Rayner Year 1 Textbook ANSWERS

Exercise 1A *page 2*

1 2^4 **2** 7^6 **3** $2^2 \times 6^3$
4 8^{-3} **5** 7^{-1} **6** $11^{\frac{1}{2}}$
7 $5^{\frac{1}{3}}$ **8** $7^{\frac{5}{2}}$ **9** n^9
10 16 **11** 1 **12** $\frac{1}{16}$
13 3 **14** 1 **15** 16
16 27 **17** 10 **18** $\frac{1}{5}$
19 x^7 **20** y^{11} **21** z^9
22 a^7 **23** r^{14} **24** d^{12}
25 s^{27} **26** t^{11} **27** $27a^6b^9$
28 k^3 **29** w^{-3} **30** x^{10}
31 $\frac{3}{4}$ **32** 0.2 **33** 10 000
34 3 **35** $\frac{2}{3}$ **36** 3

37

1 1	2 6		3 1	4 0
	5 4	6 9		1
7 2		8 9	9	
5		9		9 5
10 0	2		11 6	1

38 a^9 **39** c^6 **40** e^6
41 $10x^4y$ **42** $5t^2u^2$ **43** $\frac{1}{v^2}$
44 $3xz$ **45** $6m^2n$ **46** $15p^5qr^4$
47 $2a^2b$ **48** $6x^3y^6$ **49** $24r^{16}$
50 a 1.99 **b** 0.94 **c** 2.24
d 2.78 **e** 2.65 **f** 6.75

Exercise 1B *page 4*

1 3 **2** 2 **3** −1
4 −3 **5** 4 **6** 0
7 −1 **8** 5 **9** 6
10 a $2^{\frac{3}{2}}$ **b** 2^2 **c** $2^{\frac{4}{3}}$
d 2^6 **e** 2^{-5} **f** $2^{\frac{3}{2}}$
g $2^{\frac{1}{5}}$ **h** 2^{10} **i** $2^{\frac{5}{2}}$
j 2^0 **k** 2^{-18} **l** 2^{16}
11 a $\frac{4}{5}$ **b** $\frac{3}{7}$ **c** $\frac{2}{3}$
d $-\frac{3}{2}$ **e** $\frac{2}{3}$ **f** $-\frac{5}{4}$
12 a 3 **b** 2 **c** −7 **d** 5
13 a ±5.5 **b** 2.6 **c** $\frac{1}{121}$
d 81 **e** $\frac{1}{6}$ **f** 8
g 0.52 **h** 0.00046 **i** 6.2
14 a 4 **b** $\frac{1}{2}$ **c** 25 **d** 3 **e** $\frac{1}{4}$
f 27 **g** 9 **h** $\frac{1}{27}$ **i** 32

Exercise 1C *page 6*

1 a 1.44 **b** 4 **c** $\frac{1}{25}$ **d** 7
e 3 **f** 16 **g** $\frac{1}{27}$ **h** 16
i $\frac{1}{8}$ **j** $\frac{1}{8}$ **k** $\frac{1}{4}$ **l** 8
2 a 2 **b** 3, −2 **c** $\frac{1}{2}$ **d** 1
3 a $a^2b^3c^4$ **b** $\frac{b^3 + a}{ab(ab^3 + 1)}$
c $\frac{a\sqrt{a}b^2 + a\sqrt{b}}{a^2 + b}$ **d** $\frac{a^2\sqrt{b} + ab\sqrt{a}}{a - b}$
4 a 18 **b** $12\frac{4}{5}$ **c** 112 **d** 12
5 a $x^{\frac{5}{2}}$ **b** $x^{\frac{3}{5}}$ **c** $x^{-\frac{1}{2}}$
d $x^{\frac{5}{3}}$ **e** $x^{\frac{1}{12}}$ **f** $x^{\frac{2}{3}}$
6 a 4 **b** 9 **c** 25
d 125 **e** 27 **f** 64
8 a $4x - 4x^{\frac{1}{2}} + 1$ **b** $3x^{\frac{3}{2}} + x^{\frac{1}{2}}$
c $x^{\frac{3}{2}} + x^{-\frac{1}{2}}$ **d** $\frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$
e $5x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ **f** $2x^{-1} - \frac{1}{3}x^{-2}$
10 $x^{p^2 + q^2}$ **11** $\frac{x^{a+c^2}}{x^{c+ca}} = x^{(c-a)(c-1)}$

Exercise 1D *page 9*

1 a $3\sqrt{3}$ **b** $5\sqrt{5}$ **c** $2\sqrt{7}$
d $6\sqrt{3}$ **e** $7\sqrt{3}$ **f** $11\sqrt{2}$
2 a/f, b/h, c/e, d/g
3 a $3\sqrt{2}$ **b** $3\sqrt{7}$ **c** $2\sqrt{2}$
d $9\sqrt{3}$ **e** $5\sqrt{7}$ **f** $10\sqrt{5}$
4 a T **b** T **c** F
d T **e** F **f** T
g F **h** T **i** T
5 a $5 + 4\sqrt{2}$ **b** $3 + \sqrt{5}$ **c** $3 + 2\sqrt{2}$
d $15 - 6\sqrt{7}$ **e** $8 + 2\sqrt{15}$ **f** $7 - 4\sqrt{3}$
g 1 **h** $-1 + \sqrt{22}$ **i** 8
6 a $30\sqrt{5}$ **b** $6\sqrt{2}$ **c** $30\sqrt{10}$
d $12\sqrt{7}$ **e** $33\sqrt{6}$ **f** $56\sqrt{5}$
7 a $4\sqrt{2}$ **b** $2\sqrt{5}$ **c** $5\sqrt{3}$
d $4\sqrt{2}$ **e** $9\sqrt{3}$ **f** $10\sqrt{5}$
g $6\sqrt{5}$ **h** $\sqrt{6}$
8 a $3\sqrt{2} - 3$ **b** $2\sqrt{5} + 2$ **c** $3\sqrt{7} + 6$
d $\sqrt{5} - \sqrt{3}$ **e** $2\sqrt{7} + 2\sqrt{2}$ **f** $\sqrt{13} + 3$

Exercise 1E *page 12*

1 a $5x^2 + 6x + 6$ **b** $6x^2 + 3x - 6$
c $3x^3 + 10x + 4$ **d** $4x^2 + 5x - 2$
e $5x^3 - 2x^2 + x + 4$ **f** $6x^4 + 3x^2 - x + 3$
g $5x^2 + x - 7$
2 a $x^2 + 4x + 3$ **b** $z^2 + 3z - 10$
c $r^2 - 4r + 3$ **d** $y^2 - 9$
e $10x^2 - 17x + 3$ **f** $49x^2 + 28x + 4$
g $6x^2 - 5x + 1$ **h** $35y^2 + 8y - 3$
i $14u^2 + 9u + 1$ **j** $9k^2 - 16$
3 a $x^3 + 6x^2 + 10x + 3$ **b** $r^3 - 5r^2 + 11r - 15$
c $2t^3 + 3t^2 - 7t - 3$ **d** $6w^3 - 5w^2 + 3w - 1$
e $10y^3 + 27y^2 - 1$
f $10z^4 - 7z^3 - 20z^2 - 11z - 2$
4 a $-x^2$ **b** $5y$ **c** $-8z^3$
d $-9p^2$ **e** r^3
5 a x^2 **b** $11x$ **c** $0x^3$

6 **a** $(1x^2 - x + 4)$ **b** $(x^2 + 3x + 1)$
c $(2x^2 - x + 3)$ **d** $(3x^2 + 0x - 2)$
e $(x^2 + 5x + 2)$

7 **a** $(x + 3)(x + 4)$ **b** $(x + 3)(x + 7)$
c $(x + 5)(x - 3)$ **d** $(x - 5)(x + 1)$
e $(x + 5)^2$ **f** $(x + 3)(x - 2)$
g $(x - 7)(x + 4)$ **h** $(x - 20)(x + 12)$

8 **a** $(2x + 3)(x + 2)$ **b** $(3x + 2)(x + 4)$
c $(3a + 4)(2a + 1)$ **d** $(2y + 9)(2y + 1)$
e $(3d + 4)(6d + 1)$ **f** $(8z + 2)(z + 3)$
g $(3r - 4)(4r + 1)$ **h** $(3u - 4)(5u + 1)$
i $(5e - 7)(5e + 7)$ **j** $(4s - 5)(4s + 5)$

9 **a** $3x(x - 5)$ **b** $(x - 4)(x + 4)$
c $x(x + 5)(x + 2)$ **d** $x(x^2 + 8x + 10)$
e $x(x - 1)(x + 1)$ **f** $x(x - 4)(x + 4)$
g $a(a - b)(a + b)$ **h** $2x(2x - y)(2x + y)$
i $x(2x + 3)(x + 1)$

Exercise 1F *page 15*

1 **a** 81, 2 **b** 0, 1 **c** 64, 2
d −59, 0 **e** 0, 1 **f** −23, 0

2 $m^2 < 64$ **3** $a = 2$ **4** $p < 4\frac{1}{2}$ **5** $k > -\frac{9}{4}$

6 **a** $(x + 2)^2 - 3$ **b** $(x - 1)^2 - 4$
c $(x - 2)^2 - 5$ **d** $(x + \frac{3}{2})^2 - \frac{9}{4}$

7 **a** $12 - (x + 3)^2$ **b** $2 - (x + 1)^2$
c $30 - (x - 5)^2$

8 **a** (−3, −8) **b** (2, −6)
c (−5, −25) **d** (−2, 14)

9 **a** $2(x + 2)^2 - 3$ **b** $3(x - 1)^2 - 2$
c $2(x + \frac{1}{2})^2 - \frac{7}{2}$

11 **a** 3 **b** −2 **c** $\frac{1}{3}$

12 expression $= (x + 2)^2$

13 **a** $y = x^2 - 4x + 7$ **b** $y = x^2 + 2x + 5$

Exercise 1G *page 19*

1 **a** −2, −9 **b** −1, 5 **c** 12, 2 **d** −1, −1
e 0, 7 **f** $0, -\frac{7}{2}$ **g** −4, 4 **h** $-\frac{3}{2}, \frac{3}{2}$

2 **a** −2, −5 **b** 1, −3 **c** −2, 6
d −2, 0.4 **e** 0.75, 2 **f** $-\frac{1}{3}, \frac{5}{4}$
g **−1.41, 0.63** **h** −1.33, 2.08 **i** −2.5, 1
j −0.88, 0.38 **k** −0.69, 1.32 **l** $2, -2\frac{1}{7}$
m −9, 5 **n** 3, 15 **o** 0.23, −1.10
p 5.37, −0.37 **q** 7.20, −3.20 **r** −2.74, 4.74

3 **a** $-2 \pm \sqrt{3}$ **b** $3 \pm \sqrt{2}$ **c** $5 \pm \sqrt{5}$
d $-4 \pm \sqrt{5}$ **e** $-1 \pm \sqrt{8}$ **f** −5

4 $h = 5$ **5** 11, 14 **6** 78×32

7 **d** 4, 5, 6 **8** **c** 6.13 **9** $x^2 + 3x + 1 = 0$

10 $2x^2 - 2x - 1 = 0$ **11** 2.32 m

12 4.96 cm × 11.96 cm **13** 5.78 cm

14 9.03 m/s **15** 10.1

Exercise 1H *page 23*

1 **a** (3, 8) **b** (4, 1) **c** (5, 1)
d (1, 1) **e** (5, −7) **f** (24, 9)
g (2, 3) **h** (4, 3) **i** (−3, 5)
j (2, −3)

2 **a** (1, 2) **or (−2, −1)** **b** (−3.5, −4) **or (−3.5, 4)**
c (2, 4) or (−8, −26) **d** (−1, −2) or $(-\frac{2}{3}, -1)$
e (4, 3) **f** (2, −3) or $(-\frac{14}{11}, \frac{39}{11})$

3 **a** (1, 2) or (2, 1) **b** (2, 3) or $(-\frac{62}{7}, \frac{59}{7})$
c (1, −1) or $(\frac{7}{25}, \frac{29}{25})$ **d** (5, −2) or $(\frac{1}{11}, \frac{86}{11})$

4 (2, 3), (−3, −2)

5 $(3x + 2y)(2x + y)$, $x = \frac{1}{2}$, $y = \frac{1}{4}$

Exercise 1I *page 26*

1 **a** $w \leqslant 2$ **b** $x < -4$
c $x > 5$ **d** $x \geqslant -4$
e $x > 1$ **f** $x < \frac{1}{2}$
g $2 < y < 5$ **h** $-9 < z < -3$

2 $x > 1$ **3** 1, 2, 3, 4, 5

4 7 **5** 4, 5, 6

6 **b** **i** $-3 < x < 3$ **ii** $-5 < x < 5$
iii $x > +7$ or $x < -7$

7 **a** $-1 < x < 1$ **b** $x > 4$, $x < -4$
c $-3 \leqslant x \leqslant 3$ **d** $-4 < x < 4$
e $x > 5$, $x < -5$ **f** $x \geqslant 4$, $x \leqslant -4$

8 **a** $x \geqslant 6$ or $x \leqslant -2$ **b** $-4 < x < -2$
c $x \geqslant 6$ or $x \leqslant -1$ **d** $-3 < x < -2$
e $x \geqslant 4$ or $x \leqslant 3$ **f** $1 \leqslant x \leqslant 2$
g $\frac{3}{2} < x < 4$ **h** $-\frac{2}{3} < x < 5$
i $-4\frac{1}{2} < x < \frac{1}{2}$ **j** $x \geqslant 3$ or $x \leqslant \frac{4}{3}$
k $x \leqslant -\frac{1}{2}$, $x \geqslant \frac{3}{2}$ **l** $-2 < x < \frac{13}{3}$
m $2 < x < \frac{5}{2}$ **n** $x > \frac{5}{2}$ or $x < \frac{1}{3}$

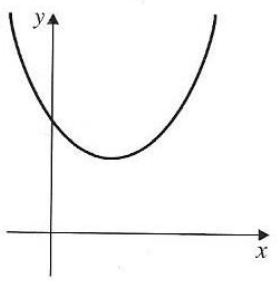
9 $x > 4$ **10** $x > 5$ **11** $x < -3$

12 $p \leqslant 4$ and $p \geqslant 20$ **13** $-1 < x < 3$ and $x > 3$

Exercise 1J *page 29*

1 **a** quadratic, negative x^2
b cubic, positive x^3
c reciprocal
d cubic, negative x^3
e quadratic, positive x^2
f exponential

2 **a**



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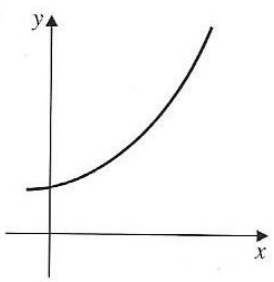

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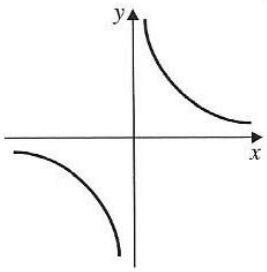

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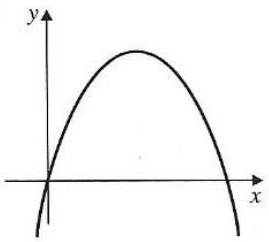

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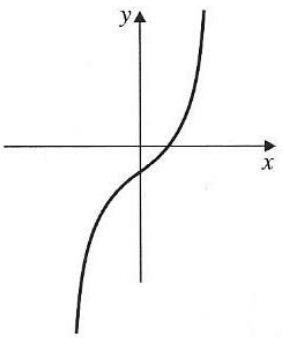

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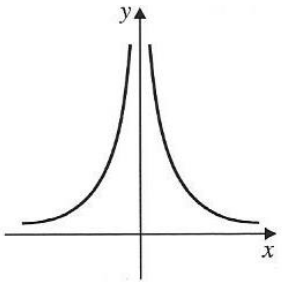
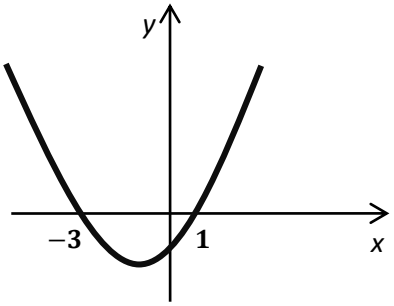


3 i c ii b iii a iv e v f vi d
4 a 1 b 2 c 1 d 3

5



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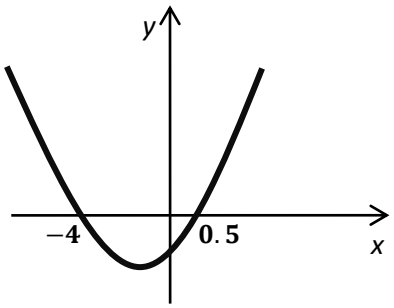

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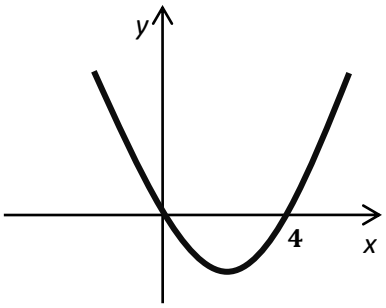

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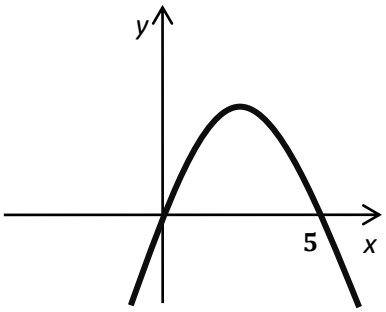

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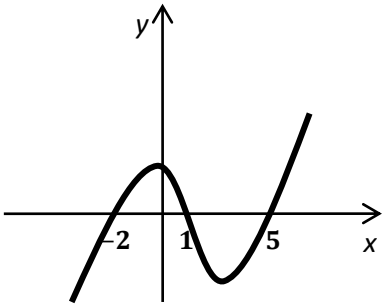

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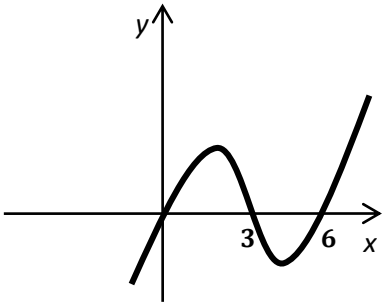

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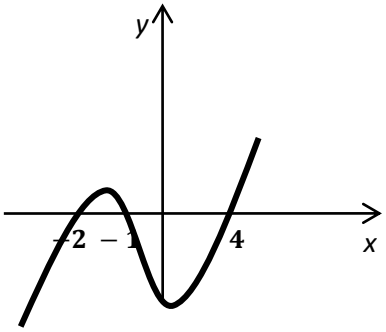

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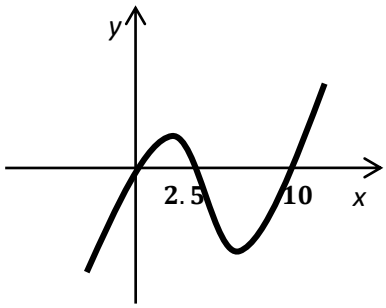
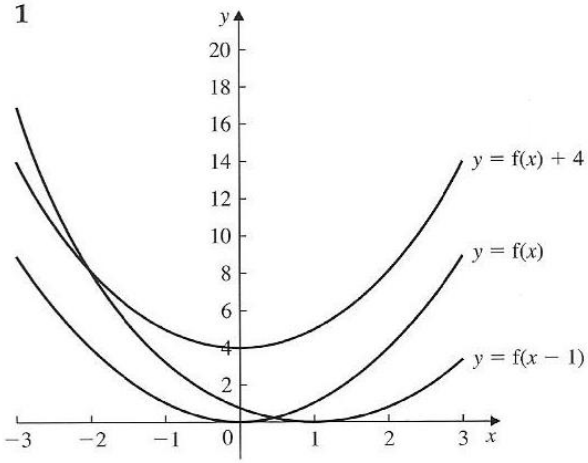


Exercise 1K *page 32*

1 a −0.4, 2.4 **b** −0.8, 3.8 **c** −1, 3
2 a −2.6, 1.6 **b** −0.75, 2.75
c −2, 2 **d** 0.7
3 a −1.65, 3.65 **b** −1.3, 2.3 **c** −1.45, 3.45
4 a 1 **b** 2 **c** 2
d 1 **e** 2 **f** 3
5 a 3.35 **b** 2.4, 7.6 **c** 4.25
6 $0 < k < 4$ **7** (1, 2)
10 $k = \pm 20$ **11** 2

Exercise 1L *page 37*

1



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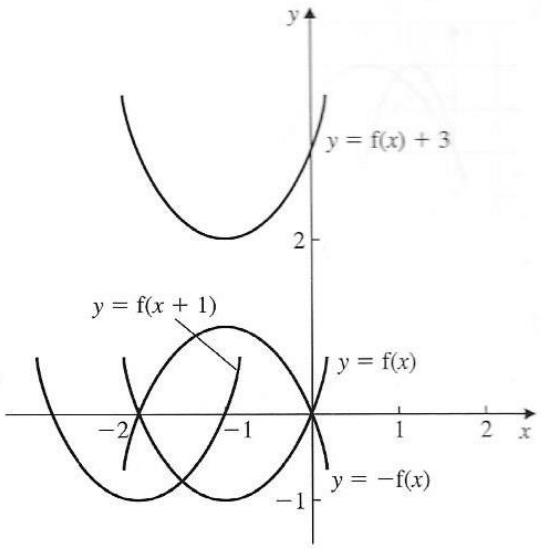
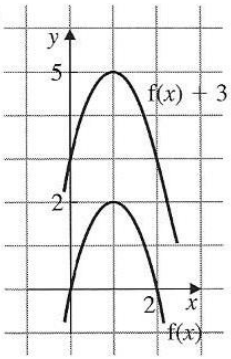


