

2017 paper 3 (Edexcel GCSE) – solutions

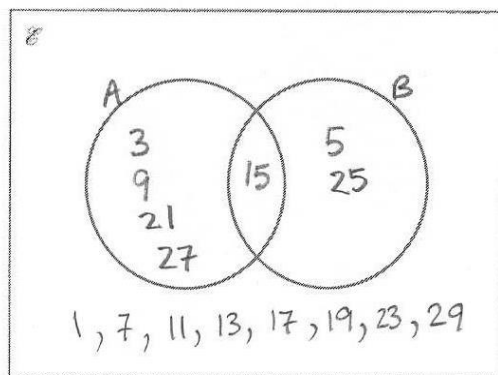
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 $\mathcal{E} = \{\text{odd numbers less than 30}\}$
 $A = \{3, 9, 15, 21, 27\}$
 $B = \{5, 15, 25\}$

(a) Complete the Venn diagram to represent this information.



A number is chosen at random from the universal set, \mathcal{E} .

(b) What is the probability that the number is in the set $A \cup B$?

$$\begin{aligned} \text{numbers in } A \cup B &= 7 \\ \text{numbers in total} &= 15 \end{aligned}$$

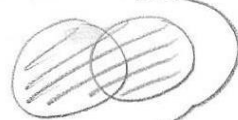
1 ~~3~~ 5
 7 ~~9~~ 11
 13 ~~15~~ 17
 19 ~~21~~ 23
 25 ~~27~~ 29

(4)

$$\frac{7}{15}$$

(Total for Question 1 is 6 marks)

$A \cup B$ means
the union



2 Solve the simultaneous equations

$$\begin{aligned} 3x + y &= -4 & \text{--- (1)} \\ 3x - 4y &= 6 & \text{--- (2)} \end{aligned}$$

① - ②

$$5y = -10$$

$$y = \frac{-10}{5}$$

$$y = -2$$

$$\begin{aligned} &+y - -4y \\ &= y + 4y \end{aligned}$$

Sub in ①

$$3x + (-2) = -4$$

$$3x - 2 = -4$$

$$3x = -4 + 2$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$\left[\begin{aligned} \text{Check in ② } 3\left(-\frac{2}{3}\right) - 4(-2) &= 6 \\ -2 + 8 &= 6 \checkmark \end{aligned} \right]$$

$$\begin{aligned} x &= -\frac{2}{3} \\ y &= -2 \end{aligned}$$

(Total for Question 2 is 3 marks)

- 3 The table shows some information about the dress sizes of 25 women.

Dress size	Number of women
8	2
10	9
12	8
14	6
	25

- (a) Find the median dress size.

25 objects $\rightarrow (12) - M - (12)$

Median = 13th value

= Dress size 12

12
(1)

3 of the 25 women have a shoe size of 7

Zoe says that if you choose at random one of the 25 women, the probability that she has either a shoe size of 7 or a dress size of 14 is $\frac{9}{25}$ because

$$\frac{3}{25} + \frac{6}{25} = \frac{9}{25}$$

- (b) Is Zoe correct?

You must give a reason for your answer.

Zoe is incorrect. 3 women have shoe size of 7, 6 have dress size of 14, but some women might have both!

(1)

(Total for Question 3 is 2 marks)

- 4 Daniel bakes 420 cakes.

He bakes only vanilla cakes, banana cakes, lemon cakes and chocolate cakes.

$\frac{2}{7}$ of the cakes are vanilla cakes.

35% of the cakes are banana cakes.

