1) $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{8.3}{\sin A} = \frac{13.5}{\sin 32}$

$\sin A = \frac{8.3 \sin 32}{13.5}$

$A = 19.0°$ (3 sf)

2)a)i) $x^2 - 6x + y^2 + 4y + 4 = 0$

$(x-3)^2 - 9 + (y+2)^2 \cancel{-4} \cancel{+4} = 0$

$(x-3)^2 + (y+2)^2 = 9$

centre $= (3, -2)$

ii) radius $= 3$

b) $(x-3)^2 + (y+2)^2 = 9$ ①

$y = kx - 3$ ②

sub ② into ① $(x-3)^2 + (kx-1)^2 = 9$

$x^2 - 6x + \cancel{9} + k^2x^2 - 2kx + 1 - \cancel{9} = 0$

$(1+k^2)x^2 - 2(3+k)x + 1 = 0$

$a = 1 + k^2, \quad b = -2(3+k), \quad c = 1$

$D = b^2 - 4ac = [2(3+k)]^2 - 4(1+k^2)(1)$

$= 4(9 + 6k + k^2) - 4(1 + k^2)$

$= 36 + 24k + \cancel{4k^2} - 4 - \cancel{4k^2}$

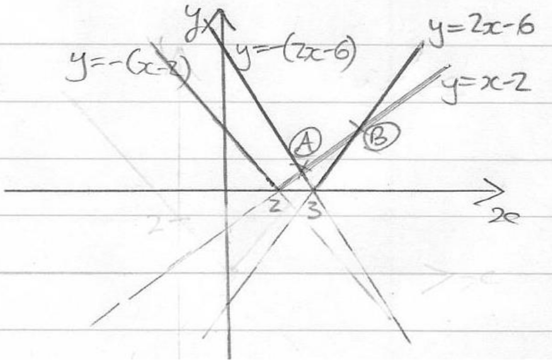
$= 24k + 32$

No intersections $\Rightarrow D < 0 \Rightarrow 24k + 32 < 0$

$k < -\frac{32}{24}$

$k < -\frac{4}{3}$

3a) $|x-2| \le |2x-6|$



Ⓐ $x - 2 = -(2x-6)$ Ⓑ $x - 2 = 2x - 6$

$3x = 8$ $\quad x = 4$

$x = \frac{8}{3}$

From sketch, $x \le \frac{8}{3}$ or $x \ge 4$

b) $y = |x-2| \rightarrow y = |x-6|$ Horiz. translation by $+4$

$\rightarrow y = |2x-6|$ Horiz. stretch by $\times \frac{1}{2}$

Note: the stretch must be AFTER the translation. Otherwise, we'd have $y = |2(x-6)|$

4a) $y = 3x \cdot \sin 2x$

$$\frac{dy}{dx} = 3x \cdot 2\cos 2x + \sin 2x \cdot 3$$

$$= 6x\cos 2x + 3\sin 2x$$

If $\frac{dy}{dx} = 0$ then $6x\cos 2x + 3\sin 2x = 0$

$(\div 3)$ $2x\cos 2x + \sin 2x = 0$

$(\div \cos 2x$, provided $\cos 2x \neq 0)$ $2x + \tan 2x = 0$ //

b) Let $f(x) = \tan 2x + 2x$

$f'(x) = 2\sec^2 2x + 2$

$$\boxed{x_{n+1} = x_n - \frac{\tan 2x_n + 2x_n}{2\sec^2 2x_n + 2}}$$

$x_0 = 1.2$

$x_1 = 0.93865\ldots$

$x_2 = 0.99215\ldots$

$x_3 = 1.01262\ldots$

$x_4 = 1.01436\ldots$

$x_5 = 1.01437\ldots$

$x_6 = 1.\underline{01437}\ldots$

Diagram shows P between $x = 0$ and $x = \frac{\pi}{2} = 1.57$, but closer to 1.57. Choose any value $0.8 \le x \le 1.5$

Converges to 1.0144 // (4 dp)

c) $h = \frac{a+b}{n} = \frac{0 + \frac{1}{2}\pi}{4} = \frac{\pi}{8}$

GOAL
$k\pi^2(\sqrt{2}+1)$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0	$\frac{3\pi\sqrt{2}}{16}$	$\frac{3\pi}{4}$	$\frac{9\pi\sqrt{2}}{16}$	0

$$\text{Integral} \approx \frac{1}{2}\left(\frac{\pi}{8}\right)\left\{\cancel{(0+0)} + 2\left(\frac{3\pi\sqrt{2}}{16} + \frac{3\pi}{4} + \frac{9\pi\sqrt{2}}{16}\right)\right\}$$

$$= \frac{\pi}{16}\left\{\frac{3\pi\sqrt{2}}{8} + \frac{3\pi}{2} + \frac{9\pi\sqrt{2}}{8}\right\} = \frac{\pi}{16}\left\{\frac{3\pi\sqrt{2}}{2} + \frac{3\pi}{2}\right\} = \frac{3\pi^2}{32}(\sqrt{2}+1) \text{ //}$$

d i) $\displaystyle\int_0^{\frac{\pi}{2}} 3x\sin 2x\,dx$

$u = 3x \quad \frac{dv}{dx} = \sin 2x$

$\frac{du}{dx} = 3 \quad v = -\frac{\cos 2x}{2}$

$= -\frac{3x\cos 2x}{2} + \int \frac{3\cos 2x}{2}\,dx$

$= \left[-\frac{3x\cos 2x}{2} + \frac{3\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$

$= \left(-\frac{3}{2}\left(\frac{\pi}{2}\right)\cos\pi + \cancel{\frac{3}{4}\sin\pi}\right) - \left(\cancel{0} + \cancel{\frac{3\sin 0}{4}}\right) = \frac{3\pi}{4}$ //

ii) Exact area $= \frac{3\pi}{4} = 2.3561\ldots$

Approx area $= \frac{3\pi^2}{32}(\sqrt{2}+1) = 2.233\ldots$

Trapezium Rule gives underestimate //

iii) From diagram, tops of trapeziums are partly above and partly below the curve, so not easy to tell over- or underestimate //

5a) LHS $= (\cot\theta + \operatorname{cosec}\theta)^2$

$= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$

$= \left(\frac{\cos\theta + 1}{\sin\theta}\right)^2$

$= \frac{\cos^2\theta + 2\cos\theta + 1}{\sin^2\theta}$

$= \frac{\cos^2\theta + 2\cos\theta + 1}{1-\cos^2\theta}$

$= \frac{\cos^2\theta + 2\cos\theta + 1}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{(1+\cos\theta)^{\cancel{2}}}{(1-\cos\theta)\cancel{(1+\cos\theta)}} = \frac{1+\cos\theta}{1-\cos\theta} = \text{RHS}$ //

$$\boxed{\begin{aligned}\text{RHS} &= \frac{1+\cos\theta}{1-\cos\theta} \\ &= \frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}\end{aligned}}$$

b) $3(\cot\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta \qquad 0 < \theta < 2\pi$

$3\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = \frac{2}{\cos\theta}$

$3\cos\theta(1+\cos\theta) = 2(1-\cos\theta)$

$3\cos\theta + 3\cos^2\theta = 2 - 2\cos\theta$

$3\cos^2\theta + 5\cos\theta - 2 = 0$

$$(3\cos\theta - 1)(\cos\theta + 2) = 0$$

$$\cos\theta = \frac{1}{3} \quad \text{or} \quad \cos\theta = -2$$

(no sols)

$$\theta = 1.23,\ 2\pi - 1.23$$

$$\theta = 1.23,\ 5.05 \,//$$

6) $$\frac{2x-1}{(2x+3)(x+1)^2} = \frac{A}{2x+3} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\boxed{2x-1 = A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)}$$

If $x=-1$, $\quad -3 = A\cancel{0} + B\cancel{0} + C(1) \Rightarrow C = -3$

If $x = -\frac{3}{2}$, $\quad -4 = \frac{A}{4} + B\cancel{(0)} + \cancel{C(0)} \Rightarrow A = -16$.

If $x = 0$, $\quad -1 = A + B(3) + C(3)$

$$-1 = -16 + 3B - 9 \Rightarrow B = 8.$$

$\therefore$ Consider $$\int_{x=0}^{\frac{1}{2}} \left(-\frac{16}{2x+3} + \frac{8}{x+1} - 3(x+1)^{-2} \right) dx$$

$$= \left[-\frac{\overset{8}{\cancel{16}} \ln(2x+3)}{\cancel{2}} + 8\ln(x+1) + \frac{3(x+1)^{-1}}{+1} \right]_0^{\frac{1}{2}}$$

$$= \left(-8\ln 4 + 8\ln\left(\tfrac{3}{2}\right) + \frac{3}{\left(\frac{3}{2}\right)}\right) - \left(-8\ln 3 + 8\cancel{\ln 1} + 3\right)$$

$$= -8\ln 4 + 8\ln\left(\tfrac{3}{2}\right) + 2 + 8\ln 3 - 3$$

$$= 8\left[\ln\left(\tfrac{3}{2}\right) + \ln 3 - \ln 4\right] - 1$$

$$= 8\left[\ln\left(\tfrac{3}{2} \times 3 \div 4\right)\right] - 1 = 8\ln\left(\tfrac{9}{8}\right) - 1 \quad (= -0.0577)$$

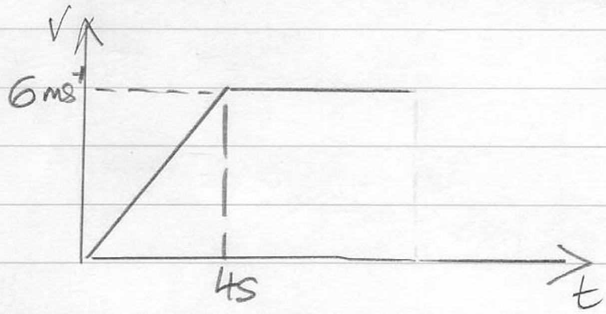
Because curve is below x-axis,

$$\text{Area required} = -\left[8\ln\left(\tfrac{9}{8}\right) - 1\right] = 1 - 8\ln\left(\tfrac{9}{8}\right)$$

$$= 1 + 8\ln\left(\tfrac{8}{9}\right) \,//$$

$$\left[p=1,\ q=8,\ r=\tfrac{8}{9}\right]$$

7a)



$$\text{Grad} = \text{accel} = \frac{v_1}{4} = 1.5 \Rightarrow v_1 = 6\ ms^{-1}$$

b) Distance travelled = area under V-T graph

$$= \frac{1}{2}(4)(6) + (6)(6) = 48\,m$$

8a)

	H	V
s	x	y
u	$\frac{5}{3}d$	u_y
v	V_x	V_y
a	0	-9.8
t	t	t

$(s = ut + \frac{1}{2}at^2)$

H: $x = \frac{5}{3}dt$ ①

V: $y = u_y t - 4.9t^2$ ②

When $t = 2.4$,

$$x = \frac{5}{3}(d)(2.4) = 4\,m$$

b) $y = 0$ when $t = 2.4$ so

$$0 = u_y(2.4) - 4.9(2.4)^2$$

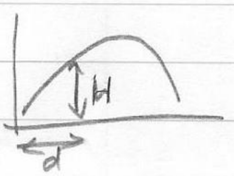
$$2.4u_y = 28.224$$

$$u_y = 11.76\ ms^{-1}$$

c) When $t = T$ goes through point $x = d$, $y = H$

Sub in ① $\quad d = \frac{5}{3}dT \Rightarrow T = \frac{3}{5}$

Sub in ② $\quad H = 11.76\left(\frac{3}{5}\right) - 4.9\left(\frac{3}{5}\right)^2$



$$H = 5.292\,m$$

d) (using $v = u + at$)

H: $V_x = \frac{5}{3}d$ ③

V: $V_y = 11.76 - 9.8t$ ④

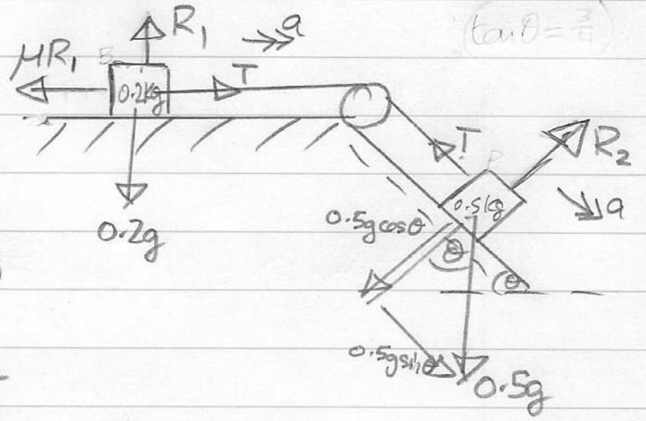
If $t = T = \frac{3}{5}$ then $V_y = 11.76 - 9.8\left(\frac{3}{5}\right) = 5.88$