3 (0, −2), (7, 0)
4 a A′(−2, −1), B′(0, −3), C′(2, 0)
b A′(0, 1), B′(2, 3), C′(4, 0)
c A′(−1, 1), B′(0, 3), C′(1, 0)

5



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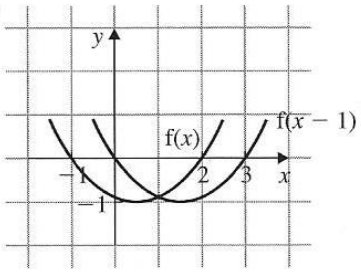

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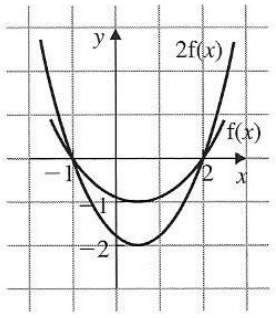

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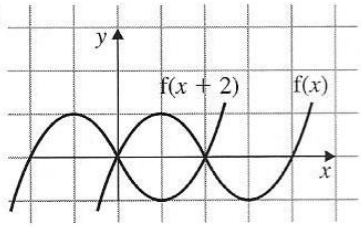

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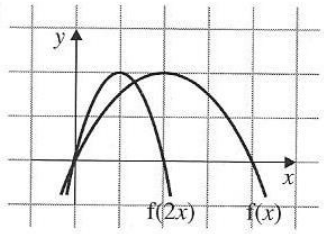

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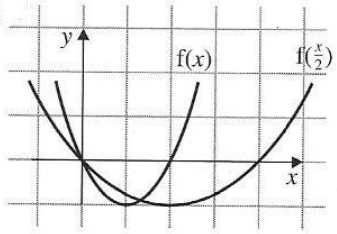

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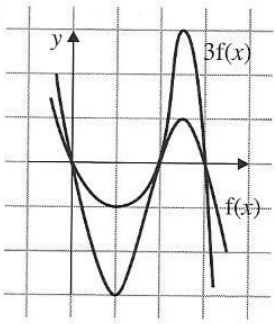

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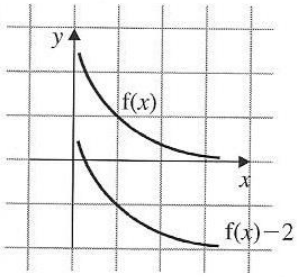

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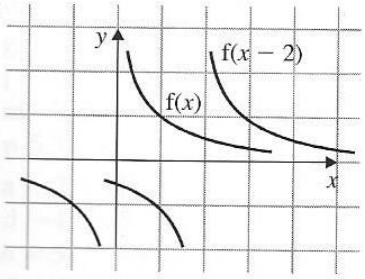

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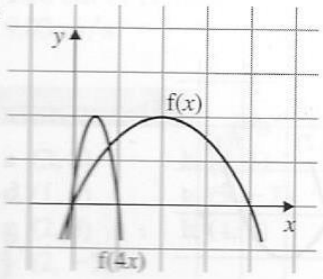

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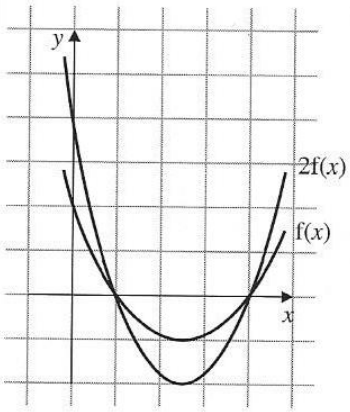

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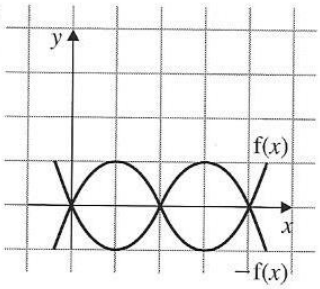
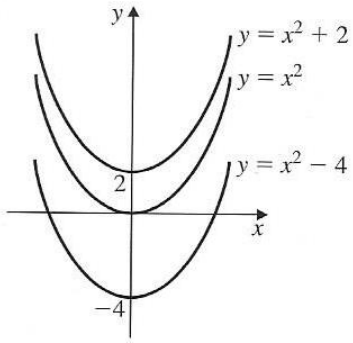


Exercise 1M *page 38*

1



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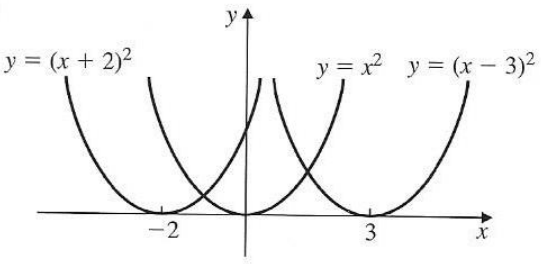

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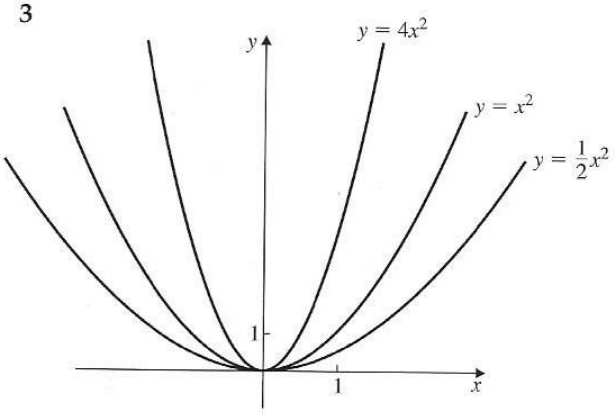

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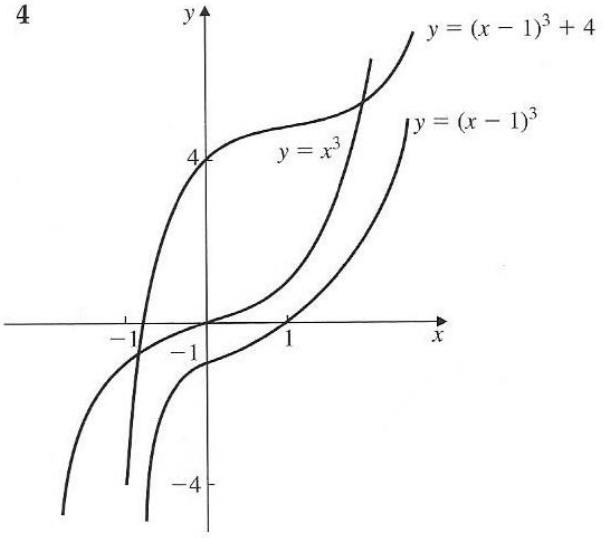

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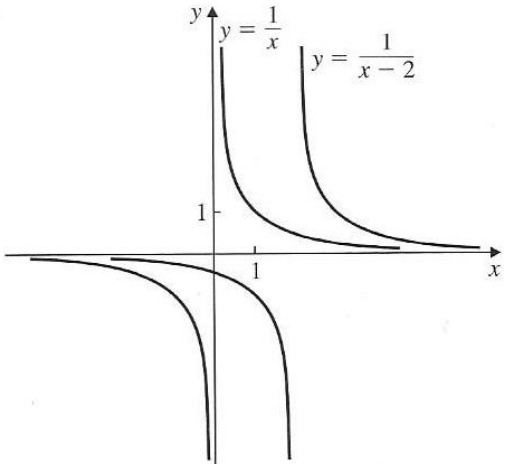
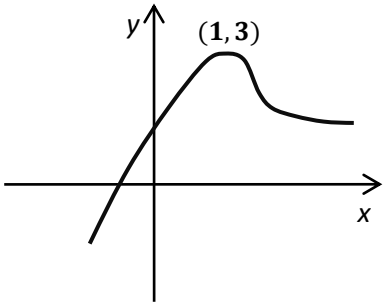


6a



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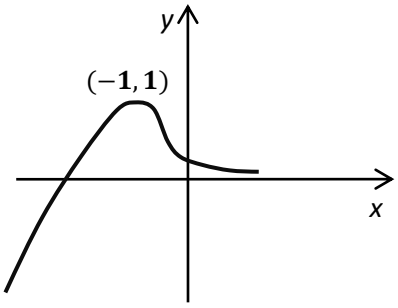

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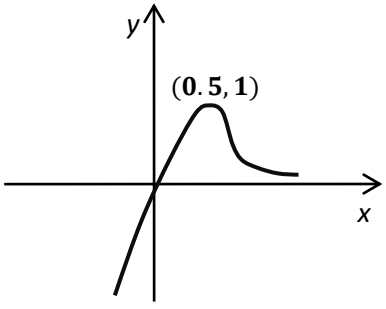

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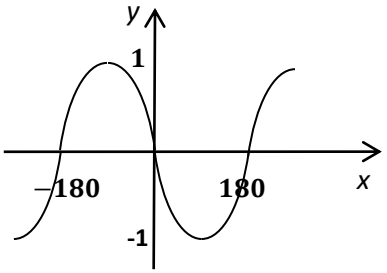
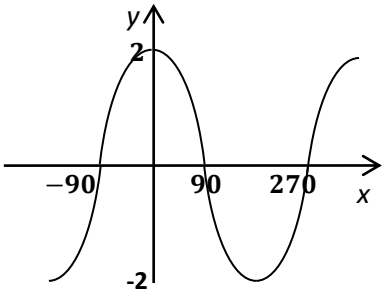


7b



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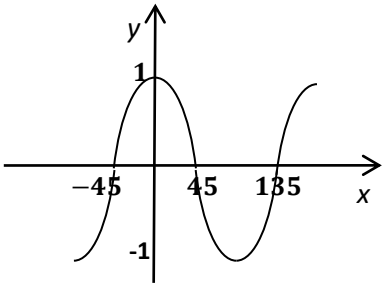

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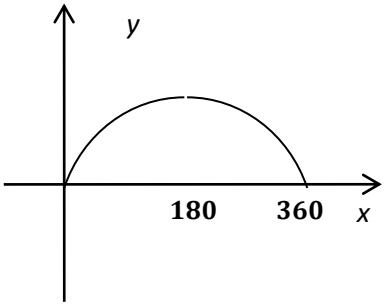

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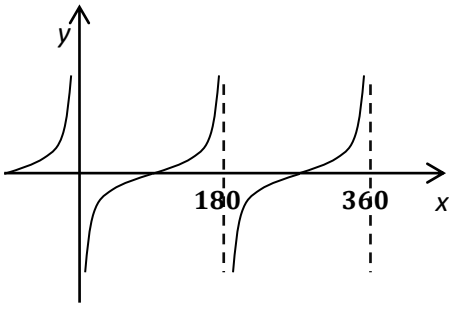

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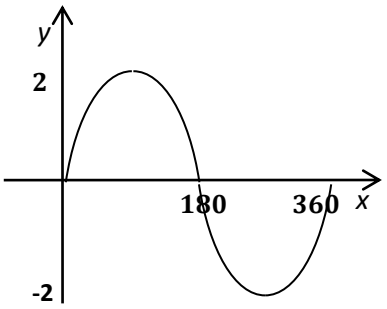

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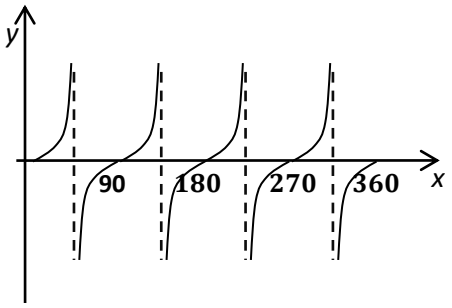
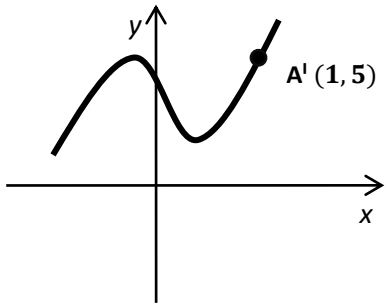
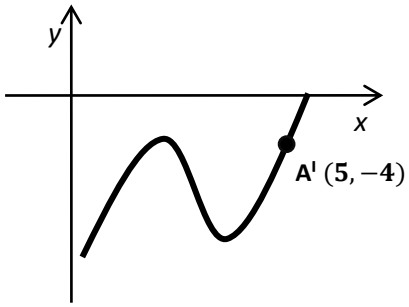


9a

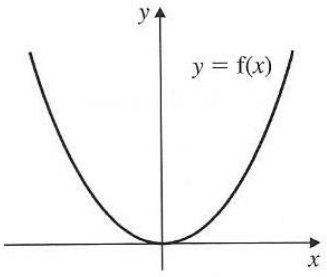


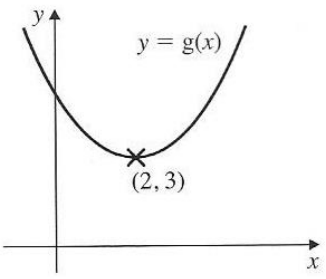
9a



10 a $y = x^2 + 3x + 5$ **b** $y = x^2 - x - 2$
c $y = -(x^2 + 3x)$

11 $a = 2, b = 3$





Review Exercise 1N *page 40*

1 a 32 **b** 9 **c** 1 **d** 64
e 4 **f** $\frac{1}{10}$ **g** $\frac{1}{16}$ **h** $\frac{1}{9}$
i $\frac{1}{8}$ **j** 1000 **k** $\frac{1}{64}$ **l** $\frac{1}{16}$

2 a 2^2 **b** 2^4 **c** 2^6
d 2^1 **e** 2^{-1} **f** 2^{-2}
g 2^0 **h** 2^{-5} **i** 2^7
3 a 2^{17} **b** 2^{-2} **c** 2^{-3}
4 a 3^3 **b** 2^{10} **c** 5^{10}
5 5
6 a 2 **b** 3 **c** 5
d 2 **e** $\frac{1}{3}$ **f** $\frac{1}{10}$
g 4 **h** 27 **i** 4
7 a 2 **b** −4 **c** 0
d 2 **e** −3 **f** −3
g −2 **h** −2 **i** 10
8 a −7 **b** $\frac{1}{4}$
9 a x^5 **b** y^2 **c** z^7
d a^9 **e** b^7 **f** 1
g d^5 **h** e^9 **i** f^3
10 a $3x\sqrt{x}$ **b** $12y^2$ **c** $2z^2$
d $2z^{-\frac{5}{2}}$ **e** $3\sqrt{w}$ **f** $r^{\frac{5}{6}}$
11 a $6\sqrt{2}$ **b** $2\sqrt{2}$ **c** $3\sqrt{5}$
d $5\sqrt{2}$ **e** $5\sqrt{3}$ **f** $2\sqrt{11}$
g $3\sqrt{2}$ **h** $6\sqrt{5}$ **i** $10\sqrt{3}$
j $8\sqrt{5}$ **k** $3\sqrt{7}$ **l** $2\sqrt{3}$
12 a $3\sqrt{2}$ **b** $\sqrt{3}$ **c** $2\sqrt{5}$
d $3\sqrt{7}$ **e** $3\sqrt{5}$ **f** $3\sqrt{11}$
g $\sqrt{2}$ **h** $5\sqrt{5}$ **i** $5\sqrt{3}$
13 a $7\sqrt{3}$ **b** $4\sqrt{2}$ **c** $3\sqrt{3}$
d $5\sqrt{3}$ **e** $8\sqrt{5}$ **f** $12\sqrt{2}$
14 a 30 **b** 20 **c** 9
d 20 **e** 84 **f** 225
g 2 **h** 3 **i** 2
15 a $3 + 2\sqrt{2}$ **b** $5 + 2\sqrt{6}$ **c** $7 - 2\sqrt{10}$
d 1
16 a $3(\sqrt{2} + 1)$ **b** $\sqrt{5} - 2$ **c** $3 + 2\sqrt{3}$
d $\frac{2\sqrt{2} - 1}{7}$ **e** $3 - 2\sqrt{2}$
17 $n = 12$
18 a $4x(x - 5)$ **b** $(x - 3)(x + 3)$
c $x(x^2 + 5x - 7)$ **d** $x(x - 1)(x + 1)$
e $(x - 4)(x + 2)$ **f** $(x - 4)(x - 3)$
19 a $x^3 + 3x^2 + 5x + 6$ **b** $2x^3 - 4x^2 + x - 2$
c $2x^3 + 5x^2 + 4x + 1$
20 a 41, 2 solutions **b** −23, no roots
c 0, 1 repeated root
21 $q < 3$
23 a 0, −12 **c** 2
24 a $(x + 3)^2 - 8$ **b** $(x - 4)^2 - 11$
c $(x + \frac{1}{2})^2 + \frac{3}{4}$ **d** $(x - 2)^2 - 4$
25 a (−4, −15) **b** (1, 4) **c** (−3, 19)
26 a $2(x + 3)^2 - 15$ **b** $3(x - 1)^2 - 1$
c $2(x + \frac{1}{2})^2 - \frac{11}{2}$
27 $3(x - 2)^2 - 11$; (2, −11)
28 2
29 a 2, −7 **b** 0, 9 **c** ±5
d 7.46, 0.54 **e** −2, 3 **f** 3.62, 1.38
30 1, 8 **31** (3, 12) or (1, 4)
32 a (3, −2), $(\frac{5}{9}, \frac{26}{9})$ **b** (1, −3), $(6, -\frac{1}{2})$
c (5, 2), (−3, −2)

33 a $x \leqslant 10$ b $x < \frac{1}{3}$

c $-4 < x < 4$ d $-\frac{1}{2} < x < 6$

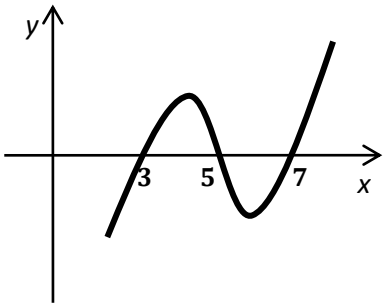
34 a $-5 < x < 8$ b $-1 < x < 3$

35 a $-3 < x < 2$ b $x < 0$ or $x > \frac{1}{2}$

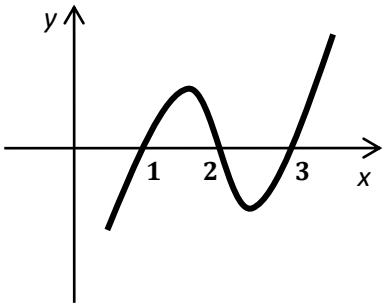
c $-3 < x < 5$ d $x \geqslant 4, x \leqslant -\frac{1}{2}$

e $-3 < x < 4$ f $x > \frac{1}{3}, x < -\frac{3}{2}$

38a

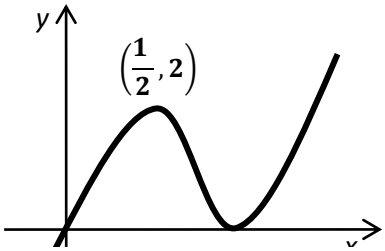


38b

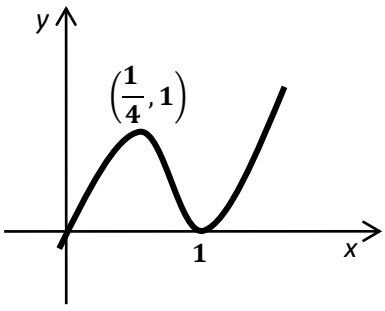


38c (-1,0), (1,0) and (3,0)

39a

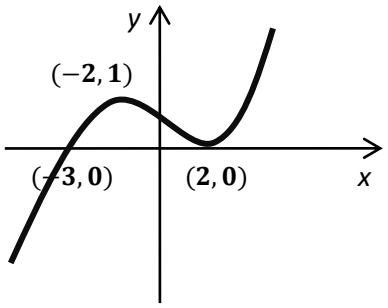


39b

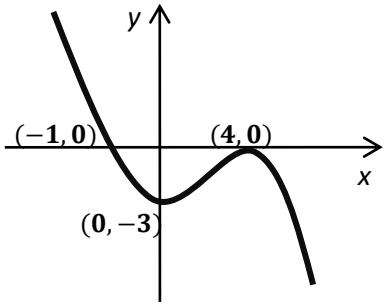


39c -4 and -2

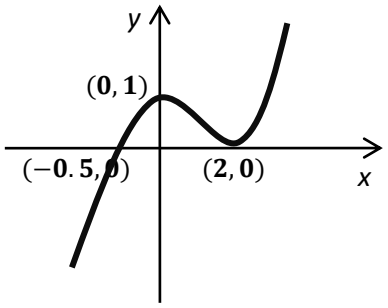
40a



40b

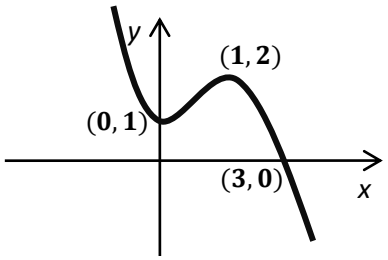


40c

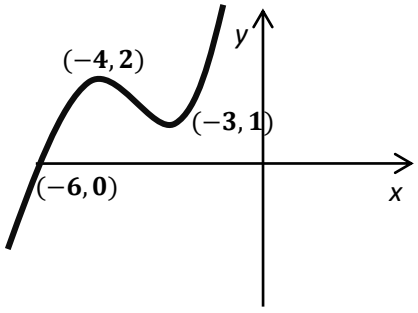


41a $(h + a, k)$ b $(0,0)$ and $\left(\frac{1}{2}a, 0\right)$

42a



42b



Examination Exercise 1 *page 45*

1 a 729 **b** $16x$

2 i 4^{-2} **ii** 4^3 **iii** $4^{\frac{3}{2}}$

3 a 32 **b** $2x^{-\frac{1}{2}}$

4 i $m = 4$ **ii** $p = 2$ **iii** $n = -1$

5 i $9 - 3\sqrt{5}$ **ii** $2\sqrt{2}$

6 a $6\sqrt{3}$ **b** $7 - 4\sqrt{3}$

7 $-3 + 2\sqrt{7}$

8 a $4 - \sqrt{5}$ **b** $7\sqrt{5}$

9 $5\sqrt{2}$

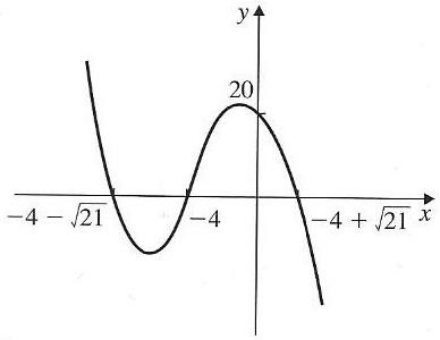
10 a $11 + 4\sqrt{7}$ **b** $\sqrt{2}$

11 $p < 3 - \sqrt{8}, p > 3 + \sqrt{8}$

12 i $x = -4 \pm \sqrt{21}$

ii $x \leqslant -4 - \sqrt{21}, x \geqslant -4 + \sqrt{21}$

iii



13 b $\frac{20}{7} \leqslant k \leqslant 4$

14 $\pm\frac{1}{2}$

15 $\frac{1}{4}, 9$

16 $-1, \frac{8}{27}$

17 i $5(x + 2)^2 - 28$

ii $x = -2$

iii 560

iv two

18 $p = -10, q = 5, r = -13$

19 $x = \frac{9}{2}, y = \frac{25}{2}$

$x = -2, y = 32$

20 i $x = -\frac{1}{2}, y = -3$

ii The line is a tangent to the curve.