The ratio of the number of lemon cakes to the number of chocolate cakes is 4:5

Work out the number of lemon cakes Daniel bakes.

cakes = V, B, L, C
 $\frac{2}{7}$ 35% (ratio 4:5)

Vanilla cakes = $\frac{2}{7}$ of 420 = 120

Banana cakes = 35% of 420 = 147

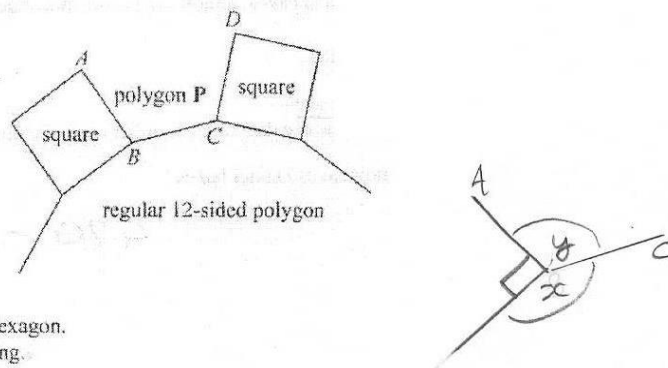
L + C cakes = 420 - (120 + 147) = 153

Lemon cakes = $\frac{4}{9}$ of 153
 = 68

Ans = 68

(Total for Question 4 is 5 marks)

- 5 In the diagram, AB , BC and CD are three sides of a regular polygon P .



Show that polygon P is a hexagon.
You must show your working.

$$\text{Exterior angle of a 12-sided polygon} = \frac{360}{12} = 30$$

$$\text{Interior angle of a 12-sided polygon} = 180 - 30 = 150$$

$$\therefore x = 150^\circ$$

$$x + y + 90 = 360 \quad (\text{angles around a point add to } 360^\circ)$$

$$y = 360 - (150 + 90)$$

$$y = 120^\circ$$

$$\therefore \text{Interior angle of polygon } P = 120^\circ$$

$$\text{Exterior angle of polygon } P = 180 - 120 = 60$$

$$\frac{360}{n} = 60 \Rightarrow n = \frac{360}{60} = 6$$

so polygon P has 6 sides

(Total for Question 5 is 4 marks)

- 6 The density of apple juice is 1.05 grams per cm^3 .

The density of fruit syrup is 1.4 grams per cm^3 .

The density of carbonated water is 0.99 grams per cm^3 .

25 cm^3 of apple juice are mixed with 15 cm^3 of fruit syrup and 280 cm^3 of carbonated water to make a drink with a volume of 320 cm^3 .

Work out the density of the drink.

Give your answer correct to 2 decimal places.

$$\text{Density} = \frac{\text{mass}}{\text{vol}}$$

liquid	density	vol	mass
AJ	1.05	25	26.25
FS	1.4	15	21
CW	0.99	280	277.2
Drink	?	320	324.45

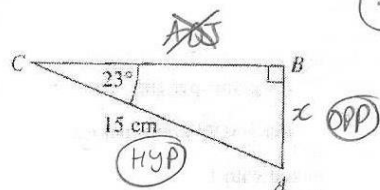
$$\left(\begin{array}{l} \text{mass} \\ = \text{density} \\ \times \text{vol} \end{array} \right)$$

$$\begin{aligned} \text{Density of drink} &= \frac{324.45}{320} \\ &= 1.01390... \end{aligned}$$

1.01 g/cm^3

(Total for Question 6 is 4 marks)

- 7 ABC is a right-angled triangle.



Calculate the length of AB .

Give your answer correct to 3 significant figures.

$$\sin 23 = \frac{\text{OPP}}{\text{HYP}}$$

$$\sin 23 = \frac{x}{15}$$

$$x = 15 \sin 23$$

$$x = 5.8609 \dots$$

$$5.86 \text{ cm}$$

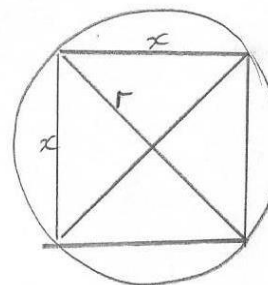
(Total for Question 7 is 2 marks)

- 8 A square, with sides of length x cm, is inside a circle. Each vertex of the square is on the circumference of the circle.