$$\text{speed} = \sqrt{V_x^2 + V_y^2} = 16$$

$$\therefore V_x^2 + 5.88^2 = 16^2$$

$$V_x^2 = 16^2 - 5.88^2$$
$$V_x = 14.88$$

Using ③ $\quad \frac{5}{3}d = 14.88 \Rightarrow d = 8.93$ //



9) a) (0.2kg ↑) $\quad R_1 = 0.2g$ ①

(0.2kg →) $\quad T - \mu R_1 = 0.2a$ ②

(0.5kg ↗) $\quad R_2 = 0.5g\cos\theta$ ③

(0.5kg ↘) $\quad 0.5g\sin\theta - T = 0.5a$ ④

$$\left.\begin{array}{l} s = 0.3 \\ u = 0 \\ v = X \\ a = ? \\ t = 0.4 \end{array}\right\} \quad \begin{array}{l} s = ut + \frac{1}{2}at^2 \\ 0.3 = \cancel{0} + \left(\frac{1}{2}\right)a(0.4)^2 \\ 0.3 = 0.08a \\ a = 3.75 \end{array}$$

$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Sub in ④ $\quad 0.5(9.8)\left(\frac{3}{5}\right) - T = 0.5(3.75)$

$$2.94 - T = 1.875$$
$$T = 1.065\,N$$ //

b) From ① $\quad R_1 = 0.2(9.8) = 1.96$

Sub in ② $\quad 1.065 - \mu(1.96) = (0.2)(3.75)$

$$1.065 - 1.96\mu = 0.75$$
$$1.96\mu = 0.315$$
$$\mu = 0.161 \text{ // } \quad (3\text{ sf})$$

10a) $\underline{v} = (pt^2 - 3t)\underline{i} + (8t + q)\underline{j}$ — ✱

$$\underline{x} \underset{int}{\overset{diff}{\rightleftarrows}} \underline{v} \underset{int}{\overset{diff}{\rightleftarrows}} \underline{a}$$

$\underline{a} = (2pt - 3)\underline{i} + (8)\underline{j}$

If $t = 0.5$, $\underline{a} = (p-3)\underline{i} + 8\underline{j}$

Using $\underline{F} = m\underline{a}$, $\quad \underline{F} = 2\left[(p-3)\underline{i} + 8\underline{j}\right]$

$$= (2p-6)\underline{i} + 16\underline{j}$$

$|F| = 20 \Rightarrow \sqrt{(2p-6)^2 + 16^2} = 20$

$$(2p-6)^2 + 256 = 400$$
$$(2p-6)^2 = 144$$
$$2p - 6 = \pm 12$$
$$2p = 18 \text{ or } -6 \Rightarrow p = 9 \text{ or } -3$$

From question $p < 0$ so $p = -3$ //

b) $\underline{v} = (-3t^2 - 3t)\underline{i} + (8t+q)\underline{j}$

Integrating,

$$\underline{x} = \left(-t^3 - \frac{3t^2}{2}\right)\underline{i} + \left(4t^2 + qt\right)\underline{j} + \underline{c}$$

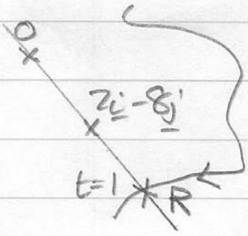
If $t=0$, $\underline{x} = 2\underline{i} - 3\underline{j}$

so $2\underline{i} - 3\underline{j} = 0\underline{i} + 0\underline{j} + \underline{c} \Rightarrow \underline{c} = 2\underline{i} - 3\underline{j}$

$$\underline{x} = \left(-t^3 - \frac{3t^2}{2} + 2\right)\underline{i} + \left(4t^2 + qt - 3\right)\underline{j}$$

c) If $t=1$, $\underline{x} = (-1 - \frac{3}{2} + 2)\underline{i} + (4 + q - 3)\underline{j} = -\frac{1}{2}\underline{i} + (1+q)\underline{j}$

Particle is on line through O and $2\underline{i} - 8\underline{j}$



$\therefore \quad -\frac{1}{2}\underline{i} + (1+q) \parallel (2\underline{i} - 8\underline{j})$

$-\frac{1}{2}\underline{i} + (1+q)\underline{j} = \lambda(2\underline{i} - 8\underline{j})$

$$\therefore \begin{cases} -\frac{1}{2} = 2\lambda & (1) \\ 1+q = -8\lambda & (2) \end{cases}$$

Sub (1) into (2) $\quad 1 + q = -\cancel{8}^{2}\left(-\frac{1}{\cancel{4}_{1}}\right)$

$1 + q = 2 \Rightarrow q = 1$ //

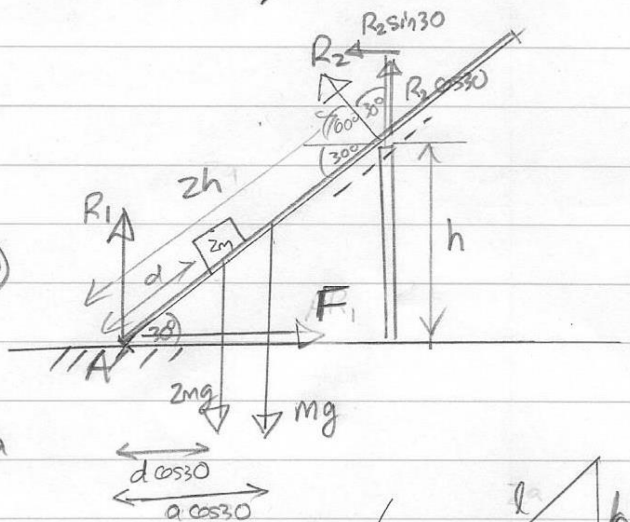
11a) $(\uparrow)$ $\quad R_1 + R_2\cos 30 = 3mg \quad (1)$

$(\rightarrow)$ $\quad F = R_2 \sin 30 \quad (2)$

(moments about A)

$(R_2)(2a) = (2mg)(d\cos 30) + (mg)(a\cos 30)$

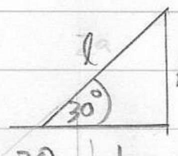
$2hR_2 = \sqrt{3}\,mgd + \frac{\sqrt{3}}{2}mga \quad (3)$



From (3) $\quad 2hR_2 = \dfrac{2\sqrt{3}mgd + \sqrt{3}mga}{2}$

$2hR_2 = \dfrac{mg(a+2d)\sqrt{3}}{2}$

$R_2 = \dfrac{mg(a+2d)\sqrt{3}}{4h}$ //



$\sin 30 = \frac{h}{l}$

$\frac{1}{2} = \frac{h}{l}$

$\therefore l = 2h$

b) Limiting equilibrium with $\mu = \frac{1}{8}\sqrt{3}$ $\therefore$ $F = \mu R_1 = \frac{1}{8}\sqrt{3}\, R_1$

Sub in ② $\frac{1}{8}\sqrt{3}\, R_1 = R_2 \sin 30$

$$\frac{1}{\cancel{8}}\sqrt{3}\, R_1 = \frac{mg(a+2d)\sqrt{3}}{\cancel{4}h} \times \frac{1}{\cancel{2}}$$

$$\cancel{\sqrt{3}}\, h R_1 = mg(a+2d)\cancel{\sqrt{3}}$$

$$R_1 = \frac{mg(a+2d)}{h}$$

Sub in ① $\frac{\cancel{mg}(a+2d)}{h} + \left(\frac{\cancel{mg}(a+2d)\sqrt{3}}{4h}\right)\left(\frac{\sqrt{3}}{2}\right) = 3\cancel{mg}$

$$\frac{a+2d}{h} + \frac{3(a+2d)}{8h} = 3$$

GOAL
$h = k(a+2d)$

$(\times 8h)$ $8a + 16d + 3(a+2d) = 24h$

$$11(a+2d) = 24h$$

$$h = \frac{11}{24}(a+2d)$$

c) Length of ladder $= 2a$ & Distance from A to top of wall $= 2h$

$$\therefore 2h \leq 2a$$

$$\cancel{2} \times \frac{11}{24}(a+2d) \leq \cancel{2}a$$

$$11a + 22d \leq 24a$$

$$22d \leq 13a$$

$$d \leq \frac{13a}{22} \qquad \therefore d_{max} = \frac{13a}{22}$$

d) Have modelled wall as smooth; to improve the model take into account some friction at the top of the wall