21 a $x < \frac{11}{2}$

b $x \leqslant -4, x \geqslant \frac{3}{2}$

22 $x < -3, x > -\frac{1}{3}$

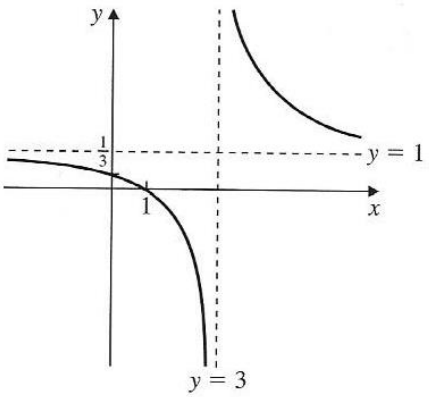
23 i $-\frac{7}{3} \leqslant x \leqslant -\frac{2}{3}$

ii $x < -2, x > 6$

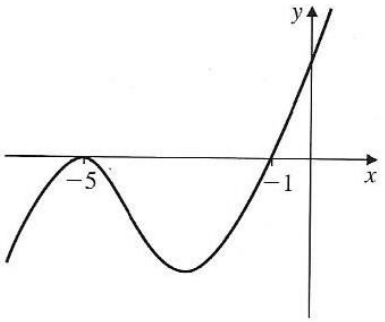
24 b $-3 < x < 2.5$

c $1.7 < x < 2.5$

25



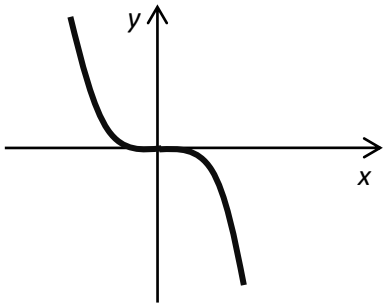
26 a



b $y = (x + 5)^2(x + 1)$

c (0, 25)

27a



27b $y = -(x - 3)^3$

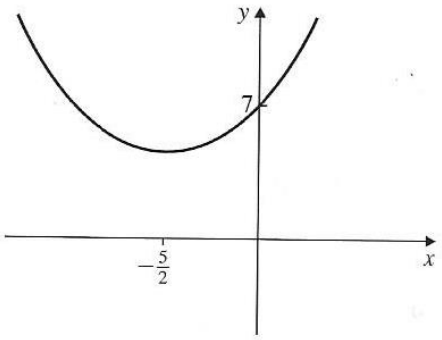
27c Vertical Stretch by factor 5

28 a $(x + \frac{5}{2})^2 + \frac{3}{4}$

b i $(-\frac{5}{2}, \frac{3}{4})$

ii $x = -\frac{5}{2}$

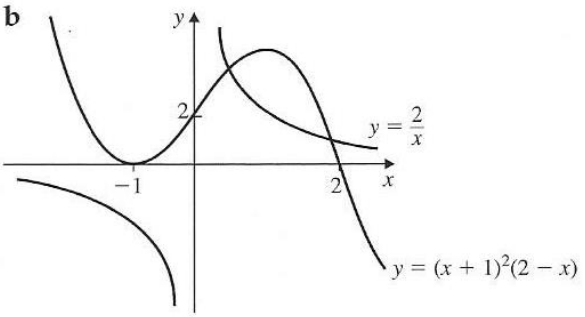
iii



c Translation of $\frac{5}{2}$ units in the negative x direction, and $\frac{3}{4}$ units in the positive y direction

29 a 4

b



c 2

Exercise 2A *page 53*

1 **a** 2 **b** 2 **c** 4 **d** -5
e 5 **f** 6 **g** -3 **h** -3
i 0 **j** infinite

2 **a** $\frac{2a-5}{a-3}$ **b** 4 **c** $\frac{5}{2}$

3 **a** $\frac{n+4}{2m-3}$ **b** -4 **c** $\frac{3}{2}$

4 **a** $(1, 5)$ **b** $(4, 7)$ **c** $(5, 14)$
d $(4, -4)$ **e** $(3, -5)$ **f** $(1, -7)$

5 $a = 4, b = -6$ **6** $\sqrt{41}$

7 **a** $\sqrt{13}$ **b** $4\sqrt{2}$ **c** $\sqrt{29}$

8 0, 7 **9** $-\frac{1}{3}$

10 $5, -\frac{1}{2}, -\frac{2}{3}, \frac{1}{5}, -\frac{4}{3}$ **11** -1

12 **b** $(6, 0)$ **c** $(4\frac{1}{2}, 3\frac{1}{2})$ **d** 29

13 3 **14** **a** $4, \frac{32}{3}$ **c** $(1\frac{1}{2}, 2\frac{1}{2})$

Exercise 2B *page 57*

2 **a** parallel **b** parallel
c perpendicular **d** perpendicular
e perpendicular **f** **parallel**
g perpendicular

3 A $y = 2x - 3$ B $y = \frac{1}{3}x + 2$ C $x + y = 6$

4 **a** $x - 3y + 7 = 0$ **b** $2x - y = 0$
c $5x - y - 9 = 0$ **d** $2x - 3y - 10 = 0$

5 **a** $y = x + 1$ **b** $y = 2x + 3$
c $y = 5x - 1$ **d** $y = 7x - 2$
e $y = -3x - 4$ **f** $y = -2x + 11$

6 **a** $y = 5x - 1$ **b** $y = -4x - 5$
c $3x - 4y + 11 = 0$ **d** $2x - 5y + 19 = 0$
e $y = 3x - 1$ **f** $5x - 2y + 26 = 0$

7 **a** 3 **b** $-\frac{1}{3}$ **c** 4 **d** $-\frac{3}{2}$

8 **a** $5x + y - 9 = 0$ **b** $x + 3y - 14 = 0$
c $3x + 4y - 34 = 0$ **d** $3x + y - 17 = 0$
e $5x + 3y + 2 = 0$ **f** $3x + 4y - 23 = 0$
g $2x + 3y - 21 = 0$

Exercise 2C *page 59*

1 $(3, 7)$ **2** $(11, -1)$
3 $x + 2y + 1 = 0$ **4** A & C
5 $m_1 m_2 = -1$ **6** 24 sq. units
7 **a** $y + 2x = 10$ **b** $\sqrt{5}$
8 $(2, -4)$
9 **a** $a = 10$ **b** $2y + 3x = 17$
10 **b** $-\frac{2}{3}$ **c** $\frac{3}{2}$ **f** $\sqrt{13}$
11 **a** $(2, 4)$ **b** B $(-6, 0)$ C $(-2, 12)$
12 $6\sqrt{2}$
13 **a** $(2, 5)$ **c** $(8, 1)$
14 **a** 10:53 **b** 104.375 km
c may be in different directions

Exercise 2D *page 64*

1 **a** $(2, 6)\ r = 2$ **b** $(5, -2), r = 5$
c $(0, 3), r = \sqrt{10}$ **d** $(p, 0), r = k$

2 **a** No **b** Yes
c No **d** Yes

3 **a** $(4, 1)\ 5$ **b** $(-3, 2)\ 3$
c $(8, 3)\ 1$ **d** $(0, -9)\ \sqrt{11}$
e $(-1, -\frac{1}{4})\ \frac{1}{4}$ **f** $(\frac{1}{4}, -2)\ \sqrt{2\frac{1}{16}}$
g $(a, b)\ \sqrt{a^2 + b^2}$

4 $(x-1)^2 + (y-3)^2 = 25$
5 $(x-5)^2 + (y-5)^2 = 25$
6 $(x+2)^2 + (y-4)^2 = 16$
7 $(x-5)^2 + (y+1)^2 = 25$
8 $(x-4)^2 + (y-5)^2 = 25$
11 **b** $(x-6)^2 + (y-6)^2 = 36$
13 $(2, 3)$ $(x-2)^2 + (y-3)^2 = 50$
14 **a** $(5, 4)$ **b** $x^2 + y^2 - 10x - 8y + 28 = 0$
15 $(x-2)^2 + (y-2)^2 = 4$
16 **b** $x^2 + y^2 - 4x - 2y + 4 = 0$
17 $x^2 + y^2 - 6x + 8y = 0$
19 **b** $(x-5)^2 + (y-4)^2 = 25$

Exercise 2E *page 68*

1 $4x + y - 6 = 0$ **2** $5x + y - 19 = 0$
3 $x - 2y + 4 = 0$ **4** $5y = x + 7$
5 **a** $x + 2y - 8 = 0$ **b** $(2, 3)$
6 $(-2, 5)$ **7** $(-1, -2)$
8 A$(4, 0)$, B$(0, -8)$, area 16 u^2
9 $\sqrt{17}$ **10** 5
11 $(x-3)^2 + (y-2)^2 = 5$
12 $(x-6)^2 + y^2 = 20$

13 Sub $y = mx + c$ into $x^2 + y^2 = a^2$
to get $x^2 + (mx + c)^2 = a^2$
i.e. $(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$
tangent ∴ single intersection ∴ $D = b^2 - 4ac = 0$
giving $(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$
rearrange to get $c^2 = a^2(1 + m^2)$

Review Exercise 2F *page 69*

1 **a** -2 **b** $\frac{1}{2}$ **c** $\frac{1}{3}$
d -5 **e** -5 **f** 6
g 0 **h** -1

2 **a** $y = 3x - 6$ **b** $y = 2x$
c $4y = x - 7$ **d** $2y = 5x + 33$
e $y = 2x - 4$ **f** $y = 5$

3 **a** $y = 3x - 2$ **b** $y = -6x - 12$
c $x + 2y + 5 = 0$ **d** $3x - 4y + 11 = 0$
e $2x - 5y + 19 = 0$ **f** $2x + 5y - 7 = 0$

4 **a** $(1, 7)$ **b** $(3, 2)$ **c** $(5, -4)$ **d** $(2\frac{1}{2}, -4)$

5 **a** $3y - 5x = -2$ **b** $(1, 1)$

6 **a** $3x - 4y + 9 = 0$ **b** $4x + 3y - 38 = 0$
c 5 units

7 **a** $y = 5x$ **b** $4x + 5y + 17 = 0$
8 **a** (3, 5)
b $-\frac{1}{2}$, 2; lines are perpendicular
9 **a** $y = 15 - 3x$ **b** A(5, 0), B(0, 15)
c $5\sqrt{10}$
10 **a** (2, 7) **b** (5, 11) **c** (−2, 10)
d (1, 14) **e** a square
11 **b** $3x + 4y - 36 = 0$, $4x - 3y - 23 = 0$
c perpendicular **d** $100 - 25\pi$ sq units
13 **a** 8 **b** −2
14 **a** (5, 1) **b** (5, 1)
15 **a** $y = -3x + 19$ **b** $3y = x + 2$
c $x = 4$ **d** (4, 2)
16 **b** 4 or −1
17 **a** (3, 2) 6 **b** (−1, −2) 3
c (−3, 1) 4 **d** (5, 3) 7
18 **a** $(x - 3)^2 + (y - 1)^2 = 4$
b $(x - \frac{1}{2})^2 + (y - \frac{1}{4})^2 = 1$
19 **a** (1, 4) $r = 5$
c normal $4x - 3y + 8 = 0$
tangent $3x + 4y - 44 = 0$
20 −23
21 $x^2 + y^2 - 18y + 56 = 0$
22 $x^2 + y^2 - 10x + 2y + 10 = 0$
23 8
25 **a** $y = \frac{1}{2}x - 1$ **b** 5 u^2
26 **a** $(x - 4)^2 + (y - 3)^2 = 13$
27 **b** $y = -3x + 10$

Examination Exercise 2 *page 72*

1 **a** $-\frac{3}{5}$ **b** $y = \frac{5}{3}x - 4$
2 **i** 5 **ii** $8y - 6x + 29 = 0$
3 $y = -\frac{1}{2}x + 3$
4 **a** $-\frac{7}{3}$
b **i** $7x + 3y - 2 = 0$ **ii** A(4, −5)
c B(−2, 9)
5 **i** $p = -1$ **ii** $m = 2, q = 3$
iii $d = 4$ or −8
6 $y = 4x + 10$, (0, 10), (−2.5, 0)
7 **a** $4x - 5y - 8 = 0$ **b** $\sqrt{41}$
c $t = 8$ **d** **20**
8 **a** **i** $\frac{4}{3}$ **ii** $4x - 3y = 27$
b D(−2, −5) **c** $k = -3$
9 **a** $y = -\frac{1}{2}x + 6$ **c** $2\sqrt{5}$
10 **a** **i** $-\frac{7}{4}$ **ii** $7x + 4y = 1$
b **i** M$(1, -\frac{3}{2})$
ii $8x - 14y = 29$
c $k = -\frac{11}{5}$ or 1
11 $(x + 1)^2 + (y - 7)^2 = 50$
12 **i** C(5, −2), $r = 5$
iii 40
13 **i** $r = 2\sqrt{5}$, C(3, 2)
ii $(-1, 0)$, $(7, 0)$, $(0, 2 \pm \sqrt{11})$
iii M(4, 5), $d = \sqrt{10}$
14 **a** **i** $(x + 3)^2 + (y - 1)^2 = 13$
ii $x^2 + y^2 + 6x - 2y - 3 = 0$
b 4 **c** **ii** $2x + 3y + 16 = 0$
15 **i** C(1, −5), $r = 3\sqrt{5}$
iii $2x + y - 12 = 0$
16 **a** $(x - 2)^2 + (y + 1)^2 = 20$
b $x - 2y - 14 = 0$
17 **ii** E(−1, 5)
iii $(x + 1)^2 + (y - 5)^2 = 17$
18 **a** $(x - 5)^2 + (y + 6)^2 = 20$
b **i** C(5, −6), $r = 2\sqrt{5}$
c $x - 2y = 7$
d AB = 4
19 **i** A
ii $x^2 + y^2 - 4x + 8y - 30 = 0$

Exercise 3A *page 77*

2 **a** $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$
b $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$
c $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
d $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$
e $16 + 32x + 24x^2 + 8x^3 + x^4$
f $1 - 3x + 3x^2 - x^3$
g $1 + 8x + 24x^2 + 32x^3 + 16x^4$
h $1 + 15x + 90x^2 + 270x^3 + 405x^4 + 243x^5$
3 **a** $16 + 32x + 24x^2 + 8x^3 + x^4$;
$16 - 32x + 24x^2 - 8x^3 + x^4$
b $32 + 48x^2 + 2x^4$
4 **a** $1 + 3x^2 + 3x^4 + x^6$ **b** $1 - 3x^2 + 3x^4 - x^6$
c $2 + 6x^4$
5 18
6 **a** 40 **b** 945

Exercise 3B *page 80*

1 **a** 6 **b** 120 **c** 20
d 90 **e** 66 **f** 1
2 **a** 10 **b** 20 **c** 35
d 60 **e** 28 **f** 780
3 **a** 1, 5, 10, 10, 5, 1
4 **a** $1 + 6x + 15x^2 + \ldots$ **b** $1 + 10x + 45x^2$
c $1 + 8x + 28x^2$ **d** $1 + 10x + 40x^2 + \ldots$
e $1 + 18x + 135x^2$ **f** $1 + 8a + 24a^2 \ldots$
5 $1 + 12x + \mathbf{60}x^2 + 160x^3$ 12, **60**, 160
6 **a** $1 + 14x + 84x^2 + 280x^3$
b $1 - 5x + 10x^2 - 10x^3$
c $1 + 18x + 144x^2 + 672x^3$
d $1 - 10x + 40x^2 - 80x^3$
e $1 - 12x + 54x^2 - 108x^3$
f $1 + 2x + \frac{3}{2}x^2 + \frac{x^3}{2} + \ldots$
8 405
9 448
10 **a** $a^5 + 5a^4b + 10a^3b^2$
b $x^{10} + 10x^9y + 45x^8y^2$
c $a^4 + 8a^3b + 24a^2b^2 + \ldots$
11 $64 + 192x + 240x^2 + \ldots$
12 **a** $1024 + 5120x + 11520x^2$
b $81 + 108x + 54x^2 + \ldots$

c $32 + 480x + 2880x^2 + \ldots$
d $32 - 80x + 80x^2$
e $64 - 48x + 12x^2$

13 b $0, -\frac{1}{2}$
14 b $0, 1, -4$

Exercise 3C *page 82*

1 $1 + 9x + 36x^2 + 84x^3 + \ldots$; 1.0090
2 $1 - 7x + 21x^2 - 35x^3$; 0.9861
3 $1 + 20x + 180x^2 + 960x^3$; 1.020
4 a 1.127 **b** 0.979
5 $1 + 11x + 50x^2$
6 a $2 + 61x + 840x^2$ **b** $3 + 35x + 168x^2$
c $1 + 8x + 29x^2$ **d** $1 - 11x + 54x^2$
e $1 + 9x + 37x^2$
7 $a = 3, n = 7$ **8** $k = 2, n = 6$
9 3, 3 **11** 70
12 15
13 $2^{10}[1 + 5x + \frac{45}{4}x^2 + 15x^3]$ 1131.13
14 $30\sqrt{30}$
15 $1 + 3x + 6x^2 + 7x^3$
16 $1 + 6x + 27x^2 + 80x^3$
17 $n = 7$
18 $a^4 + 4a^2b + 2b^2$

Exercise 3D *page 85*

1 a 0.118 **b** 0.0595 **c** 0.999
2 a 0.0168 **b** 0.00786 **c** 0.279
3 a 0.315 **b** 0.0746 **c** 0.401
4 a 0.222 **b** 0.978
5 a 0.193 **b** 0.0290
6 a 0.282 **b** 0.526
7 a 0.115 **b** 0.00759
8 a 0.177 **b** 0.997
9 a 0.405 **b** 0.925
10 a 0.000285 **b** 0.00569

Review Exercise 3E *page 86*

1 a $1 + 4x + 4x^2$ **b** $1 + 15x + 75x^2$
c $1 - 15x + 90x^2$ **d** $16 + 96x + 216x^2$
e $729 - 1458x + 1215x^2$ **f** $1 - 14x + 84x^2$
2 a $2 + 24x^2$ **b** $-(72x + 54x^3)$
c $8x + 8x^3$
3 a $1 + 30x + 405x^2 + 3240x^3$
b i 1.03041 **ii** 0.737
4 a $1\,259\,712x^2$ **b** $-22\,680x^3$
c $-56x$
5 a $x^3 + 6x^2 + 12x + 8$ **b** $15\sqrt{3} + 26$
6 b $608\sqrt{3}$
7 $k = 2, n = 5$
9 $1 + 4x + 18x^2 + 40x^3$
10 b $0, 2, -5$
11 a 0.238 **b** 0.252
12 a 0.172 **b** 0.999
13 a 0.337 **b** 0.00542
14 a $\frac{1}{81}$ **b** $\frac{16}{81}$ **c** $\frac{8}{27}$
15 a $\frac{2}{3}$ **b** $\frac{17}{81}$
16 a 0.0523 **b** 0.236