The area of the circle is 49 cm^2 .

Work out the value of x .

Give your answer correct to 3 significant figures.



$$\text{Area of circle} = \pi r^2$$

$$49 = \pi r^2$$

$$r^2 = \frac{49}{\pi}$$

$$r = \sqrt{\frac{49}{\pi}} = 3.94932 \dots$$

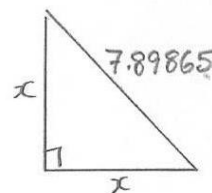
$$\text{Diameter} = 2 \times 3.949 \dots = 7.89865 \dots$$

$$x^2 + x^2 = 7.89865^2$$

$$2x^2 = 62.3887 \dots$$

$$x^2 = 31.194 \dots$$

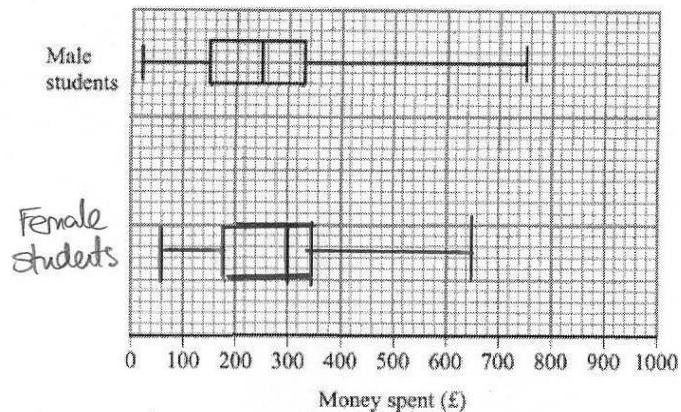
$$x = 5.58519 \dots$$



$$5.59$$

(Total for Question 8 is 4 marks)

- 9 The box plot shows information about the distribution of the amounts of money spent by some male students on their holidays.



- (a) Work out the interquartile range for the amounts of money spent by these male students.

$$LQ = 150$$

$$UQ = 330$$

$$IQR = UQ - LQ \\ = 330 - 150$$

$$= 180$$

The table below shows information about the distribution of the amounts of money spent by some female students on their holidays.

	Smallest	Lower quartile	Median	Upper quartile	Largest
Money spent (£)	60	180	300	350	650

- (b) On the grid above, draw a box plot for the information in the table.

Chris says,

"The box plots show that the female students spent more money than the male students."

- (c) Is Chris correct?

Give a reason for your answer.

Chris is not correct.

On average, female students spent more.

Some male students spent more than some female students.

(Total for Question 9 is 5 marks)

- 10 Naohy invests £6000 for 5 years.

The investment gets compound interest of $x\%$ per annum.

At the end of 5 years the investment is worth £8029.35

Work out the value of x .

Let multiplier = M

Start = £6000

After 5 years = £6000 $\times M^5 = 8029.35$

$$M^5 = \frac{8029.35}{6000} = 1.338225$$

$$\therefore M = \sqrt[5]{1.338225} = 1.059999908$$

$$\therefore \text{interest rate} = 5.99999\% \\ = 6.00\% \text{ (3 sf)}$$

$$6.00\%$$

(Total for Question 10 is 3 marks)

- 11 Jeff is choosing a shrub and a rose tree for his garden.
At the garden centre there are 17 different types of shrubs and some rose trees.

Jeff says,

"There are 215 different ways to choose one shrub and one rose tree."

Could Jeff be correct?

You must show how you get your answer.

