1) $x^2 + 8x + y^2 - 2y - 7 = 0$

$(x+4)^2 - 16 + (y-1)^2 - 1 - 7 = 0$

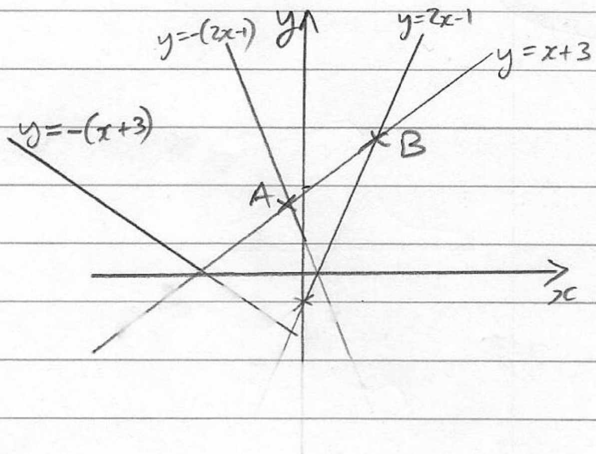
$(x+4)^2 + (y-1)^2 = 24$

i) $C = (-4, 1)$

ii) radius $= \sqrt{24} = 2\sqrt{6}$

2) $|2x-1| = |x+3|$

sketch $y = |2x-1|$ and $y = |x+3|$



(A) $-(2x-1) = x+3$

$-2x + 1 = x + 3$

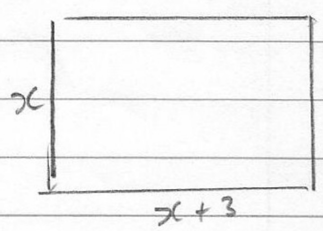
$-3x = 2 \Rightarrow x = -\frac{2}{3}$

(B) $2x - 1 = x + 3$

$x = 4$

Ans: $x = -\frac{2}{3}$ or $x = 4$

3) Let width $= x$.



Length $\geq 14.5 \Rightarrow x + 3 \geq 14.5$

$x \geq 11.5$ — (1)

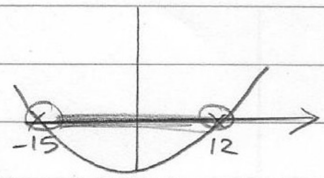
Area $< 180 \Rightarrow x(x+3) < 180$

$x^2 + 3x - 180 < 0$

$(x+15)(x-12) < 0$

CV $x = -15$, $x = 12$

$-15 < x < 12$ — (2)



Width of flower bed must satisfy both (1) & (2)

$\therefore 11.5 \leq x < 12$

4) i) a) $fg(x) = f(g(x))$

$= f(x^2+2)$

$= (x^2+2)^3$

b) $gf(x) = g(f(x))$

$= g(x^3)$

$= (x^3)^2 + 2$ (or $x^6 + 2$)

4ii)
$$fg(x) - gf(x) = 24$$
$$(x^2+2)^3 - (x^6+2) = 24 \quad \text{— } \circledast$$
Now $(x^2+2)^3 = (x^2)^3 + {}^3C_1(x^2)^2(2) + {}^3C_2(x^2)(2)^2 + 2^3$
$$= x^6 + 6x^4 + 12x^2 + 8$$
Sub in $\circledast$ $\cancel{x^6} + 6x^4 + 12x^2 + 8 - \cancel{x^6} - 2 - 24 = 0$
$$6x^4 + 12x^2 - 18 = 0$$
$\div 6$ $\quad x^4 + 2x^2 - 3 = 0$ $\quad$ Let $X = x^2$
$$X^2 + 2X - 3 = 0$$
$$(X+3)(X-1) = 0$$
$$X = -3 \text{ or } X = 1$$
$$x^2 = -3 \text{ or } x^2 = 1$$
(no sol) $\quad x = \pm 1$ //

5i)

x	0	2	4
y	$\frac{1}{2}$	$\frac{2-\sqrt{2}}{2}$	$\frac{1}{4}$

$\therefore$ Integral $\approx \frac{1}{\cancel{2}}(\cancel{2})\left\{\left(\frac{1}{2}+\frac{1}{4}\right) + \cancel{2}\left(\frac{2-\sqrt{2}}{\cancel{2}}\right)\right\}$

$$= \frac{3}{4} + 2 - \sqrt{2} = \frac{11}{4} - \sqrt{2} \;//$$

Note: Not a good estimate, 9% error!

ii)
$$\int_{x=0}^{4} \frac{1}{2+\sqrt{x}}\,dx$$
$$= \int \left(\frac{1}{2+\sqrt{x}}\right)(2u\,du)$$
$$= \int_{u=0}^{2} \frac{2u}{2+u}\,du$$
$$= \int \frac{2u}{v}\,dv$$
$$= \int_{v=2}^{4} \frac{2v-4}{v}\,dv$$
$$= \int_{v=2}^{4} \left(2 - \frac{4}{v}\right)dv$$
$$= \left[2v - 4\ln v\right]_2^4$$
$$= (8 - 4\ln 4) - (4 - 4\ln 2)$$
$$= 8 - 4\ln(2^2) - 4 + 4\ln 2 = 4 - 4\ln 2 \;//$$

Let $x = u^2$

$\frac{dx}{du} = 2u$

$(dx = 2u\,du)$

If $x=0$, $u=0$

If $x=4$, $u=2$

Let $v = 2+u$

$\frac{dv}{du} = 1$

$(du = dv)$

If $u=0$, $v=2$

If $u=2$, $v=4$

$u = v-2$

$2u = 2v-4$

5iii) $\therefore \quad \frac{11}{4} - \sqrt{2} \approx 4 - 4\ln 2$

$$(\times 4) \quad 11 - 4\sqrt{2} \approx 16 - 16\ln 2$$

$$16\ln 2 \approx 4\sqrt{2} + 5$$

$$\ln 2 \approx \frac{5}{16} + \frac{\sqrt{2}}{4} \qquad \left(k = \frac{-7}{16}\right)$$

6i) $\sin\left(2\theta + \frac{\pi}{4}\right) = 3\cos\left(2\theta + \frac{\pi}{4}\right)$

$$\sin 2\theta \cos\frac{\pi}{4} + \cos 2\theta \sin\frac{\pi}{4} = 3\cos 2\theta \cos\frac{\pi}{4} - 3\sin 2\theta \sin\frac{\pi}{4}$$

$$\frac{\cancel{\sqrt{2}}}{\cancel{2}}\sin 2\theta + \frac{\cancel{\sqrt{2}}}{\cancel{2}}\cos 2\theta = \frac{3\cancel{\sqrt{2}}}{\cancel{2}}\cos 2\theta - \frac{3\cancel{\sqrt{2}}}{\cancel{2}}\sin 2\theta$$

$$4\sin 2\theta = 2\cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2}{4} \Rightarrow \tan 2\theta = \frac{1}{2}$$

ii) From (i) $\quad \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{1}{2}$

Let $t = \tan\theta$, $\quad \frac{2t}{1-t^2} = \frac{1}{2}$

$$4t = 1 - t^2$$

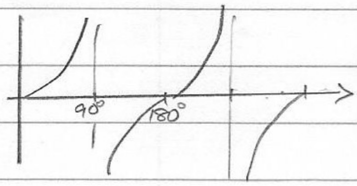
$$t^2 + 4t - 1 = 0$$

$$t = \frac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2}$$

$$t = \frac{-4 \pm \sqrt{20}}{2} = \frac{-\cancel{4}^{2} \pm \cancel{2}\sqrt{5}}{\cancel{2}} = -2 \pm \sqrt{5}$$

$\tan\theta = -2 + \sqrt{5} \quad$ or $\quad \tan\theta = -2 - \sqrt{5}$

$= 0.236\ldots \qquad = -4.236$



Given that $90° \le \tan\theta \le 180°$ (ie. obtuse)

then $\tan\theta < 0$ (see graph)

$\therefore \tan\theta = -2 - \sqrt{5}$

7) $(2x-1)^3 \frac{dy}{dx} + 4y^2 = 0$

(Q7 contd)

$$(2x-1)^3\frac{dy}{dx} = -4y^2$$

$$\int \frac{1}{y^2}\,dy = \int \frac{-4}{(2x-1)^3}\,dx$$

$$\int y^{-2}\,dy = \int -4(2x-1)^{-3}\,dx$$

$$-y^{-1} = +2(2x-1)^{-2} + c$$

GS $$\frac{1}{y} = -\frac{2}{(2x-1)^2} + A.$$

If $x=1, y=1$ then $1 = -\frac{2}{(1)^2} + A \Rightarrow A = 3.$

$$\frac{1}{y} = -\frac{2}{(2x-1)^2} + 3.$$

$$= \frac{-2 + 3[(2x-1)^2]}{(2x-1)^2}$$

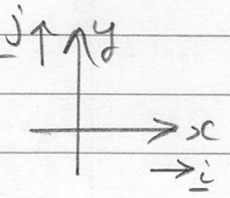
$$= \frac{-2 + 3[4x^2-4x+1]}{4x^2-4x+1}$$

$$= \frac{-2 + 12x^2 - 12x + 3}{4x^2-4x+1}$$

$$\frac{1}{y} = \frac{12x^2-12x+1}{4x^2-4x+1}$$

$$\therefore \quad y = \frac{4x^2-4x+1}{12x^2-12x+1}$$ $(a=4,\ b=12)$

Section B: Mechanics



8) i) Force due to gravity = weight

$= -mg\underline{j}$

$= -(5)(9.8)\underline{j}$

$= -49\underline{j}$

$(\underline{F}=m\underline{a})$ $\quad -49\underline{j} + (15\underline{i}-8\underline{j}) + (-7\underline{i}-2\underline{j}) = (5)(\underline{a})$

$\therefore 5\underline{a} = 8\underline{i} - 59\underline{j}$

$\underline{a} = 1.6\underline{i} - 11.8\underline{j}$ // or $\begin{pmatrix}1.6\\-11.8\end{pmatrix}$

ii) $\underline{s} = ?$
$\underline{u} = 0$
$\underline{v} = \times$
$\underline{a} = 1.6\underline{i} - 11.8\underline{j}$
$t = 10$

$\underline{s} = \cancel{\underline{u}t} + \frac{1}{2}\underline{a}t^2$

$\underline{s} = \frac{1}{2}(1.6\underline{i} - 11.8\underline{j})(10^2)$

$\underline{s} = 80\underline{i} - 590\underline{j}$

Initially at point $\underline{r}_0 = 2\underline{i} + 45\underline{j}$

$\therefore \underline{r} = \underline{r}_0 + \underline{s}$

$= (2\underline{i} + 45\underline{j}) + (80\underline{i} - 590\underline{j})$

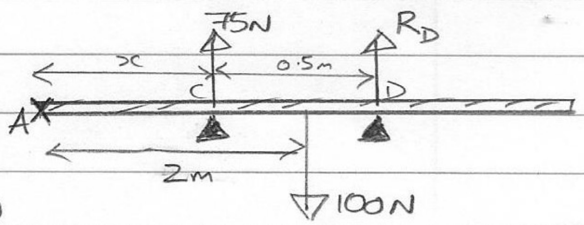
$= 82\underline{i} - 545\underline{j}$ //

9 i) (Res ↑) $\quad 75 + R_D = 100 \quad$ ①

(Moments about A)

$(75)(x) + (R_D)(x+0.5) = (100)(2)$

$75x + R_D(x+0.5) = 200 \quad$ ②



From ①, $R_D = 25$ N //

ii) Sub in ② $\quad 75x + 25(x+0.5) = 200$

$75x + 25x + 12.5 = 200$

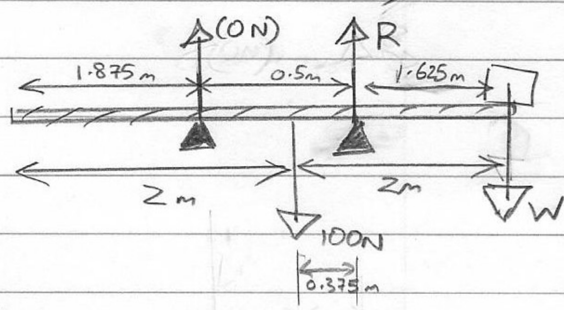
$100x = 187.5 \Rightarrow x = 1.875$ m //

iii) Tilting about D ⇒ reaction force at C = zero.

(Moments about D)

$(W)(1.625) = (100)(0.375)$

$W = 23.1$ N // (3 s.f.)

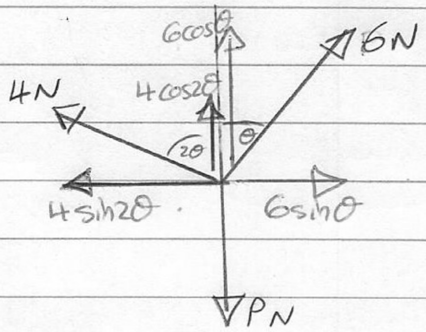


iv) a) Doesn't allow for the width of the rock //

b) Doesn't allow for bending of the plank //

10) i) (Res ↑) $4\cos 2\theta + 6\cos\theta = P$ — ①

(Res →) $4\sin 2\theta = 6\sin\theta$ — ②



From ② $4\sin 2\theta - 6\sin\theta = 0$

$4(2\sin\theta\cos\theta) - 6\sin\theta = 0$

$8\sin\theta\cos\theta - 6\sin\theta = 0$

$2\sin\theta(4\cos\theta - 3) = 0$

$\sin\theta = 0$ or $\cos\theta = \frac{3}{4}$

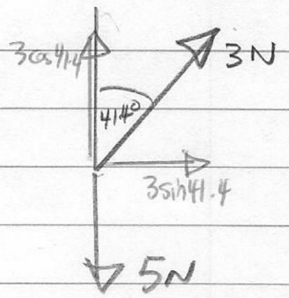
(not applicable) $\theta = 41.4°$ (3 sf.)

ii) From ① $P = 4\cos(82.8°) + 6\cos(41.4°)$

$P = 5$ N

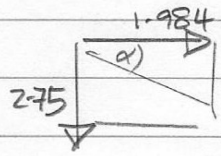
iii) a) Resultant force $= \underline{R} = 3\sin 41.4\underline{i} + 3\cos 41.4\underline{j} - 5\underline{j}$

$= 1.984\underline{i} - 2.75\underline{j}$



Magnitude $= \sqrt{1.984^2 + 2.75^2}$

$= 3.39$ N (3sf)



b) Angle with horizontal $= \alpha$

$= \tan^{-1}\left(\frac{2.75}{1.984}\right) = 54.2°$

Direction of resultant = to right, 54.2° below horizontal

11 i) $v = kt + 0.03t^2$ — (*)

$a = \frac{dv}{dt} = k + 0.06t$.

If $t = 20$, $a = 1.3$ so $1.3 = k + 0.06(20)$

$k = 0.1$

ii) $v = 0.1t + 0.03t^2$ — ①

Integrating, $x = \frac{0.1t^2}{2} + \frac{0.03t^3}{3} + c$

$x = 0.05t^2 + 0.01t^3 + c$

Let $x = 0$ when $t = 0$, so $0 = \cancel{0} + \cancel{0} + c \Rightarrow c = 0$.

$x = 0.05t^2 + 0.01t^3$ — ②

Phase 1: Before $t = 20$ (var. accel.)

If $t = 20$, $v = 0.1(20) + 0.03(20)^2$

$= 14$

(Q11ii contd) If $t=20$, $x = 0.05(20)^2 + 0.01(20)^3$
$= 100$

Car changes direction when $v=0$, ie. $0.1t + 0.03t^2 = 0$

$t(0.1 + 0.03t) = 0$

$t=0$ or $t = -\frac{10}{3}$

$\therefore$ ~~Does~~ Doesn't change direction between $t=0$ & $t=20$,
so distance covered $= 100$ m.

Phase II : After $t=20$ (const. accel.)

$s = ?$
$u = 14$
$v = 25$
$a = 1.3$
$t = \times$

$v^2 = u^2 + 2as$

$25^2 = 14^2 + 2(1.3)s$

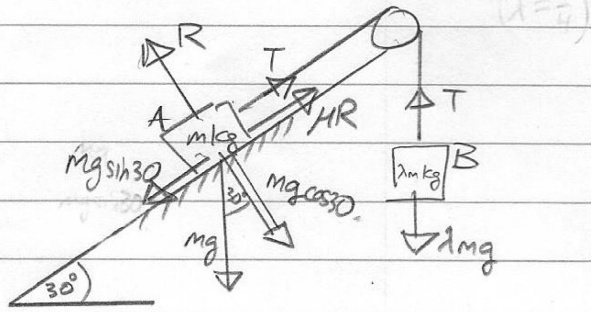
$s = \frac{25^2 - 14^2}{2(1.3)} = 165$ m

$\therefore$ Total distance covered $= 100 + 165$
$= 265$ m

12ia) (A, res ↖) $R = mg\cos 30$ — ①

(A, res ↙) $mg\sin 30 = T + \mu R$ — ②

(B, res ↑) $T = \lambda mg$ — ③



Sub ① and ③ into ②

$\cancel{m}\cancel{g}\sin 30 = \lambda \cancel{m}\cancel{g} + \mu . \cancel{m}\cancel{g}\cos 30$

$\frac{1}{2} = \frac{1}{4} + \frac{\mu\sqrt{3}}{2}$

$(\times 4)$ $2 = 1 + 2\mu\sqrt{3}$

$2\mu\sqrt{3} = 1 \Rightarrow \mu = \frac{\sqrt{3}}{6}$

b) If $\lambda \neq \frac{1}{4}$, then $\frac{1}{2} = \lambda + \frac{\mu\sqrt{3}}{2}$

$(\times 2)$ $1 = 2\lambda + \sqrt{3}\mu$

$2\lambda = 1 - \sqrt{3}\mu$

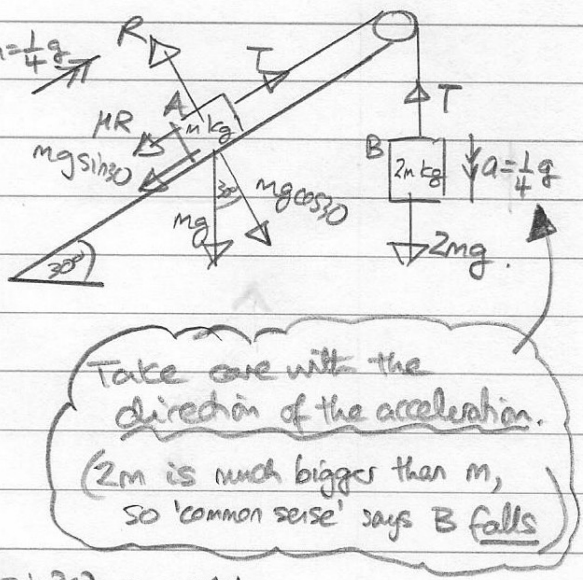
$\lambda = \frac{1 - \sqrt{3}\mu}{2}$

For a rough plane $\mu > 0$, so therefore $\lambda < \frac{1}{2}$

12ii) (A, res ↖) $\quad R = mg\cos 30 \quad -①$

(A, F=ma ↗) $\quad T - \mu R - mg\sin 30 = m\left(\frac{1}{4}g\right) \quad -②$

(B, F=ma ↓) $\quad 2mg - T = 2m\left(\frac{1}{4}g\right) \quad -③$



Rearrange ③ $\quad T = 2mg - \frac{1}{2}mg$

$$T = \frac{3mg}{2}$$

Sub this (and ①) into ②

$$\frac{3\cancel{mg}}{2} - \mu(\cancel{mg}\cos 30) - \cancel{mg}\sin 30 = \frac{\cancel{mg}}{4}$$

$$\frac{3}{2} - \mu\frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{1}{4}$$

$(\times 4)$ $\quad 6 - 2\mu\sqrt{3} - 2 = 1$

$$2\mu\sqrt{3} = 3$$

$$\mu = \frac{\sqrt{3}}{2}$$