Examination Exercise 3 *page 88*

1 $243 - 810x + 1080x^2$
2 $81x^4 - 216x^3 + 216x^2 - 96x + 16$
3 $a = 2, b = \frac{5}{3}, c = \frac{20}{27}$
4 $p = 3, q = 5$
5 6000
6 i $243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$
ii $486 + 2160x^2 + 480x^4$
7 i 10 **ii** -720
8 a $1 + 10ax + 45a^2x^2 + 120a^3x^3$
b $a = \frac{3}{4}$
9 a $243 + 405bx + 270b^2x^2$
b $b = 3$
10 i $8 + 12y + 6y^2 + y^3$
ii $b = 16, q = 12$
11 i $a = \frac{1}{2}$ **ii** $4096 + 3072x$
12 a $64 - 576x + 2160x^2$
b $64 - 544x +$ **1872** x^2
13 a $1 + \frac{8}{x} + \frac{16}{x^2}$
b $1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$
c 30
14 i $64 + 960x + 6000x^2$
ii $c = -11$
15 i $32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$
ii 1560
16 20,000
17 i $a = 243$ **ii** $k = \frac{3}{2}$ **iii** $\frac{1215}{4}$
18 $\frac{1792}{6561}$

Exercise 4A *page 95*

1 $2x$ **2** $5x^4$ **3** 5
4 $7x^6$ **5** $3x^2 + 4x^3$ **6** $2x + 5x^4$
7 $6x^5 + 4$ **8** $3x^2 - 2x$ **9** $4x^3 + 2x + 1$
10 $4x$ **11** $12x^2$ **12** $30x^4$
13 7 **14** $6x + 4$ **15** $30x^2 - 7$
16 $40x^9 - 18x^8$ **17** $2x + 4$ **18** $2x - 3$
19 $4x$ **20** $3x^2 + 2x - 6$ **21** $12x^3 - 3x^2$
22 100 **23** -3 **24** $-2x$
25 $-4x$ **26** $3x^2 - 17$ **27** $15x^2 - 4x + 4$
28 $2x - 11$ **29** $100\,x^{99}$ **30** x^9
31 x **32** $2x^7$ **33** $\frac{4}{3}x + 3$
34 $\frac{9}{2}x^5$ **35** $0.02x$ **36** $\frac{1}{2}x^4$
37 $1 + 3x^2$ **38** $x + x^2 + x^3$ **39** $-17x^{16}$
41 1 **42** -13 **43** 13
44 $14x + 4$ **45** $2x^7 - 3x^2$ **46** 5
47 -4 **48** (2, 3)
49 (1, 1) **50** 1, -3

Exercise 4B *page 98*

1 a $5x^4$ **b** $7x^6$ **c** $6x^2$
d $20x^3$ **e** 3 **f** 0

g $8x^{\frac{1}{3}}$ **h** $-2x^{-2}$ **i** $x^{-\frac{2}{3}}$

j $-14x^{-3}$ **k** $3x^{-\frac{1}{2}}$ **l** $\dfrac{-5}{x^2}$

2 a $3x^2 + 2x$ **b** $4x^3 + 1$

c $6x + 2$ **d** $18x^2 + 30x^5$

e $3x^{-\frac{1}{2}} - 3x^{\frac{1}{2}}$ **f** $4x + 5x^{-2}$

g $-9x^{-4} - 4x^{-3}$ **h** $2x^{-\frac{1}{2}} + 3$

i x **j** $-\dfrac{1}{x^2} + \dfrac{2}{x^3}$

3 a $3x^2 + 10x + \dfrac{3}{x^2}$ **b** $2x^{-\frac{2}{3}}$

c $3x^{\frac{1}{2}}$ **d** $\dfrac{-2}{x^3} + \dfrac{3}{x^2}$

e $\frac{5}{2}x^{\frac{3}{2}}$ **f** $x^{-\frac{2}{3}} + x^{-\frac{1}{2}}$

4 a $2x + 1$ **b** $6x^2 - 2x$ **c** $2x + 2$

d $8x - 4$ **e** $8x + 4$ **f** $4x^3 + 4x$

g $5x^4 + 7x^6$ **h** $2x - 2$ **i** $18x + 12$

j $16x^3 - 8x$ **k** $1 - x^{-\frac{1}{2}}$ **l** $2x - 2x^{-3}$

5 16 **6** 1 **7** $-\frac{1}{4}$

8 a 2 **b** 10

9 $(3, -4)$

10 $(-1, 12), (3, -20)$

11 a $2x - 6$ **b** $x = 3$ **c** $(3, -8)$

12 a $3x^2 - 18x + 15$ **b** $x = 1, 5$

c $(1, 2)$ $(5, -30)$

13 a $a = 2, b = 8, c = -1$ **b** $(-2, -9)$

15 a $1, 2, -5, -6$ **c** $(-3, 0)$ $(-1, 0)$

16 a $2x + 5$ **b** $9x^2 - 2$

c $2x$ **d** $-2x^{-2} + 3$

e $12x^{-3} - 36x^{-4}$ **f** $\frac{7}{2}x^{-\frac{1}{2}} = \dfrac{7}{2\sqrt{x}}$

g $6x^2 + 4$ **h** $-8x^{-5} + 3x^{-2}$

17 a $\dfrac{5}{2\sqrt{x}}$ **b** $\frac{4}{3}x^{\frac{1}{3}}$

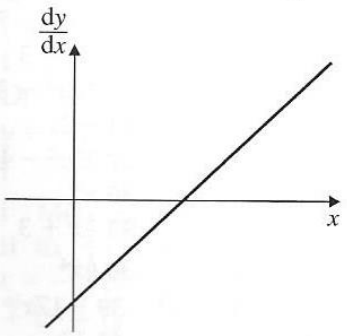
c $-\dfrac{5}{x^2}$ **d** $1 - \frac{1}{2}x^{-\frac{1}{2}}$

e $1 + \dfrac{1}{\sqrt{x}}$ **f** $4x^{-\frac{1}{3}}$

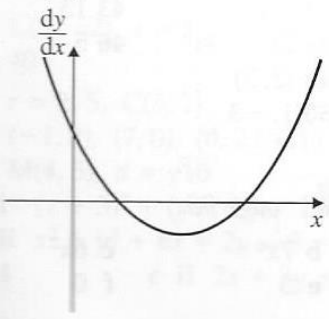
g $\dfrac{1}{\sqrt{x}} - x^{-\frac{4}{3}}$ **h** $\frac{3}{2}\sqrt{x} + \dfrac{1}{2\sqrt{x}}$

i $\frac{1}{5}x^{-\frac{4}{5}} + 4x^{-\frac{3}{5}}$ **j** $\frac{5}{2}x^{\frac{3}{2}} - \frac{1}{6}x^{-\frac{7}{6}}$

18 a



b



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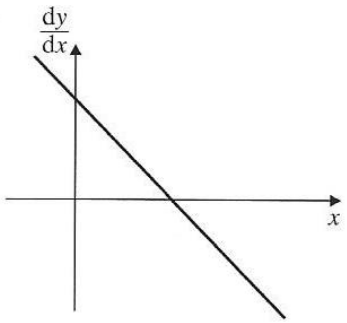
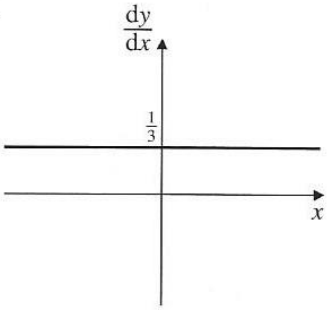
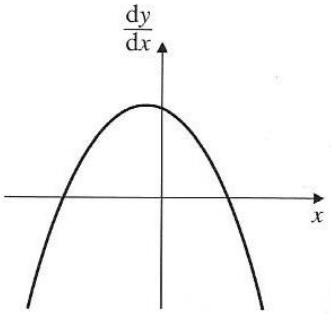


d



e



19 A → E, B → F, C → D

Exercise 4C *page 105*

1 $y = 7x - 3$ **2** $y = -7x - 3$

3 a $10y = x + 99$ **b** $(-99, 0)$

4 $y + x + 6 = 0$ **5** $4y + x = 2$

6 $x = 1, 2$; tangents are $y = 2$ and $y = 1$

7 a $y = 6x - 18$, $y = -6x - 18$

b $(0, -18)$

8 a $y = 4x - 3$ **b** $y = 12x + 4$

c $3y + 4x = 24$ **d** $3y = 2x + 18$

e $y = 3x - 7$ **f** $y = 7x - 4$

g $y = -2$ **h** $y + 6x = 24$

9 a $4y + x = 29$ **b** $6y + x = 37$

c $6y + x = 33$ **d** $3y + 4x = 136$

e $2y + 3x = 17$ **f** $y = -x$

g $y = 2 - x$ **h** $3y + 2x = 14$

10 a -16 **b** 2

c $y = 2x + 6$ **d** $16y + x - 127 = 0$

11 a $y = -24x + 13$

12 a 1, 3 **c** $y = -2x + 2$; $y = 2x - 6$

13 a $c = -26$ **b** $(-4.2, 6.04)$

14 a $x = 1, -\frac{7}{3}$ **b** $(1, -15)$

15 b $(0, 1)$

16 a $2y + x + 10 = 0$ **b** $\left(\frac{3}{2}, -\frac{23}{4}\right)$

17 $a = 2, b = 13$ **18** $c = 12, d = 1$

19 $a = 2, b = 48$ **20** $c = 2, d = 5$

21 a -16 **b** $y + 16x = 12$

c $4\frac{1}{2}$ sq units

22 a $(1, 5), (-4, 0)$ **b** $y = -4x - 16$

c $(-\frac{3}{2}, -10)$

23 a $(\frac{3}{2}, -\frac{3}{2})$ $(-2, -5)$

b $6y = 8x - \frac{27}{2}$ $y = -6x - 17$

c $\left(-\frac{1}{4}, -\frac{31}{2}\right)$

24 b $p = 1$ c $y = x$

EXERCISE 4D *page 113*

1 -4; min. **2** 26; max.
3 3; max. **4** -8; min.
5 -1; min. **6** 4; max.
7 $\frac{1}{4}$; min. **8** $5\frac{1}{4}$; max.
9 -70, min.; **38**, max.
10 a $4x + 3$, 4 b $6x^2 + 10x - 2$, $12x + 10$
c $3x^2$, $6x$ d $x^{-\frac{1}{2}}$, $-\frac{1}{2}x^{-\frac{3}{2}}$
e $-3x^{-2}$, $6x^{-3}$ f $-2x^{-3} - 3x^{-4}$, $6x^{-4} + 12x^{-5}$
11 $\left(\frac{3}{2}, -\frac{7}{2}\right)$ **12** (1, 6) max; (2, 5) min.
13 2, min.; 29, max. **14** 10, max.; 6, min.
15 $-3\frac{1}{3}$, min.; $1\frac{1}{6}$, max. **16** $7\frac{4}{27}$, max.; 7, min.
17 2, min.; -2, max. **18** 4, min.; -4, max.
19 3, max.; 7, min. **20** -1, max.; 3, min.
21 c $a = 3, b = -45$ d minimum
22 c (4, 17) max., (16, 1) min.
23 a (2, -15) min., (-2, 17) max.
c $-15 \leqslant k \leqslant 17$
24 a (0, 0) min., (-1, 1) max.
b $0 < k < 1$

EXERCISE 4E *page 116*

1 36 cm^2 **2** 225 cm^2

3a. $h = \frac{16}{r^2}$ b. $A = 2\pi r^2 + 2\pi rh$ d. 24π cm^2

4. 54π cm^2 5. 32 cm

6. 40 m 7. 90 m

8a. $h = \frac{108}{x^2}$ c. 108 cm^2

9a. $60 - 2x$ b. $x(60 - 2x)$

c. $x = 15$ m, $A = 450$ m^2

10a. 12 c. 212 cm

11a. 5

12a. $h = \frac{192 - r^2}{2r}$ c. 512π cm^3

13b. 16 cm^3 14b. $2\sqrt{3}$ cm^2

15a. $5x - x^2$ b. $x = y = \frac{5}{2}$

16. 2

17a. $V = 108\pi h - \frac{1}{3}\pi h^3$ b. $432\pi\sqrt{3}$ cm^3

18 a. $V = \frac{4\pi}{3}x^2 + 16\pi x + \frac{256\pi}{3x}$ b. 144π cm^3

EXERCISE 4F *page 120*

1 a $x > 3$ b $x < 1$ and $x > 4$
c $1 < x < 4$
2 b $x > 2$ **3** b $x > 2$ and $x < 1$
4 a $3x^2 - 6x - 9$ c $x > 3$ and $x < -1$
5 $x < 2$
7 a $12x^2 - 2x - 4$ b $-\frac{1}{2} < x < \frac{2}{3}$

EXERCISE 4F
(alternative answers for EDEXCEL board)

1 a $x \geqslant 3$ b $x \leqslant 1$ and $x \geqslant 4$
c $1 \leqslant x \leqslant 4$
2 b $x \geqslant 2$ **3** b $x \geqslant 2$ and $x \leqslant 1$
4 a $3x^2 - 6x - 9$ c $x \geqslant 3$ and $x \leqslant -1$
5 $x \leqslant 2$
7 a $12x^2 - 2x - 4$ b $-\frac{1}{2} \leqslant x \leqslant \frac{2}{3}$

REVIEW EXERCISE 4G *page 121*

1 $2x + 1$ **2** $2x - 4$
3 $4x - 1$ **4** $6x - 16$
5 $3x^2 - 10x$ **6** $3x^2 + 22x$
7 $2x + 6$ **8** $2x - 8$
9 $8x - 12$ **10** $4x^3 - 6x^2 + 2x$
11 $3x^2 + 20x + 25$ **12** $4x + 12$
13 $-3x^{-4}$ **14** $-4x^{-5}$

15 $-2x^{-3} + 2x$ **16** $-x^{-2} + 3x^2$

17 $-\frac{5}{x^6}$ **18** $-\frac{4}{x^5}$

19 $-\frac{6}{x^3}$ **20** $-\frac{12}{x^4}$

21 $-\frac{3}{x^2} - \frac{4}{x^3}$ **22** $-\frac{4}{x^2} + \frac{9}{x^4}$

23 $3x^2 - \frac{3}{x^4}$ **24** $2x + \frac{2}{x^3}$

25 $2x - \frac{2}{x^3}$ **26** $3x^2 + 1$

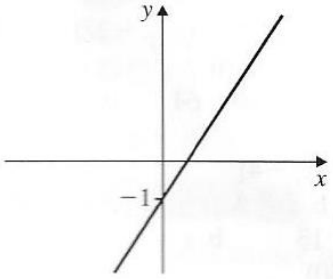
27 $2x$ **28** $-\frac{1}{x^2} - \frac{2}{x^3}$

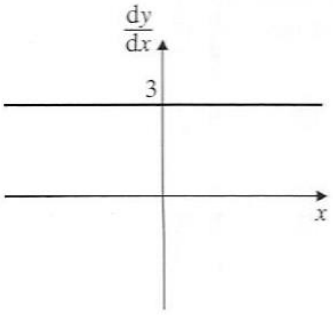
29 $2x - \frac{2}{x^3}$ **30** $8x - \frac{2}{x^3}$

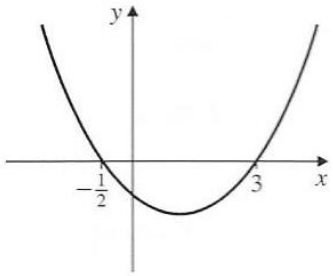
33 13 **34** 5 **35** 2 **36** -3

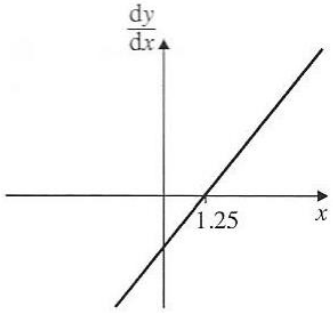
37 $\frac{3}{4}$ **38** -7 **39** -3

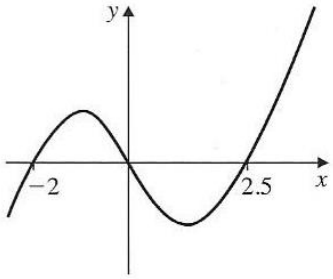
40 a

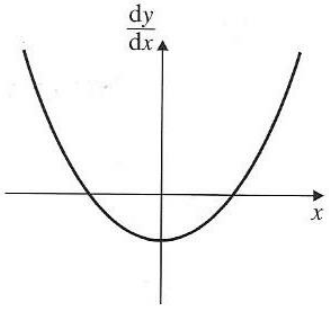












41 $-7, 7$ **42** 2

43 $(3, -2)$ **44** $-1, 1$

45 $2; \frac{5}{2}$ **46** 5

47 $(\frac{1}{2}, 4), (-\frac{1}{2}, -4)$ **48** $(1, 4)$

49 b $(5, 21)$ **c** $y = 9x - 24$

50 a $3x^2 - 18x + 17$ **b** $y = 2x + 2$

51 $(2, 68), (\frac{4}{3}, \frac{369}{4})$ **52** $(1, 6), (2, 5)$

53 b $y = 9x - 6$ **c** $(-\frac{7}{3}, -27)$

54 a $1, -6, 5, 1, 4, -4, -16$

b $6y = x + 30, 4y = x - 64$

55 $(3, 9), (-3, 9)$

56 $(2\sqrt{2}, -4), (-2\sqrt{2}, -4)$

57 a $2x - 6$ **b** $x = 3$ **c** $(3, -8)$

58 a $3x^2 - 18x + 15$ **b** $x = 1, 5$

c $(1, 2), (5, -30)$

59 a $(-3, -10)$ **b** $(2, -10)$

c $(1, 2), (2, 1)$ **d** $(-1, 0), (2, -27)$

60 b $(5, 21)$ **c** $y = 9x - 24$

61 a $3x^2 - 18x + 17$ **b** $y = 2x + 2$

c $(5, -20)$

62 a $9x^2 - 30x + 66$ **b** $(2, 68), (\frac{4}{3}, \frac{364}{9})$

63 a $6x^2 - 18x + 12$ **b** **(1, 6) max. (2, 5) min.**

64 b $y = 9x - 6$ **c** $(-\frac{7}{3}, -27)$

65 $(-1, -1)$

66 c a = 21, b = 23

67 a $(1, -1)$ min. $(-1, 3)$ max.

68 b $(1, 2)$

69 c $(32, \frac{1}{64})$ **d** max.

70 a $(16, -32)$ min.

71 a $(4, 4)$

73 b $(1, 0)$

75 a $a = 9$ **c** $\left(3, \frac{8\sqrt{3}}{3}\right)$

76 b $x = 1, 2$ **b** $x < 1, x > 2$

Note: For Edexcel, answer to 76b is $x \leq 1, x \geq 2$

77 a $L = 16x + 4h$ **b** $h = 24 - 4x$

d $x = 4$, $V_{max} = 384\ \text{cm}^3$

78 $x = 10$, $V_{max} = 1333\frac{1}{3}\ \text{cm}^3$

79 $a = 2$, $b = -15$

Examination Exercise 4 *page 127*

1 $12x^2 + x^{-\frac{1}{2}}$
2 $6x + 6$
3 i $18x^2 - 2x^{-\frac{3}{2}} + 5$
ii $36x + 3x^{-\frac{5}{2}}$
4 i $-10x^{-6}$ **ii** $\frac{1}{3}x^{-\frac{2}{3}}$
5 i $-12x^{-3} + 2$ **ii** $36x^{-4}$
6 i $-10x^{-3} + \frac{1}{4}x^{-2} + 1$
ii $30x^{-4} - \frac{1}{2}x^{-3}$
9 a $x - \frac{3}{2} + 6x^{-2}$
b $7x - 2y + 27 = 0$
10 $y = 2x - 9$
11 $2x + 8y - 37 = 0$
12 $4x - 6y - 29 = 0$
13 $16x - 24y + 9 = 0$
14 a $-2x^{-3} + 4$
b $x + 6y + 19 = 0$
c $y = -12x + 12$
15 i $(1.3, -4.3)$, $(-1.3, 4.3)$
ii $(-\sqrt{5}, 0)$, $(0, 0)$, $(\sqrt{5}, 0)$
iii $y = -2x - 2$, $x = -2$
16 $(3, 12)$, $(-3, -12)$
17 iii $y = 4x + 23$
iv Yes
18 a ii $\frac{4}{3} < x < 2$ **[Edexcel: $\frac{4}{3} \leq x \leq 2$]**
b ii R(43, 3)
19 $B(\frac{9}{2}, -2)$
20 a $6 - 3x^{\frac{1}{2}}$
b i $(4, 8)$ **ii** $x = 4$
c i $8x + 12y = 99$ **ii** $(4, \frac{67}{12})$
21 i $4x + y - 32 = 0$ **ii** $(6.5, 6)$
iii $x = 3$ **iv** $3 > x$
22 a $\frac{3}{4}t^2 - 3$
b i $-2.25\ \text{m}^3\text{s}^{-1}$ **ii** decreasing
c i $t = 2$ **ii** minimum
23 a i $5x^4 - 6x + 1$
ii $20x^3 - 6$
b $y = 12x + 12$
24 $x < -\sqrt{2}, x > \sqrt{2}$ **[Edexcel: $x \leq -\sqrt{2},\ x \geq \sqrt{2}$]**
25 $x < -4, x > 1$ **[Edexcel: $x \leq -4,\ x \geq 1$]**
26 a $12x^{-1} + x^{\frac{3}{2}}$
b i $-12x^{-2} + \frac{3}{2}x^{\frac{1}{2}}$
ii $4x + 9y - 115 = 0$
iii $k = \frac{6}{5}$
27 i $(-1, 7)$
ii Minimum
28 i $(-2, -48)$ **ii** Minimum **iii** $x > -2$
29 a P(4, −28) **b** Minimum
30 b $x = -\sqrt{2}$ **c** $-48x^{-5}$
d P is a maximum, Q is a minimum
31 i $4x^3 - 16x + 60$
iv 92 **v** Minimum
32 a $18 + 6x - 12x^2$ **b** $x = \frac{3}{2}$
c i −30 **ii** Maximum
33 i $a = 13$ **ii** Minimum **iii** $x = \frac{1}{3}$
34 a $\frac{1}{2}t^3 - 2t$
b i $-\frac{3}{2}\text{m/s}$ **ii** decreasing
c i 4 **ii** Minimum
35 i $k = -5$ **ii** Minimum **iii** A(−2, −27)
36 i $(1, 9)$ **ii** −11
37 a i $h = \dfrac{48 - r^2}{2r}$
b i $24\pi - \dfrac{3\pi}{2}r^2$
ii $r = 4$, maximum
38 a ii $V = 12x - \frac{9}{4}x^3$ **b** $12 - \frac{27}{4}x^2$
c ii $-\frac{27}{2}x$, maximum
39 c P = 8 **d** $y = 21\,\text{cm}$
40 b $1737\,\text{cm}^3$