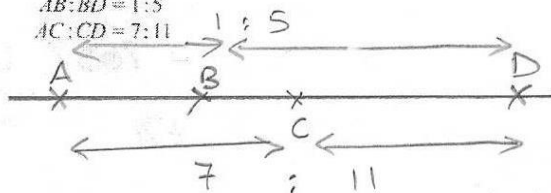
If there were x rose trees,
number of ways to choose 1 shrub & 1 rose tree $= 17 \times x$
But 17 does not divide exactly into 215
So Jeff cannot be correct. //

(Total for Question 11 is 2 marks)

- 12 The points A , B , C and D lie in order on a straight line.

$$\begin{aligned} AB:BD &= 1:5 \\ AC:CD &= 7:11 \end{aligned}$$

Work out $AB:BC:CD$



$$AB = \frac{1}{6} \text{ of } AD$$

$$CD = \frac{11}{18} \text{ of } AD$$

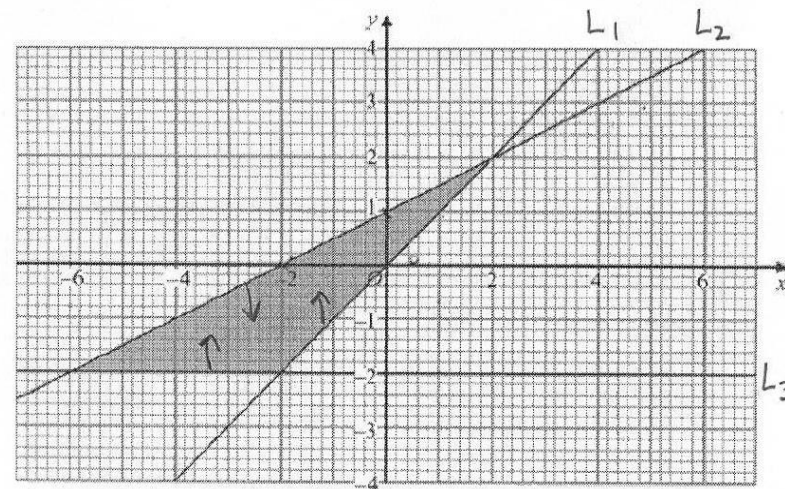
$$BC = 1 - \left(\frac{1}{6} + \frac{11}{18}\right) = \frac{2}{9} \text{ of } AD$$

$$\begin{aligned} AB:BC:CD &= \frac{1}{6} : \frac{2}{9} : \frac{11}{18} \\ &= 3:4:11 \end{aligned}$$

$$3:4:11 //$$

(Total for Question 12 is 3 marks)

13



Write down the three inequalities that define the shaded region.

$$\text{Line } L_1 \begin{cases} \text{gradient} = \frac{3}{3} = 1 \\ \text{y-int} = 0 \end{cases} \Rightarrow y = 1x + 0 \text{ or } y = x$$

$$\text{Line } L_2 \begin{cases} \text{gradient} = \frac{1}{2} \\ \text{y-int} = 1 \end{cases} \Rightarrow y = \frac{1}{2}x + 1$$

$$\text{Line } L_3 \text{ is horizontal} \Rightarrow y = -2$$

$$\text{Inequalities are } y \geq x, y \leq \frac{1}{2}x + 1, y \geq -2 //$$

(Total for Question 13 is 4 marks)

14 (a) Simplify $\frac{x^2 - 16}{2x^2 - 5x - 12}$

$$= \frac{(x-4)(x+4)}{(2x+3)(x-4)}$$

$$\begin{array}{r} 12 \\ 1 \overline{) 12} \\ 2 \\ 3 \end{array}$$

$$\frac{x+4}{2x+3}$$

(b) Make v the subject of the formula $w = \frac{15(t-2v)}{v}$

$$wv = 15(t-2v)$$

$$wv = 15t - 30v$$

$$wv + 30v = 15t$$

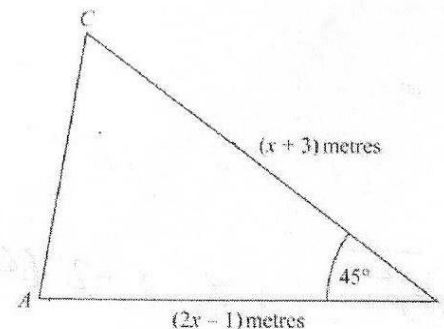
$$v(w+30) = 15t$$

$$v = \frac{15t}{w+30}$$

$$v = \frac{15t}{w+30}$$

(Total for Question 14 is 6 marks)

15



The area of triangle ABC is $6\sqrt{2} \text{ m}^2$.

