s^{-1} from 2 m above ground. Find its speed on landing. – 3 marks. Answer: $\sqrt{103.2} = 10.2$ m s^{-1}. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). A ball is projected vertically upwards at 26 m/s from ground level. Using g=9.8, find time to greatest height, greatest height and total flight time. State the model assumptions.

Answer: t=2.6531 s; height=34.49 m; total=5.3061 s. Constant g, negligible air resistance, returns to launch level.

- Use a consistent vertical-motion model: 1
- Find time to greatest height: 1
- Find greatest height: 2
- Find total flight time: 1
- State relevant model assumptions: 1

Q4 (5 marks). A car starts at 8 m/s and accelerates at 2 m/s^2 over 40 m. Find its final speed and travel time; explain why a negative quadratic time root is rejected.

Answer: v=14.967 m/s; t=3.4833 s; elapsed time must be nonnegative.

- Use an appropriate constant-acceleration equation for final speed: 2
- Find the travel time: 2
- Explain rejection of a negative elapsed time: 1

Q5 (5 marks). A stone is dropped from rest from a height of 60 m. Find time to impact and impact speed. Then explain what changes if its initial velocity is downward rather than zero.

Answer: t=3.4993 s; speed=34.293 m/s. Nonzero u enters both motion equations; downward initial velocity reduces the time.

- Find the impact time: 2
- Find the impact speed: 1
- Explain the changed initial-velocity model and its effect on time: 2

Q6 (6 marks). Two particles start at the same point. A has u=6 m/s and a=2 m/s^2; B has u=12 m/s and a=0. Find when they next meet, their displacement then, and each velocity. Explain why equal positions do not mean equal velocities.

Answer: $6t+t^2=12t$; next meet t=6 s at 72 m. Velocities 18 and 12 m/s; position equality constrains values, not gradients.

- Form both position functions and equate them: 2
- Find the next meeting time and displacement: 2
- Find both velocities: 1
- Explain why equal position need not imply equal velocity: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (11 marks). Two-stage motion. Take $g = 9.8\ \text{m s}^{-2}$.

- (a) A lift accelerates up from rest at 2 m s^{-2} for 3 s. Find its speed. – 2 marks. Answer: 6 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (b) It then moves at constant speed for 5 s. Find the total distance so far. – 2 marks. Answer: 9 + 30 = 39 m. Award for: Correct method (1); accurate answer (1).
- (c) It then decelerates to rest in 4 s. Find the total distance. – 2 marks. Answer: 39 + 12 = 51 m. Award for: Correct method (1); accurate answer (1).
- (d) A particle released from rest falls 10 m. Find its speed. – 2 marks. Answer: 14 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (e) It then enters water and decelerates uniformly to rest in 2 m. Find the deceleration. – 3 marks. Answer: 49 m s^{-2}. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (11 marks). A stone is thrown vertically upwards at 14.7 m/s from a point 19.6 m above the ground. Take g = 9.8 m/s^2.

- (a) Find the greatest height of the stone above the ground. – 3 marks. Award for: $v^2 = u^2 + 2as$ and add 19.6
- (b) Find the time taken to reach the ground. – 4 marks. Award for: Displacement −19.6, solve quadratic
- (c) Find the speed at impact. – 2 marks. Award for: v = u + at

- (d) Find the time for which the stone is above its point of projection. – 2 marks. Award for: Displacement zero

Answers: (a) 11.025 + 19.6 ≈ 30.6 m. (b) $t^2 - 3t - 4 = 0$, t = 4 s. (c) 24.5 m/s. (d) 3 s.

Q3 (exam style) (9 marks). A particle moves along a straight line with constant acceleration. It passes A at 4 m/s and B, 6 s later, at 16 m/s.

- (a) Find the acceleration and the distance AB. – 3 marks. Award for: Both suvat results
- (b) M is the midpoint of AB. Find the speed at M. – 3 marks. Award for: $v^2 = u^2 + 2as$ with s = 30
- (c) Find the time taken from A to M. – 2 marks. Award for: v = u + at
- (d) Explain why this time is not 3 s. – 1 marks. Award for: Accelerating, so second half covered faster

Answers: (a) 2 m/s², 60 m. (b) √136 ≈ 11.7 m/s. (c) ≈ 3.83 s. (d) The particle is speeding up, so it takes longer to cover the first half.

Q4 (exam style) (8 marks). Car A passes point O at 20 m/s and decelerates uniformly at 0.5 m/s². At the same instant car B starts from rest at O and accelerates at 1.5 m/s² in the same direction.

- (a) Find the time at which B catches A, checking that A is still moving. – 4 marks. Award for: Equate displacements, solve, check velocity
- (b) Find the distance from O at this time. – 1 marks. Award for: Substitute
- (c) Find the greatest distance between the cars before B catches A. – 3 marks. Award for: Form separation and maximise

Answers: (a) $20t - 0.25t^2 = 0.75t^2$, t = 20 s; A's velocity is then 10 m/s > 0. (b) 300 m. (c) $d = 20t - t^2$, maximum 100 m at t = 10.

Week 14: Differentiation: gradients, tangents and normals

Pearson P1 chapter 12.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Differentiating powers of x.

- (a) Given $y = x^5$, find $\frac{dy}{dx}$. – 2 marks. Answer: $5x^4$. Award for: Correct method (1); accurate answer (1).
- (b) Given $y = 4x^3 - 6x + 2$, find $\frac{dy}{dx}$. – 2 marks. Answer: $12x^2 - 6$. Award for: Correct method (1); accurate answer (1).
- (c) Given $y = \frac{3}{x^2}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $-\frac{6}{x^3}$. Award for: Correct method (1); accurate answer (1).
- (d) Given $y = 8\sqrt{x}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $\frac{4}{\sqrt{x}}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Gradients, tangents and normals.

- (a) Find the gradient of $y = x^2 - 3x$ at $x = 4$. – 2 marks. Answer: 5. Award for: Correct method (1); accurate answer (1).
- (b) Find the gradient of $y = 2x^3$ at $(1, 2)$. – 2 marks. Answer: 6. Award for: Correct method (1); accurate answer (1).
- (c) Find the equation of the tangent to $y = x^2 + 1$ at $(2, 5)$. – 2 marks. Answer: $y = 4x - 3$. Award for: Correct method (1); accurate answer (1).
- (d) Find the equation of the normal to $y = x^2 + 1$ at $(2, 5)$. – 2 marks. Answer: $x + 4y - 22 = 0$. Award for: Correct method (1); accurate answer (1).
- (e) Find the point on $y = x^2 - 4x$ where the gradient is 2. – 3 marks. Answer: $(3, -3)$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). For $f(x) = x^3 - 9x^2 + 2x$, find f'(x), f''(x), and the tangent at x=1. Explain which derivative controls the local gradient.

Answer: f'=$3x^2 - 18x + 2$; f''=$6\,(x - 3)$; tangent $y = 7 - 13x$.

- Find the first and second derivatives: 2
- Find the point and tangent gradient: 1
- Form the tangent equation: 1
- Identify the first derivative as the local gradient: 1

Q4 (5 marks). Differentiate $y = 9x^{3/2} + \frac{4}{x} - 2\sqrt{x}$, for x>0. Find its gradient at x=4 and explain why the stated domain is needed.

Answer: Derivative $\frac{27\sqrt{x}}{2} - \frac{4}{x^2} - \frac{1}{\sqrt{x}}$; at 4=$\frac{105}{4}$. Denominator and fractional powers constrain x.

- Differentiate the three terms correctly: 3
- Evaluate the gradient: 1
- Explain the domain restriction: 1

Q5 (5 marks). For $y=x^2+9x$, use first principles to derive dy/dx. State the tangent and normal slopes at x=1, then find the normal equation.

Answer: [f(x+h)-f(x)]/h=2x+9+h, so derivative 2x+9; tangent slope 11; normal slope -1/11, through (1,10).

- Form and simplify the difference quotient: 2
- Take the limit to obtain the derivative: 1
- Find tangent and normal slopes: 1
- Form the normal equation: 1

Q6 (5 marks). The curve $y=x^3-9x$ has a tangent of gradient 27. Find all possible x-coordinates and the corresponding points, explaining why there can be two.

Answer: $3x^2-a=3a \Rightarrow x=\pm 2\sqrt{3}$. y=(a/3)x. Two roots of the quadratic gradient equation.

- Differentiate and form the gradient equation: 2

- Find both x-coordinates: 1
- Find the corresponding y-coordinates: 1
- Explain why two points are possible: 1

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (11 marks). Second derivatives and increasing functions.

- (a) Find $\frac{d^2y}{dx^2}$ for $y = x^4 - 3x^2$. – 2 marks. Answer: $12x^2 - 6$. Award for: Correct method (1); accurate answer (1).
- (b) Find $f''(x)$ for $f(x) = \frac{1}{x}$. – 2 marks. Answer: $\frac{2}{x^3}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the set of x for which $f(x) = x^2 - 6x$ is increasing. – 2 marks. Answer: $x > 3$. Award for: Correct method (1); accurate answer (1).
- (d) Show that $f(x) = x^3 + 3x$ is increasing for all x. – 2 marks. Answer: $f'(x) = 3x^2 + 3 > 0$. Award for: Correct method (1); accurate answer (1).
- (e) Find the interval on which $f(x) = x^3 - 3x^2$ is decreasing. – 3 marks. Answer: $0 < x < 2$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). The curve C has equation $y = x^3 - 5x^2 + 3x + 7$.

- (a) Find dy/dx. – 2 marks. Award for: Differentiate each term
- (b) Find the equation of the tangent at the point P where x = 4. – 3 marks. Award for: Point, gradient and equation
- (c) Find the x-coordinate of the other point on C where the tangent has the same gradient as at P, and its y-coordinate. – 3 marks. Award for: Solve quadratic and substitute
- (d) The normal at P meets the x-axis at Q. Find the coordinates of Q. – 2 marks. Award for: Perpendicular gradient and intercept

Answers: (a) $3x^2 - 10x + 3$. (b) P(4, 3), gradient 11: $y = 11x - 41$. (c) $3x^2 - 10x - 8 = 0$, $x = -2/3$, $y = 67/27$. (d) Q(37, 0).

Q3 (exam style) (10 marks). The curve C has equation $y = \frac{(2x-1)^2}{\sqrt{x}}$, x > 0.

- (a) Write y in the form $4x^{3/2} + px^{1/2} + qx^{-1/2}$. – 2 marks. Award for: Expand and divide
- (b) Find dy/dx. – 2 marks. Award for: Power rule for each term
- (c) Find the gradient of C at x = 1/4. – 2 marks. Award for: Correct substitution
- (d) Show that stationary points satisfy $12x^2 - 4x - 1 = 0$ and find the stationary point. – 4 marks. Award for: Multiply through, solve, reject negative root, find y

Answers: (a) 4x^(3/2) − 4x^(1/2) + x^(−1/2). (b) 6x^(1/2) − 2x^(−1/2) − ½x^(−3/2). (c) −5. (d) (6x + 1)(2x − 1) = 0, x = 1/2, point (1/2, 0).

Q4 (exam style) (10 marks).

- (a) Prove, from first principles, that the derivative of $2x^2 - 3x$ is 4x − 3. – 4 marks. Award for: Difference quotient, expansion, cancel h, limit
- (b) Find the point on $y = 2x^2 - 3x$ at which the normal has gradient 1/5. – 3 marks. Award for: Tangent gradient −5 and coordinates
- (c) Find the coordinates of the point where this normal meets the curve again. – 3 marks. Award for: Equation of normal and solve with curve

Answers: (a) $[2(x + h)^2 - 3(x + h) - 2x^2 + 3x]/h = 4x + 2h - 3 \to 4x - 3$. (b) $(-1/2, 2)$. (c) $y = x/5 + 2.1$; $20x^2 - 32x - 21 = 0$, other point (2.1, 2.52).

Week 15: Stationary points and optimisation

Pearson P1 chapter 12.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Stationary points. Find the coordinates of each stationary point and determine its nature.

- (a) $y = x^2 - 8x + 3$ – 2 marks. Answer: $(4, -13)$, minimum. Award for: Correct method (1); accurate answer (1).
- (b) $y = 5 + 6x - x^2$ – 2 marks. Answer: $(3, 14)$, maximum. Award for: Correct method (1); accurate answer (1).
- (c) $y = x^3 - 3x$ – 2 marks. Answer: $(1, -2)$ min, $(-1, 2)$ max. Award for: Correct method (1); accurate answer (1).
- (d) $y = 2x^3 - 9x^2 + 12x$ – 2 marks. Answer: $(1, 5)$ max, $(2, 4)$ min. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Optimisation.

- (a) Two positive numbers have sum 20. Find the maximum of their product. – 2 marks. Answer: 100. Award for: Correct method (1); accurate answer (1).
- (b) A rectangle has perimeter 40 cm. Show that its area is $A = x(20 - x)$. – 2 marks. Answer: width $20 - x$. Award for: Correct method (1); accurate answer (1).
- (c) Hence find the maximum area. – 2 marks. Answer: 100 cm^2. Award for: Correct method (1); accurate answer (1).
- (d) A box with square base side x cm and open top has volume 500 cm^3. Show the surface area is $S = x^2 + \frac{2000}{x}$. – 2 marks. Answer: height $\frac{500}{x^2}$; four sides $4x \cdot \frac{500}{x^2}$. Award for: Correct method (1); accurate answer (1).

Q3 (7 marks). For $y = x^3 - 9x$, find and classify every stationary point using the second derivative. Sketch the curve with stationary points and intercepts.

Answer: x=±sqrt(3); points (-sqrt(3),6sqrt(3)) maximum and (sqrt(3),-6sqrt(3)) minimum; roots 0,±sqrt(9).

- Differentiate and solve for both stationary x-values: 2

- Find both y-coordinates: 1
- Use the second derivative to classify both: 2
- Sketch with stationary points and all intercepts: 2

Q4 (6 marks). An open-top box has square base side x and volume 300 cm^3. Form surface area as a function of x and find its minimum, proving it is a minimum.

Answer: h=300/x^2; S=x^2+1200/x; x^3=600; S"=2+2400/x^3>0; minimum S=$60\sqrt[3]{45}$.

- Express the height and surface area in terms of x: 2
- Differentiate and solve the stationary equation: 2
- Establish a minimum: 1
- Find the minimum surface area: 1

Q5 (6 marks). A rectangle lies under y=7-x^2 above the x-axis, symmetric about the y-axis. Express area using its positive vertex coordinate x, then maximise it and state the physical domain.

Answer: A=2x(7-x^2), 0<x<sqrt(7); max at x=sqrt((7)/3); max area=$\frac{28\sqrt{21}}{9}$.

- Form the area and physical domain: 2
- Differentiate and solve for the admissible stationary point: 2
- Establish that it maximises the area: 1
- State the maximum area: 1

Q6 (5 marks). A cost model is C(x)=x^3-9x^2+29x+10 for x≥0. Show that C is increasing everywhere. Explain why having no stationary points is not alone a sufficient argument.

Answer: C'=3(x-3)^2+2>0. Need the derivative's sign, not merely absence of roots; a decreasing function can also have none.

- Differentiate: 1
- Complete the square or give another valid positivity argument: 2
- Conclude increasing on the whole domain: 1
- Explain why absence of stationary points alone is insufficient: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (11 marks). Rates and gradients in context.

- (a) The volume of water in a tank is $V = 2t^2 + 5t$ litres. Find $\frac{dV}{dt}$ at $t = 3$. – 2 marks. Answer: 17 litres per second. Award for: Correct method (1); accurate answer (1).
- (b) A ball's height is $h = 20t - 5t^2$. Find its velocity at $t = 1$. – 2 marks. Answer: 10 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (c) Find the maximum height of the ball. – 2 marks. Answer: 20 m. Award for: Correct method (1); accurate answer (1).

- (d) The cost is $C = x^2 - 40x + 500$. Find the value of x that minimises C. – 2 marks. Answer: $x = 20$. Award for: Correct method (1); accurate answer (1).
- (e) Find the minimum cost. – 3 marks. Answer: 100. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). A machine's energy use per batch is modelled by $E(x) = 2x^3 - 27x^2 + 108x + 240$, for 2 ≤ x ≤ 8. Here x is a speed setting and E is measured in kJ.

- (a) Show that $E'(x) = 6(x - 3)(x - 6)$. Suggested allocation: 3 marks. Award for differentiating the cubic terms (2) and correct factorisation (1).
- (b) Find both stationary values of x and classify them. Suggested allocation: 4 marks. Award for setting the derivative to zero and finding x = 3, 6 (2), then a valid second-derivative or sign test and both classifications (2).
- (c) Check the endpoints and find the least energy use in the permitted interval. Suggested allocation: 2 marks. Award for evaluating E(2), E(8) and E(6) (1), then comparing and concluding 348 kJ at x = 6 (1).
- (d) Give one reason not to use the model for x > 8. Suggested allocation: 1 mark. Award for recognising that extrapolation beyond the stated interval is not supported by the model.

Answers: (a) $E'(x) = 6x^2 - 54x + 108 = 6(x - 3)(x - 6)$. (b) $E''(x) = 12x - 54$, so $E''(3) = -18$: local maximum at x = 3; $E''(6) = 18$: local minimum at x = 6. (c) E(2) = 364 kJ, E(8) = 400 kJ and E(6) = 348 kJ. The derivative signs show that the only interior minimum is at x = 6, and its value is below both endpoint values; the least energy use is 348 kJ. (d) The model is specified only for 2 ≤ x ≤ 8, so predictions beyond that interval may be unreliable.

Q3 (exam style) (9 marks). $f(x) = x^3 - 6x^2 + 9x + 1$.