Exercise 5A *page 137*

1 $x^2 + 2x + 1$ **2** $x^2 + 4x + 3$ **3** $2x^2 + x - 1$
4 $x^2 + 3x - 2$ **5** $x^2 + 1$ **6** $x^3 - 7$

Exercise 5B *page 139*

1 a 18 **b** 1 **c** 2 **d** −12
e -8 **f** 12 **g** -8 **h** 1
2 a A = 1, B = 5 **b** A = 2, B = 3
c A = 3, B = 2 **d** A = 3, B = 1
e A = 3, B = 2 **f** A = 3, B = −1
g A = 1, B = 2 **h** A = 3, B = −1
3 a 2, 11, 5 **b** 1, 5, 6
c 1, 2, 1 **d** 2, 3, 2
e 4, −1, 3
4 a $(x + 1)(x - 3)(x + 2)$
b $(x - 1)(x + 3)(x - 2)$
c $(x - 1)(x + 2)(x - 1)$
d $x(x + 3)(x - 2)$
e $(x - 1)(x^2 + 2x + 2)$
f $(x - 2)(x^2 + 3x + 4)$
g $(x - 3)(x^2 - 2x - 1)$
h $(x - 1)(x + 1)(x + 4)$
i $(x + 3)(2x^2 - 4x + 1)$
j $(x + 2)(4x^2 + 2x - 3)$
5 a −4 **b** 4 **c** −4 **d** −1
6 a $a = -9, b = 13$ **b** $a = 3, b = -7$
c $a = -34$, b = 50 **d** $a = 4, b = -5$
e $a = 2, b = -2$
7 $a = 19$, f$(x) = (x + 4)(x + 3)(x + 1)$
8 $(x + 1)(x^2 + 2x - 1)$
9 $x = 2, 3, -1$
10 a $1, \frac{2}{5}, 1\frac{1}{2}$ **b** $2, \frac{1}{3}, \frac{3}{4}$
c $0, \frac{1}{2}, 2\frac{1}{2}$ **d** $0, -1, 1$
11 a $-3, -\frac{1}{2}, -\frac{1}{3}$ **b** $1, \frac{2}{5}, -\frac{3}{2}$
c $-2, \frac{1}{2}, \frac{4}{3}$ **d** $0, -\frac{3}{2}, \frac{3}{2}$
e $-3, \frac{2}{3}, \frac{5}{2}$ **f** $2, \frac{1}{5}, \frac{2}{3}$

Exercise 5C *page 141*

1 a $(x - 1)(x^2 + 3x + 1)$
b $(x + 2)(2x^2 + 4x + 3)$
c $(x - 3)(3x - 1)(2x + 1)$
d $(x + 1)(5x - 2)(3x - 4)$
e $(x - 2)(4x - 3)(2x - 5)$
f $x(2x - 1)(3x - 2)$
g $x(x - 2)^2$ **h** $(x + 1)^2(2x - 3)$
2 a $a = -9, b = 10$ **b** $a = -8, b = 17$
c $a = 3, b = -8$ **d** $a = 12, b = 1$
3 c $\frac{1}{2}, \frac{3}{4}, -\frac{1}{3}$
4 $a = 0, b = -7, c = -6$
5 $a = 6, b = 3, c = -15, d = 6$
6 a $y = (x + 4)(x - 1)(x - 2)$
b $y = (x - 1)(x + 1)(x - 3)$
c $y = -(x + 1)(x - 2)(x - 3)$
d $y = 2(x - 1)^2(x + 2)$
e $y = (x - 1)(x - 3)(x - 5)$
f $y = (x + 2)^2(x - 1)$

Exercise 5D *page 145*

1 a 6 **b** 5 **c** 15
2 a 6 **b** 18 **c** 14
d 41 **e** 6 **f** 14
3 $a = 2$ **4** $b = -10$
5 $c = 2$ **6** 35
7 $a = 1, 3\frac{1}{8}$ **8** $a = 2, b = 7$
9 $a = -2$ **10** $a = 3, b = 8$
11 $a = 1$ **12** $a = 4, b = -1$
13 b $a = 6, b = -5$
14 b $a = 6, b = -17$ **d** $\frac{3}{4}, -2, \frac{1}{2}$
15 b $a = 6, b = -4$
d f$(x) = (2x - 1)(3x - 1)(x + 1)$
16 b f$(x) = (2x - 1)(x^2 + x + 1) + 2$
17 a $a = -3$ **b** $(x - 1)(3x^2 + 2) + 7$
18 a $(x - 1)(x^2 + 2x + 3) + 1$
b $(x - 2)(x^2 - x + 5) + 10$
c $(x + 1)(2x^2 + x + 1) + 5$
19 a $a = -10$ **b** $b > -4$, but $b \neq 14$
20 a, b $p = 6, q = 10$

Review Exercise 5E *page 147*

1 a $x^2 + 2x - 5$ remainder 16
b $x^2 - 3x + 6$ remainder −2
c $x^2 - x + 2$ remainder 0
2 a $(x - 2)(x^2 + 3x + 4)$
b $(x - 2)(x + 1)^2$
c $(x + 2)(x + 3)(x - 3)$
3 $a = -3$ **4** $a = 5$
5 $2, -4, \frac{1}{2}$ **6** $x = 3$
7 a 0 **c** $x > 1$ and $-2 < x < 1$
8 6
9 a −2 **b** −43 **c** −5 **d** $\frac{4}{9}$
10 $a = 2$, remainder = −12
11 a quotient $= x^2 + 4x + 13$, remainder = 25
b g$(x) = x^2 + 4x + 13, R = 25$
12 −5
13 b $(x + 2)(x + 3)(x - 3)$ **c** −2, +3
14 b $(x + 2)(x + 1)(x - 2)$
c 2 or −1 (repeated)
d −1 or $\pm\sqrt{5}$

Examination Exercise 5 *page 148*

1 b $(x - 2)(2x + 1)(x - 2)$
2 ii $(x + 3)(x^2 - 3x + 5)$
3 i $(x - 1)(x^3 + x + 10)$ **ii** $x = -2$
iii $(x - 1)(x + 2)(x^2 - 2x + 5)$
4 i $(x - 2)$ **ii** $x = 2, x = \frac{-3 \pm \sqrt{29}}{2}$
5 a $a = 8$
b $(x - 2)(2x + 3)(3 - 2x)$
c −12
6 i $(x + 3)(2x - 1)(x - 4)$
ii $x = -3, 0, 4.5$
7 a 16
b ii $(x - 3)(x + 2)(x - 3)$
8 b i 36
ii $(x + 2)(x^2 - 5x + 10)$
iii $x = -2$

9 $k = 3$
10 $k = 2, x = -1$
11 **b** $3c + d + 5 = 0$ **c** $c = -14, d = 37$
12 **b** $a = 9, b = -6$
13 **i** $b = 3$ **ii** $(x - 2)$ and $(x + 3)$
14 **i** 36
ii $(x - 2)(2x^2 + 7x - 3)$
iii 3 real roots
15 **b** **i** -12 **ii** $(x - 3)(x^2 - x + 1)$
c $x = 3$
16 **b** $(x - 3)(2x - 3)(x + 2)$

Exercise 6A *page 153*

1 **a** $\frac{x^3}{3} + c$ **b** $\frac{x^5}{5} + c$
c $\frac{x^8}{8} + c$ **d** $\frac{x^2}{2} + c$
e $2x^2 + c$ **f** $2x^3 + c$
g $\frac{x^3}{3} + c$ **h** $5x^2 + c$
i $7x + c$ **j** $x^5 + c$
k $x^2 + x + c$ **l** $2x^2 - 3x + c$
m $-\frac{x^2}{2} + c$ **n** $-\frac{x^3}{3} + c$
o $4x - \frac{x^3}{3} + c$ **p** $x^2 - \frac{x^4}{4} + c$

2 **a** $\frac{x^2}{2} + c$ **b** $2x^2 + 5x + c$
c $\frac{x^3}{3} - x + c$ **d** $6x + c$
e $2x^2 - x^3 + c$ **f** $\frac{x^3}{3} + 3x^2 + 7x + c$

3 **a** $x^3 + c$ **b** $2x^4 + c$
c $2x^5 + c$ **d** $\frac{x^4}{2} + x^2 + c$
e $\frac{x^8}{4} + 2x^3 + c$ **f** $\frac{x^{10}}{2} + x + c$
g $\frac{2x^3}{3} + \frac{x^2}{2} + c$ **h** $\frac{3x^5}{5} + \frac{x^4}{4} + c$
i $-4x^{-1} + c$ **j** $-3x^{-2} + c$
k $-2x^{-2} - x^{-5} + c$ **l** $-\frac{3}{x} - \frac{5}{2x^2} + c$
m $\frac{x^3}{3} - \frac{1}{x} + c$ **n** $-\frac{3}{x} + \frac{x^3}{9} + c$
o $\frac{x^5}{5} + x + c$

4 $y = x^2 + 3x + c$
5 $A = t^2 - 2t + c$
6 $S = t^3 - 3t + c$
7 **a** False **b** True **c** False

8 **a** $\frac{2}{5}x^{1.5} + c$ **b** $\frac{10}{23}x^{2.3} + c$
c $\frac{5}{31}x^{6.2} + c$ **d** $\frac{3}{4}x^{\frac{4}{3}} + c$
e $\frac{10}{11}x^{\frac{11}{10}} + c$ **f** $\frac{3}{8}x^{\frac{8}{3}} + c$

9 **a** $2x^{\frac{3}{2}} + c$ **b** $6x^{\frac{4}{3}} + c$ **c** $\frac{4}{3}x^{\frac{3}{2}} + c$
d $\frac{2}{3}x^{\frac{3}{2}} + 2x + c$ **e** $2x^{\frac{5}{2}} + c$ **f** $\frac{12}{5}x^{\frac{5}{2}} + c$
g $\frac{3}{4}x^{\frac{4}{3}} - 3x + c$ **h** $\frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$
i $\frac{x^2}{2} + \frac{4}{3}x^{\frac{3}{2}} + x + c$

11 **a** $2x^3 + 7x + c$ **b** $x^6 + x^2 + c$
c $-6x^{-1} - x^{-2} + c$ **d** $-\frac{2}{x} - \frac{5}{2x^2} + c$
e $x + \frac{4}{x} + c$ **f** $-\frac{4}{x} - \frac{2}{x^2} - \frac{1}{3x^3} + c$

12 **a** $\frac{3}{4}x^{\frac{4}{3}} + c$ **b** $-\frac{1}{2x} - \frac{1}{x^2} + c$
c $\frac{x^2}{2} - x + c$ **d** $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$
e $\frac{3}{5}x^{\frac{5}{3}} + c$ **f** $x + \frac{1}{x} + c$
g $\frac{3}{5}x^{\frac{5}{3}} - \frac{3}{2}x^{\frac{2}{3}} + c$ **h** $\frac{x^2}{6} + \frac{2}{3x} + c$
i $\frac{1}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + c$

Exercise 6B *page 156*

1 **a** $y = x^2 + 3x + 1$
b $y = 2\sqrt{x} + 2x^2 - 36$
c $y = 2x^3 + x^2 + 3x - 16$
2 $y = x^2 + 3x - 1$
3 $y = x^3 + 4x^2 - 3x + 10$
4 $A = \frac{t^3}{3} - \frac{t^2}{2} + 10$
5 **a** $y = 2x^2 - x - 2$ **b** yes
6 **a** $y = 3 + 5x - 2x^2$ **b** $(3, 0)$ and $(-\frac{1}{2}, 0)$
7 **a** $y = 12 + x - x^2$ **b** $(-3, 0)$
8 $y = \frac{\pi}{2}x^2 + \frac{x}{2} + \pi$
9 $y = x^2 - 6x + 10$
10 $y = 2x^2 - 3x + 4$
11 $y = 4x^2 + x - 10$
12 **a** $A = 6$ **b** $y = 1 + 3x^2 - 2x^3$
13 $y = x^3 + 2x^2 - x + 1$
14 $y = x^3 - x^2 + 2x + 3$
15 $y = 2x\sqrt{x} + 8\sqrt{x} + 10$

Exercise 6C *page 158*

1 **a** 6 **b** 26 **c** $\frac{5}{6}$
d 34 **e** 18 **f** $49\frac{3}{5}$
4 **a** 52 **b** 6 **c** $-\frac{3}{8}$
d 15 **e** 24 **f** 2
g 52 **h** 45 **i** $\frac{1}{6}$
j -12 **k** $\frac{1}{6}$ **l** $-\frac{5}{72}$
m $-\frac{11}{24}$ **n** $-\frac{5}{12}$ **o** $30\frac{4}{5}$
5 12 **6** -7 **7** 4 **8** 4

Exercise 6D *page 160*

1 $9u^2$ **2** $45\frac{1}{3}u^2$
3 $3\frac{3}{4}u^2$ **4** $10\frac{2}{3}u^2$
5 $\frac{2}{3}u^2$
6 **b** 0
c areas above and below axis cancel
7 **b** $1\frac{1}{3}u^2$

Note
u^2 stands for 'units squared' - OPTIONAL

Note
u^2 stands for 'units squared' - OPTIONAL

8 $\frac{3}{4}u^2$
9 **a** 1, 5 **b** $10\frac{2}{3}u^2$
10 **a** $-3, 1$ **b** $10\frac{2}{3}u^2$
11 **a** $-2, 3$ **b** $20\frac{5}{6}u^2$
12 **a** $x = 1, 2$ **b** area $\frac{1}{6}$
13 **a** $x = -3, -1$ **b** $1\frac{1}{3}u^2$
14 **a** $-11\frac{1}{3}$ **c** area is below x-axis
15 **a** $-4, 4, 0$ 16 $2\frac{2}{3}u^2$

Exercise 6E *page 163*

1 $4\frac{1}{2}$ 2 $4\frac{1}{2}$
3 $10\frac{2}{3}$ 4 $\frac{1}{48}$
5 **a** $y = 8x - 16$
6 **b** $\frac{7}{2}$ **d** $2\frac{29}{48}$ (=2.60 to 2 d.p.)
7 **a** $(x-1)(x-2)(x-3)$ **c** $\frac{1}{2}$
8 **a** $-1, 2, 3$ **c** $11\frac{5}{6}$
9 $21\frac{1}{12}$
10 **a** $y = -x + 5$ **b** $(3, 2)$ **d** $1\frac{1}{3}$

Review Exercise 6F *page 164*

1 $\frac{x^4}{4} + c$ 2 $\frac{x^3}{3} + c$
3 $\frac{x^5}{5} + \frac{x^2}{2} + c$ 4 $\frac{x^6}{6} + 3x + c$
5 $\frac{x^3}{3} - 4x + c$ 6 $\frac{x^7}{7} + \frac{3x^2}{2} + c$
7 $\frac{x^4}{2} - x^3 + c$ 8 $\frac{x^5}{5} - \frac{x^2}{2} + c$
9 $\frac{x^3}{6} - 7x + c$ 10 $\frac{x^2}{6} + \frac{1}{4}x + c$
11 $\frac{3}{5}x^5 - 3x^2 + c$ 12 $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + c$
13 $x - \frac{7}{2}x^2 + c$ 14 $\frac{x^3}{3} + \frac{3}{2}x^2 + 2x + c$
15 $\frac{2}{3}x^3 + \frac{5}{2}x^2 - 3x + c$ 16 $\frac{x^4}{4} - x^3 + c$
17 $\frac{x^4}{4} - \frac{x^2}{2} + c$ 18 $\frac{4}{3}x^3 + 2x^2 + x + c$
19 $-x^{-1} + c$ 20 $-\frac{1}{2}x^{-2} + c$
21 $\frac{x^3}{3} + 4x + c$ 22 $x + x^2 + c$
23 $\frac{x^4}{4} + x^2 + c$ 24 $5x - x^2 + \frac{x^3}{3} + c$
25 $\frac{x^2}{4} + \frac{x}{3} + c$ 26 $\frac{\pi}{2}x^2 + 10x + c$
27 $x^3 + \frac{x^2}{8} + c$ 28 $33x^3 - 99x + c$
29 $-\frac{1}{x} + c$ 30 $\frac{2}{3}x^{\frac{3}{2}} + c$
31 $\pi x^2 + c$ 32 $\frac{x^{101}}{101} + c$
33 $\frac{x^3}{3} + \frac{x^2}{2} + c$ 34 $x^3 + 6x^2 + c$
35 $\frac{x^3}{3} - 9x + c$ 36 $y = x^2 - 3$
37 $y = x^2 + 4x - 2$ 38 $y = x^3 - x + 10$
39 $y = x^3 + 3x^2 + 10$
40 $y = x^2 - 3x + 4$ 41 $y = x^2 + 2x - 15$
42 $y = 2x^2 - 7x + 1$ 43 $y = 3x^2 - 4x - 10$
44 $y = x^3 - 2x + 1$ 45 $y = 2x^3 + 5$
46 $v = t^2 + 5t - 3$ 47 $z = y^3 - 4y + 1$
48 11 49 4
50 $2x^{\frac{3}{2}}$ 51 $\frac{4}{5}x^{\frac{5}{4}}$
52 $\frac{2}{3}x^{\frac{3}{2}} + 5x + c$ 53 $\frac{2}{5}x^{\frac{5}{2}}$
54 $\frac{2}{3}x^{\frac{3}{2}}$ 55 $\frac{x^2}{2} - \frac{4}{3}x^{\frac{3}{2}} + x$
56 $\frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}}$ 57 $x + 9x$
58 $-\frac{5}{x} - \frac{1}{2x^2}$ 61 $\frac{1}{x} - \frac{2}{\sqrt{x}}$
62 **a** $a = 3$ **b** $y = x^3 - 3x^2 + 5$
63 6 64 44 65 15 66 -12
67 8 68 1 69 21 70 $\frac{1}{2}$
71 $8\frac{2}{3}$ square units 72 6 73 9
74 36 75 $20\frac{5}{6}$ 76 $20\frac{5}{6}$ 77 $1\frac{1}{3}$ 78 $1\frac{1}{3}$
79 **b** $\left(-\frac{9}{2}, -\frac{7}{4}\right)$

Examination Exercise 6 *page 166*

1 $2x^{\frac{7}{2}} + c$
2 $2x^3 - 2x^{-1} + 5x + c$
3 **a** $2x^3 - 5x^2 - 12x$
4 **a** $p = \frac{1}{2}, q = 2$ **b** $y = 4x^{\frac{3}{2}} + x^3 - 6$
5 $y = -9x^{-2} + 2x + 1$
6 2.5
7 **a** $\frac{2}{3}x^{\frac{3}{2}} + 18x^{\frac{1}{2}} - 72$ **b** $x = 1, x = 81$
8 **a** $y = 9x - 5$
b $y = 2x^5 - 2x^3 + 5x - 1$
9 **i** $k = \frac{3}{2}$ **ii** $y = x^3 - \frac{3}{4}x^2 + 2$
10 $y = 4x^{\frac{3}{2}} - 7x - 3$
11 29
12 **a** $\frac{1}{3}x^3 + 6x^{\frac{1}{2}} + c$
b i $\frac{1}{4} - \frac{2}{a^3}$ **ii** $\frac{1}{4}$
13 **a** $\frac{5}{4}x^4 - 3x^2 + x + c$
b i $-12x^{-2} + c$ **ii** $a = 2$
14 $\frac{194}{3}$
15 **i** $\frac{1}{2}x^2 + 3x - \frac{8}{3}x^{-3} + c$ **ii** $\frac{41}{6}$
16 **a** $4x^{\frac{5}{2}} - 10x^2 + c$ **b** 226
17 **i** B(4, 9) **ii** $\frac{1}{3}x^3 - \frac{3}{2}x^2 + 5x + c$
iii $\frac{32}{3}$
18 **i** $\frac{369}{10}$ **ii** $\frac{189}{10}$
19 **a ii** A$(-2, 5)$, C$(3, 10)$
b $\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{5}{2}x^2 + 7x + c$ **c** $\frac{16}{3}$
20 **i** 8 **ii** 6
21 **a** L$(1, 0)$, M$(4, 0)$ **c** $\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + c$
d $\frac{37}{6}$
22 **ii** $\frac{37}{30}$
23 **a** A$(2, 8)$, B$(9, 1)$ **b** $\frac{343}{6}$
24 **a** 0.0197 **b** 24.8 m^3