Calculate the value of x .

Give your answer correct to 3 significant figures.

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$6\sqrt{2} = \frac{1}{2} (x+3)(2x-1) \left(\frac{\sqrt{2}}{2}\right)$$

$$6\sqrt{2} = \frac{\sqrt{2} (x+3)(2x-1)}{4}$$

$$24\sqrt{2} = \sqrt{2} (x+3)(2x-1)$$

$$24 = 2x^2 - x + 6x - 3$$

$$2x^2 + 5x - 27 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 - 4(2)(-27)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{241}}{4}$$

$$= 2.63 \text{ or } -5.13$$

$$\begin{array}{r} 27 \\ 3 \overline{) 27} \\ 9 \end{array}$$

From diagram, x must be positive!

$$\text{Ans } x = 2.63$$

(Total for Question 15 is 5 marks)

16 Using $x_{n+1} = -2 - \frac{4}{x_n^2}$
with $x_0 = -2.5$

(a) find the values of x_1 , x_2 and x_3

$$x_0 = -2.5$$

$$x_1 = -2 - \frac{4}{(-2.5)^2} = -2.64$$

$$x_2 = \dots = -2.57392\dots$$

$$x_3 = \dots = -2.60376\dots$$

$$x_1 = -2.64$$

$$x_2 = -2.57$$

$$x_3 = -2.60$$

(3)

(b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^4 + 2x^2 + 4 = 0$

If continued, the sequence x_1, x_2, x_3, \dots will gradually converge to a solution of the equation.

(2)

(Total for Question 16 is 5 marks)

17 A train travelled along a track in 110 minutes, correct to the nearest 5 minutes.

Jake finds out that the track is 270 km long.

He assumes that the track has been measured correct to the nearest 10 km.

(a) Could the average speed of the train have been greater than 160 km/h?
You must show how you get your answer.



$$D = 265 \rightarrow 275 \text{ km}$$

$$S = ? \text{ (km/h)}$$

$$T = 107.5 \rightarrow 112.5 \text{ mins}$$

$$\text{or } \frac{107.5}{60} \rightarrow \frac{112.5}{60} \text{ hours}$$

$$S_{\max} = \frac{D_{\max}}{T_{\min}}$$

$$S_{\max} = \frac{275}{\left(\frac{107.5}{60}\right)}$$

$$= 153.488\dots \text{ km/h}$$

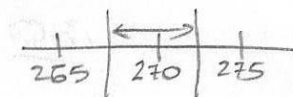
No, the speed can not be greater than 160 km/h

(4)

Jake's assumption was wrong.

The track was measured correct to the nearest 5 km.

(b) Explain how this could affect your decision in part (a).



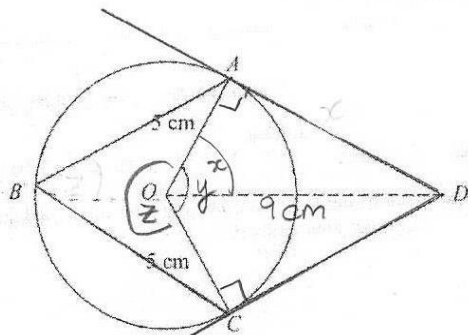
$$\text{If } D \text{ is } 267.5 \rightarrow 272.5 \text{ km}$$

$$\text{then } S_{\max} = \frac{272.5}{\left(\frac{107.5}{60}\right)} = 152.09\dots \text{ km/h}$$

The maximum average speed is slightly reduced

(1)

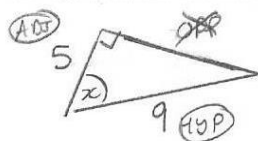
(Total for Question 17 is 5 marks)



A, B and C are points on a circle of radius 5 cm, centre O.
DA and DC are tangents to the circle.
DO = 9 cm

Work out the length of arc ABC.
Give your answer correct to 3 significant figures.