- (a) Find the coordinates of the stationary points and determine their nature. – 5 marks. Award for: Differentiate, solve, coordinates and nature test
- (b) Find the set of values of x for which f is decreasing. – 2 marks. Award for: Interval between roots
- (c) State the values of k for which f(x) = k has three distinct real roots. – 2 marks. Award for: Use stationary values

Answers: (a) (1, 5) maximum, (3, 1) minimum. (b) 1 ≤ x ≤ 3. (c) 1 < k < 5.

Q4 (exam style) (9 marks). A sheet of card measures 24 cm by 15 cm. A square of side x cm is cut from each corner and the sides are folded up to make an open box.

- (a) Show that the volume is $V = 4x^3 - 78x^2 + 360x$. – 2 marks. Award for: Dimensions and expansion
- (b) State the possible values of x. – 1 marks. Award for: 0 < x < 7.5
- (c) Find the maximum volume. – 5 marks. Award for: Differentiate, solve, reject invalid root, compute
- (d) Justify that this volume is a maximum. – 1 marks. Award for: Second derivative or sign change

Answers: (a) x(24 − 2x)(15 − 2x). (b) 0 < x < 7.5. (c) V' = 12(x − 3)(x − 10); x = 3, V = 486 cm^3. (d) V''(3) = −84 < 0.

Week 16: Integration and boundary conditions

Pearson P1 chapter 13.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (11 marks). Indefinite integration.

- (a) Find $\int 4x^3\,dx$. – 2 marks. Answer: $x^4 + c$. Award for: Correct method (1); accurate answer (1).
- (b) Find $\int \left(6x^2 - 2x + 5\right)dx$. – 2 marks. Answer: $2x^3 - x^2 + 5x + c$. Award for: Correct method (1); accurate answer (1).
- (c) Find $\int \frac{1}{x^2}\,dx$. – 2 marks. Answer: $-\frac{1}{x} + c$. Award for: Correct method (1); accurate answer (1).
- (d) Find $\int 3\sqrt{x}\,dx$. – 2 marks. Answer: $2x^{\frac{3}{2}} + c$. Award for: Correct method (1); accurate answer (1).
- (e) Find $\int (x+2)^2\,dx$. – 3 marks. Answer: $\frac{x^3}{3} + 2x^2 + 4x + c$. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Finding the constant of integration.

- (a) $\frac{dy}{dx} = 2x$ and $y = 5$ when $x = 1$. Find y. – 2 marks. Answer: $y = x^2 + 4$. Award for: Correct method (1); accurate answer (1).
- (b) $\frac{dy}{dx} = 3x^2 - 4$, curve through $(2, 3)$. Find y. – 2 marks. Answer: $y = x^3 - 4x + 3$. Award for: Correct method (1); accurate answer (1).
- (c) $f'(x) = 6x + 1$, $f(0) = -2$. Find $f(x)$. – 2 marks. Answer: $f(x) = 3x^2 + x - 2$. Award for: Correct method (1); accurate answer (1).
- (d) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$, curve through $(4, 1)$. Find y. – 2 marks. Answer: $y = 2\sqrt{x} - 3$. Award for: Correct method (1); accurate answer (1).
- (e) The gradient of a curve is $x^2 - 1$ and it passes through $(3, 10)$. Find its equation. – 3 marks. Answer: $y = \frac{x^3}{3} - x + 4$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). Find $\int (4x^2 - 4x + 3)\,dx$. Then find the particular function passing through (1,5), and check it by differentiation.

Answer: General $\frac{4x^3}{3} - 2x^2 + 3x$+C; C=$\frac{8}{3}$. Derivative recovers the integrand.

- Integrate all terms and include the arbitrary constant: 2
- Apply the boundary condition: 1
- Give the particular function: 1
- Check by differentiation: 1

Q4 (4 marks). Integrate $4\sqrt{x} + \frac{3}{x^2}$, for x>0. Explain why the same power rule does not integrate 1/x.

Answer: $\frac{8}{3}x^{3/2} - \frac{3}{x} + C$; power rule denominator n+1 vanishes at n=-1.

- Rewrite powers and integrate both terms, including the constant: 3
- Explain the failure of the power-rule expression when the exponent is minus one: 1

Q5 (4 marks). A curve has second derivative 24x and gradient 2 at x=0. It passes through (0,4). Find y and dy/dx, accounting for both integration constants.

Answer: $y'=12x^2+2$; $y=4x^3+2x+4$.

- Integrate once and use the gradient condition: 2
- Integrate again and use the position condition: 2

Q6 (5 marks). A particle has velocity $v=4t^2+2$ for $t\geq 0$ and s=3 at t=0. Find displacement function and displacement from t=1 to t=3. Explain why the initial position cancels in a displacement change.

Answer: $s=\frac{4}{3}t^3 + 2t + 3$; change=$\frac{116}{3}$ m. Subtraction cancels the constant.

- Integrate velocity and use the initial position: 2
- Evaluate the displacement change between the stated times: 2
- Explain cancellation of the initial-position constant: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Evaluate each definite integral.

- (a) Evaluate $\int_1^3 2x\,dx$, giving an exact answer. – 2 marks. Answer: 8. Award for: Correct method (1); accurate answer (1).
- (b) Evaluate $\int_0^2 \left(3x^2 + 1\right) dx$, giving an exact answer. – 2 marks. Answer: 10. Award for: Correct method (1); accurate answer (1).
- (c) Evaluate $\int_1^4 \frac{1}{x^2}\,dx$, giving an exact answer. – 2 marks. Answer: $\frac{3}{4}$ (about 0.75). Award for: Correct method (1); accurate answer (1).
- (d) Evaluate $\int_0^9 \sqrt{x}\,dx$, giving an exact answer. – 2 marks. Answer: 18. Award for: Correct method (1); accurate answer (1).
- (e) Evaluate $\int_0^3 (x-1)(x+3)\,dx$, giving an exact answer. – 3 marks. Answer: 9. Award for: Correct method (2); accurate answer (1).
- (f) Evaluate $\int_{-1}^2 \left(x^3 - x\right) dx$, giving an exact answer. – 3 marks. Answer: $\frac{9}{4}$ (about 2.25). Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). The curve C passes through (1, 22) and has gradient $\frac{dy}{dx} = \frac{(x+3)^2}{\sqrt{x}}$, x > 0.

- (a) Write dy/dx as a sum of powers of x. – 2 marks. Award for: Expand and divide

- (b) Find the equation of C. – 4 marks. Award for: Integrate each term, find constant
- (c) Find the y-coordinate when x = 9. – 1 marks. Award for: Substitute
- (d) Explain why the gradient is undefined at x = 0. – 1 marks. Award for: Division by √x

Answers: (a) x^(3/2) + 6x^(1/2) + 9x^(−1/2). (b) y = (2/5)x^(5/2) + 4x^(3/2) + 18x^(1/2) − 2/5. (c) 258.8. (d) The x^(−1/2) term involves division by zero.

Q3 (exam style) (8 marks). A curve y = f(x) has $f''(x) = 6x - 4$ and a stationary point at (1, 5).

- (a) Find f′(x). – 3 marks. Award for: Integrate and use f′(1) = 0
- (b) Find f(x). – 2 marks. Award for: Integrate and use f(1) = 5
- (c) Find the other stationary point and determine the nature of both. – 3 marks. Award for: Solve f′ = 0 and test with f″

Answers: (a) $3x^2 - 4x + 1$. (b) $x^3 - 2x^2 + x + 5$. (c) (1/3, 139/27) maximum; (1, 5) minimum.

Q4 (exam style) (8 marks). Water flows into a tank at a rate $\frac{dV}{dt} = 40 - 6\sqrt{t}$ litres per minute, 0 ≤ t ≤ 25. Initially the tank holds 200 litres.

- (a) Find V in terms of t. – 3 marks. Award for: Integrate and use initial value
- (b) Find the volume when t = 25. – 1 marks. Award for: Substitute
- (c) Show that the tank is filling throughout the 25 minutes. – 2 marks. Award for: Rate zero only at t ≈ 44.4
- (d) Find the mean rate of inflow over the 25 minutes. – 2 marks. Award for: Change in volume over time

Answers: (a) V = 40t − 4t^(3/2) + 200. (b) 700 L. (c) 40 − 6√t = 0 at t = 400/9 > 25, so the rate is positive. (d) 20 L/min.

Week 17: Definite integrals and areas

Pearson P1 chapter 13.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Areas under curves.

- (a) Find the area between $y = x^2$, the x-axis and $x = 0$, $x = 3$. – 2 marks. Answer: 9. Award for: Correct method (1); accurate answer (1).
- (b) Find the area between $y = 4 - x^2$ and the x-axis. – 2 marks. Answer: $\frac{32}{3}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the area under $y = \frac{4}{x^2}$ from $x = 1$ to $x = 4$. – 2 marks. Answer: 3. Award for: Correct method (1); accurate answer (1).
- (d) Find the area enclosed by $y = x(x - 4)$ and the x-axis. – 2 marks. Answer: $\frac{32}{3}$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Areas between curves and lines.

- (a) Find where $y = x^2$ meets $y = 2x$. – 2 marks. Answer: $x = 0,\ x = 2$. Award for: Correct method (1); accurate answer (1).
- (b) Find the area enclosed by $y = x^2$ and $y = 2x$. – 2 marks. Answer: $\frac{4}{3}$. Award for: Correct method (1); accurate answer (1).
- (c) Find the area enclosed by $y = x^2 - 4x + 5$ and $y = 5$. – 2 marks. Answer: $\frac{32}{3}$. Award for: Correct method (1); accurate answer (1).
- (d) Find the area enclosed by $y = 6x - x^2$ and $y = x$. – 2 marks. Answer: $\frac{125}{6}$. Award for: Correct method (1); accurate answer (1).
- (e) Find the area between $y = x^2 + 1$, $y = x + 3$. – 3 marks. Answer: $\frac{9}{2}$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). Find the area between y=x(5-x) and the x-axis. Identify the appropriate limits and explain the sign of the integrand between them.

Answer: Roots 0,5; positive between; area $\frac{125}{6}$.

- Find the intercepts and justify the sign between them: 2
- Set up the correct definite integral: 1
- Integrate and evaluate the exact area: 2

Q4 (7 marks). Evaluate $\int_{-1}^{2}(5x - 1)\,dx$. Then find the total area between the line and x-axis over that interval, explaining why the answers differ.

Answer: Signed $\frac{9}{2}$; total area $\frac{117}{10}$. Split at x=1/5; negative contributions must be reversed.

- Evaluate the signed integral: 2
- Locate the sign-change point: 1
- Set up the split total-area calculation: 1
- Evaluate the total area: 2
- Explain the difference between signed integral and area: 1

Q5 (5 marks). Find the finite area enclosed by $y=x^2$ and y=5x. Show the intersection calculation and explain which curve is upper.

Answer: Intersections x=0,5; line upper; area $\frac{125}{6}$.

- Find both intersections: 1
- Identify the upper curve: 1
- Form the difference integral: 1
- Evaluate the exact area: 2

Q6 (4 marks). Find k>0 such that $\int_0^k 5x^2\,dx = 45$. Explain why there is exactly one positive k and verify by substitution.

Answer: k^3=27; k=3. For k>0 the accumulated integral is strictly increasing.

- Integrate to obtain an equation for k: 1

- Solve for positive k: 1
- Justify uniqueness on the positive domain: 1
- Verify the result: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Definite integrals with unknowns.

- (a) Given $\int_0^k 2x\,dx = 16$, $k > 0$, find k. – 2 marks. Answer: 4. Award for: Correct method (1); accurate answer (1).
- (b) Given $\int_1^a 3x^2\,dx = 26$, find a. – 2 marks. Answer: 3. Award for: Correct method (1); accurate answer (1).
- (c) Find k if $\int_0^2 (kx + 1)\,dx = 10$. – 2 marks. Answer: 4. Award for: Correct method (1); accurate answer (1).
- (d) Show $\int_1^4 \frac{1}{\sqrt{x}}\,dx = 2$. – 2 marks. Answer: $[2\sqrt{x}]_1^4 = 4 - 2$. Award for: Correct method (1); accurate answer (1).
- (e) Find p if the area under $y = x^2$ from 0 to p is 72. – 3 marks. Answer: 6. Award for: Correct method (2); accurate answer (1).
- (f) Find a if $\int_a^{2a} x\,dx = 6$, $a > 0$. – 3 marks. Answer: 2. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). The curve C has equation $y = x^3 - 7x^2 + 10x$.

- (a) Find the x-coordinates of the points where C meets the x-axis. – 1 marks. Award for: Factorise
- (b) Find the total area of the regions enclosed between C and the x-axis. – 6 marks. Award for: Two separate integrals, signs handled, total
- (c) Explain why $\int_0^5 y\,dx$ does not give this area. – 1 marks. Award for: Negative region cancels

Answers: (a) 0, 2, 5. (b) 16/3 + 63/4 = 253/12. (c) The region below the axis contributes negatively; the integral is −125/12.

Q3 (exam style) (10 marks). The line y = x + 4 meets the curve $y = 16 + 2x - x^2$ at A and B.

- (a) Find the coordinates of A and B. – 3 marks. Award for: Equate, solve and find y
- (b) Find the area of the region enclosed between the line and the curve. – 5 marks. Award for: Integral of difference with limits
- (c) Find the coordinates of the maximum point of the curve and show it lies above the line. – 2 marks. Award for: Vertex and comparison

Answers: (a) (−3, 1), (4, 8). (b) ∫(12 + x − x²) dx from −3 to 4 = 343/6. (c) (1, 17); the line gives 5 at x = 1.

Q4 (exam style) (8 marks). The curve C has equation $y = 2x + \frac{8}{x^2}$, x > 0.

- (a) Find the coordinates of the stationary point of C and show that it is a minimum. – 4 marks. Award for: Differentiate, solve, coordinates, nature
- (b) Find the area of the region bounded by C, the x-axis and the lines x = 1 and x = 4. – 4 marks. Award for: Integrate negative power and evaluate

Answers: (a) (2, 6), y'' = $48/x^4 > 0$. (b) $[x^2 - 8/x]$ from 1 to 4 = 21.

Week 18: Exponentials and logarithms

Pearson P1 chapter 14.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Laws of logarithms.

- (a) Write $\log_2 32 = 5$ in index form. – 2 marks. Answer: $2^5 = 32$. Award for: Correct method (1); accurate answer (1).
- (b) Evaluate $\log_3 81$. – 2 marks. Answer: 4. Award for: Correct method (1); accurate answer (1).
- (c) Evaluate $\log_4 \frac{1}{16}$. – 2 marks. Answer: -2. Award for: Correct method (1); accurate answer (1).
- (d) Write $\log 6 + \log 5 - \log 3$ as a single logarithm. – 2 marks. Answer: $\log 10$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Exponential and logarithmic equations. Give answers to 3 significant figures where needed.

- (a) Solve $2^x = 50$. – 2 marks. Answer: $x = 5.64$. Award for: Correct method (1); accurate answer (1).
- (b) Solve $3^{2x} = 20$. – 2 marks. Answer: $x = 1.36$. Award for: Correct method (1); accurate answer (1).
- (c) Solve $e^x = 7$. – 2 marks. Answer: $x = \ln 7 = 1.95$. Award for: Correct method (1); accurate answer (1).
- (d) Solve $\ln(2x + 1) = 3$. – 2 marks. Answer: $x = \frac{e^3 - 1}{2} = 9.54$. Award for: Correct method (1); accurate answer (1).
- (e) Solve $\log_2 x + \log_2(x - 2) = 3$. – 3 marks. Answer: $x = 4$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). Solve $\log_2(x - 1) + \log_2(x + 1) = 8$. State domain restrictions before combining logarithms and reject invalid roots.

Answer: x>1; x^2-1=2^8; x=$\sqrt{257}$ only.

- State the initial domain: 1
- Combine logarithms and form an algebraic equation: 2
- Solve and reject inadmissible roots: 2

Q4 (6 marks). A model is N=600e^(-0.2t), t in hours. Find N after 5 hours, its half-life and when N first falls below 120. Explain why the model never reaches zero.

Answer: N(5)=220.73; half-life=3.4657 h; threshold t>8.0472 h; exponential always positive.

- Calculate the amount after the stated time: 1
- Find the half-life: 2
- Solve the threshold inequality: 2
- Explain why zero is never reached: 1

Q5 (4 marks). A model y=Ab^x has y=60 at x=0 and y=240 at x=2. Find A,b (b>0), predict y at x=3 and linearise the relationship using logarithms.

Answer: A=60, b=2; prediction 480; ln y=ln(60)+x ln2.

- Determine A: 1
- Determine the positive base b: 1
- Make the requested prediction: 1
- Give the logarithmic linearisation: 1

Q6 (5 marks). Solve $e^{2x} - 7e^{x} + 6 = 0$. Explain the substitution and the condition it imposes. Verify all real x solutions.

Answer: Let u=e^x>0; (u-1)(u-6)=0; x=0 or ln(6).

- Make the exponential substitution with its positivity condition: 1
- Solve the quadratic: 2
- Convert to all real x-solutions: 1
- Verify the solutions: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Exponential models and linear forms.

- (a) A population is $P = 200e^{0.05t}$. Find P when $t = 10$. – 2 marks. Answer: 330. Award for: Correct method (1); accurate answer (1).
- (b) Find when the population reaches 500. – 2 marks. Answer: $t = 20 \ln 2.5 = 18.3$. Award for: Correct method (1); accurate answer (1).
- (c) Find $\frac{dP}{dt}$ and its value when $t = 0$. – 2 marks. Answer: $10e^{0.05t}$; 10. Award for: Correct method (1); accurate answer (1).
- (d) Show that $y = ax^n$ gives a straight line when $\log y$ is plotted against $\log x$. – 2 marks. Answer: $\log y = \log a + n \log x$. Award for: Correct method (1); accurate answer (1).
- (e) A graph of $\log y$ against x is a line with gradient 0.3 and intercept 1.2. Given $y = ab^x$, find a and b (logs base 10). – 3 marks. Answer: $a = 10^{1.2} = 15.8$, $b = 10^{0.3} = 2.00$. Award for: Correct method (2); accurate answer (1).

- (f) A car's value is $V = 18000e^{-0.15t}$. Find the value after 4 years and interpret 18000. – 3 marks. Answer: 9880; initial value. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks).

- (a) Solve $3^{2x} - 10 \times 3^x + 9 = 0$. – 3 marks. Award for: Substitution, quadratic and both solutions
- (b) Solve $\log_2(x + 5) - \log_2(x - 1) = 2$. – 3 marks. Award for: Combine logs, solve, check validity
- (c) Solve $5^{x+1} = 2^{3x}$, giving x to 3 significant figures. – 3 marks. Award for: Take logs, collect x terms, evaluate

Answers: (a) x = 0, 2. (b) x = 3. (c) x = ln 5/(ln 8 − ln 5) ≈ 3.42.

Q3 (exam style) (8 marks). The value, £V, of a car t years after purchase is modelled by $V = 18000e^{-0.15t} + 1500$.

- (a) State the value when new. – 1 marks. Award for: t = 0
- (b) Find the value after 4 years. – 1 marks. Award for: Substitute
- (c) Find when the value first falls below £6000. – 3 marks. Award for: Rearrange and use ln
- (d) Find dV/dt when t = 4 and interpret it. – 2 marks. Award for: Differentiate and interpret rate
- (e) State the long-term value predicted by the model. – 1 marks. Award for: Limit

Answers: (a) £19 500. (b) ≈ £11 379. (c) t > ln 4/0.15 ≈ 9.24 years. (d) −2700e^(−0.6) ≈ −1482: losing about £1482 per year. (e) £1500.

Q4 (exam style) (8 marks). A quantity y is believed to follow $y = ab^x$. A graph of $\log_{10}$ y against x is a straight line through (0, 1.2) and (5, 2.7).

- (a) Show that $y = ab^x$ gives a straight line when $\log_{10}$ y is plotted against x. – 2 marks. Award for: Take logs and identify gradient and intercept
- (b) Find a and b. – 3 marks. Award for: Gradient and intercept converted
- (c) Predict y when x = 8 and comment on its reliability. – 2 marks. Award for: Prediction and extrapolation comment
- (d) Explain how you could test whether $y = kx^n$ is a better model. – 1 marks. Award for: Plot log y against log x

Answers: (a) log y = log a + x log b. (b) a = 10^1.2 ≈ 15.8, b = 10^0.3 ≈ 2.00. (c) 10^3.6 ≈ 3980; extrapolation may be unreliable. (d) Plot log y against log x and check linearity.

Week 19: Two-dimensional vectors

Pearson P1 chapter 11.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (11 marks). Vector basics. $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = -2\mathbf{i} + 5\mathbf{j}$.

- (a) Find $\mathbf{a} + \mathbf{b}$. – 2 marks. Answer: $\mathbf{i} + 9\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (b) Find $2\mathbf{a} - 3\mathbf{b}$. – 2 marks. Answer: $12\mathbf{i} - 7\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (c) Find $|\mathbf{a}|$. – 2 marks. Answer: 5. Award for: Correct method (1); accurate answer (1).
- (d) Find a unit vector in the direction of $\mathbf{a}$. – 2 marks. Answer: $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$. Award for: Correct method (1); accurate answer (1).
- (e) Find the angle $\mathbf{a}$ makes with $\mathbf{i}$. – 3 marks. Answer: 53.1°. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Position vectors. A is $(1, 2)$, B is $(7, -6)$.

- (a) Find $\overrightarrow{AB}$. – 2 marks. Answer: $6\mathbf{i} - 8\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (b) Find the distance AB. – 2 marks. Answer: 10. Award for: Correct method (1); accurate answer (1).
- (c) Find the position vector of the midpoint of AB. – 2 marks. Answer: $4\mathbf{i} - 2\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (d) Find the position vector of P on AB with $AP : PB = 1 : 3$. – 2 marks. Answer: $2.5\mathbf{i}$. Award for: Correct method (1); accurate answer (1).
- (e) Show that $\mathbf{p} = 4\mathbf{i} - 6\mathbf{j}$ is parallel to $\mathbf{q} = -6\mathbf{i} + 9\mathbf{j}$. – 3 marks. Answer: $\mathbf{q} = -\frac{3}{2}\mathbf{p}$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A=(1,7), B=(5,10). Find AB, its magnitude and a unit vector from A to B. Find C such that B is the midpoint of AC.

Answer: AB=(4,3), magnitude5, unit=(4/5,3/5); C=(9,13).

- Find the displacement vector: 1
- Find its magnitude and unit vector: 2
- Use the midpoint condition to find C: 2

Q4 (5 marks). A particle has position $\mathbf{r} = (2 + 3t)\mathbf{i} + (7 - 4t)\mathbf{j}$. Find its position at t=2, its speed and when its j component is zero. Explain why speed is not the sum of components.

Answer: (8,-1); velocity=(3,-4), speed5; t=$\frac{7}{4}$. Magnitude uses Pythagoras.

- Find the position at the given time: 1
- Find velocity and hence speed: 2
- Find the time the j-component vanishes: 1
- Explain why speed is a magnitude, not a component sum: 1

Q5 (4 marks). Find λ such that (λ,9) is parallel to (2,3). Find its magnitude and explain whether a negative scalar multiple would point in the same direction.

Answer: λ=6; vector=((9)/3)(2,3); magnitude=$3\sqrt{13}$. A negative multiple points oppositely.

- Use a common scalar factor to determine lambda: 2
- Find the magnitude: 1
- Explain the direction of a negative multiple: 1

Q6 (5 marks). A=(0,0), B=(7,2), C=(14,4). Prove collinearity using vectors. Find the point dividing AB in ratio 2:1 from A, then explain the position-vector formula used.

Answer: AC=2AB, hence collinear; point $\left(\frac{14}{3}, \frac{4}{3}\right)$. Position A+(2/3)(B-A).

- Use scalar multiples to prove collinearity: 2
- Find the division point: 2
- Explain the position-vector formula: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Vectors in geometry and motion.

- (a) Find the vector of magnitude 10 in the direction of $6\mathbf{i} + 8\mathbf{j}$. – 2 marks. Answer: $6\mathbf{i} + 8\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (b) Write the vector of magnitude 4 at 30° above $\mathbf{i}$ in component form. – 2 marks. Answer: $2\sqrt{3}\mathbf{i} + 2\mathbf{j}$. Award for: Correct method (1); accurate answer (1).
- (c) In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$. Find $\overrightarrow{AB}$. – 2 marks. Answer: $\mathbf{b} - \mathbf{a}$. Award for: Correct method (1); accurate answer (1).
- (d) M is the midpoint of AB. Find $\overrightarrow{OM}$. – 2 marks. Answer: $\frac{1}{2}(\mathbf{a} + \mathbf{b})$. Award for: Correct method (1); accurate answer (1).
- (e) A ship moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1} from the origin. Find its position after 2 hours. – 3 marks. Answer: $6\mathbf{i} + 8\mathbf{j}$. Award for: Correct method (2); accurate answer (1).
- (f) Find its speed. – 3 marks. Answer: 5 km h^{-1}. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). Relative to O, $\overrightarrow{OA} = 4\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 10\mathbf{i} - 6\mathbf{j}$.

- (a) Find $|\overrightarrow{AB}|$ and a unit vector in the direction of $\overrightarrow{AB}$. – 3 marks. Award for: AB, magnitude, unit vector
- (b) Find the angle between $\overrightarrow{AB}$ and i. – 2 marks. Award for: Correct angle and direction
- (c) P lies on AB with AP : PB = 1 : 3. Find $\overrightarrow{OP}$. – 2 marks. Award for: Ratio division
- (d) OABC is a parallelogram. Find $\overrightarrow{OC}$. – 2 marks. Award for: OC = AB

Answers: (a) 6i − 8j, magnitude 10, unit 0.6i − 0.8j. (b) 53.1° below i. (c) 5.5i. (d) 6i − 8j.

Q3 (exam style) (8 marks). In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of AB, N is the midpoint of OA and P lies on OM with OP : PM = 2 : 1.

- (a) Find $\overrightarrow{OM}$ in terms of a and b. – 1 marks. Award for: Midpoint
- (b) Find $\overrightarrow{OP}$. – 1 marks. Award for: Ratio
- (c) Find $\overrightarrow{BP}$ and $\overrightarrow{BN}$. – 3 marks. Award for: Both vectors
- (d) Hence prove that B, P and N are collinear and find BP : PN. – 3 marks. Award for: Scalar multiple and common point

Answers: (a) ½(a + b). (b) ⅓(a + b). (c) BP = ⅓a − ⅔b, BN = ½a − b. (d) BP = ⅔BN with B common, so BP : PN = 2 : 1.

Q4 (exam style) (8 marks). At t = 0, ship A is at (2i + 3j) km moving with velocity (3i + 4j) km/h, and ship B is at (10i − j) km moving with velocity (−i + 6j) km/h.

- (a) Write down the position vectors of A and B after t hours. – 2 marks. Award for: Both expressions
- (b) Show that $\overrightarrow{AB} = (8 - 4t)\mathbf{i} + (2t - 4)\mathbf{j}$. – 1 marks. Award for: Subtract
- (c) Show that the ships collide and find when and where. – 3 marks. Award for: AB = 0 at the same t, position
- (d) Find the speed of each ship. – 2 marks. Award for: Magnitudes

Answers: (a) A: (2 + 3t)i + (3 + 4t)j; B: (10 − t)i + (6t − 1)j. (b) Subtract. (c) Both components vanish at t = 2; position 8i + 11j. (d) 5 km/h and √37 ≈ 6.08 km/h.

Week 20: Forces and Newton's laws

Pearson S1 chapter 10.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (11 marks). Newton's second law. Take $g = 9.8$ m s^{-2}.

- (a) A resultant force of 12 N acts on a 3 kg mass. Find the acceleration. – 2 marks. Answer: 4 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (b) Find the weight of a 5 kg mass. – 2 marks. Answer: 49 N. Award for: Correct method (1); accurate answer (1).
- (c) A 2 kg mass is pulled by 15 N against resistance 5 N. Find the acceleration. – 2 marks. Answer: 5 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) A 1200 kg car accelerates at 0.5 m s^{-2} against resistance 400 N. Find the driving force. – 2 marks. Answer: 1000 N. Award for: Correct method (1); accurate answer (1).

- (e) A 4 kg particle is lifted by a rope with acceleration 2 m s^{-2}. Find the tension. – 3 marks. Answer: 47.2 N. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Forces as vectors.

- (a) Find the resultant of $(3\mathbf{i} + 2\mathbf{j})$ N and $(\mathbf{i} - 5\mathbf{j})$ N. – 2 marks. Answer: $(4\mathbf{i} - 3\mathbf{j})$ N. Award for: Correct method (1); accurate answer (1).
- (b) Find its magnitude. – 2 marks. Answer: 5 N. Award for: Correct method (1); accurate answer (1).
- (c) A 2 kg particle has resultant force $(6\mathbf{i} + 8\mathbf{j})$ N. Find its acceleration vector. – 2 marks. Answer: $(3\mathbf{i} + 4\mathbf{j})$ m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) Find the magnitude of that acceleration. – 2 marks. Answer: 5 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (e) Forces $(a\mathbf{i} + 2\mathbf{j})$, $(3\mathbf{i} + b\mathbf{j})$ and $(-5\mathbf{i} - 7\mathbf{j})$ N are in equilibrium. Find a, b. – 3 marks. Answer: $a = 2,\ b = 5$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A 8 kg particle on a smooth horizontal surface is pulled by a horizontal force 32 N. Draw its force diagram, find its acceleration and normal reaction, and explain why the reaction is not an extra horizontal force.

Answer: a=4 m/s^2; R=78.4 N upward, weight downward. Reaction perpendicular to surface.

- Draw a correct force diagram: 2
- Use horizontal Newton's second law to find acceleration: 1
- Use vertical equilibrium to find reaction: 1
- Explain the direction of the reaction: 1

Q4 (5 marks). A lift carries a particle of mass 10 kg. Find the support force when accelerating (i) upward at 2 m/s^2, (ii) downward at 2 m/s^2. Interpret the difference.

Answer: Up 118 N; down 78 N. Apparent weight changes with acceleration.

- Form and solve the upward-acceleration force equation: 2
- Form and solve the downward-acceleration force equation: 2
- Interpret the change in support force: 1

Q5 (3 marks). A particle of mass 8 kg is acted on by forces (6,8) N and (-2,4) N. Find resultant force, acceleration vector and acceleration magnitude.

Answer: Resultant (4,12) N; acceleration (4/8,12/8); magnitude $\frac{\sqrt{10}}{2}$ m/s^2.

- Find the resultant force vector: 1
- Find the acceleration vector: 1
- Find its magnitude: 1

Q6 (4 marks). A car of mass 800 kg has driving force 4000 N and resistance 800 N. Find acceleration, then speed after 6 s from rest. Explain how resistance would change the result if it increased with speed.

Answer: Resultant=3200 N, acceleration4 m/s^2; speed24 m/s. If resistance increases, acceleration decreases and the constant-acceleration prediction overestimates speed.

- Find resultant force and acceleration: 2
- Find the later speed: 1
- Explain the effect of increasing resistance on the prediction: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Force diagrams and modelling.

- (a) Draw a force diagram for a box pulled along rough horizontal ground by a horizontal rope. – 2 marks. Answer: weight, normal reaction, tension, friction. Award for: Correct method (1); accurate answer (1).
- (b) A 3 kg box on smooth ground is pulled by a horizontal 9 N force from rest. Find its speed after 4 s. – 2 marks. Answer: 12 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (c) State what 'light' and 'inextensible' mean for a string. – 2 marks. Answer: massless; same acceleration/tension throughout. Award for: Correct method (1); accurate answer (1).
- (d) A 0.5 kg ball falls with air resistance 1.9 N. Find its acceleration. – 2 marks. Answer: 6 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (e) A 1000 kg car brakes from 20 m s^{-1} to rest in 40 m. Find the braking force. – 3 marks. Answer: 5000 N. Award for: Correct method (2); accurate answer (1).
- (f) Explain why the normal reaction on a lift floor changes when the lift accelerates. – 3 marks. Answer: resultant force must equal mass × acceleration. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). A box of mass 12 kg is pulled from rest across a horizontal floor by a horizontal rope with tension 60 N. Resistance to motion is 24 N.

- (a) Find the acceleration. – 2 marks. Award for: F = ma
- (b) Find the speed and the distance travelled after 4 s. – 2 marks. Award for: suvat
- (c) The rope then breaks. Find the further distance travelled before the box stops. – 3 marks. Award for: Deceleration and distance
- (d) State one modelling assumption. – 1 marks. Award for: Valid assumption

Answers: (a) 3 m/s^2. (b) 12 m/s, 24 m. (c) Deceleration 2 m/s^2, distance 36 m. (d) e.g. box as particle; resistance constant.

Q3 (exam style) (8 marks). A lift of mass 600 kg carries a person of mass 75 kg. Take g = 9.8 m/s^2.

- (a) The lift accelerates upwards at 0.8 m/s^2. Find the tension in the cable. – 2 marks. Award for: Whole system
- (b) Find the reaction of the floor on the person. – 2 marks. Award for: Person alone
- (c) Later, moving upwards, the lift decelerates and the reaction on the person is 690 N. Find the deceleration and the tension. – 4 marks. Award for: Person equation then whole system

Answers: (a) 675 × 10.6 = 7155 N. (b) 795 N. (c) 0.6 m/s^2; T = 675 × 9.2 = 6210 N.

Q4 (exam style) (8 marks). Forces (3i + 4j) N, (pi − 7j) N and (−5i + qj) N act on a particle of mass 2 kg.

- (a) The particle is in equilibrium. Find p and q. – 2 marks. Award for: Resultant zero
- (b) The third force is removed. Find the magnitude of the acceleration and its direction relative to i. – 3 marks. Award for: Resultant, acceleration, angle
- (c) The particle starts from rest. Find its displacement vector after 4 s. – 2 marks. Award for: s = ½at^2
- (d) Find its distance from the starting point. – 1 marks. Award for: Magnitude

Answers: (a) p = 2, q = 3. (b) a = 2.5i − 1.5j, |a| ≈ 2.92 m/s^2, 31.0° below i. (c) 20i − 12j m. (d) ≈ 23.3 m.

Week 21: Connected particles and pulleys

Pearson S1 chapter 10.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Connected particles. Take $g = 9.8$ m s^{-2}.

- (a) A car of mass 1000 kg tows a trailer of mass 200 kg with driving force 1800 N and no resistance. Find the acceleration. – 2 marks. Answer: 1.5 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (b) Find the tension in the tow bar. – 2 marks. Answer: 300 N. Award for: Correct method (1); accurate answer (1).
- (c) Repeat with resistances 200 N on the car and 100 N on the trailer: find the acceleration. – 2 marks. Answer: 1.25 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) Find the new tension. – 2 marks. Answer: 350 N. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). A 4 kg block on a smooth table is joined by a string over a pulley at the edge to a 1 kg hanging mass ($g = 9.8$).

- (a) Write the equation of motion for each particle. – 2 marks. Answer: $T = 4a$; $9.8 - T = a$. Award for: Correct method (1); accurate answer (1).
- (b) Find the acceleration. – 2 marks. Answer: 1.96 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (c) Find the tension. – 2 marks. Answer: 7.84 N. Award for: Correct method (1); accurate answer (1).
- (d) Find the speed after the system moves 0.5 m from rest. – 2 marks. Answer: 1.4 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (e) Find the force on the pulley. – 3 marks. Answer: $7.84\sqrt{2} = 11.1$ N. Award for: Correct method (2); accurate answer (1).

Q3 (7 marks). Masses 9 kg and 11 kg hang over a smooth pulley on a light inextensible string. Find acceleration and tension, drawing separate force diagrams. State the common-acceleration assumption.

Answer: $a=0.98$ m/s^2, heavier mass down; $T=97.02$ N. Equations $11g-T=11a$ and $T-9g=9a$; string taut/inextensible.

- Draw separate force diagrams: 2
- Form both equations of motion: 2
- Solve for acceleration and tension: 2
- State the linked-acceleration assumption: 1

Q4 (5 marks). A 9 kg mass on a smooth horizontal table is connected over a smooth pulley to a hanging 18 kg mass. Find acceleration, tension and distance moved from rest in 2 s.

Answer: $a=2g/3=6.5333$; $T=58.8$ N; distance=13.067 m. No obstruction or loss of string tension.

- Form the equations for both particles: 2
- Solve for acceleration and tension: 2
- Find the distance from rest: 1

Q5 (5 marks). Two particles of masses 9 and 10 kg lie on a smooth horizontal line and are connected by a light string. A force 114 N pulls the heavier particle. Find acceleration and tension, and explain why the whole-system equation alone cannot give tension.

Answer: $a=6$ m/s^2; $T=54$ N. Tension internal, cancels in whole-system force balance.

- Use the whole-system equation to find acceleration: 2
- Use one particle to obtain tension: 2
- Explain cancellation of internal tension in the system equation: 1

Q6 (4 marks). A light string passes over a pulley. Explain the separate consequences of 'light', 'inextensible' and 'smooth pulley'. A student sets tension equal to each hanging weight while the masses accelerate; explain the error.

Answer: Light string: tension uniform along a free segment; smooth ideal pulley gives equal tension across it; inextensible taut string links acceleration magnitudes. During acceleration resultant is ma, so tension need not equal weight.

- Explain the consequence of a light string: 1
- Explain the consequence of an inextensible taut string: 1
- Explain the consequence of a smooth pulley: 1
- Use Newton's second law to explain why tension need not equal weight: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Pulleys and strings that break ($g = 9.8$).

- (a) Masses 2 kg and 6 kg hang over a smooth pulley and are released from rest. Find the acceleration. – 2 marks. Answer: 4.9 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (b) Find the speed after 1 s. – 2 marks. Answer: 4.9 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (c) At this moment the string breaks. Find the further time for the 2 kg mass to reach its highest point. – 2 marks. Answer: 0.5 s. Award for: Correct method (1); accurate answer (1).
- (d) Find how much higher it rises after the string breaks. – 2 marks. Answer: 1.225 = 1.23 m. Award for: Correct method (1); accurate answer (1).
- (e) A truck pulls a 500 kg trailer at 0.8 m s^{-2} with trailer resistance 150 N. Find the tension. – 3 marks. Answer: 550 N. Award for: Correct method (2); accurate answer (1).
- (f) State why the tension is the same throughout a light inextensible string. – 3 marks. Answer: light: no mass to accelerate; inextensible: same acceleration. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). Particles A (3 kg) and B (5 kg) hang from a light inextensible string over a smooth pulley. They are released from rest with B 1.6 m above the ground. Take g = 9.8 m/s^2.

- (a) Find the acceleration and the tension. – 5 marks. Award for: Two equations and solve
- (b) Find the speed of B as it reaches the ground. – 2 marks. Award for: suvat
- (c) B does not rebound. Find the further height risen by A. – 2 marks. Award for: String slack, A under gravity
- (d) Find the greatest height of A above its starting point. – 1 marks. Award for: Add

Answers: (a) a = 2.45 m/s^2, T = 36.75 N. (b) 2.8 m/s. (c) 0.4 m. (d) 2.0 m.

Q3 (exam style) (8 marks). A car of mass 1200 kg tows a trailer of mass 400 kg with a light rigid towbar on a horizontal road. Resistances are 300 N on the car and 100 N on the trailer. The driving force is 2400 N.

- (a) Find the acceleration. – 2 marks. Award for: Whole system
- (b) Find the tension in the towbar. – 2 marks. Award for: Trailer equation
- (c) The driving force is removed and a braking force B N acts on the car, giving a deceleration of 2 m/s^2. Find B and the force in the towbar, stating whether it is a tension or a thrust. – 4 marks. Award for: Whole system and trailer, interpret sign

Answers: (a) 1.25 m/s^2. (b) 600 N. (c) B = 2800 N; thrust of 700 N.

Q4 (exam style) (8 marks). Block P of mass 4 kg rests on a smooth horizontal table and is connected by a light string over a smooth pulley at the edge to particle Q, of mass m kg, hanging freely. From rest, Q falls 0.9 m in 1.5 s.

- (a) Find the acceleration. – 2 marks. Award for: suvat
- (b) Find m. – 4 marks. Award for: Equations for P and Q
- (c) Find the magnitude and direction of the force exerted on the pulley by the string. – 2 marks. Award for: Resultant of two perpendicular tensions

Answers: (a) 0.8 m/s^2. (b) T = 3.2 N, m(9.8 − 0.8) = 3.2, m = 16/45 ≈ 0.356 kg. (c) 3.2√2 ≈ 4.53 N at 45° below the horizontal.

Week 22: Variable acceleration

Pearson S1 chapter 11.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Variable acceleration: differentiation. $s = t^3 - 6t^2 + 9t$ m.

- (a) Find v in terms of t. – 2 marks. Answer: $v = 3t^2 - 12t + 9$. Award for: Correct method (1); accurate answer (1).
- (b) Find a in terms of t. – 2 marks. Answer: $a = 6t - 12$. Award for: Correct method (1); accurate answer (1).
- (c) Find the velocity when $t = 2$. – 2 marks. Answer: -3 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (d) Find the times when the particle is at rest. – 2 marks. Answer: 1 s and 3 s. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Variable acceleration: integration.

- (a) $v = 4t + 3$. Find s given $s = 0$ at $t = 0$. – 2 marks. Answer: $s = 2t^2 + 3t$. Award for: Correct method (1); accurate answer (1).
- (b) $a = 6t$, $v = 2$ when $t = 0$. Find v. – 2 marks. Answer: $v = 3t^2 + 2$. Award for: Correct method (1); accurate answer (1).
- (c) Hence find s when $t = 2$, given $s = 0$ at $t = 0$. – 2 marks. Answer: 12 m. Award for: Correct method (1); accurate answer (1).
- (d) $v = 3t^2 - 2t$. Find the displacement from $t = 1$ to $t = 3$. – 2 marks. Answer: 18 m. Award for: Correct method (1); accurate answer (1).
- (e) $a = 4 - 2t$, starting from rest. Find the maximum velocity. – 3 marks. Answer: 4 m s^{-1} at $t = 2$. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A particle has velocity $v(t) = t^2 - 4t + 3$ for 0≤t≤4. Find acceleration, stationary times and intervals of forward/backward motion.

Answer: Acceleration $2t - 4$; v=0 at 1,3; forward [0,1) and (3,4], backward (1,3).

- Differentiate to find acceleration: 1
- Find stationary times: 2
- Use velocity signs to state the motion intervals: 2

Q4 (6 marks). For the same velocity $t^2 - 4t + 3$, find displacement and total distance over 0≤t≤4. Explain why a split integral is necessary.

Answer: Displacement $\frac{4}{3}$ m; distance 4 m. Reverse sign of negative velocity contribution on (1,3).

- Integrate and evaluate displacement: 2
- Split at the stationary times with the correct signs: 2
- Evaluate total distance: 1
- Explain why splitting is required: 1

Q5 (5 marks). A particle has acceleration 18t and initial velocity -3 m/s. Find v(t), its first positive stationary time, and displacement by t=2.

Answer: v=9t²-3; stationary t=$\frac{\sqrt{3}}{3}$; displacement 18 m.

- Integrate acceleration and use the initial velocity: 2
- Find the first positive stationary time: 1
- Integrate velocity for the stated displacement: 2

Q6 (6 marks). A particle has position $s = t^3 - 9t + 2$. Find the time t≥0 of minimum position, that minimum, and the speed at t=0. Distinguish 'at rest' from 'zero acceleration'.

Answer: t=sqrt(3); minimum 2-6sqrt(3); initial speed 9. Rest means v=0; zero acceleration means dv/dt=0.

- Differentiate position and find the admissible stationary time: 2
- Establish and evaluate the minimum position: 2
- Find initial speed: 1
- Distinguish rest from zero acceleration: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Motion problems with calculus.

- (a) $v = 12t - 3t^2$. Find the time when the acceleration is zero. – 2 marks. Answer: 2 s. Award for: Correct method (1); accurate answer (1).
- (b) Find the maximum velocity. – 2 marks. Answer: 12 m s^{-1}. Award for: Correct method (1); accurate answer (1).
- (c) Find when the particle returns to rest. – 2 marks. Answer: 4 s. Award for: Correct method (1); accurate answer (1).
- (d) Find the displacement over the first 4 s. – 2 marks. Answer: 32 m. Award for: Correct method (1); accurate answer (1).
- (e) Explain why $\int v\,dt$ can differ from total distance. – 3 marks. Answer: velocity changes sign; negative displacements cancel. Award for: Correct method (2); accurate answer (1).

- (f) $s = 2t^3 - 3t^2$. Find the acceleration when the velocity is 12 m s^{-1}. – 3 marks. Answer: $t = 2$; $a = 18$ m s^{-2}. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). A particle moves in a straight line with velocity $v = 2t^2 - 14t + 20$ m/s, t ≥ 0.

- (a) Find the times when the particle is at rest. – 2 marks. Award for: Solve v = 0
- (b) Find the acceleration when t = 3. – 2 marks. Award for: Differentiate
- (c) Find the total distance travelled in the first 5 seconds. – 5 marks. Award for: Integrate and split at t = 2

Answers: (a) t = 2, 5. (b) −2 m/s^2. (c) 52/3 + 27/3 = 79/3 ≈ 26.3 m.

Q3 (exam style) (7 marks). A particle starts from rest at O and moves in a straight line with acceleration (6 − 2t) m/s^2, 0 ≤ t ≤ 6.

- (a) Find v in terms of t. – 2 marks. Award for: Integrate with v(0) = 0
- (b) Find the maximum speed. – 2 marks. Award for: a = 0
- (c) Find the distance from O when t = 6. – 2 marks. Award for: Integrate again
- (d) Explain why the particle does not return to O during this interval. – 1 marks. Award for: v ≥ 0

Answers: (a) 6t − t^2. (b) 9 m/s at t = 3. (c) 36 m. (d) v = t(6 − t) ≥ 0, so it never moves back.

Q4 (exam style) (10 marks). A particle moves along a line with displacement $s = t^3 - 9t^2 + 24t$ m from O, t ≥ 0.

- (a) Find expressions for v and a. – 2 marks. Award for: Differentiate twice
- (b) Find the positions where the particle is instantaneously at rest. – 3 marks. Award for: Solve v = 0 and substitute
- (c) Find the distance travelled in the first 5 s. – 3 marks. Award for: Split at turning points
- (d) Find the speed when the acceleration is zero. – 2 marks. Award for: Solve a = 0, substitute

Answers: (a) v = 3t^2 − 18t + 24, a = 6t − 18. (b) s = 20 (t = 2), s = 16 (t = 4). (c) 20 + 4 + 4 = 28 m. (d) t = 3, v = −3, speed 3 m/s.

Week 23: Distance, displacement and changes of direction

Pearson S1 chapter 11.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Distance and displacement.

- (a) A particle moves from $x = 2$ to $x = -3$. Find its displacement. – 2 marks. Answer: -5 m. Award for: Correct method (1); accurate answer (1).

- (b) It then returns to $x = 4$. Find the total distance. – 2 marks. Answer: 12 m. Award for: Correct method (1); accurate answer (1).
- (c) $v = 2t - 6$. Find when the particle changes direction. – 2 marks. Answer: 3 s. Award for: Correct method (1); accurate answer (1).
- (d) Find the displacement from $t = 0$ to $t = 5$. – 2 marks. Answer: -5 m. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Changes of direction with constant acceleration.

- (a) $u = 10$, $a = -2$. Find when the velocity is zero. – 2 marks. Answer: 5 s. Award for: Correct method (1); accurate answer (1).
- (b) Find the displacement at $t = 8$. – 2 marks. Answer: 16 m. Award for: Correct method (1); accurate answer (1).
- (c) Find the distance travelled in the first 8 s. – 2 marks. Answer: 25 + 9 = 34 m. Award for: Correct method (1); accurate answer (1).
- (d) $u = -4$, $a = 1$. Find when the particle returns to its start. – 2 marks. Answer: 8 s. Award for: Correct method (1); accurate answer (1).
- (e) For that motion find the greatest distance from the start before returning. – 3 marks. Answer: 8 m. Award for: Correct method (2); accurate answer (1).

Q3 (5 marks). A particle has velocity $v = t^2 - 5t + 4$ m/s for 0 ≤ t ≤ 5, starting at O. Find the times at which it changes direction and its displacement from O at each of these times.

Answer: t = 1 and t = 4; s = 11/6 m and −8/3 m.

- Solve v = 0: 2
- Integrate to find s: 1
- Evaluate both displacements: 2

Q4 (5 marks). A particle has velocity v = 6 − 2t m/s for 0 ≤ t ≤ 8. Find its displacement after 8 s and the total distance travelled, explaining why the interval must be split.

Answer: Displacement −16 m; distance 9 + 25 = 34 m, split at t = 3 where v changes sign.

- Displacement by integration: 2
- Identify t = 3: 1
- Total distance: 2

Q5 (5 marks). A particle moves with constant acceleration −2 m/s² and initial velocity 10 m/s. Find its displacement and the distance travelled after 8 s, and explain why the suvat value of s alone does not give the distance.

Answer: s = 16 m; it stops at t = 5 (s = 25), returns 9 m, so distance 34 m. s is displacement, which cancels motion in opposite directions.

- Displacement: 1
- Time at rest: 1

- Distance: 2
- Explanation: 1

Q6 (5 marks). A velocity–time graph has v = 4 for 0 ≤ t ≤ 3, then decreases linearly to v = −2 at t = 6, then stays at −2 until t = 8. Find the time at which the direction changes, the displacement at t = 8 and the total distance travelled.

Answer: t = 5; displacement 12 + 4 − 1 − 4 = 11 m; distance 21 m.

- Time of direction change: 1
- Areas of each region: 2
- Displacement: 1
- Distance: 1

Practice 2 (Day 4) – total 41 marks

Q1 (basic skills) (14 marks). Using graphs for distance.

- (a) A v–t graph goes from (0, 6) to (3, 0) to (5, -4). Find the displacement. – 2 marks. Answer: $9 - 4$ = 5 m. Award for: Correct method (1); accurate answer (1).
- (b) Find the total distance. – 2 marks. Answer: 13 m. Award for: Correct method (1); accurate answer (1).
- (c) $v = t^2 - 5t + 4$. Find the times of rest. – 2 marks. Answer: 1 s, 4 s. Award for: Correct method (1); accurate answer (1).
- (d) Find $\int_0^1 v\,dt$. – 2 marks. Answer: $\frac{11}{6}$ m. Award for: Correct method (1); accurate answer (1).
- (e) Find $\int_1^4 v\,dt$. – 3 marks. Answer: $-\frac{9}{2}$ m. Award for: Correct method (2); accurate answer (1).
- (f) Hence find the distance travelled from $t = 0$ to $t = 4$. – 3 marks. Answer: $\frac{11}{6} + \frac{9}{2} = \frac{19}{3}$ m. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (10 marks). A particle moves along a straight line with velocity $v = 3t^2 - 16t + 16$ m/s, 0 ≤ t ≤ 5. It starts at O.

- (a) Find the times when the particle is at rest. – 2 marks. Award for: Factorise
- (b) Find the displacement from O at t = 5. – 2 marks. Award for: Integrate
- (c) Find the total distance travelled. – 4 marks. Award for: Split at both turning times
- (d) Find the maximum speed for 0 ≤ t ≤ 5, justifying your answer. – 2 marks. Award for: Compare stationary value with endpoints

Answers: (a) t = 4/3, 4. (b) $s = t^3 - 8t^2 + 16t$; 5 m. (c) 256/27 + 256/27 + 5 = 647/27 ≈ 24.0 m. (d) Speeds: 16 (t = 0), 16/3 (t = 8/3), 11 (t = 5); maximum 16 m/s.

Q3 (exam style) (8 marks). Particles A and B leave O at the same time along the same line. A has velocity (8 − 2t) m/s and B has velocity t m/s.

- (a) Find when they are next at the same position, and that position. – 3 marks. Award for: Equate displacements
- (b) Find the distance travelled by A by then. – 3 marks. Award for: Account for change of direction at t = 4
- (c) Find the greatest separation before they meet. – 2 marks. Award for: Maximise difference

Answers: (a) t = 16/3 s, 128/9 m from O. (b) 16 + 16/9 = 160/9 m. (c) 32/3 m at t = 8/3.

Q4 (exam style) (9 marks). A particle starts at O with velocity 5 m/s and moves with acceleration (2t − 6) m/s^2.

- (a) Find v in terms of t. – 2 marks. Award for: Integrate
- (b) Find the interval in which the particle moves in the negative direction. – 2 marks. Award for: Solve v < 0
- (c) Find the displacement after 6 s. – 2 marks. Award for: Integrate
- (d) Find the total distance travelled in 6 s. – 3 marks. Award for: Split at t = 1 and t = 5

Answers: (a) $t^2 - 6t + 5$. (b) $1 < t < 5$. (c) −6 m. (d) 7/3 + 32/3 + 7/3 = 46/3 ≈ 15.3 m.

Week 24: Correlation, regression and data interpretation

Pearson S1 chapter 1–4.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (11 marks). Correlation and regression basics.

- (a) Describe the correlation shown when points lie close to a line sloping down. – 2 marks. Answer: strong negative correlation. Award for: Correct method (1); accurate answer (1).
- (b) State what is meant by an explanatory variable. – 2 marks. Answer: the independent variable, plotted on the x-axis. Award for: Correct method (1); accurate answer (1).
- (c) The regression line of y on x is $y = 2.4 + 0.8x$. Estimate y when $x = 10$. – 2 marks. Answer: 10.4. Award for: Correct method (1); accurate answer (1).
- (d) Interpret the gradient 0.8 in context-free terms. – 2 marks. Answer: y increases by 0.8 for each unit increase in x. Award for: Correct method (1); accurate answer (1).
- (e) Explain why estimating y for $x = 50$ may be unreliable if data lie in $0 \le x \le 20$. – 3 marks. Answer: extrapolation. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (14 marks). Using regression in context. Hours revised h and test score s: $s = 32 + 4.5h$, for $1 \le h \le 10$.

- (a) Estimate the score for 6 hours. – 2 marks. Answer: 59. Award for: Correct method (1); accurate answer (1).
- (b) Interpret 32. – 2 marks. Answer: estimated score with no revision. Award for: Correct method (1); accurate answer (1).

- (c) Interpret 4.5. – 2 marks. Answer: each extra hour adds about 4.5 marks. Award for: Correct method (1); accurate answer (1).
- (d) Comment on using the line for 15 hours. – 2 marks. Answer: extrapolation; unreliable. Award for: Correct method (1); accurate answer (1).
- (e) $r = 0.91$. Describe the correlation. – 3 marks. Answer: strong positive. Award for: Correct method (2); accurate answer (1).
- (f) Explain why the line of h on s would be needed to predict hours from a score. – 3 marks. Answer: regression of y on x minimises errors in y only. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). A fitted model for a dataset is y=5+1.8x, where x is hours of study and y a score. Predict y at x=5. Interpret slope/intercept, and explain why prediction at x=100 might be unreliable if data cover 0≤x≤8.

Answer: Prediction 14. Slope 1.8 score units/hour; intercept 5 is prediction at zero study, not necessarily causal. x=100 extrapolates far beyond the data.

- Make the numerical prediction: 1
- Interpret slope and intercept in context: 2
- Explain the extrapolation risk using the observed range: 1

Q4 (4 marks). A dataset has correlation -0.35. Explain its sign, compare its strength with correlation 0.2, give a plausible non-causal interpretation and explain why reversing x,y does not reverse its sign.

Answer: Negative linear association, magnitude 0.35, stronger than |0.2|. Confounding/common influence may explain it. Correlation is symmetric in x,y.

- Interpret the sign: 1
- Compare strengths using absolute values: 1
- Give a plausible non-causal interpretation: 1
- Explain symmetry on interchanging the variables: 1

Q5 (4 marks). For x values 1,2,3,4 and y values 7,9,11,13, find the means and exact linear relation. State the correlation and explain what happens to it if 100 is added to every y.

Answer: Mean x=2.5, mean y=10; y=2x+5; r=1, unchanged by translation.

- Find both means: 1
- Find the exact linear relation: 1
- State the correlation: 1
- Explain invariance under translation: 1

Q6 (4 marks). For the official Large Data Set used in your course, select one quantitative variable. Record its units and one data-quality issue, choose an appropriate display, and explain one comparison you could make. Do not invent numerical observations.

Answer: Tutor checks against the current official dataset: valid variable/units; real missing-data or recording issue; suitable display; justified comparison. Award for evidence, not invented facts.

- Select a genuine quantitative variable and give its units: 1
- Identify an evidenced data-quality issue: 1
- Choose an appropriate display: 1
- Explain a defensible comparison: 1

Practice 2 (Day 4) – total 37 marks

Q1 (basic skills) (14 marks). Testing correlation and log models.

- (a) State hypotheses to test for positive correlation. – 2 marks. Answer: $H_0 : \rho = 0,\ H_1 : \rho > 0$. Award for: Correct method (1); accurate answer (1).
- (b) $n = 10$, $r = 0.62$; the 5% critical value is 0.5494. State the conclusion. – 2 marks. Answer: reject H_0: evidence of positive correlation. Award for: Correct method (1); accurate answer (1).
- (c) $n = 12$, $r = -0.41$; the 5% one-tailed critical value is 0.4973. State the conclusion for negative correlation. – 2 marks. Answer: do not reject H_0. Award for: Correct method (1); accurate answer (1).
- (d) Given $\log y = 0.5 + 0.2x$, write y in the form ab^x. – 2 marks. Answer: $y = 3.16 \times 1.58^x$. Award for: Correct method (1); accurate answer (1).
- (e) Given $\log y = 1.1 + 2 \log x$, write y in the form ax^n. – 3 marks. Answer: $y = 12.6x^2$. Award for: Correct method (2); accurate answer (1).
- (f) State one reason the product-moment correlation coefficient may be misleading. – 3 marks. Answer: relationship may be non-linear, or outliers. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (7 marks). For 10 days with maximum temperature x °C between 14 and 28, ice-cream sales y have regression line y = 12.4 + 3.6x and PMCC r = 0.87.

- (a) Interpret the gradient of the line. – 1 marks. Award for: Context
- (b) Estimate sales on a 20 °C day. – 1 marks. Award for: Substitute
- (c) Comment on using the line for a 35 °C day. – 1 marks. Award for: Extrapolation
- (d) Interpret the value of r. – 1 marks. Award for: Strong positive linear
- (e) The manager says hotter weather causes higher sales. Comment. – 1 marks. Award for: Correlation not causation
- (f) The temperatures are converted to °F. State the effect on r. – 1 marks. Award for: Unchanged by linear coding
- (g) Explain how removing two unusually low rainy-day values would be likely to affect r. – 1 marks. Award for: Likely to increase

Answers: (a) Each extra °C is associated with about 3.6 more sales. (b) 84.4. (c) Extrapolation outside 14–28. (d) Strong positive linear correlation. (e) Correlation does not prove causation. (f) r unchanged. (g) r would probably increase.

Q3 (exam style) (8 marks). A scatter diagram of $\log_{10}$ P against t is approximately linear, with regression line $\log_{10}$ P = 0.52 + 0.035t.

- (a) Show that this gives $P = ab^t$ and find a and b. – 4 marks. Award for: Exponentiate and evaluate
- (b) Interpret b in context. – 1 marks. Award for: Growth factor per unit time
- (c) Predict P when t = 20. – 2 marks. Award for: Substitute
- (d) Comment on predicting P when t = 80. – 1 marks. Award for: Extrapolation

Answers: (a) a = 10^0.52 ≈ 3.31, b = 10^0.035 ≈ 1.084. (b) P grows by about 8.4% per unit of time. (c) 10^1.22 ≈ 16.6. (d) Likely outside the data range: unreliable.

Q4 (exam style) (8 marks). Group A: n = 25, mean 54.2, standard deviation 8.1. Group B: n = 30, $\sum y = 1740$, $\sum y^2 = 103950$.

- (a) Find the mean and standard deviation of Group B. – 3 marks. Award for: Mean, variance, sd
- (b) Compare the two groups. – 2 marks. Award for: Location and spread in context
- (c) Find the mean of all 55 values. – 2 marks. Award for: Combined total
- (d) A value of 92 is recorded in Group B. Using mean ± 3 standard deviations, decide whether it is an outlier. – 1 marks. Award for: Compute boundary and conclude

Answers: (a) 58, √101 ≈ 10.0. (b) B is higher on average but more variable. (c) 3095/55 ≈ 56.3. (d) 58 + 30.1 = 88.1 < 92, so it is an outlier.

Week 25: Statistics consolidation

Pearson S1 chapter 1–7.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 6] Find the median of 4, 7, 7, 9, 10, 12. – 2 marks. Answer: 8. Award for: Correct method (1); accurate answer (1).
- (b) [Week 10] $P(A) = 0.3$, $P(B) = 0.6$, independent. Find $P(A \cap B)$. – 2 marks. Answer: 0.18. Award for: Correct method (1); accurate answer (1).
- (c) [Week 11] $X \sim B(12, 0.5)$. Find $P(X \geq 10)$. – 2 marks. Answer: 0.0193. Award for: Correct method (1); accurate answer (1).
- (d) [Week 24] For $y = 5 - 1.2x$, estimate y when $x = 3$. – 2 marks. Answer: 1.4. Award for: Correct method (1); accurate answer (1).

Answers: (a) 8; (b) 0.18; (c) 0.0193; (d) 1.4

Q2 (retrieval from Week 10) (6 marks). A bag contains 12 red and 5 blue counters. Two are drawn without replacement. Find P(both red), P(exactly one red) and P(second red). Explain why the draws are not independent.

Answer: $\frac{33}{68}$, $\frac{15}{34}$, $\frac{12}{17}$. Composition changes after the first draw.

- Find the probability of two reds without replacement: 2
- Include both orders for exactly one red: 2
- Find the probability that the second counter is red: 1
- Explain the dependence of the draws: 1

Q3 (retrieval from Week 11) (6 marks). A device is claimed to work with probability 0.5 independently on each trial. In 29 trials it works 9 times. Test at 5% whether the success probability has fallen. State hypotheses, probability and conclusion.

Answer: H0:p=0.5; H1:p<0.5. X~B(29,0.5), P(X≤9)=0.030714≤0.05; reject H0, evidence of a fall, assuming independent representative trials.

- State both hypotheses using the population parameter: 1
- State the null binomial model: 1
- Calculate the correct cumulative tail probability: 2
- Compare with the significance level: 1
- Give the conclusion in context: 1

Q4 (retrieval from Week 24) (4 marks). A fitted model for a dataset is y=9+1.8x, where x is hours of study and y a score. Predict y at x=5. Interpret slope/intercept, and explain why prediction at x=100 might be unreliable if data cover 0≤x≤8.

Answer: Prediction 18. Slope 1.8 score units/hour; intercept 9 is prediction at zero study, not necessarily causal. x=100 extrapolates far beyond the data.

- Make the numerical prediction: 1
- Interpret slope and intercept in context: 2
- Explain the extrapolation risk using the observed range: 1

Q5 (retrieval from Week 6) (6 marks). For the five values 9, 10, 11, 12, 18, find mean, median and population variance. Explain the effect of adding 10 to every observation on mean and variance.

Answer: Mean 12, median 11, variance 10. Adding 10 increases mean by 10 and leaves variance unchanged.

- Find mean and median: 2
- Calculate population variance with a valid method: 2
- Explain the translation effect on mean and variance: 2

Q6 (retrieval from Week 24) (4 marks). A dataset has correlation -0.55. Explain its sign, compare its strength with correlation 0.2, give a plausible non-causal interpretation and explain why

reversing x,y does not reverse its sign.

Answer: Negative linear association, magnitude 0.55, stronger than |0.2|. Confounding/common influence may explain it. Correlation is symmetric in x,y.

- Interpret the sign: 1
- Compare strengths using absolute values: 1
- Give a plausible non-causal interpretation: 1
- Explain symmetry on interchanging the variables: 1

Q7 (retrieval from Week 6) (5 marks). A grouped table has intervals 0≤x<10, 10≤x<30, 30≤x<50 and frequencies 9, 18, 9. Find frequency densities and a midpoint estimate of the mean. State one limitation.

Answer: Densities $\frac{9}{10}, \frac{9}{10}, \frac{9}{20}$; estimated mean $\frac{85}{4}$=21.25. Individual values within classes are unknown.

- Find frequency densities for all three intervals: 2
- Use class midpoints and frequencies to estimate the mean: 2
- State a valid limitation: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (16 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 6] Find the mean of 2, 4, 9, 13. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (b) [Week 10] $X \sim B(8, 0.25)$. Find $P(X = 2)$. – 2 marks. Answer: 0.311. Award for: Correct method (1); accurate answer (1).
- (c) [Week 11] State the hypotheses to test whether a dice shows six less often than expected. – 2 marks. Answer: $H_0 : p = \frac{1}{6},\ H_1 : p < \frac{1}{6}$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 24] Describe the correlation for that line. – 2 marks. Answer: negative. Award for: Correct method (1); accurate answer (1).
- (e) [Week 1] Simplify $\sqrt{18} \times \sqrt{8}$. – 2 marks. Answer: 12. Award for: Correct method (1); accurate answer (1).
- (f) [Week 2] Solve $x^2 + 2x - 15 = 0$. – 2 marks. Answer: $x = -5, x = 3$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 3] State where $y = (x + 1)(x - 5)$ meets the axes. – 2 marks. Answer: $x = -1, 5$; $y = -5$. Award for: Correct method (1); accurate answer (1).
- (h) [Week 4] Find the gradient of $2x - 5y = 10$. – 2 marks. Answer: $\frac{2}{5}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 7; (b) 0.311; (c) $H_0 : p = \frac{1}{6},\ H_1 : p < \frac{1}{6}$; (d) negative; (e) 12; (f) $x = -5, x = 3$; (g) $x = -1, 5$; $y = -5$; (h) $\frac{2}{5}$

Q2 (exam style) (8 marks). In a survey of 50 households, the number of cars X was: 0 cars: 8; 1 car: 22; 2 cars: 15; 3 cars: 5.

- (a) Find the mean and standard deviation of X. – 3 marks. Award for: Sums and formulae
- (b) A household is chosen at random. Given that it has at least one car, find the probability that it has exactly two. – 2 marks. Award for: Conditional
- (c) Two different households are chosen at random. Find the probability that they have 4 cars in total. – 3 marks. Award for: Without replacement, all cases

Answers: (a) 1.34, ≈ 0.863. (b) 15/42 = 5/14. (c) (2 × 22 × 5 + 15 × 14)/(50 × 49) = 43/245.

Q3 (exam style) (8 marks). A drug is claimed to work for 70% of patients. A doctor suspects it is less effective and treats 20 patients.

- (a) Find the critical region for a test at the 5% level. – 3 marks. Award for: Hypotheses and boundary
- (b) 11 patients improve. State the conclusion. – 2 marks. Award for: Conclusion in context
- (c) State the actual significance level. – 1 marks. Award for: Probability
- (d) The 20 patients all came from one clinic. Comment. – 1 marks. Award for: Sampling issue
- (e) Find P(X ≥ 15) under the null hypothesis. – 1 marks. Award for: Probability

Answers: (a) X ≤ 10. (b) 11 is not in the region: insufficient evidence. (c) 0.0480. (d) The sample may not be representative. (e) 0.4164.

Q4 (exam style) (8 marks). Daily maximum temperatures, x °C, at Heathrow for 31 days give $\sum x = 682$ and $\sum x^2 = 15382$. For Camborne over the same days, the mean is 19.2 °C and the standard deviation is 2.1 °C.

- (a) Find the mean and standard deviation for Heathrow. – 3 marks. Award for: Formulae
- (b) Compare the two locations. – 2 marks. Award for: Location and spread
- (c) Explain why one month of data may not represent the whole year. – 1 marks. Award for: Seasonality
- (d) The Heathrow data are coded using t = 2x − 40. Find the mean and standard deviation of t. – 2 marks. Award for: Coding

Answers: (a) 22, ≈ 3.49. (b) Heathrow is warmer on average and more variable. (c) Seasonal variation. (d) 4, ≈ 6.98.

Week 26: Year 1 Pure synthesis

Pearson P1 chapter 1–14.

Practice 1 (Day 2) – total 43 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 1] Simplify $(2x^2)^3$. – 2 marks. Answer: $8x^6$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 2] Write $x^2 - 8x + 3$ in the form $(x + b)^2 + c$. – 2 marks. Answer: $(x - 4)^2 - 13$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 3] $y = f(x)$ has a minimum at $(-1, 4)$. State the minimum of $y = f(x + 2)$. – 2 marks. Answer: $(-3, 4)$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 4] Find the distance between $(1, 1)$ and $(4, 5)$. – 2 marks. Answer: 5. Award for: Correct method (1); accurate answer (1).

Answers: (a) $8x^6$; (b) $(x - 4)^2 - 13$; (c) $(-3, 4)$; (d) 5

Q2 (retrieval from Week 1) (5 marks). (a) Simplify $10\sqrt{12} - \sqrt{27} + 2\sqrt{3}$. (b) Rationalise $\frac{10}{2+\sqrt{3}}$. (c) Verify (b) by multiplication.

Answer: (a) $19\sqrt{3}$. (b) $20 - 10\sqrt{3}$. (c) Product with the original denominator is 10.

- Simplify the surds, with correct intermediate simplification: 2
- Rationalise using the conjugate and simplify: 2
- Verify by multiplying by the original denominator: 1

Q3 (retrieval from Week 2) (6 marks). For $f(x) = x^2 - 20x + 96$, (a) complete the square, (b) find the roots and minimum, (c) sketch, labelling intercepts and vertex.

Answer: $f(x) = (x - 10)^2 - 4$; roots 8,12; minimum -4 at x=10; vertical intercept 96.

- Complete the square: 2
- Find both roots: 1
- State the minimum value and its x-coordinate: 1
- Sketch with correct shape, intercepts and vertex: 2

Q4 (retrieval from Week 3) (7 marks). Starting with $y = x^2$, describe a sequence of transformations to $y = -2(x - 10)^2 + 11$. State vertex, range and intercept with the vertical axis; sketch.

Answer: Translate right 10, stretch vertically by 2, reflect in x-axis, translate up 11; vertex (10,11); y≤11; y-intercept -189.

- Describe the four transformations, with a valid order: 3
- State vertex and range: 2
- Find the vertical intercept: 1
- Sketch the transformed parabola consistently: 1

Q5 (retrieval from Week 4) (6 marks). A circle has centre (10,2) and radius 5. Find its equation, its intersections with y=2, and the tangent at (13,6).

Answer: $(x-10)^2+(y-2)^2=25$; (5,2),(15,2). Tangent $3(x-13)+4(y-6)=0$.

- Write the circle equation: 1
- Find the two intersections: 2
- Use the radius to obtain the tangent gradient: 1
- Find the tangent equation: 2

Q6 (retrieval from Week 7) (6 marks). Expand $(1+10x)^5$ through the x^3 term. Find the coefficient of x^2 in $(1+10x)^5(1-x)$. Explain why terms beyond x^2 cannot affect that coefficient.

Answer: $10000x^3+1000x^2+50x+1$; coefficient 950; multiplication by 1 or -x cannot lower degree.

- Expand through the cubic term: 3
- Collect the requested product coefficient: 2
- Explain why higher powers cannot contribute: 1

Q7 (retrieval from Week 8) (5 marks). A triangle has sides 13 and 15 with included angle 60°. Find its area exactly and its third side. Explain why these data determine the side uniquely.

Answer: Area $\frac{195\sqrt{3}}{4}$; side $\sqrt{199}$; SAS fixes it.

- Use the area formula to obtain an exact area: 2
- Use the cosine rule to find the third side: 2
- Explain uniqueness from the given SAS data: 1

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (14 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 1] Expand and simplify $(x+2)(x^2-3x+1)$. – 2 marks. Answer: x^3-x^2-5x+2. Award for: Correct method (1); accurate answer (1).
- (b) [Week 2] Find the discriminant of $2x^2+3x+5$ and state the number of real roots. – 2 marks. Answer: -31; no real roots. Award for: Correct method (1); accurate answer (1).
- (c) [Week 3] State the asymptotes of $y=\frac{3}{x-2}$. – 2 marks. Answer: $x=2,\ y=0$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 4] State the centre and radius of $x^2+(y-4)^2=7$. – 2 marks. Answer: $(0,4)$, $\sqrt{7}$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 7] Expand $(1+3x)^3$. – 2 marks. Answer: $1+9x+27x^2+27x^3$. Award for: Correct method (1); accurate answer (1).
- (f) [Week 8] Find the area of a triangle with sides 4 and 9 enclosing 60°. – 2 marks. Answer: $9\sqrt{3}\approx 15.6$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 9] Solve $\cos x=0.3$ for $0\le x\le 360^\circ$. – 2 marks. Answer: $72.5^\circ, 287.5^\circ$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $x^3 - x^2 - 5x + 2$; (b) -31; no real roots; (c) $x = 2,\ y = 0$; (d) $(0, 4)$, $\sqrt{7}$; (e) $1 + 9x + 27x^2 + 27x^3$; (f) $9\sqrt{3} \approx 15.6$; (g) $72.5°, 287.5°$

Q2 (exam style) (9 marks). The curve $y = x^2 - 6x + 10$ and the line y = 2x − 2 meet at A and B.

- (a) Find the coordinates of A and B. – 3 marks. Award for: Equate and solve
- (b) Find the area enclosed between the line and the curve. – 4 marks. Award for: Integral of difference
- (c) Find the point on the curve where the tangent is parallel to the line. – 2 marks. Award for: Gradient equals 2

Answers: (a) (2, 2), (6, 10). (b) 32/3. (c) (4, 2).

Q3 (exam style) (8 marks).

- (a) Solve $2\log_3 x - \log_3(x - 2) = 2$. – 4 marks. Award for: Log laws, quadratic, both checked
- (b) Solve $3^{2x+1} = 4^x$, giving x to 3 significant figures. – 3 marks. Award for: Logs and rearrange
- (c) Explain why any solution to (a) must satisfy x > 2. – 1 marks. Award for: Domain of logs

Answers: (a) $x^2 = 9(x - 2)$: x = 3 or 6. (b) x = −ln 3/(ln 9 − ln 4) ≈ −1.35. (c) $\log_3(x - 2)$ requires x − 2 > 0.

Q4 (exam style) (8 marks).

- (a) Show that $3\sin^2\theta - \cos\theta - 1 = 0$ can be written as $3\cos^2\theta + \cos\theta - 2 = 0$. – 2 marks. Award for: Identity
- (b) Hence solve the equation for 0° ≤ θ < 360°. – 4 marks. Award for: Factorise and all solutions
- (c) Hence solve $3\sin^2 2x - \cos 2x - 1 = 0$ for 0° ≤ x < 180°. – 2 marks. Award for: Adjust interval

Answers: (a) $3(1 - c^2) - c - 1 = 0$. (b) θ = 48.2°, 180°, 311.8°. (c) x = 24.1°, 90°, 155.9°.

Week 27: Year 1 Pure assessment and repair

Pearson P1 chapter 1–14.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 1] Rationalise the denominator of $\frac{1}{\sqrt{7}-2}$. – 2 marks. Answer: $\frac{2}{3} + \frac{\sqrt{7}}{3}$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 2] Solve $x^2 - 9 \le 0$. – 2 marks. Answer: $-3 \le x \le 3$. Award for: Correct method (1); accurate answer (1).

- (c) [Week 3] Find where $y = x^2 - 1$ meets $y = 3$. – 2 marks. Answer: $(\pm 2, 3)$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 4] Find the line through $(0, 3)$ parallel to $y = -2x + 1$. – 2 marks. Answer: $y = -2x + 3$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $\frac{2}{3} + \frac{\sqrt{7}}{3}$; (b) $-3 \leq x \leq 3$; (c) $(\pm 2, 3)$; (d) $y = -2x + 3$

Q2 (retrieval from Week 1) (5 marks). For positive x, y, simplify $\frac{(11x^{3/2}y^{-1})^2}{x^{1/2}y^{-3}}$. Evaluate it at x=4, y=2. Explain why the positivity condition matters.

Answer: $121x^{5/2}y$; value 7744. Fractional powers/denominators require domain care.

- Apply the outer power correctly: 1
- Combine the powers of x: 1
- Combine the powers of y and the numerical coefficient: 1
- Evaluate at the given values: 1
- Explain the real-domain and nonzero-denominator restrictions: 1

Q3 (retrieval from Week 2) (5 marks). Solve $(x - 10)(x - 13) < 0$. Then solve $x^2 - 22x + k = 0$ for the values of k that give (i) two distinct real roots, (ii) a repeated root.

Answer: 10<x<13; k<121 for two distinct roots; k=121 for a repeated root.

- Find the correct interval for the strict inequality: 2
- Form the discriminant condition: 1
- Give the condition for two distinct roots: 1
- Give the condition for a repeated root: 1

Q4 (retrieval from Week 3) (6 marks). Sketch $y = (x + 11)(x - 1)^2$, identifying roots, repeated roots and the y-intercept. State the sign on each interval between roots and end behaviour.

Answer: Roots -11,1 (double); y-intercept 11; negative for x<-11, positive for x>-11 except zero at 1. Falls to left, rises to right; touches at 1.

- Identify the simple and repeated roots: 2
- Find the y-intercept: 1
- Give signs between roots: 1
- Show both end behaviours and correct crossing/touching in the sketch: 2

Q5 (retrieval from Week 4) (6 marks). A=(11,1), B=(15,7). Find AB's length and midpoint, then its perpendicular bisector. Decide whether C=(16,2) is equidistant from A and B.

Answer: Length $2\sqrt{13}$; midpoint (13,4); bisector y-4=-(2/3)(x-13). CA^2=26, CB^2=26, so yes.

- Find length and midpoint: 2
- Find the perpendicular-bisector gradient and equation: 2
- Compare the distances from C and conclude: 2

Q6 (retrieval from Week 7) (5 marks). Find the coefficient of x^3 in $(2+x)^{14}$. State the general term used, and check a nearby coefficient independently using Pascal's relation.

Answer: Coefficient 745472; term $\binom{14}{r}2^{14-r}x^r$. Adjacent coefficients follow binomial coefficients.

- State an appropriate general term: 1
- Obtain the cubic coefficient: 2
- Check a nearby coefficient using Pascal's relation: 2

Q7 (retrieval from Week 9) (5 marks). Solve $2\sin^2 x - 3\sin x + 1 = 0$ for $0° \le x < 360°$. Show the factorisation, list every solution and verify by substitution.

Answer: $(2\sin x - 1)(\sin x - 1) = 0$; $x = 30°, 90°, 150°$. Check each gives zero.

- Factorise the quadratic in the trigonometric function: 2
- Find every solution in the interval: 2
- Verify the solutions: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (14 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 1] Evaluate $9^{-\frac{3}{2}}$. – 3 marks. Answer: $\frac{1}{27}$. Award for: Correct method (2); accurate answer (1).
- (b) [Week 2] Solve $y = 2x$ and $x^2 + y = 8$. – 3 marks. Answer: $(2, 4)$ and $(-4, -8)$. Award for: Correct method (2); accurate answer (1).
- (c) [Week 3] The minimum of $y = f(x)$ is at $(3, -2)$. State the minimum of $y = 3f(x)$. – 3 marks. Answer: $(3, -6)$. Award for: Correct method (2); accurate answer (1).
- (d) [Week 4] Find the centre and radius of $x^2 + y^2 + 8x - 2y = 8$. – 3 marks. Answer: $(-4, 1)$, 5. Award for: Correct method (2); accurate answer (1).
- (e) [Week 7] Find the coefficient of x^2 in $(1+x)^9$. – 2 marks. Answer: 36. Award for: Correct method (1); accurate answer (1).

Answers: (a) $\frac{1}{27}$; (b) $(2, 4)$ and $(-4, -8)$; (c) $(3, -6)$; (d) $(-4, 1)$, 5; (e) 36

Q2 (exam style) (9 marks). The circle C has equation $x^2 + y^2 - 10x - 4y + 4 = 0$.

- (a) Find the centre and radius. – 2 marks. Award for: Complete the square
- (b) Show that the line $4x + 3y = 51$ is a tangent to C. – 4 marks. Award for: Substitute and repeated root
- (c) Find the point of contact. – 1 marks. Award for: Coordinates
- (d) Find the equation of the other tangent to C that is parallel to this line. – 2 marks. Award for: Diametrically opposite point

Answers: (a) (5, 2), r = 5. (b) $25x^2 - 450x + 2025 = 0$ has discriminant 0. (c) (9, 5). (d) $4x + 3y = 1$.

Q3 (exam style) (8 marks).

- (a) Expand $(1-2x)^6$ in ascending powers of x up to the term in x^3. – 3 marks. Award for: Coefficients
- (b) Find the coefficient of x^3 in $(3+x)^2(1-2x)^6$. – 3 marks. Award for: Combine terms
- (c) Use (a) to estimate 0.98^6, stating the value of x. – 2 marks. Award for: x = 0.01

Answers: (a) 1 − 12x + 60x² − 160x³. (b) −1440 + 360 − 12 = −1092. (c) 0.88584.

Q4 (exam style) (9 marks). $f(x) = x^3 - 3x^2 + 4$.

- (a) Show that (x + 1) is a factor and factorise f(x) fully. – 3 marks. Award for: Factor theorem and factorise
- (b) Sketch y = f(x). – 2 marks. Award for: Shape with roots and intercept
- (c) Use calculus to find the stationary points and their nature. – 3 marks. Award for: Differentiate and test
- (d) Prove that f(x) ≥ 0 for all x ≥ −1. – 1 marks. Award for: Product of non-negative factors

Answers: (a) (x + 1)(x − 2)². (b) Root −1, touches at 2, y-intercept 4. (c) (0, 4) maximum, (2, 0) minimum. (d) x + 1 ≥ 0 and (x − 2)² ≥ 0.

Week 28: Year 1 applied assessment and repair

Pearson S1 chapter 1–11.

Practice 1 (Day 2) – total 40 marks

Q1 (basic skills) (10 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 12] $u = 2$, $a = 3$, $t = 4$. Find s. – 2 marks. Answer: 32 m. Award for: Correct method (1); accurate answer (1).
- (b) [Week 13] A ball is dropped from 20 m. Taking $g = 9.8$, find its time to land. – 2 marks. Answer: 2.02 s. Award for: Correct method (1); accurate answer (1).
- (c) [Week 20] Find the acceleration of a 5 kg mass under resultant 20 N. – 2 marks. Answer: 4 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (d) [Week 21] A 900 kg car tows a 300 kg trailer with driving force 2400 N. Find the acceleration. – 2 marks. Answer: 2 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (e) [Week 22] $s = 5t^2 - t^3$. Find v. – 2 marks. Answer: $10t - 3t^2$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 32 m; (b) 2.02 s; (c) 4 m s^{-2}; (d) 2 m s^{-2}; (e) $10t - 3t^2$

Q2 (retrieval from Week 6) (5 marks). A distribution has lower quartile 12, median 18 and upper quartile 24. Use the 1.5 IQR rule to find outlier fences. Classify -12 and 57. Explain why a box plot cannot establish a causal relationship.

Answer: IQR=12, fences -6 and 42. -12 is an outlier; 57 is an outlier. A box plot shows a distribution, not causation.

- Calculate the IQR and both fences: 2
- Classify the two observations correctly: 2
- Explain why the display does not establish causation: 1

Q3 (retrieval from Week 11) (5 marks). For X~B(32,0.5), find the lower-tail critical region at 5% and the actual significance level. Explain why it need not equal exactly 5%.

Answer: X≤10; actual level 0.025051. Next cumulative probability 0.055092>.05; discreteness prevents exact equality.

- Find the boundary count: 1
- Check the next count to establish the boundary: 1
- State the critical region: 1
- Find its actual significance level: 1
- Explain the effect of discreteness: 1

Q4 (retrieval from Week 24) (4 marks). For x values 1,2,3,4 and y values 14,16,18,20, find the means and exact linear relation. State the correlation and explain what happens to it if 100 is added to every y.

Answer: Mean x=2.5, mean y=17; y=2x+12; r=1, unchanged by translation.

- Find both means: 1
- Find the exact linear relation: 1
- State the correlation: 1
- Explain invariance under translation: 1

Q5 (retrieval from Week 12) (5 marks). A particle accelerates uniformly from 12 to 20 m/s in 4 s, then continues at 20 m/s for 3 s. Sketch its velocity-time graph, find acceleration and total displacement.

Answer: Acceleration 2 m/s^2; displacement 124 m, by trapezium plus rectangle. Graph continuous with a straight rising segment then horizontal.

- Sketch and label both stages of the velocity-time graph: 2
- Find the acceleration: 1
- Calculate both displacement contributions and their total: 2

Q6 (retrieval from Week 13) (6 marks). A ball is projected vertically upwards at 34 m/s from ground level. Using g=9.8, find time to greatest height, greatest height and total flight time. State the model assumptions.

Answer: t=3.4694 s; height=58.98 m; total=6.9388 s. Constant g, negligible air resistance, returns to launch level.

- Use a consistent vertical-motion model: 1
- Find time to greatest height: 1

- Find greatest height: 2
- Find total flight time: 1
- State relevant model assumptions: 1

Q7 (retrieval from Week 20) (5 marks). A 12 kg particle on a smooth horizontal surface is pulled by a horizontal force 48 N. Draw its force diagram, find its acceleration and normal reaction, and explain why the reaction is not an extra horizontal force.

Answer: a=4 m/s^2; R=117.6 N upward, weight downward. Reaction perpendicular to surface.

- Draw a correct force diagram: 2
- Use horizontal Newton's second law to find acceleration: 1
- Use vertical equilibrium to find reaction: 1
- Explain the direction of the reaction: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (16 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 12] $u = 20$, $v = 8$, $t = 6$. Find a. – 2 marks. Answer: -2 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (b) [Week 13] A ball is thrown up at 9.8 m s^{-1}. Taking $g = 9.8$, find its greatest height. – 2 marks. Answer: 4.9 m. Award for: Correct method (1); accurate answer (1).
- (c) [Week 20] Find the tension when a 2 kg mass is lifted at constant speed. – 2 marks. Answer: 19.6 N. Award for: Correct method (1); accurate answer (1).
- (d) [Week 21] Find the tension in the tow bar. – 2 marks. Answer: 600 N. Award for: Correct method (1); accurate answer (1).
- (e) [Week 22] Find the acceleration at $t = 1$. – 2 marks. Answer: 4 m s^{-2}. Award for: Correct method (1); accurate answer (1).
- (f) [Week 6] Given $n = 8$, $\sum x = 48$, $\sum x^2 = 320$, find the standard deviation. – 2 marks. Answer: $\sqrt{40 - 36} = 2$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 10] $X \sim B(12, 0.4)$. Find $P(X \leq 4)$. – 2 marks. Answer: 0.438. Award for: Correct method (1); accurate answer (1).
- (h) [Week 11] $X \sim B(30, 0.1)$. Find $P(X = 0)$. – 2 marks. Answer: 0.0424. Award for: Correct method (1); accurate answer (1).

Answers: (a) -2 m s^{-2}; (b) 4.9 m; (c) 19.6 N; (d) 600 N; (e) 4 m s^{-2}; (f) $\sqrt{40 - 36} = 2$; (g) 0.438; (h) 0.0424

Q2 (exam style) (8 marks). A seed company claims that 85% of its seeds germinate. A gardener plants 30 seeds and 21 germinate.

- (a) Test at the 5% level whether the claim is too high. – 5 marks. Award for: Hypotheses, probability, conclusion
- (b) Find the critical region. – 2 marks. Award for: Boundary

- (c) State one assumption required for the test. – 1 marks. Award for: Independence

Answers: (a) P(X ≤ 21) = 0.0278 < 0.05: reject H_0, evidence that the germination rate is lower. (b) X ≤ 21. (c) Seeds germinate independently.

Q3 (exam style) (8 marks). A particle moves with constant acceleration from A to B to C. AB = 24 m is covered in 2 s and BC = 60 m in the next 3 s.

- (a) Find the speed at A and the acceleration. – 5 marks. Award for: Two equations and solve
- (b) Find the speed at C. – 1 marks. Award for: v = u + at
- (c) Find the distance from A at which the speed is 20 m/s. – 2 marks. Award for: $v^2 = u^2 + 2as$

Answers: (a) u = 8.8 m/s, a = 3.2 m/s^2. (b) 24.8 m/s. (c) 50.4 m.

Q4 (exam style) (8 marks). A truck of mass 2000 kg tows a car of mass 1000 kg with a horizontal rope. Resistances are 800 N on the truck and 400 N on the car. The rope tension is 1600 N.

- (a) Find the acceleration and the driving force. – 4 marks. Award for: Car then truck
- (b) The rope breaks when the speed is 15 m/s. Find the distance travelled by the car before stopping. – 3 marks. Award for: Deceleration and distance
- (c) State one assumption. – 1 marks. Award for: Valid

Answers: (a) 1.2 m/s^2, 4800 N. (b) Deceleration 0.4 m/s^2, 281.25 m. (c) e.g. rope light and inextensible.

Week 29: Year 2 bridge

Pearson P1 chapter 4, 7, 12–14.

Practice 1 (Day 2) – total 42 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 5] Find the remainder when $x^3 - 4x + 7$ is divided by $(x + 1)$. – 2 marks. Answer: 10. Award for: Correct method (1); accurate answer (1).
- (b) [Week 14] Given $y = 6x^2 - 5x$, find $\frac{dy}{dx}$. – 2 marks. Answer: $12x - 5$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 18] Evaluate $\log_2 64$. – 2 marks. Answer: 6. Award for: Correct method (1); accurate answer (1).
- (d) [Week 16] Find $\int \left(8x^3 - 3\right) dx$. – 2 marks. Answer: $2x^4 - 3x + c$. Award for: Correct method (1); accurate answer (1).

Answers: (a) 10; (b) $12x - 5$; (c) 6; (d) $2x^4 - 3x + c$

Q2 (retrieval from Week 3) (8 marks). For $y = \frac{6}{x-2} + 1$, give asymptotes, domain and range. Find intercepts and solve y>1, explaining graphically.

Answer: Asymptotes x=2,y=1; domain x≠2, range y≠1; x-intercept -4, y-intercept -2; y>1 for x>2.

- State both asymptotes: 2

- State domain and range: 2
- Find both intercepts: 2
- Solve the inequality and explain it using the graph: 2

Q3 (retrieval from Week 5) (6 marks). For $p(x) = x^3 + 3x^2 - 16x + 12$, verify that x=1 is a root, divide by x-1 and factorise fully. Solve p(x)≥0 using a sign diagram.

Answer: $p(x) = (x - 1)(x + 6)(x - 2)$; quotient $x^2 + 4x - 12$; -6≤x≤1 or x≥2.

- Verify the stated root: 1
- Divide by the stated factor: 2
- Factorise fully: 1
- Use a sign diagram to obtain all intervals, including the endpoints: 2

Q4 (retrieval from Week 14) (5 marks). For $f(x) = x^3 - 6x^2 + 2x$, find f'(x), f''(x), and the tangent at x=1. Explain which derivative controls the local gradient.

Answer: f'=$3x^2 - 12x + 2$; f''=$6\,(x - 2)$; tangent $y = 4 - 7x$.

- Find the first and second derivatives: 2
- Find the point and tangent gradient: 1
- Form the tangent equation: 1
- Identify the first derivative as the local gradient: 1

Q5 (retrieval from Week 18) (5 marks). Solve $\log_2(x - 1) + \log_2(x + 1) = 9$. State domain restrictions before combining logarithms and reject invalid roots.

Answer: x>1; x²-1=2^9; x=$3\sqrt{57}$ only.

- State the initial domain: 1
- Combine logarithms and form an algebraic equation: 2
- Solve and reject inadmissible roots: 2

Q6 (retrieval from Week 16) (5 marks). Find $\int (6x^2 - 4x + 3)\,dx$. Then find the particular function passing through (1,5), and check it by differentiation.

Answer: General $2x^3 - 2x^2 + 3x$+C; C=2. Derivative recovers the integrand.

- Integrate all terms and include the arbitrary constant: 2
- Apply the boundary condition: 1
- Give the particular function: 1
- Check by differentiation: 1

Q7 (retrieval from Week 17) (5 marks). Find the area between y=x(6-x) and the x-axis. Identify the appropriate limits and explain the sign of the integrand between them.

Answer: Roots 0,6; positive between; area 36.

- Find the intercepts and justify the sign between them: 2
- Set up the correct definite integral: 1
- Integrate and evaluate the exact area: 2

Practice 2 (Day 4) – total 39 marks

Q1 (exam style) (14 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 5] Show $(x-3)$ is a factor of x^3-2x^2-2x-3. – 2 marks. Answer: $27-18-6-3=0$. Award for: Correct method (1); accurate answer (1).
- (b) [Week 14] Given $y=\frac{2}{x}$, find $\frac{dy}{dx}$. – 2 marks. Answer: $-\frac{2}{x^2}$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 18] Write $3\log x-\log y$ as a single logarithm. – 2 marks. Answer: $\log\frac{x^3}{y}$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 16] Find $\int x^{\frac{1}{2}}\,dx$. – 2 marks. Answer: $\frac{2x^{\frac{3}{2}}}{3}+c$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 17] Find the area under $y=3x^2$ from 1 to 2. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (f) [Week 15] Find the stationary point of $y=x^2+6x$. – 2 marks. Answer: $(-3,-9)$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 7] Prove that the difference of two consecutive squares is odd. – 2 marks. Answer: $(n+1)^2-n^2=2n+1$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $27-18-6-3=0$; (b) $-\frac{2}{x^2}$; (c) $\log\frac{x^3}{y}$; (d) $\frac{2x^{\frac{3}{2}}}{3}+c$; (e) 7; (f) $(-3,-9)$; (g) $(n+1)^2-n^2=2n+1$

Q2 (exam style) (9 marks). $f(x)=x^3-4x$.

- (a) Sketch y = f(x). – 2 marks. Award for: Roots and shape
- (b) Sketch y = f(x + 2) and find where it meets the y-axis. – 2 marks. Award for: Translation
- (c) Find the area enclosed between y = f(x) and the x-axis for 0 ≤ x ≤ 2. – 3 marks. Award for: Integral sign
- (d) Find the equation of the tangent at x = −2. – 2 marks. Award for: Gradient and point

Answers: (a) Roots −2, 0, 2. (b) Through the origin. (c) 4. (d) y = 8x + 16.

Q3 (exam style) (8 marks). The number of people who have heard a rumour after t days is modelled by $N=2000-1800e^{-0.3t}$.

- (a) Find the initial value of N. – 1 marks. Award for: t = 0
- (b) Find dN/dt when t = 5. – 2 marks. Award for: Differentiate
- (c) Find when N = 1500. – 3 marks. Award for: Logs
- (d) State the limiting value. – 1 marks. Award for: Limit
- (e) Show that dN/dt = 0.3(2000 − N). – 1 marks. Award for: Substitute

Answers: (a) 200. (b) 540e^(−1.5) ≈ 120 per day. (c) t = ln 3.6/0.3 ≈ 4.27 days. (d) 2000. (e) 540e^(−0.3t) = 0.3 × 1800e^(−0.3t).

Q4 (exam style) (8 marks). The curve C has equation y = 3 cos(x − 60°), 0° ≤ x ≤ 360°.

- (a) Find the maximum and minimum points. – 2 marks. Award for: Both
- (b) Find the y-intercept. – 1 marks. Award for: Value
- (c) Solve 3 cos(x − 60°) = −1.5. – 3 marks. Award for: Adjusted interval
- (d) Describe the transformations from y = cos x. – 2 marks. Award for: Translation and stretch

Answers: (a) (60°, 3), (240°, −3). (b) 1.5. (c) x = 180°, 300°. (d) Translate 60° right, stretch vertically by 3.

Week 30: Readiness check and summer consolidation

Pearson P1 chapter 1–14.

Practice 1 (Day 2) – total 43 marks

Q1 (basic skills) (8 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 7] Disprove: $2^n + 1$ is prime for every positive integer n. – 2 marks. Answer: $n = 3$: 9. Award for: Correct method (1); accurate answer (1).
- (b) [Week 8] Write down the exact value of $\cos 30^\circ$. – 2 marks. Answer: $\frac{\sqrt{3}}{2}$. Award for: Correct method (1); accurate answer (1).
- (c) [Week 9] Solve $\tan x = -2$ for $0 \leq x \leq 360^\circ$. – 2 marks. Answer: $116.6^\circ, 296.6^\circ$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 14] Find the gradient of $y = x^3$ at $x = -2$. – 2 marks. Answer: 12. Award for: Correct method (1); accurate answer (1).

Answers: (a) $n = 3$: 9; (b) $\frac{\sqrt{3}}{2}$; (c) $116.6^\circ, 296.6^\circ$; (d) 12

Q2 (retrieval from Week 1) (6 marks). Solve $(7)^{2x-1} = (7)^5$. Then solve $x^{2/3} = 49$, over the real numbers. Explain why the second equation has two solutions.

Answer: x=3; the second gives $\{-343, 343\}$. Cube roots may be positive or negative before squaring.

- Equate exponents and solve the exponential equation: 2
- Use the real cube root to reduce the fractional-power equation: 1
- Find both real roots: 2
- Explain why squaring a cube root permits both signs: 1

Q3 (retrieval from Week 2) (6 marks). Find all intersections of $y = x^2$ and $y = 7x + 8$. Check each solution in both equations and sketch their relative positions.

Answer: x=-1,8; points (-1,1),(8,64); line above parabola between the roots.

- Equate the expressions and solve for both x-values: 2
- Find both intersection coordinates: 1
- Check in both original equations: 1
- Sketch the relative positions correctly: 2

Q4 (retrieval from Week 3) (5 marks). A point (2,7) lies on y=f(x). Find its corresponding point on (i) y=3f(x-1), (ii) y=f(2x)+4. Explain why the horizontal coordinate is not multiplied by 2 in (ii).

Answer: (i) (3,21); (ii) (1,11). To obtain the old input 2, the new x must satisfy 2x=2.

- Find the first transformed point: 2
- Find the second transformed point: 2
- Explain the inverse horizontal-coordinate change: 1

Q5 (retrieval from Week 4) (5 marks). Find intersections of $(x-7)^2+y^2=25$ with y=3. Find the chord length. Explain why y=6 gives no intersection.

Answer: (3,3),(11,3); length 8. Distance 6 from centre exceeds radius 5.

- Substitute the line and find both intersection points: 3
- Find the chord length: 1
- Explain non-intersection for the other line: 1

Q6 (retrieval from Week 8) (8 marks). Sketch $y=7\sin(2x)-1$ for 0°≤x≤360°. State amplitude, period, range and all x for which y=-1.

Answer: Amplitude 7, period 180°, range [-8,6]; x=0°,90°,180°,270°,360°.

- Sketch the correct curve over the full interval: 3
- State amplitude, period and range: 3
- Find every requested x-value including endpoints: 2

Q7 (retrieval from Week 9) (5 marks). Solve cos(2x)=1/2 for 0°≤x<1260°. State the transformed interval first and explain your method for ensuring no roots are missed.

Answer: 0°≤2x<2520°. Solutions x=30°+180°k or 150°+180°k, for k=0,...,6.

- State the transformed interval: 1
- Find the two solution families: 2
- Restrict them to the given interval without missing roots: 1
- Explain the completeness method: 1

Practice 2 (Day 4) – total 40 marks

Q1 (exam style) (15 marks). Mixed basic skills. Each part revisits the week shown in brackets.

- (a) [Week 7] Find the first three terms of $(2 - x)^4$. – 3 marks. Answer: $16 - 32x + 24x^2$. Award for: Correct method (2); accurate answer (1).
- (b) [Week 8] Find c when $a = 5$, $b = 8$, $C = 60°$. – 2 marks. Answer: 7. Award for: Correct method (1); accurate answer (1).
- (c) [Week 9] Solve $4\sin^2 x = 3$ for $0 \le x \le 360°$. – 2 marks. Answer: $60°, 120°, 240°, 300°$. Award for: Correct method (1); accurate answer (1).
- (d) [Week 14] Find the tangent to $y = x^2$ at $(3, 9)$. – 2 marks. Answer: $y = 6x - 9$. Award for: Correct method (1); accurate answer (1).
- (e) [Week 15] Find the stationary points of $y = x^3 - 12x + 1$. – 2 marks. Answer: $(2, -15)$, $(-2, 17)$. Award for: Correct method (1); accurate answer (1).
- (f) [Week 16] $\frac{dy}{dx} = 4x$, curve through $(1, 7)$. Find y. – 2 marks. Answer: $y = 2x^2 + 5$. Award for: Correct method (1); accurate answer (1).
- (g) [Week 17] Find the area enclosed by $y = x(3 - x)$ and the x-axis. – 2 marks. Answer: $\frac{9}{2}$. Award for: Correct method (1); accurate answer (1).

Answers: (a) $16 - 32x + 24x^2$; (b) 7; (c) $60°, 120°, 240°, 300°$; (d) $y = 6x - 9$; (e) $(2, -15)$, $(-2, 17)$; (f) $y = 2x^2 + 5$; (g) $\frac{9}{2}$

Q2 (exam style) (9 marks). The equation $(k - 1)x^2 + 4x + (k + 2) = 0$ has no real roots.

- (a) Show that $k^2 + k - 6 > 0$. – 3 marks. Award for: Discriminant
- (b) Find the set of possible values of k. – 3 marks. Award for: Solve quadratic inequality
- (c) Explain why k = 1 needs separate consideration. – 1 marks. Award for: Linear case
- (d) For k = 3, find the minimum value of $2x^2 + 4x + 5$. – 2 marks. Award for: Completed square

Answers: (a) 16 − 4(k − 1)(k + 2) < 0. (b) k < −3 or k > 2. (c) The equation is linear with a root. (d) $2(x + 1)^2 + 3$: minimum 3.

Q3 (exam style) (8 marks). The curve C has equation $y = 3x^2 - 2x^3$.

- (a) Find the stationary points and their nature. – 3 marks. Award for: Differentiate and test
- (b) Find the area enclosed by C and the x-axis. – 4 marks. Award for: Roots and integral
- (c) State where C is increasing. – 1 marks. Award for: Interval

Answers: (a) (0, 0) minimum, (1, 1) maximum. (b) 27/32. (c) 0 < x < 1.

Q4 (exam style) (8 marks). A particle of mass 3 kg starts from rest at (2i + 5j) m and moves under a constant force (6i − 9j) N.

- (a) Find the acceleration. – 1 marks. Award for: F = ma
- (b) Find the speed after 4 s. – 2 marks. Award for: Velocity

- (c) Find the position after 4 s. – 2 marks. Award for: Displacement
- (d) Find when the particle crosses the x-axis. – 2 marks. Award for: j-component
- (e) Find the bearing of the direction of motion. – 1 marks. Award for: Angle

Answers: (a) 2i − 3j. (b) √208 ≈ 14.4 m/s. (c) 18i − 19j. (d) t = √(10/3) ≈ 1.83 s. (e) 146.3°.

Week 31: Algebraic methods and partial fractions

Pearson P2 chapter 1.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (8 marks). Algebraic fractions.

- (a) Simplify $\frac{x^2-9}{x^2+x-6}$. – 2 marks. Answer: $\frac{x-3}{x-2}$. Award for: Correct method (1); accurate answer (1).
- (b) Write as a single fraction $\frac{2}{x+1}+\frac{3}{x-2}$. – 2 marks. Answer: $\frac{5x-1}{(x-2)(x+1)}$. Award for: Correct method (1); accurate answer (1).
- (c) Simplify $\frac{x+2}{x^2-1}\times\frac{x-1}{x^2+2x}$. – 2 marks. Answer: $\frac{1}{x(x+1)}$. Award for: Correct method (1); accurate answer (1).
- (d) Simplify $\frac{x^2-4}{3x}\div\frac{x+2}{6x^2}$. – 2 marks. Answer: $2x\,(x-2)$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (11 marks). Partial fractions with distinct linear factors.

- (a) Express in partial fractions $\frac{5}{(x+1)(x-4)}$. – 2 marks. Answer: $-\frac{1}{x+1}+\frac{1}{x-4}$. Award for: Correct method (1); accurate answer (1).
- (b) Express in partial fractions $\frac{3x+1}{(x-1)(x+1)}$. – 2 marks. Answer: $\frac{1}{x+1}+\frac{2}{x-1}$. Award for: Correct method (1); accurate answer (1).
- (c) Express in partial fractions $\frac{x+7}{x^2+x-2}$. – 2 marks. Answer: $-\frac{5}{3(x+2)}+\frac{8}{3(x-1)}$. Award for: Correct method (1); accurate answer (1).
- (d) Express in partial fractions $\frac{4x-1}{2x^2-x-1}$. – 2 marks. Answer: $\frac{2}{2x+1}+\frac{1}{x-1}$. Award for: Correct method (1); accurate answer (1).
- (e) Express in partial fractions $\frac{x^2+x+4}{x(x+1)(x-2)}$. – 3 marks. Answer: $\frac{4}{3(x+1)}+\frac{5}{3(x-2)}-\frac{2}{x}$. Award for: Correct method (2); accurate answer (1).

Q3 (6 marks). Resolve $\frac{17x+7}{(x-1)(x+2)}$ into partial fractions. State excluded x-values, recombine to check, and explain why denominators must retain their original restrictions.

Answer: $\frac{9}{x+2}+\frac{8}{x-1}$; x≠1,-2. Recombination verifies numerator; simplification does not restore undefined original inputs.

- Set up the decomposition: 1
- Find both coefficients: 2
- State exclusions: 1

- Recombine to check: 1
- Explain why original restrictions remain: 1

Q4 (4 marks). Divide $x^3 + 8x^2 + 2x + 1$ by x+1, giving quotient and remainder. Use the remainder theorem to check.

Answer: Quotient $x^2 + 7x - 5$; remainder 6=p(-1).

- Carry out polynomial division and give the quotient: 2
- Give the remainder: 1
- Check using the remainder theorem: 1

Q5 (5 marks). Prove by contradiction that sqrt(2) is irrational. State precisely where the assumption of lowest terms is used. Then explain why an example-based argument cannot prove irrationality.

Answer: Assume sqrt2=p/q in lowest positive integer terms. $p^2=2q^2 \Rightarrow$ p even; writing p=2r gives $q^2=2r^2 \Rightarrow$ q even, contradiction to coprimality. Finite decimal checks are not proof.

- Assume a rational representation in lowest terms: 1
- Deduce that the numerator is even: 1
- Deduce that the denominator is even: 1
- State the contradiction with lowest terms: 1
- Explain why numerical examples are not a proof: 1

Q6 (5 marks). Resolve $\frac{9x-8}{x(x-1)}$ into partial fractions and solve the expression=0, respecting exclusions.

Answer: 8/x+1/(x-1); root x=$\frac{8}{9}$; exclude0,1.

- Set up and solve for partial-fraction coefficients: 3
- Solve the rational expression equal to zero: 1
- Respect and state denominator exclusions: 1

Practice 2 (Day 4) – total 38 marks

Q1 (basic skills) (14 marks). Repeated factors and improper fractions.

- (a) Express in partial fractions $\frac{x+5}{(x+1)^2}$. – 2 marks. Answer: $\frac{1}{x+1} + \frac{4}{(x+1)^2}$. Award for: Correct method (1); accurate answer (1).
- (b) Express in partial fractions $\frac{3x+2}{x(x+1)^2}$. – 2 marks. Answer: $-\frac{2}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{x}$. Award for: Correct method (1); accurate answer (1).
- (c) Express in partial fractions $\frac{x^2+1}{(x-1)(x+1)^2}$. – 2 marks. Answer: $\frac{1}{2(x+1)} - \frac{1}{(x+1)^2} + \frac{1}{2(x-1)}$. Award for: Correct method (1); accurate answer (1).
- (d) Express in partial fractions $\frac{x^2+2}{(x+1)(x-1)}$. – 2 marks. Answer: $1 - \frac{3}{2(x+1)} + \frac{3}{2(x-1)}$. Award for: Correct method (1); accurate answer (1).

- (e) Express in partial fractions $\frac{x^3}{x^2-1}$. – 3 marks. Answer: $x + \frac{1}{2(x+1)} + \frac{1}{2(x-1)}$. Award for: Correct method (2); accurate answer (1).
- (f) Express in partial fractions $\frac{5x^2+2x+1}{x^2(x+1)}$. – 3 marks. Answer: $\frac{4}{x+1} + \frac{1}{x} + \frac{1}{x^2}$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks).

- (a) Express $\frac{4x^2+x+1}{x(x+1)^2}$ in the form $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$. – 4 marks. Award for: Correct identity, substitution and comparison of coefficients
- (b) Use algebraic division to write $\frac{2x^3+3x^2-1}{x^2+x}$ as a polynomial plus a proper fraction, and simplify your answer fully. – 4 marks. Award for: Division, remainder, factorise and cancel

Answers: (a) A = 1, B = 3, C = −4. (b) 2x + 1 + (−x − 1)/(x(x + 1)) = 2x + 1 − 1/x.

Q3 (exam style) (8 marks).

- (a) Prove by contradiction that there are no positive integers a and b such that $a^2 - b^2 = 1$. – 4 marks. Award for: Assume, factorise, deduce both factors are 1, contradiction
- (b) Prove by contradiction that $\log_2 3$ is irrational. – 4 marks. Award for: Assume p/q, form 2^p = 3^q, parity contradiction

Answers: (a) (a − b)(a + b) = 1 with positive integers forces a + b = 1 = a − b, so b = 0: contradiction. (b) 2^(p/q) = 3 gives 2^p = 3^q; the left is even and the right odd (p, q ≥ 1): contradiction.

Q4 (exam style) (8 marks). $f(x) = \frac{x^2+8x+9}{(x+1)(x+3)}$, x > −1.

- (a) Show that f(x) can be written as $A + \frac{B}{x+1} + \frac{C}{x+3}$ and find A, B and C. – 4 marks. Award for: Division or identity, then partial fractions
- (b) Hence explain why f is a decreasing function for x > −1. – 2 marks. Award for: Each fraction decreases as x increases
- (c) State the range of f. – 2 marks. Award for: Limits as x → −1 and x → ∞

Answers: (a) A = 1, B = 1, C = 3. (b) 1/(x + 1) and 3/(x + 3) both decrease since their positive denominators increase. (c) f(x) > 1.

Week 32: Functions, domains and inverses

Pearson P2 chapter 2.

Practice 1 (Day 2) – total 41 marks

Q1 (basic skills) (8 marks). Composite and inverse functions. $f(x) = 2x + 3$, $g(x) = x^2$.

- (a) Find $fg(2)$. – 2 marks. Answer: 11. Award for: Correct method (1); accurate answer (1).
- (b) Find $gf(x)$. – 2 marks. Answer: $(2x + 3)^2$. Award for: Correct method (1); accurate answer (1).

- (c) Find $f^{-1}(x)$. – 2 marks. Answer: $\frac{x-3}{2}$. Award for: Correct method (1); accurate answer (1).
- (d) Solve $fg(x) = 21$. – 2 marks. Answer: $x = \pm 3$. Award for: Correct method (1); accurate answer (1).

Q2 (basic skills) (8 marks). Domain, range and modulus.

- (a) State the range of $f(x) = x^2 + 3$, $x \in \mathbb{R}$. – 2 marks. Answer: $f(x) \geq 3$. Award for: Correct method (1); accurate answer (1).
- (b) State the range of $f(x) = e^x - 1$. – 2 marks. Answer: $f(x) > -1$. Award for: Correct method (1); accurate answer (1).
- (c) Solve $|x - 3| = 5$. – 2 marks. Answer: $x = 8,\ x = -2$. Award for: Correct method (1); accurate answer (1).
- (d) Solve $|2x + 1| < 7$. – 2 marks. Answer: $-4 < x < 3$. Award for: Correct method (1); accurate answer (1).

Q3 (6 marks). Let f(x)=sqrt(x-9), g(x)=2x+1. Find fg(x), gf(x), with full domains, and solve fg(x)=3. State your convention for composition.

Answer: fg=f∘g=sqrt(2x+1-9), x≥4; gf=2sqrt(x-9)+1, x≥9; solution x=$\frac{17}{2}$.

- State the composition convention: 1
- Find both compositions: 2
- Give the full domain of each: 2
- Solve the composition equation within its domain: 1

Q4 (8 marks). Let f(x)=(x-9)^2+1 on x≥9. Find its inverse, its domain/range and the fixed point(s) of f if any. Explain why the restriction is needed.

Answer: Inverse 9+sqrt(x-1), x≥1; inverse range x≥9. Fixed points solve x^2-(19)x+82=0 subject to x≥9; roots $\frac{19}{2} - \frac{\sqrt{33}}{2}$, $\frac{\sqrt{33}}{2} + \frac{19}{2}$; only the upper root satisfies the restricted domain. Restriction makes f one-to-one.

- Rearrange and choose the correct inverse branch: 2
- Give inverse domain and range: 2
- Solve the fixed-point quadratic: 2
- Filter using the original domain: 1
- Explain the need for a one-to-one restriction: 1

Q5 (5 marks). Solve |2x-9|=11. Then solve |2x-9|<11. Sketch to justify both answers.

Answer: Equation x=-1,10; inequality -1<x<10.

- Solve both branches of the modulus equation: 2
- Solve the strict modulus inequality: 2
- Sketch to justify both results: 1

Q6 (6 marks). For $f(x) = \frac{1}{x-9} + 2$, find domain, range and inverse. Explain the symmetry between the two graphs.

Answer: Domain x≠9, range y≠2; inverse 9+1/(x-2), x≠2. Graphs reflect in y=x.

- State the original domain and range: 2
- Rearrange to find the inverse: 2
- Give the inverse domain: 1
- Explain reflection in y=x: 1

Practice 2 (Day 4) – total 40 marks

Q1 (basic skills) (14 marks). Transformations of graphs. $y = f(x)$ passes through $A(2, -4)$ and has an asymptote $y = 1$.

- (a) State the image of A on $y = f(x) - 3$. – 2 marks. Answer: $(2, -7)$. Award for: Correct method (1); accurate answer (1).
- (b) State the image of A on $y = f(2x)$. – 2 marks. Answer: $(1, -4)$. Award for: Correct method (1); accurate answer (1).
- (c) State the image of A on $y = |f(x)|$. – 2 marks. Answer: $(2, 4)$. Award for: Correct method (1); accurate answer (1).
- (d) State the image of A on $y = 3f(x + 1)$. – 2 marks. Answer: $(1, -12)$. Award for: Correct method (1); accurate answer (1).
- (e) State the asymptote of $y = 2f(x) + 1$. – 3 marks. Answer: $y = 3$. Award for: Correct method (2); accurate answer (1).
- (f) State the image of A on $y = f(|x|)$ for $x < 0$. – 3 marks. Answer: $(-2, -4)$. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (9 marks). f(x) = 2x + 3, x ∈ ℝ, and $g(x) = \frac{5}{x-1}$, x > 1.

- (a) Find fg(x) and state its range. – 3 marks. Award for: Composition and range
- (b) Find $g^{-1}(x)$ and state its domain. – 3 marks. Award for: Rearrangement and domain
- (c) Solve fg(x) = 8. – 2 marks. Award for: Solve
- (d) Explain why gf(−1) is not defined. – 1 marks. Award for: f(−1) outside domain of g

Answers: (a) 10/(x − 1) + 3, range fg(x) > 3. (b) g^{-1}(x) = 1 + 5/x, x > 0. (c) x = 3. (d) f(−1) = 1, which is not in the domain x > 1.

Q3 (exam style) (8 marks). f(x) = |2x − 4| − 3.

- (a) Sketch y = f(x), giving the coordinates of the vertex and the points where it meets the axes. – 3 marks. Award for: V-shape and key points
- (b) Solve f(x) = x/2. – 4 marks. Award for: Both branches, check validity
- (c) Find the values of k for which f(x) = k has exactly two solutions. – 1 marks. Award for: k > −3

Answers: (a) Vertex (2, −3), x-intercepts 0.5 and 3.5, y-intercept 1. (b) x = 0.4 or x = 14/3. (c) k > −3.

Q4 (exam style) (9 marks). $f(x) = x^2 - 6x + 11$, x ≥ 3.

- (a) State the range of f. – 1 marks. Award for: Completed square
- (b) Find $f^{-1}(x)$, stating its domain. – 3 marks. Award for: Inverse via completed square, choose positive root
- (c) Solve $f(x) = f^{-1}(x)$. – 4 marks. Award for: Use y = x and select valid root
- (d) Explain why the domain of f must be restricted for an inverse to exist. – 1 marks. Award for: Many-to-one otherwise

Answers: (a) f(x) ≥ 2. (b) 3 + √(x − 2), x ≥ 2. (c) $x^2 - 7x + 11 = 0$, x = (7 + √5)/2. (d) Without restriction f is many-to-one.

Week 33: Sequences, series and convergence

Pearson P2 chapter 3.

Practice 1 (Day 2) – total 39 marks

Q1 (basic skills) (11 marks). Arithmetic series.

- (a) Find the 20th term of $5, 8, 11, \ldots$ – 2 marks. Answer: 62. Award for: Correct method (1); accurate answer (1).
- (b) Find the sum of the first 20 terms of that series. – 2 marks. Answer: 670. Award for: Correct method (1); accurate answer (1).
- (c) An arithmetic series has first term 7 and 10th term 34. Find the common difference. – 2 marks. Answer: 3. Award for: Correct method (1); accurate answer (1).
- (d) Find $\sum_{r=1}^{30}(2r + 1)$. – 2 marks. Answer: 960. Award for: Correct method (1); accurate answer (1).
- (e) How many terms of $4 + 9 + 14 + \ldots$ are needed for a sum over 500? – 3 marks. Answer: 14. Award for: Correct method (2); accurate answer (1).

Q2 (basic skills) (11 marks). Geometric series.

- (a) Find the 8th term of $3, 6, 12, \ldots$ – 2 marks. Answer: 384. Award for: Correct method (1); accurate answer (1).
- (b) Find the sum of the first 8 terms. – 2 marks. Answer: 765. Award for: Correct method (1); accurate answer (1).
- (c) A geometric series has $a = 81$, $r = \frac{1}{3}$. Find the sum to infinity. – 2 marks. Answer: 121.5. Award for: Correct method (1); accurate answer (1).
- (d) Explain why $2 + 4 + 8 + \ldots$ has no sum to infinity. – 2 marks. Answer: $|r| = 2 > 1$. Award for: Correct method (1); accurate answer (1).
- (e) The 2nd term is 12 and the 5th term is 96. Find r and a. – 3 marks. Answer: $r = 2,\ a = 6$. Award for: Correct method (2); accurate answer (1).

Q3 (4 marks). An arithmetic sequence starts at 10 with common difference 3. Find the 20th term and sum of the first 20 terms. Find the least n for which the nth term exceeds 100.

Answer: u20=67; S20=770; n>31, least integer 32.

- Find the requested term: 1
- Find the finite sum: 1
- Set up the threshold inequality: 1
- Select the least admissible integer n: 1

Q4 (5 marks). A geometric series starts at 10 with ratio 0.8. Find S10 and its sum to infinity. Find the least n with the remainder after n terms less than 0.1.

Answer: S10=44.631; S∞=50; need 50(0.8)^n<0.1, least n=28.

- Find the finite geometric sum: 1
- Find the infinite sum: 1
- Form the remainder inequality: 1
- Use logarithms correctly: 1
- Choose the least integer satisfying the strict inequality: 1

Q5 (4 marks). A sequence satisfies u1=10, u(n+1)=0.5u(n)+3. Find its next three terms and its possible limit. Explain why solving L=0.5L+3 alone is not a proof of convergence.

Answer: Next 8, 7, $\frac{13}{2}$; limit6. Differences from6 halve, proving convergence.

- Calculate the three next terms: 2
- Find the possible limit: 1
- Explain why a fixed-point calculation alone does not prove convergence: 1

Q6 (4 marks). Evaluate $\sum_{r=1}^{14}(3r+2)$. Then derive the expression for an arbitrary upper limit n, explaining each use of a sum formula.

Answer: General 3n(n+1)/2+2n; value 343.

- Evaluate the specified finite sum: 2
- Derive the general expression: 1
- Explain the sum-formula steps: 1

Practice 2 (Day 4) – total 39 marks

Q1 (basic skills) (14 marks). Sequences and recurrence relations.

- (a) $u_{n+1} = 2u_n - 3$, $u_1 = 5$. Find u_2, u_3, u_4. – 2 marks. Answer: 7, 11, 19. Award for: Correct method (1); accurate answer (1).
- (b) Find u_1 to u_4 for $u_n = n^2 + 1$. – 2 marks. Answer: 2, 5, 10, 17. Award for: Correct method (1); accurate answer (1).

- (c) Describe the sequence $u_n = (-1)^n$. – 2 marks. Answer: periodic, order 2. Award for: Correct method (1); accurate answer (1).
- (d) Is $u_n = \frac{n}{n+1}$ increasing? Explain. – 2 marks. Answer: yes; $u_{n+1} - u_n = \frac{1}{(n+1)(n+2)} > 0$. Award for: Correct method (1); accurate answer (1).
- (e) $u_{n+1} = ku_n + 2$, $u_1 = 1$, $u_3 = 14$. Find the possible values of k. – 3 marks. Answer: $k^2 + 2k - 12 = 0 \Rightarrow k = -1 \pm \sqrt{13}$. Award for: Correct method (2); accurate answer (1).
- (f) Find $\sum_{r=1}^{10} 3^r$. – 3 marks. Answer: 88572. Award for: Correct method (2); accurate answer (1).

Q2 (exam style) (8 marks). Monthly loan repayments form an arithmetic sequence: the first is £150 and each increases by £12.

- (a) Find the 20th payment. – 2 marks. Award for: nth term
- (b) Find the total of the first 20 payments. – 2 marks. Award for: Arithmetic sum
- (c) Find the number of payments needed for the total to first exceed £10 000. – 4 marks. Award for: Inequality, quadratic, interpret integer

Answers: (a) £378. (b) £5280. (c) $6n^2 + 144n - 10000 > 0$ gives n > 30.5, so 31 payments.

Q3 (exam style) (8 marks). A geometric series has second term 12 and fifth term 3/2.

- (a) Find the common ratio and the first term. – 3 marks. Award for: r^3 and a
- (b) Find the sum to infinity. – 1 marks. Award for: a/(1 − r)
- (c) Find the least n for which the sum of the first n terms exceeds 47.9. – 4 marks. Award for: Inequality with logs, integer answer

Answers: (a) r = 1/2, a = 24. (b) 48. (c) $0.5^n < 1/480$, n > 8.9, so n = 9.

Q4 (exam style) (9 marks). A sequence is defined by $u_{n+1} = ku_n - 4$, $u_1 = 3$, where k is a constant.

- (a) Find u_2 and u_3 in terms of k. – 2 marks. Award for: Both terms
- (b) Given that $u_3 = 16$, find the possible values of k. – 3 marks. Award for: Form and solve quadratic
- (c) For k = −2, find $\sum_{r=1}^{4} u_r$. – 2 marks. Award for: Terms and sum
- (d) For 0 < k < 1 the sequence converges. Find its limit in terms of k. – 2 marks. Award for: L = kL − 4

Answers: (a) 3k − 4, $3k^2 - 4k - 4$. (b) k = 10/3 or k = −2. (c) 3 − 10 + 16 − 36 = −27. (d) −4/(1 − k).