Exercise 7A *page 173*

3 a $x = 68.3°$, $y = 62.7°$, $z = 9.18$
b $x = 51.0°$, $y = 10.3°$, $z = 6.22$
c $x = 23.3°$, $y = 56.7°$, $z = 73.8$
d $x = 66.0°$, $y = 2.74$, $z = 3.18$
4 50.6° **5** 39.1° **6** 8.52

Exercise 7B *page 176*

1 a 8.03 cm **b** 56.6 cm
c 20.6 cm **d** 12.8 cm
2 a 45.6° **b** 98.1°
c 72.0° **d** 75.7°
3 98.9° (or 81.1°) **4** 11.9 cm
5 3.28 cm **6** 112.5°
7 4.58 km **8** 16.5 km, 057°
9 c 4

Exercise 7C *page 178*

2 49.0 km
3 a 158 m **b** 88 m
4 a 10.4 cm **b** 24.0 mm
c 18.8° **d** 26.7°
e $e = 61.3°$, $f = 36.7°$, $g = 21.1$ cm
5 b $x^2 + 2x - 35 = 0$ **c** 5
6 a 15.5 km **b** 4.82 km^2
7 a 23.0 m **b** 69.3°
8 c $2\frac{6}{7}$ **d** 44.4°
9 64.1° or 115.9°

Exercise 7D *page 180*

1 a 29.7 cm^2 **b** 13.8 cm^2 **c** 8.36 cm^2
2 43.3 cm^2 **3** 38.5 cm^2 **4** 166 cm^2
5 10.7 cm **6** 22.1 cm **7** 9.66 cm
8 6.71 cm^2 **9** 9.34 cm

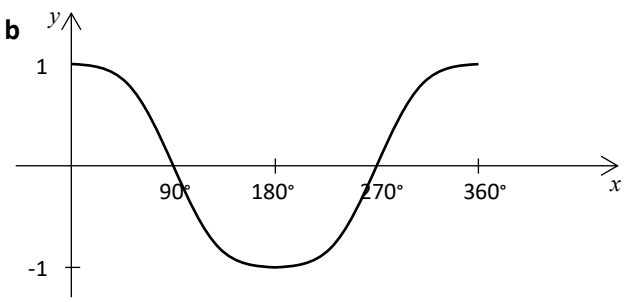
Exercise 7E *page 182*

1 a

x	0°	30°	60°	90°	120°	150°	180°
y	1	0.866	0.5	0	-0.5	-0.866	-1

x	210°	240°	270°	300°	330°	360°
y	-0.866	-0.5	0	0.5	0.866	1

b

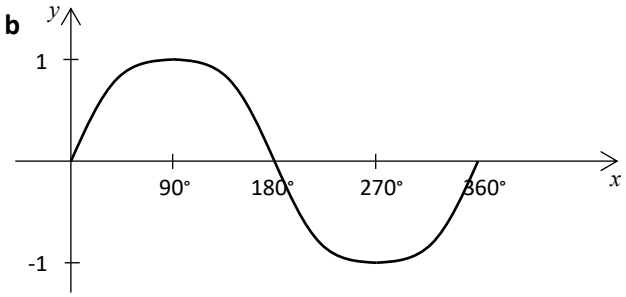


2 a

x	0°	30°	60°	90°	120°	150°	180°
y	0	0.5	0.866	1	0.866	0.5	0

x	210°	240°	270°	300°	330°	360°
y	-0.5	-0.866	1	-0.866	-0.5	0

b



3 a

x	0°	30°	60°	90°	120°	150°	180°
y	0	0.577	1.73	-	-1.73	-0.577	0

x	210°	240°	270°	300°	330°	360°
y	0.577	1.73	-	-1.73	-0.577	0

90° and 270° omitted because tan 90° and tan 270° have no values

b graph shown in textbook

c tan 89° = 57.2…, tan 89.9° = 572.9… etc.

4 $\sin 70° = \sin 110°$, $\cos 60° = \cos 300°$, $\sin 50° = \sin 130°$
5 $\tan 45° = \tan 225°$, $\sin 180° = \cos 270°$, $\cos 30° = \cos 330°$, $\tan 60° = \tan 240°$
6 162° **7** 153°
8 a 140° **b** 110° **c** 50°
9 290° **10** 315°
11 a 350° **b** 304° **c** 60°
12 220° **13** 160° **14** 82°
15 315° **16** 240° **17** 250°
18 $\sin 270° = \cos 180°$, $\tan 200° = \tan 20°$, $\sin 90° = \cos 360°$, $\tan 150° = \tan 330°$

Exercise 7F *page 185*

1 58.0°, 122.0° **2** 20.5°, 159.5°
3 53.1°, 306.9°
4 45°, 225°
5 a 19.8°, 160.2° **b** 72°, 108°
c 30°, 150° **d** 60°, 120°

6 a 46.1°, 133.9° **b** 72.5°, 287.5°
c 78.7°, 258.7° **d** 220.5°, 319.5°
8 a 30°, 150° **b** 53.1°, −53.1°
c 45°, −135° **d** 116.6°, 296.6°
e 166.0°, 346.0°
10 a 60°, 300° **b** 60°, 240°
c 60°, 120° **d** 30°, 210°
e 45°, 315° **f** 45°, 135°
11 30°, 150°, 210°, 330°
12 45°, 135°, 225°, 315°
13 a 90°, 450°, 810°, etc.
b 90°, 270°, 450°, 630°, etc.
14 a 0 **b** 1
15 a $0 < x < 180°$ **b** $90° < x < 270°$
16 a $a + b$ **b** $a - b$
c $b = 3$ **d** 30°, 150°

Exercise 7G *page 188*

1 110° **2** 26.9°, 313.1°
3 a −90°, 30° **b** 30°, 270°
c 90°, 270° **d** −135°, −45°
4 b 8.7°, 81.3°, 188.7°, 261.3°
5 a 112.5°, 202.5° **b** 30°, 60°
c 20°, 100° **d** 52.5°, 82.5°
6 a error **b** ok
c error **d** ok
7 a 20°, 140° **b** 0°, 180°, 360°
c 75°, 255° **d** 120°, 240°
e 30°, 150°, 210°, 330° **f** 170°, 350°
g 30°, 270° **h** 30°, 150°, 270°

Exercise 7H *page 190*

1 a 0, 2 **b** $0, \frac{1}{3}$ **c** −1, 1
d $-\frac{1}{2}, 3$ **e** −3, 2
2 a 0°, 30°, 150°, 180°, 360°
b 90°, 180°, 270°
c 0°, 180°, 360°, 45°, 225°
d 19.5°, 90°, 160.5°, 270°
e 0°, 180°, 360°, 60°, 300°
f 0°, 180°, 360°, 14.5°, 165.5°
g 75.5°, 284.5°, 90°, 270°
h 19.5°, 160.5°, 90°, 270°
3 a 19.5°, 160.5°, 90°
b 90°, 210°, 330°
c 60°, 300°
d 63.4°, 243.4°, 108.4°, 288.4°
e 30°, 150°, 270°
f 70.5°, 289.5°, 120°, 240°
g 148.3°, 211.7°
h 90°, 14.5°, 165.5°
4 max 4, min −6
5 a 7 **b** 270°
6 a 1, $\theta = 90°$ **b** 5, $\theta = 360°$
c 4, $\theta = 270°$ **d** 5, $\theta = 180°$
e 7, $\theta = 45°$ **f** 1, $\theta = 90°$
g 1, $\theta = 90°$ **h** 4, $\theta = 90°$
i 6, $\theta = 100°$

Exercise 7I *page 192*

1 a 1 **b** $\cos\theta$ **c** 5
d $4\cos^2 x$ **e** $3\sin x$ **f** $\sin x$

2 a 0°, 60°, 300°, 360°
b 19.5°, 90°, 160.5°
c 41.8°, 138.2°, 270°
d 0°, 14.5°, 165.5°, 180°, 360°

3 a 26.6°
b 80.5°, 260.5°
c ±90°, 45°, −135°
d 0°, 180°, 360°, 63.4°, 243.4°
e 20.5°, 159.5°

4 a $\frac{4}{5}$ **b** $\frac{4}{3}$

5 a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{2}}$

6 a $-\frac{12}{13}$ **b** $-\frac{5}{12}$

7 a $-\frac{4}{5}$ **b** $-\frac{3}{5}$

8 b 14.5°, 165.5° **c** $14.5° < x < 165.5°$

9 b $\theta = 63.4°,\ y = 0.894$
$\theta = 243.4°,\ y = -0.894$

10 a 19.5°, 160.5°
b 15°, 75°, 195°, 255°
c 90°, 270°, 18.4°, 198.4°
d 0°, 90°, 180°, 270°, 360°
e 0°, 180°, 360°

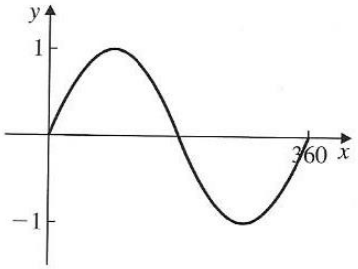
11 $\frac{p^2}{9} + \frac{q^2}{49} = 1$

12 $k = 3,\ \theta = 61.9°$

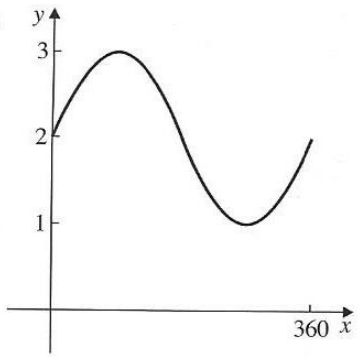
13 $x = 30°,\ y = 30°$ or $x = 30°,\ y = 90°$

Exercise 7J *page 196*

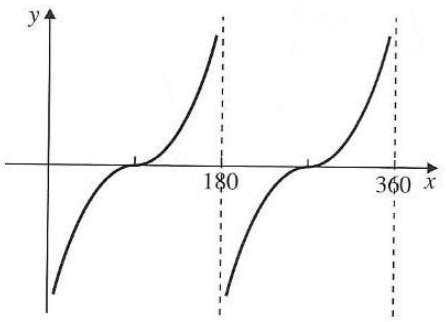
1 a



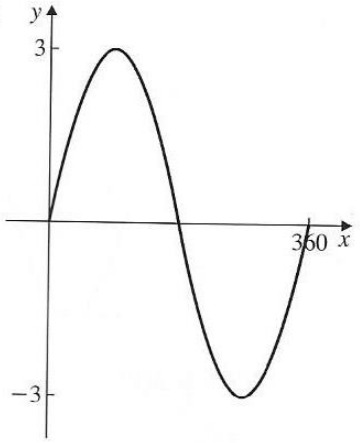
b



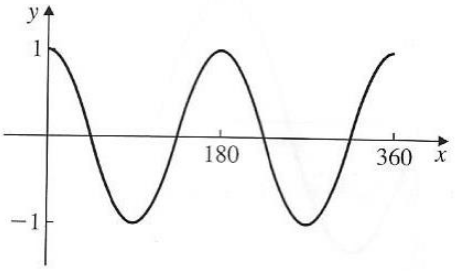
c



d

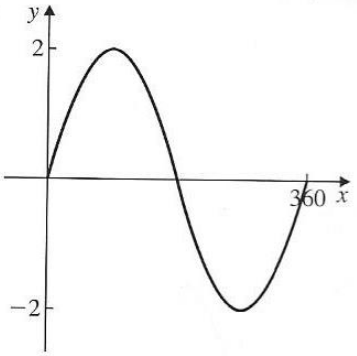


e

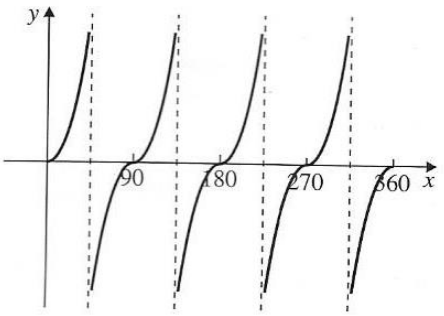


2 A is W, B is V, C is U, D is X, E is Z, F is Y

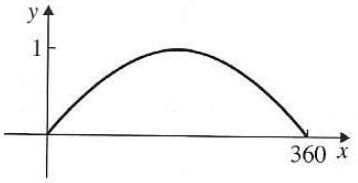
3 a



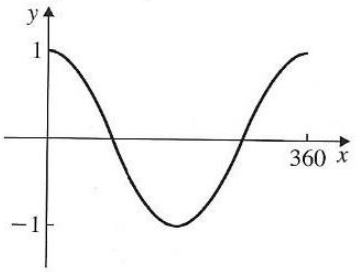
b $\tan 2x$



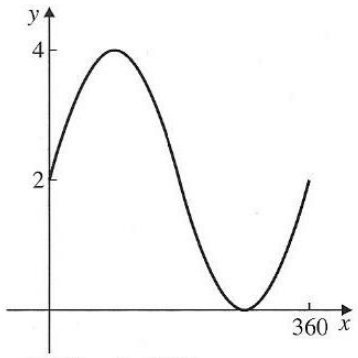
c $\sin\frac{1}{2}x$



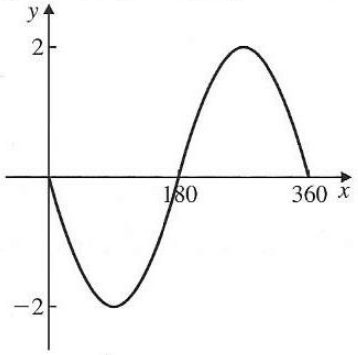
d $\sin(x + 90°)$



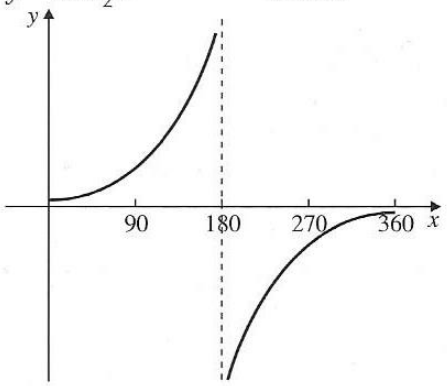
e $2 \sin x + 2$



4 a $y = 2 \sin (x + 180°)$ **b** 270°

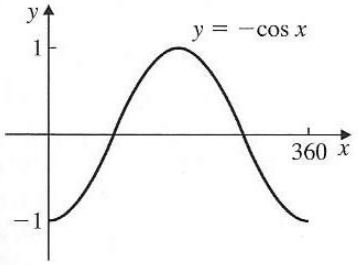


5 a $y = \tan \frac{1}{2}x$ **b** one

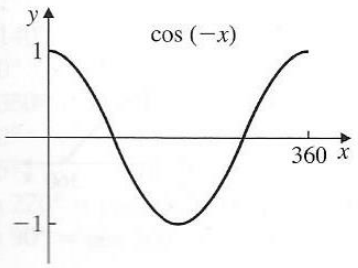


b one solution

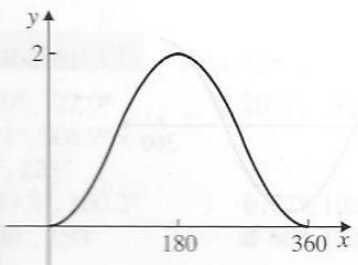
6 a



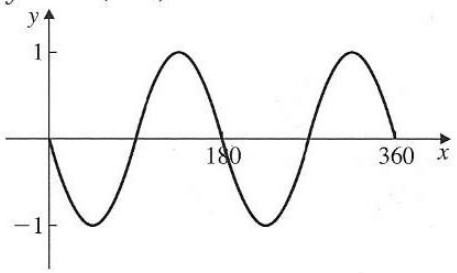
b



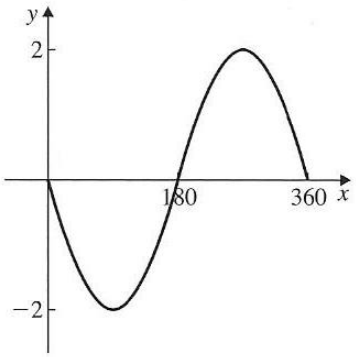
c



d $y = \sin (-2x)$



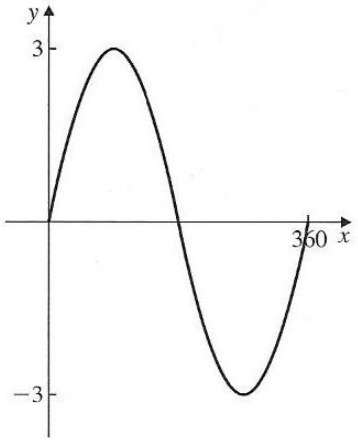
e $y = -2 \sin x$



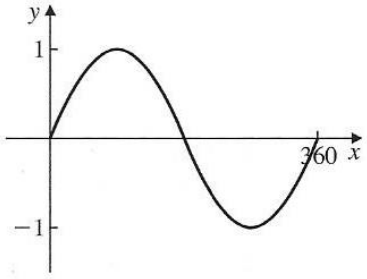
7 b 2 solutions **8 b** 2 solutions

Review Exercise 7K *page 197*

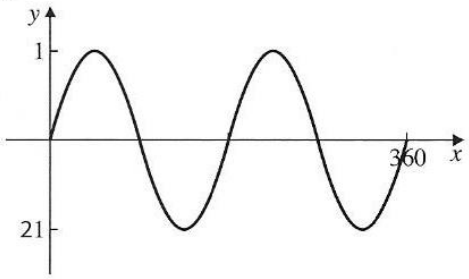
1 a 12.5 **b** 6.68
c 14.8 **d** 9.27
e 7.81 **f** 6.05
2 99.9° **3** 39.5°
4 137.8° **5** 39.1
6 a 046.5° **b** 45°, 095.0°
7 a 21.1 cm² **b** 26.5 cm²
8 26.3 cm **9** 153°, 513° etc.
10 $\sin 30° = \cos 60°$
$\cos 45° = \sin 45°$
$\tan 45° = \tan 225°$
$\cos 55° = \cos 305°$
11 a $\sqrt{3}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$
d $-\frac{1}{2}$ **e** -1 **f** $\frac{\sqrt{3}}{2}$
12 a 221.8°, 318.2° **b** 138.6°, 221.4°
c 145.0°, 325.0° **d** 210°, 330°
e 60°, 300° **f** −135°, 45°
g 135°, 225° **h** 60°, 120°
13 a 30°, 330° **b** 90°, 210°
c 75°, 195° **d** −90°, 90°
14 48.6°, 131.4°, 210°, 330°
15 a



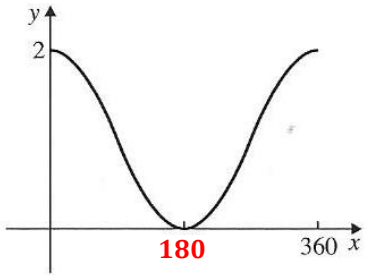
b $y = \cos(x - 90°)$



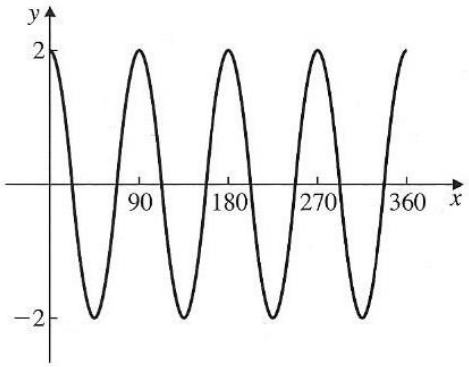
c $y = \sin 2x$



d $y = 1 + \cos x$



e $y = 2\cos 4x$



$6x^3y^6$

16 a 90° **b** 60°, 180°, 300°
c 30°, 150°, **−160.5°, −19.5°**
d **±120°, ±48.2°**
e −30°, −150°
17 a **−165°, −75°, 15°, 105°**
b 120°, 180, 300°
c 30°, 90°, 210°, 270°
18 a 100°, 160°, 280°, 340°
b 30°, 150°, 210°, 330°
c 48.6°, 131.4°, 210°, 330°
d 71.6°, 251.6°
e 0°, 180, 360°, 26.6°, 206.6°

Examination Exercise 7 *page 200*

1 i 11.6 cm **ii** 49.2 cm² **iii** 6.49 cm
2 i 36.3 cm² **ii** 9.82 cm **iii** 14.5 cm
3 a i $\frac{5\sqrt{29}}{29}$ **ii** $\frac{2\sqrt{10}}{7}$
b $-\frac{3\sqrt{3}}{8}$

4 10
5 $\cos\theta$
6 ii $x =$ **71.6°**, 117°, 252°, 297°
7 ii $\theta = 38.2°, 142°$
8 $\theta = 0°, 60°, 180°, 300°, 360°$
9 a $\frac{7}{2}$
b ii $x = 60°, 132°, 228°, 300°$
10 $\theta = 48.6°, 131°, 270°$
11 ii $x = 48.2°, 120°, 240°, 312°$
12 a $1, \pm\sqrt{3}$
b $\theta = 45°, 60°, 120°, 225°, 240°, 300°$
13 $x = 51°, 129°, 231°, 309°$
14 i $x = 9.22°, 99.2°$
ii $x = 0°, 41.8°, 138°, 180°$
15 $\theta = 9.4°, 80.3°$
16 $x = 93.2°, 356.8°$
17 i $x = -83.7°, 96.3°$
b $\theta = 72°, 288°$
18 a $x = 49°, 229°$
b translation 30 units in the negative x direction
19 i $\theta = 123.4°, 176.6°$
ii $x = 70.5°, 289.5°$
20 b $x = 0°, 39.2°, 90°, 140.8°, 180°$

Exercise 8A *page 206*

1 a $\log_5 125 = 3$ **b** $\log_2 32 = 5$
c $\log_{10} 10\,000 = 4$ **d** $\log_a 8 = 3$
e $\log_7 49 = 2$ **f** $\log_x 10 = n$

2 a 2	**b** 3	**c** 2
d 5	**e** 0	**f** −1
g −2	**h** $\frac{1}{2}$	**i** $-\frac{1}{2}$
j 3	**k** $\frac{3}{2}$	**l** 2

3 a $x = \log_{10} 7$ **b** $x = \log_5 3$ **c** $x = \log_8 2$

4 a log 21	**b** log 20	**c** log 64
d log 72	**e** log 121	**f** log 2
g log 8	**h** log 162	

5 a $3\log a + \log b$ **b** $\log b + 2\log c$
c $\frac{1}{2}\log a$ **d** $5\log c$
e $\log a + 2\log b - 3\log c$
f $4\log a + 3\log b - \frac{1}{2}\log c$

6 a 5	**b** 4	**c** −2
d 0	**e** 2	**f** 3
7 a 1.23	**b** 2.40	**c** 1.58
8 a 4.09	**b** 5.22	**c** 2.05
d 2.84	**e** 0.850	**f** 0.880
g 1.25	**h** 4.51	**i** 1.11
j −1.10	**k** **1.53**	

Exercise 8B *page 208*

1 a 5	**b** 2	**c** $\frac{1}{2}$
d 0	**e** $\frac{1}{2}$	**f** $\frac{1}{4}$
g −3	**h** $-\frac{1}{2}$	**i** $1\frac{1}{2}$

2 a $\log\left(\frac{x+4}{x}\right)$ **b** $\log\left(\frac{x+1}{x+2}\right)$
c $\log_3\left(\frac{(x+3)(x+5)}{(x+1)}\right)$ **d** $\log_7(3x+2)(x-3)$
e $\log_2(x-1)(x+2)$

3 a $\log_3\left(\frac{x+4}{x}\right)$ b $\frac{1}{2}$

4 a $\frac{1}{12}$ b $\frac{1}{6}$ c $\frac{10}{11}$ d 9
e 8 f 3, 6 g 4

5 a 36 b 16 c 6

6 a 1 b 2.01 c 200 000
d 33 e 5 764 804 f $1.495\ (=5^{\frac{1}{4}})$
g $2.47(=\sqrt[3]{15})$ h 100 i $\sqrt{5}$

7 1.89

8 $k = \dfrac{\log 3}{\log 7}$

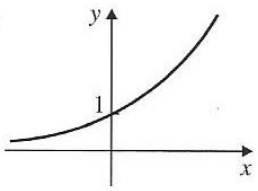
9 $k = \dfrac{\log 5}{\log 8}$

10 a $3\log x + \log y = \log 3$ b $y = \dfrac{3}{x^3}$

11 a $3\log x - \log y + \log 5 = 0$ b $y = 5x^3$

Exercise 8C *page 212*

1



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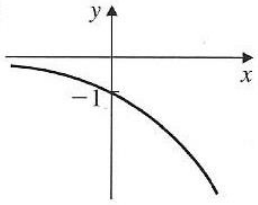

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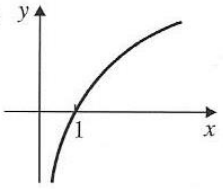

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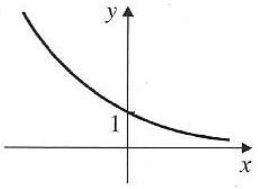

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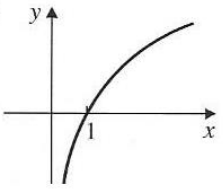

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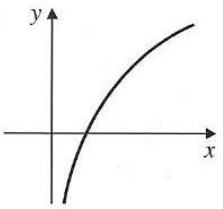

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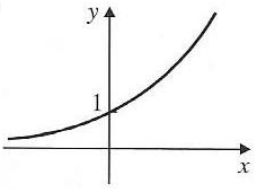

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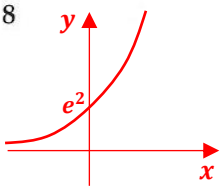

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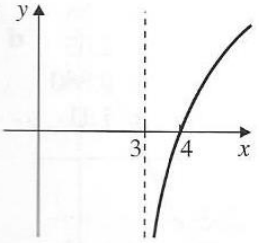
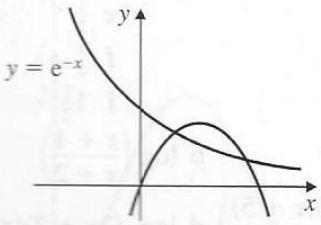


10 a



b (1, 1)
c 2

11 a e^{2x} b e^{6x} c e^{3x} d 1
e $\ln 2x$ f $\ln 3$ g $3\ln x$ h 0
i 1 j e^{x+2} k $\frac{1}{2}$ l x
m a n 7 o e p 0

12 1.95 **13** 1.04
14 1.50 **15** 3.40
16 0.193 **17** 1.03
18 0.521 **19** $\frac{1}{3}$
20 218 **21** 285
22 20 **23** ± 3
24 1.43

25 a $\ln\left(\frac{x+2}{x}\right)$ b $\dfrac{2}{e^4 - 1}$

26 a $\ln\left(\frac{x}{y}\right)$ b 0 c $\ln 2$ d x

27 a 0.65 b −2.39 c 0, $\ln 2$ d $\ln 4$ e 4

28 $x = e^3$, $y = e^2$

29 $x = 10$

30 a 25°C b 0.9 min

Exercise 8D *page 215*

1 a 1.73 b 4.25 c 3.86
d 2.94 e 0.928 f −1.30
g 1.95 h $\frac{1}{3}$ i $e^3(=20.1)$
j $\frac{e^5}{2}(=74.2)$ k $e^{-\frac{1}{2}}(=0.607)$ l $2e^4(=109)$

2 26.8 **3** 7330 **4** 1390

5 a 1000 b 45.8

6 a −3.94 b 1.82 c 120

7 a −0.0693 b 5°C

8 a 1000 g b 1660 years

9 a 2000 c 218

10 a 0.0563 b 8

11 a $\frac{1}{3}\ln\left(\frac{m}{a}\right)$ b $\ln\left(\frac{b}{10v}\right)$ c $\sqrt{\ln\frac{P}{150}}$

Exercise 8E *page 220*

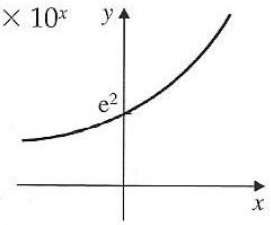
1 $\log_{10} y = x + 2$, $y = 100 \times 10^x$

2 $y = 10{,}000 \times 100^x$

3 $y = 10{,}000 \times 3.16^x$

4 $y = \dfrac{1}{10{,}000} \times 4.64^x$

5 $y = 10^8 \times \left(\dfrac{1}{100}\right)^x = \dfrac{10^8}{100^x}$



6 $P = 500 \times 2.3^t$

7 a $N = 350 \times 7.9^t$
b Dish is a finite size, bacteria cannot continue multiplying exponentially indefinitely.

8 a $M = 510 \times 2.0^{-t}$ b 90 g

9 a $T = 150 \times 2.2^{-x}$ b 150°C

10 $\log y = 3\log x + 1$, $y = 10x^3$

11 $y = 10x^2$

12 $y = 10^5 \times x^{-\frac{5}{8}}$

13 $\log y = \frac{1}{8}\log x + 2$
b $a = 100$, $b = \frac{1}{8}$

14 a $y = 0.6x^{3.5}$
b i 6,000,000 ii 13.2

15 a $y = 0.04 \times x^{1.5}$
b i 3.6 ii 63

16 a $N = 5x^{0.7}$

b No. There could be several reasons e.g. change of season.

Review Exercise 8F *page 224*

1 a $\log 10$ **b** $\log 5$ **c** $\log 9$
d $\log 300$ **e** $\log 81$ **f** $\log 6$
g $\log 32$ **h** $\log 2$ **i** $\log 3$

2 a 2 **b** 4 **c** 0
d -3 **e** 4 **f** -1

3 a $a = \log_{10} 4$ **b** $a = \log_6 11$ **c** $a = \log_7 3$

4 a $\log a + 2 \log b$ **b** $2 \log c + \log a$
c $\frac{1}{2} \log c$ **d** $2 \log a + \log b - \log c$
e $3 \log c - \log a$ **f** $\frac{1}{2} \log b - \log a$

5 a 1.76 **b** 1.89 **c** 1.92 **d** 3.32

6 a $\frac{5}{7}$ **b** $\frac{23}{20}$ **c** 1 **d** $\sqrt{7}$

7 a $\frac{x+2}{x}$ **b** $\frac{2}{7}$

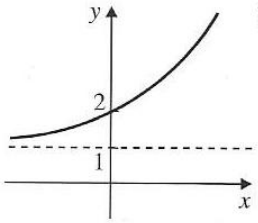
8 a $\frac{b}{3}$ **b** $\frac{2}{3}b + 1$

9 a $3b$ **b** $4b + 1$ **c** $2b + 2$

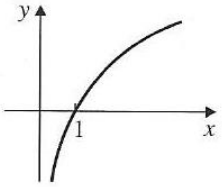
11 b $x = \frac{1}{3}$ or 2

12 $\frac{1}{8}$

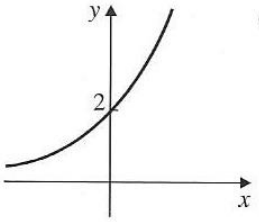
13 a



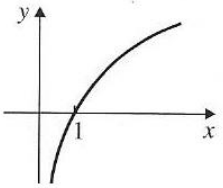
b



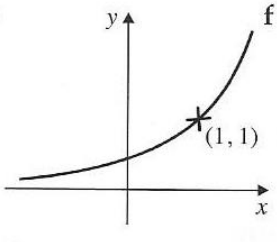
c



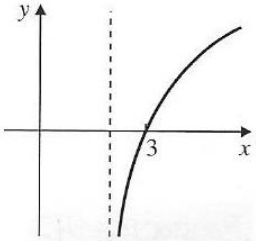
d



e



f



14 a $\frac{1}{3} \ln 7$ (=0.65) **b** $\ln \left(\frac{2}{3}\right)$
c $(\ln 5) - 1$

15 a $e^2 - 1$ **b** $2e^3$ **c** $\frac{1}{3}e^{\frac{1}{2}}$ **d** e^3

16 $\sqrt{\frac{3e}{4}}$

17 0.0707

18 333.5°C

19 b $x = 0, 2$

20 a 300 **b** 22.0

21 a $\frac{1}{2} \ln \frac{3}{5}$ (= −0.2554) **b** 21.6

22 a $\frac{1}{2} \ln \frac{m}{A}$ **b** $\ln \left(\frac{a}{4s}\right)$
c $\sqrt{\ln \frac{v}{20}}$

23 a 22.3 **b** 69.3

24 i 50 m/s **ii** 4.58 s

25 a $y = 215 \times 22^x$ **b** $y = 100{,}000x^{\frac{2}{7}}$
c $y = 10^6 \times x^{-0.75}$ **d** $y = 10^7 \times 10^{-x}$

26 a $P = 5010 \times 1.05^t$
b 12,057 **c** 12 years

27 a $y = 3.5x^{2.5}$
b i 610,000 **ii** 61

Examination Exercise 8 *page 227*

1 i 0 **ii** 18 **iii** $\frac{1}{2}$

2 i $1 \log_3 (ax^2)$ **ii** $x = \frac{3\sqrt{a}}{a}$

3 $\frac{7}{2} 1 \log_a x$

4 a $2 + a$ **b** $5a - 4$ **c** 2.498

5 $1 \log_{10} x = \frac{1 \log_{10} (y - a)}{b}$

6 a $k = \frac{x^2}{5}$ **b** $y = 2^{3b+6}$

7 a $x = 8$ **b** $x = -\frac{1}{4}, \frac{3}{2}$

8 $x = Na^{-\frac{3}{2}}$

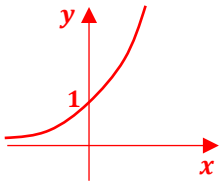
9 b $x = 9, 25$

10 $x = 12$

11 a $x = 1.21$ **b** $x = 1, 1.21$

12 a $x = 2.32$ **b** $a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$

13 a



b 2.59

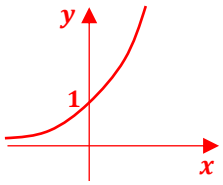
14 a $x = 3.97$ **b** $x = \frac{5}{8}$

15 a $w = 5.68$
b $x = \frac{1}{3}, y = 9$
$x = 3, y = 1$

16 b $x = 1.23$ **c** $y = 9^{-x}$

17 ii $x = 3, 3^x = 27$

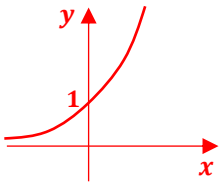
18 a



b 0, 0.56

19 a $n = 17.1$ **b** $x = 32, y = 8$

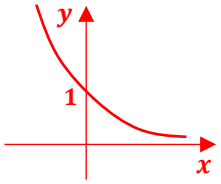
20 a



20 b translation 5 units in the negative y direction.

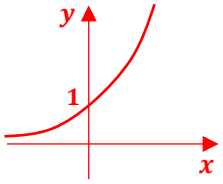
c ii $x = 2.32$

21 a



21 b $x = -0.322$ **c** $y = \frac{a}{b^2}$

22 a



22 b i $y = 4$ **ii** $x = 0.712$

23 $q = 3$

24 $x = 0.518$

25 $x = \dfrac{\log_{10} 486}{\log_{10} 144}$

26 a $\log_{10} P = \frac{1}{200}t + 5$

b $a = 100{,}000, b = 1.01$

c i initial population

ii the proportional increase of population each year.

d 300,000

e population cannot increase indefinitely, population is likely to fluctuate, no model is likely to remain accurate for 100 years.

27 $\log_{10} y = 3\log_{10} x + 2$

$y = 100x^3$

28 $y = 100x^3$

29 i $\log_{10} y = (\log_{10} q)x + \log_{10} P$

ii $p = 380, q = 0.63$

iii week 7. Model may not hold true outside of the given data.

30 i $m = b, c = \log_{10} a$
ii $-0.15, 0, 0.23, 0.36, 0.56, 0.67, 0.78, 0.91, 1.08, 1.2$
iii $h = 0.6 \times 10^{0.3t}$
iv answer between 8.0 and 26.1
v glacier will eventually melt completely.

31 i gradient $= k \log_{10} 2$
intercept $= \log_{10} A$
ii $k = 0.66, A = 100$
iii 6,300,000
iv limited space, growth cannot continue indefinitely.

32 i the rate of change of m is proportional to m. In this case it is $2m$.
ii initial membership
iii week 3
iv when the graph reaches 60,000 the graph becomes constant.

33 i $-0.03e^{-0.03t}$
ii As $e^{-0.03t} > 0$, so $-0.03e^{-0.03t} < 0$
$\therefore$ negative gradient
$\therefore$ decreasing function
iii A 70 B **38.4**
iv The data appears to be decreasing at a rate close to that predicted by the model.

34 a 200 **b** $t = 10 \ln 5$

35 a 20° **b** $\lambda = \dfrac{\ln 2}{40}$ **c** 93

36 ii $d = 65 \times 0.983^n$
iii $n = 180$
iv $k = 0.00745$
$d = 42.1°$

Exercise 9A *page 236*

1 a $2\mathbf{a} + \mathbf{b}$ **b** $2\mathbf{a} + 2\mathbf{b}$ **c** $-\mathbf{a} - \mathbf{b}$
d $4\mathbf{a} + 2\mathbf{b}$ **e** $2\mathbf{a} - 2\mathbf{b}$ **f** $2\mathbf{a} + \mathbf{b}$

2 a $\overrightarrow{CO}$ **b** $\overrightarrow{TN}$ **c** $\overrightarrow{FT}$ **d** $\overrightarrow{KC}$

3 a $-\mathbf{a}$ **b** $\mathbf{a} + \mathbf{b}$ **c** $2\mathbf{a} - \mathbf{b}$ **d** $-\mathbf{a} + \mathbf{b}$

4 a $\mathbf{a} + \mathbf{b}$ **b** $\mathbf{a} - 2\mathbf{b}$ **c** $-\mathbf{a} + \mathbf{b}$ **d** $-\mathbf{a} - \mathbf{b}$

5 a $-\mathbf{a} - \mathbf{b}$ **b** $3\mathbf{a} - \mathbf{b}$
c $2\mathbf{a} - \mathbf{b}$ **d** $-2\mathbf{a} + \mathbf{b}$

6 a $\mathbf{a} - 2\mathbf{b}$ **b** $\mathbf{a} - \mathbf{b}$
c $2\mathbf{a}$ **d** $-2\mathbf{a} + 3\mathbf{b}$

7 a and **d**

8 a AD, BE, CF

Exercise 9B *page 238*

1 a $\mathbf{a}$ **b** $-\mathbf{a} + \mathbf{b}$
c $2\mathbf{b}$ **d** $-2\mathbf{a}$
e $-2\mathbf{a} + 2\mathbf{b}$ **f** $-\mathbf{a} + \mathbf{b}$
g $\mathbf{a} + \mathbf{b}$ **h** $\mathbf{b}$
i $-\mathbf{b} + 2\mathbf{a}$ **j** $-2\mathbf{b} + \mathbf{a}$

2 a $\mathbf{a}$ **b** $-\mathbf{a} + \mathbf{b}$
c $3\mathbf{b}$ **d** $-2\mathbf{a}$
e $-2\mathbf{a} + 3\mathbf{b}$ **f** $-\mathbf{a} + \frac{3}{2}\mathbf{b}$
g $\mathbf{a} + \frac{3}{2}\mathbf{b}$ **h** $\frac{3}{2}\mathbf{b}$
i $-\mathbf{b} + 2\mathbf{a}$ **j** $-3\mathbf{b} + \mathbf{a}$

3 $\frac{1}{2}\mathbf{s} - \frac{1}{2}\mathbf{t}$

4 $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

5 $\mathbf{a} + \mathbf{c} - \mathbf{b}$

6 $2\mathbf{m} + 2\mathbf{n}$

7 a $\mathbf{b} - \mathbf{a}$ **b** $\mathbf{b} - \mathbf{a}$
c $2\mathbf{b} - 2\mathbf{a}$ **d** $\mathbf{b} - 2\mathbf{a}$
e $\mathbf{b} - 2\mathbf{a}$ **f** $2\mathbf{b} - 3\mathbf{a}$

8 a $\mathbf{y} - \mathbf{z}$ **b** $\frac{1}{2}\mathbf{y} - \frac{1}{2}\mathbf{z}$
c $\frac{1}{2}\mathbf{y} + \frac{1}{2}\mathbf{z}$ **d** $-\mathbf{x} + \frac{1}{2}\mathbf{y} + \frac{1}{2}\mathbf{z}$
e $-\frac{2}{3}\mathbf{x} + \frac{1}{3}\mathbf{y} + \frac{1}{3}\mathbf{z}$ **f** $\frac{1}{3}\mathbf{x} + \frac{1}{3}\mathbf{y} + \frac{1}{3}\mathbf{z}$

Exercise 9C *page 241*

1 a 5 **b** $5\sqrt{10}$ **c** $\sqrt{61}$ **d** $\sqrt{61}$

2 a $5, \frac{1}{5}(4\mathbf{i} + 3\mathbf{j})$ **b** $13, \frac{1}{13}(-5\mathbf{i} + 12\mathbf{j})$
c $25, \frac{1}{25}(7\mathbf{i} - 24\mathbf{j})$

3 a $\begin{pmatrix}2\\3\end{pmatrix}$ **b** $\begin{pmatrix}5\\-4\end{pmatrix}$ **c** $\begin{pmatrix}0\\5\end{pmatrix}$ **d** $\begin{pmatrix}-2\\4\end{pmatrix}$ **e** $\begin{pmatrix}1\\0\end{pmatrix}$

4 a $\frac{\sqrt{2}}{4}(2\mathbf{i} + 2\mathbf{j})$ **b** $\frac{\sqrt{5}}{10}(4\mathbf{i} - 2\mathbf{j})$
c $\frac{\sqrt{5}}{15}(3\mathbf{i} + 6\mathbf{j})$ **d** $\frac{\sqrt{5}}{15}(6\mathbf{i} - 3\mathbf{j})$

5 $a = 5$

6 $12\mathbf{i} + 16\mathbf{j}$

7 $20\mathbf{i} - 48\mathbf{j}$

8 $a = 3$

9 $b = 4$

10 $n = 1$

11 a $-2\mathbf{i} - 8\mathbf{j}$ **b** $2\mathbf{i} + 8\mathbf{j}$

12 a $2\mathbf{i} + 6\mathbf{j}$ **b** $\mathbf{i} + 3\mathbf{j}$

13 $\begin{pmatrix}7\\3\end{pmatrix}$

14 $3\mathbf{i} + \mathbf{j}$

15 a $\begin{pmatrix}3\\3\end{pmatrix}$ **b** $3\sqrt{2}$ **c** $\begin{pmatrix}-5\\1\end{pmatrix}$

16 a $\begin{pmatrix}-6\\-9\end{pmatrix}$ **b** $\begin{pmatrix}-12\\-13\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-4\end{pmatrix}$
d 5 **e** $\begin{pmatrix}\mathbf{-1}\\\mathbf{0}\end{pmatrix}$

17 a $\begin{pmatrix}3\\4\end{pmatrix}$ **b** $3\sqrt{17}$ **c** (7, 6)
d (31, 12) **e** 2 : 1

Exercise 9D *page 245*

1 a $(5\mathbf{i} + \mathbf{j})$ N **b** 11.3°

2 $\sqrt{13}$ N

3 a $3\sqrt{2}$ N, $2\sqrt{5}$ N **b** $\sqrt{26}$ N

4 $a = 7, b = -2$

5 a $(0.5\mathbf{i} + \mathbf{j})\,\text{m s}^{-2}$ **b** $(-\mathbf{i} - 1.5\mathbf{j})\,\text{m s}^{-2}$
c $\sqrt{5}\,\text{m s}^{-2}$

6 a $(-7\mathbf{i} + 3.5\mathbf{j})$ N **b** 3.5 N

7 a $(8\mathbf{i} + \mathbf{j})$ N **b** $(1.6\mathbf{i} + 0.2\mathbf{j})\,\text{m s}^{-2}$

8 a $(9\mathbf{i} - 2\mathbf{j})$ N **b** $6.15\,\text{m s}^{-2}$

9 $(3\mathbf{i} - \frac{2}{3}\mathbf{j})\,\text{m s}^{-2}$

10 a $\sqrt{13}\,\text{m s}^{-1}$ **b** $\sqrt{17}\,\text{m s}^{-1}$
c $5\sqrt{2}\,\text{km h}^{-1}$ **d** $\sqrt{29}\,\text{km h}^{-1}$

11 A by $0.334\,\text{m s}^{-1}$

12 **a** 372 m **b** 20 km **c** 1460 m
d 5.15 km **e** 6.18 km

13 58.3 km

14 12.5 s

15 8 s

16 21.0 km

17 **a** $(\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-2}$ **b** $\sqrt{5}\,\text{m s}^{-2}$

18 $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-2}$

19 **a** $(4\mathbf{i} - 5\mathbf{j})$ m **b** $\sqrt{41}$ m
c 87.3°

20 **a** $\sqrt{13}$ m, $2\sqrt{10}$ m, $\sqrt{41}$ m
b 74.7° **c** $11\,\text{m}^2$

21 $10\,\text{m}^2$

22 **a** $\sqrt{13}$ km **b** 124°

23 **a** $\overrightarrow{OA} = (4\mathbf{i} + 3\mathbf{j})$ km, $\overrightarrow{OB} = (5\mathbf{i} - 12\mathbf{j})$ km
b **i** $(\mathbf{i} - 15\mathbf{j})$ km **ii** 15.0 km

24 **a** $(9\mathbf{i} - 6\mathbf{j})$ km **b** $3\sqrt{13}$ km
c 124°

Review Exercise 9E *page 247*

1 **a** $-\mathbf{c}$ **b** $\mathbf{c} + \mathbf{d}$ **c** $2\mathbf{c} - \mathbf{d}$ **d** $\mathbf{d} - \mathbf{c}$

2 **a** $\mathbf{c} - 2\mathbf{d}$ **b** $\mathbf{c} - \mathbf{d}$ **c** $2\mathbf{c}$ **d** $3\mathbf{d} - 2\mathbf{c}$

3 **a** **i** $-2\mathbf{a} + 2\mathbf{b}$ **ii** $-2\mathbf{b} + 2\mathbf{c}$
iii **b** **iv** $\mathbf{c} - \mathbf{a}$ **v** $\mathbf{c} - \mathbf{a}$
b parallel and equal
c parallelogram

4 **a** **i** $-\mathbf{b} + \mathbf{a}$ **ii** $-\frac{1}{3}\mathbf{b} + \frac{1}{3}\mathbf{a}$
iii $-\frac{1}{6}\mathbf{a} + \frac{2}{3}\mathbf{b}$ **iv** $-\frac{1}{2}\mathbf{a} + 2\mathbf{b}$
b $\overrightarrow{CE} = 3\overrightarrow{CD}$

5 **a** 13 **b** $\sqrt{2}$ **c** 3 **d** $\sqrt{2}$ **e** 5

6 **a** $\sqrt{5}$ **b** 2

7 **a** $\frac{\sqrt{5}}{5}(\mathbf{i} + 2\mathbf{j})$ **b** $\frac{\sqrt{10}}{20}(6\mathbf{i} - 2\mathbf{j})$

8 **a** $\mathbf{i} - 6\mathbf{j}$ **b** $-\mathbf{i} + 6\mathbf{j}$

9 **a** $\begin{pmatrix}3\\3\end{pmatrix}$ **b** $3\sqrt{2}$ **c** $\begin{pmatrix}-7\\1\end{pmatrix}$

10 12.5

11 $a = 4, b = 7$

12 $\lambda = 4$

13 $\sqrt{89}$, 032°

14 **a** $5\,\text{m s}^{-1}$ **b** 9 km

15 **a** $13\mathbf{i} + 9\mathbf{j}$ **b** $2\mathbf{i} + 15\mathbf{j}$
c $\sqrt{34}$ **d** 59°

16 $13.5\,\text{m}^2$

17 **a** 15, $4\sqrt{2}$ **b** 8° **c** 6

18 **a** **i** $\sqrt{58}$ **ii** 067° **b** 30 mins

Examination Style Exercise 9 *page 250*

2 **a** $\frac{3}{10}\mathbf{a} + \frac{9}{20}\mathbf{b}$ **b** 3 : 7

3 **d**

4 **a** $\sqrt{53}$ **b** $-\mathbf{i} + 22\mathbf{j}$ **c** 73.3°

5 $\sqrt{97}$, 114°

6 $3\sqrt{2}$, 135°

7 **a** $-\mathbf{i} - 4\mathbf{j}$ **b** $-5\mathbf{i} + 6\mathbf{j}$

8 **a** $\frac{5\sqrt{146}}{146}$ **b** 33

9 7

10 B by $2.65\,\text{ms}^{-1}$

12 **a** $30\sqrt{13}$ m
b **i** 50 s **ii** 180 m

Note: In Question 11, vertices should be $(-5\mathbf{i} + 2\mathbf{j})$ ... not $(-4\mathbf{i} + 2\mathbf{j})$ ...

Exercise 10A *page 253*

1 ⇔	**2** ⇐	**3** ⇐	**4** ⇔
5 ⇒	**6** ⇐	**7** ⇐	**8** ⇒
9 ⇔	**10** ⇔	**11** ⇒	**12** ⇔
14 S	**15** A	**16** S	**17** A
18 S	**19** A	**20** N	**21** A
22 S	**23** A	**24** S	**26** A

Ex 10B solutions (Gurney, Raynes & Williams PM1)

1) Let odd $= 2n+1$ and even $= 2m$. ← (Note: Not $2n+1$ and $2n$)

$\text{Sum} = 2n+1+2m$

$= 2(n+m)+1.$

$\therefore$ Sum is always odd //

2) Let 1st $= 2n$ and 2nd $= 2m$.

$\text{Difference} = 2n-2m$

$= 2(n-m)$

$\therefore$ Difference is always even //

3) Let integers $= n, n+1, n+2$.

$\text{Sum} = n+n+1+n+2$

$= 3n+3 = 3(n+1)$

$\therefore$ Sum is always a multiple of 3 //

4) Let 1st $= 2n+1$ and 2nd $= 2m+1$.

$\text{Product} = (2n+1)(2m+1)$

$= 4nm + 2n + 2m + 1$

$= 2(2nm + n + m) + 1$

$\therefore$ Product is always odd //

5) $a^2 + 4a + 9 = (a+2)^2 - 4 + 9$

$= (a+2)^2 + 5.$

$(a+2)^2 \geq 0$ for all a, so $(a+2)^2 + 5 > 0$.

$\therefore$ $a^2+4a+9 > 0$ for all a //

6) $b^2 - 6b + 13 = (b-3)^2 - 9 + 13$

$= (b-3)^2 + 4$

$(b-3)^2 \geq 0$ for all b, so $(b-3)^2 + 4 > 0$

$\therefore$ $b^2 - 6b + 13 > 0$ for all b //

7)(a) $4x^2 + 4x + 1 = 4[x^2 + x] + 1$

$= 4[(x+\frac{1}{2})^2 - \frac{1}{4}] + 1$

$= 4(x+\frac{1}{2})^2$

$(x+\frac{1}{2})^2 \geq 0$ for all x

$\therefore$ $4x^2 + 4x + 1 \geq 0$ for all x //

b) $4n^2 + 4n + 1 = 4(n+\frac{1}{2})^2$

So if $n = -\frac{1}{2}$, $4n^2 + 4n + 1 = 0$ //

8) $x^2 - 6x + 10 + k^2 = 0$

$a=1,\ b=-6,\ c=10+k^2$

$D = b^2-4ac = 36-4(1)(10+k^2)$

$= 36-40-4k^2$

$= -4k^2-4.$

$k^2 \geq 0$ for all k, so $-4k^2-4 < 0$ for all k.

As $D<0$ for all k, no real roots //

9) $x^2 + 10x + 28 + a^2 = 0$

$a=1,\ b=10,\ c=28+a^2$

$D = b^2-4ac = 100-4(1)(28+a^2)$

$= 100-112-4a^2$

$= -4a^2-12$

$a^2 \geq 0$ for all a, so $-4a^2-12 < 0$ for all a

As $D<0$ for all a, no real roots //

10) Prove that there is no real value of b for which the following equation has DISTINCT real roots:

$x^2 + b^2 = 4(x-1)$

$x^2 - 4x + b^2 + 4 = 0$

$a=1,\ b=-4,\ c=b^2+4$

$D = 16-4(1)(b^2+4)$

$= 16-4b^2-16 = -4b^2$

$b^2 \geq 0$ for all b, so $-4b^2 \leq 0$ for all b

As $D \leq 0$ for all b, equation cannot have distinct roots //

Note: If $b=0$, equation has a single real root.

11) $x^2 + (\lambda-6)x + \lambda^2 + 13 = 0$

$a=1,\ b=\lambda-6,\ c=\lambda^2+13$

$D = (\lambda-6)^2 - 4(1)(\lambda^2+13)$

$= \lambda^2-12\lambda+36-4\lambda^2-52$

$= -3\lambda^2-12\lambda-16$

$= -3[\lambda^2+4\lambda]-16$

$= -3[(\lambda+2)^2-4]-16 = -3(\lambda+2)^2-4$

$(\lambda+2)^2 \geq 0$ for all λ, so $-3(\lambda+2)^2-4 < 0$ for all λ

As $D<0$ for all λ, no real roots //

12a) Rearrange as "$x^2 - 2x + 1 \geq 0$".

$LHS = (x-1)^2 - 1 + 1$

$= (x-1)^2$

$(x-1)^2 \geq 0$ for all x

$\therefore x^2 - 2x + 1 \geq 0$ for all x

$\therefore x^2 + 1 \geq 2x$ for all x //

b) If $x = 1$, then $LHS = x^2 + 1 = 1^2 + 1 = 2$

$RHS = 2x = 2(1) = 2$

} LHS $\not>$ RHS //

13) A triangle number is defined as a number of the form $\frac{n(n+1)}{2}$, where $n \in \mathbb{Z}$.

Prove that the sum of any two CONSECUTIVE triangle numbers is a square number.

Let 1st $= \frac{n(n+1)}{2}$ and 2nd $= \frac{(n+1)(n+2)}{2}$

$$\text{Sum} = \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2}$$

$$= \frac{n^2 + n + n^2 + 3n + 2}{2}$$

$$= \frac{2n^2 + 4n + 2}{2}$$

$$= n^2 + 2n + 1$$

$$= (n+1)(n+1) = (n+1)^2$$

$\therefore$ Sum is always a square number

Note:

Triangle numbers = 1, 3, 6, 10, 15 ...

$1 + 3 = 4 = 2^2$

$3 + 6 = 9 = 3^2$ ...

but $3 + 15$ is not a square.

Sum must be CONSECUTIVE

14)a) Let odd number $= 2n + 1$.

Square $= (2n+1)^2 = 4n^2 + 4n + 1$.

$= 4n(n+1) + 1$.

If n = even then $n+1$ = odd

If n = odd then $n+1$ = even

So $n(n+1)$ is either even × odd or odd × even $\therefore$ always even

So $4n(n+1)$ is always a multiple of 8

So Square of odd number always 'multiple of 8, plus 1' //

14b) Let 1st odd $= 2n+1$, 2nd odd $= 2m+1$

Difference of squares $= (2n+1)^2 - [(2m+1)^2]$

$= 4n^2 + 4n + 1 - [4m^2 + 4m + 1]$

$= 4n^2 + 4n - 4m^2 - 4m$

$= 4[n^2 + n - m^2 - m]$

$= 4[n^2 - m^2 + n - m]$

$= 4[(n-m)(n+m) + (n-m)]$

$= 4(n-m)[n+m+1]$

If n & m both even, then $n-m$ = even

If n & m both odd, then $n-m$ = even

If $\{n$ = even & m = odd$\}$ or $\{n$ = odd & m = even$\}$ then $n+m+1$ = even

In any case, $4(n-m)(n+m+1)$ will be a multiple of 8 //

15a) $(5n+1)^2 - [(5n-1)^2] = 25n^2 + 10n + 1 - [25n^2 - 10n + 1]$

$= \cancel{25n^2} + 10n + \cancel{1} - \cancel{25n^2} + 10n - \cancel{1}$

$= 20n$

∴ Expression always a multiple of 20 //

b) If $n \in \mathbb{R}$, then n can be <u>any</u> <u>real</u> <u>number</u>, e.g. 0.1.

If $n = 0.1$, $(5n+1)^2 - (5n-1)^2$

$= (1.5)^2 - (-0.5)^2 = 2$ (not a multiple of 20) //

16a) $(an+1)^2 - [(an-1)^2] = a^2n^2 + 2an + 1 - [a^2n^2 - 2an + 1]$

$= \cancel{a^2n^2} + 2an + \cancel{1} - \cancel{a^2n^2} + 2an - \cancel{1}$

$= 4an = 4(an)$

∴ Expression is a multiple of 4 //

b) $(an+b)^2 - [(an-b)^2] = a^2n^2 + 2anb + b^2 - [a^2n^2 - 2anb + b^2]$

$= \cancel{a^2n^2} + 2anb + \cancel{b^2} - \cancel{a^2n^2} + 2anb - \cancel{b^2}$

$= 4anb = 4(anb)$

∴ Expression is a multiple of 4 //

17) Let 1st square $= n^2$ and 2nd $= (n+1)^2$

Sum $= n^2 + (n+1)^2 = n^2 + n^2 + 2n + 1$

$= 2n^2 + 2n + 1$

$= 2(n^2+n)+1$

$\therefore$ Sum of consecutive squares is always odd //

18) Let squares $= n^2$, $(n+1)^2$ and $(n+2)^2$

$\text{Sum} = n^2 + (n+1)^2 + (n+2)^2$

$= n^2 + n^2 + 2n + 1 + n^2 + 4n + 4$

$= 3n^2 + 6n + 5$

$= 3(n^2 + 2n + 1) + 2$

$\therefore$ Sum cannot be divided by 3 //

19) a) sequence $= \overset{4}{5}, \overset{4}{9}, \overset{4}{13}, \overset{4}{17}, \overset{4}{21}, \ldots$

$4n = 4, 8, 12, 16, 20 \ldots$

n^{th} term $= 4n+1$ //

b) Let 1st $= 4n+1$, 2nd $= 4m+1$

$\text{Product} = (4n+1)(4m+1)$

$= 16nm + 4n + 4m + 1$

$= 4(4nm + n + m) + 1.$

$\therefore$ Product also a term within the sequence //

20a) sequence $= 1, \overset{3}{4}, \overset{3}{7}, \overset{3}{10}, \overset{3}{13} \ldots$

$3n = 3, 6, 9, 12, 15 \ldots$

n^{th} term $= 3n-2$ //

b) Let 1st $= 3n-2$, 2nd $= 3m-2$

$\text{Product} = (3n-2)(3m-2)$

$= 9nm - 6n - 6m + 4$

$= 9nm - 6n - 6m + 6 - 2$

$= 3(3nm - 2n - 2m + 2) - 2$

$\therefore$ Product also a term within the sequence //

21a) Evens between 30 & 40 inclusive $= 30, 32, 34 \ldots 40.$

Primes $= 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, \ldots$

So $30 = 7+23$, $32 = 3+29$, $34 = 3+31$, $36 = 5+31$,

$38 = 7+31$, $40 = 3+37$ //

21b) 2 cannot be written as the sum of two primes //

c) $5 = 2+3$, where 5 = odd and 2 & 3 are primes //

22a) Odds between 20 and 30 = 21, 23, 25, 27, 29.

$21 = 7+7+7$, $23 = 3+7+13$, $25 = 5+7+13$,

$27 = 7+7+13$, $29 = 5+7+17$ //

b) 3 cannot be written as the sum of three primes //

23)

a	b	c	$ab+bc+ca$	$a^2+b^2+c^2$	LHS ≤ RHS?
1	1	1	$1+1+1=3$	$1^2+1^2+1^2=3$	✓
1	1	2	$1+2+2=5$	$1^2+1^2+2^2=6$	✓
1	1	3	$1+3+3=7$	$1^2+1^2+3^2=11$	✓
1	2	2	$2+4+2=8$	$1^2+2^2+2^2=9$	✓
1	2	3	$2+6+3=11$	$1^2+2^2+3^2=14$	✓
1	3	3	$3+9+3=15$	$1^2+3^2+3^2=19$	✓
2	2	2	$4+4+4=12$	$2^2+2^2+2^2=12$	✓
2	2	3	$4+6+6=16$	$2^2+2^2+3^2=17$	✓
2	3	3	$6+9+6=21$	$2^2+3^2+3^2=22$	✓
3	3	3	$9+9+9=27$	$3^2+3^2+3^2=27$	✓

Have considered all cases where $a \le b \le c$.

Other cases will give the same results

e.g. $a=3, b=2, c=1$ will give the same results as $a=1, b=2, c=3$

ie. $ab+bc+ca = 11$ and $a^2+b^2+c^2 = 14$.

Exam-Style Ex 10 (Gurney, Raynes & Williams PM1)

1 i) $\Leftarrow$

ii) $\Rightarrow$

2 i) $\Leftrightarrow$

ii) $\Leftarrow$

3 i) $\Leftarrow$

ii) $\Leftrightarrow$

4 i) A

ii) S

iii) S

Note for Q1 i) Consider $n^2+1=4$
this gives $n=\pm\sqrt{3}$
n is not an odd integer

5) If $x-7=0$ then $x=7$
If $x^2=49$ then $x=\pm 7$
$\}$ ∴ statement is false //

6 i) $(n+3)^2-n^2 = n^2+6n+9-n^2$
$= 6n+9 = 3(2n+3)$
$2n$ is always even, so $2n+3$ is always odd.
3 times an odd number is always odd.
∴ $(n+3)^2-n^2$ is never even //

ii) $(n+6)^2-n^2 = n^2+12n+36-n^2$
$= 2[6n+18]$
∴ $(n+6)^2-n^2$ is ALWAYS even //

7) $(2n+5)^2-[(2n-5)^2]$
$= 4n^2+20n+25-[4n^2-20n+25]$
$= 4n^2+20n+25-4n^2+20n-25$
$= 40n = 20[2n]$
∴ expression is a multiple of 20 //

8) $n^2-12n+39$
$=(n-6)^2-36+39 = (n-6)^2+3$
$(n-6)^2 \geq 0$ for all n
∴ Expression >0 for all n //

9) $4n^2-12n+11$
$= 4[n^2-3n]+11$
$= 4[(n-1.5)^2-2.25]+11$
$= 4(n-1.5)^2-9+11 = 4(n-1.5)^2+2$

$(n-1.5)^2 \geq 0$ for all n

$\therefore$ Expression > 0 for all n //

10) $x^2 + (\lambda-3)x + \lambda^2 + 5 = 0$

THIS QUESTION IS FLAWED — DISCRIMINANT IS $D = -3(\lambda+1)^2 + 32$, may be positive or negative, e.g. if $\lambda = -1$, $D = 32$

11) a) [Show $2ab \leq a^2 + b^2$]

Rewrite $RHS - LHS = (a^2+b^2) - 2ab$

$= a^2 - 2ab + b^2$

$= (a-b)^2$

Clearly $(a-b)^2 \geq 0$

$\therefore \quad a^2 - 2ab + b^2 \geq 0$

$\therefore \quad a^2 + b^2 \geq 2ab$ //

b) For example if $a=2$, $b=2$ & $c=1$

$LHS = 3abc = 3(2)(2)(1) = 12$

$RHS = a^2+b^2+c^2 = 4+4+1 = 9$

$12 \neq 9$

$\therefore$ By counterexample, statement is wrong //

12)

$1 \to 4$ have 2	$36 \to 49$ have 37
$4 \to 9$ have 5	$49 \to 64$ have 53
$9 \to 16$ have 11	$64 \to 81$ have 67
$16 \to 25$ have 17	$81 \to 100$ have 83
$25 \to 36$ have 29	

$\therefore$ Statement is true (for the 10 square numbers)

13) $4^2 + 4^2 = 32$ (not a square)

$4^2 + 5^2 = 41$ (")

$4^2 + 6^2 = 52$ (")

$4^2 + 7^2 = 65$ (")

$5^2 + 5^2 = 50$ (")

$5^2 + 6^2 = 61$ (")

$5^2 + 7^2 = 74$ (")

$6^2 + 6^2 = 72$ (")

$6^2 + 7^2 = 85$ (")

$7^2 + 7^2 = 98$ (")

Therefore there is no Pythagorean Triple between 3^2+4^2 and 6^2+8^2 //