SOH CAH TOA



$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\cos x = \frac{5}{9}$$

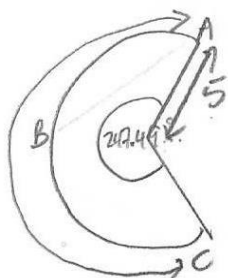
$$x = \cos^{-1}\left(\frac{5}{9}\right)$$

$$x = 56.251\dots^\circ$$

$$\therefore y = 2 \times 56.251$$

$$y = 112.5\dots^\circ$$

$$z = 360 - 112.5 = 247.4979\dots^\circ$$



$$\text{Arc} = \frac{\text{angle}}{360^\circ} \times 2\pi r$$

$$= \frac{247.4979\dots}{360} \times 2\pi(5)$$

$$= 21.578\dots$$

$$\underline{21.6 \text{ cm}}$$

(Total for Question 18 is 5 marks)

$$19 \text{ Solve } 2x^2 + 3x - 2 > 0$$

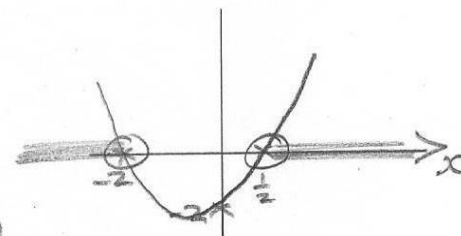
$$(2x-1)(x+2) > 0$$

$$\text{CV, } 2x-1=0 \text{ or } x+2=0$$

$$2x=1$$

$$x = \frac{1}{2} \text{ or } x = -2$$

$$x < -2 \text{ and } x > \frac{1}{2}$$



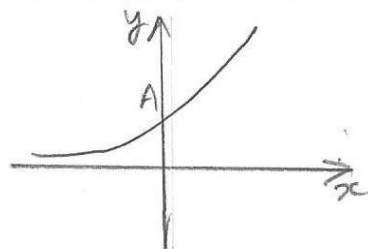
$$x < -2, x > \frac{1}{2}$$

(Total for Question 19 is 3 marks)

- 20 The equation of a curve is $y = a^x$
 A is the point where the curve intersects the y -axis.

(a) State the coordinates of A .

$$\text{If } x=0, \quad y=a^0 \\ y=1$$



(1) //

The equation of circle C is $x^2 + y^2 = 16$

The circle C is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to give circle B .

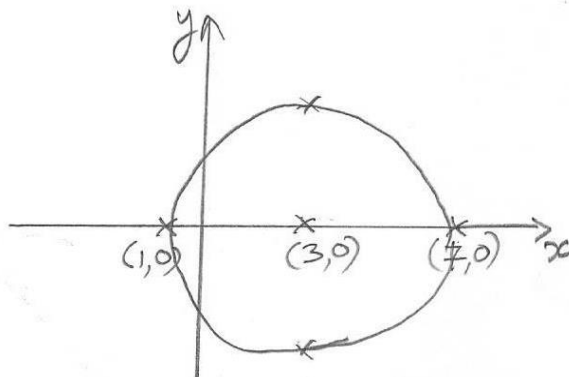
(b) Draw a sketch of circle B . ✓

Label with coordinates
the centre of circle B ✓
and any points of intersection with the x -axis. ✓

C has centre $(0,0)$ and radius 4

$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ = translation +3 to right

$\therefore B$ has centre $(3,0)$ and radius 4



(3)

(Total for Question 20 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS