Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
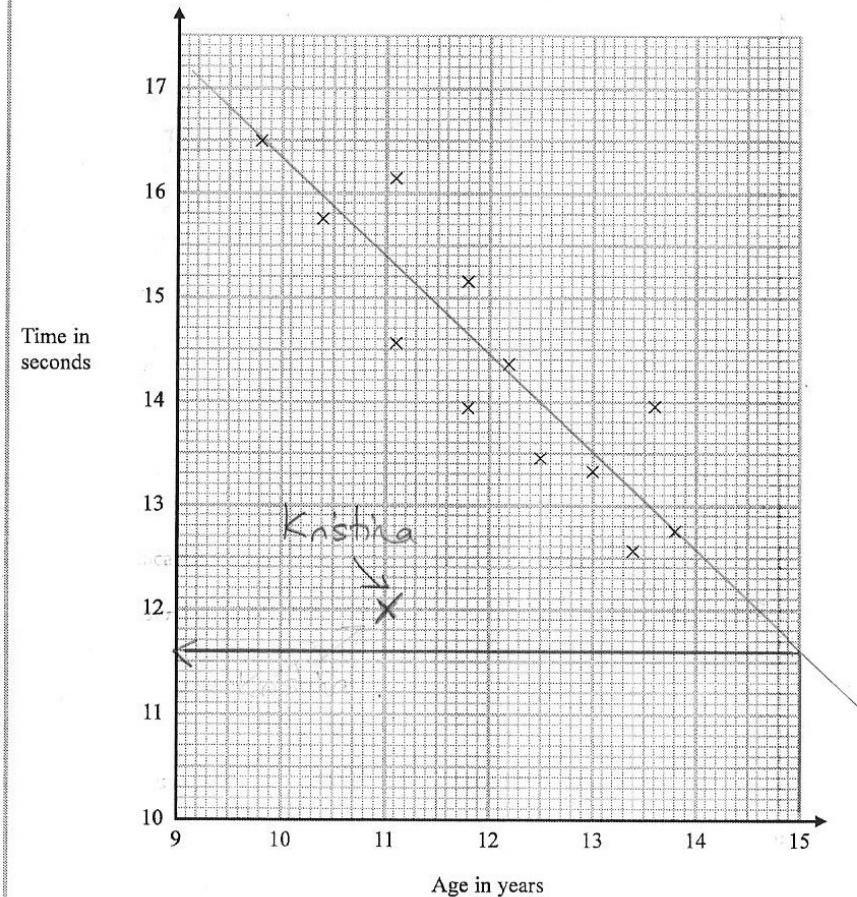- Check your answers if you have time at the end.

1   The scatter diagram shows information about 12 girls.

It shows the age of each girl and the best time she takes to run 100 metres.

(a) Write down the type of correlation.

Negative

(1)

Kristina is 11 years old.
Her best time to run 100 metres is 12 seconds.

The point representing this information would be an outlier on the scatter diagram.

(b) Explain why.

The point for Kristina is far away from the
line of best fit (shown)

(1)

Debbie is 15 years old.
Debbie says,
    "The scatter diagram shows I should take less than 12 seconds to run 100 metres."

(c) Comment on what Debbie says.

This is outside the range of given data,
so is an unreliable prediction (extrapolation)

(1)

(Total for Question 1 is 3 marks)
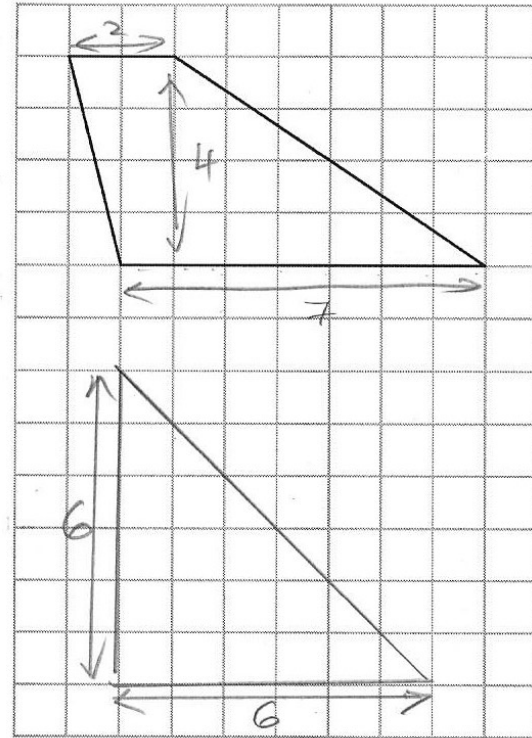
2   Expand and simplify $5(p + 3) - 2(1 - 2p)$

$$= 5p + 15 - 2 + 4p$$

$$= 9p + 13$$

$9p + 13$

(Total for Question 2 is 2 marks)

3   Here is a trapezium drawn on a centimetre grid.

On the grid, draw a triangle equal in area to this trapezium.

Area of trapezium $= \frac{1}{2}(a+b)h = \frac{1}{2}(2+7)(4)$

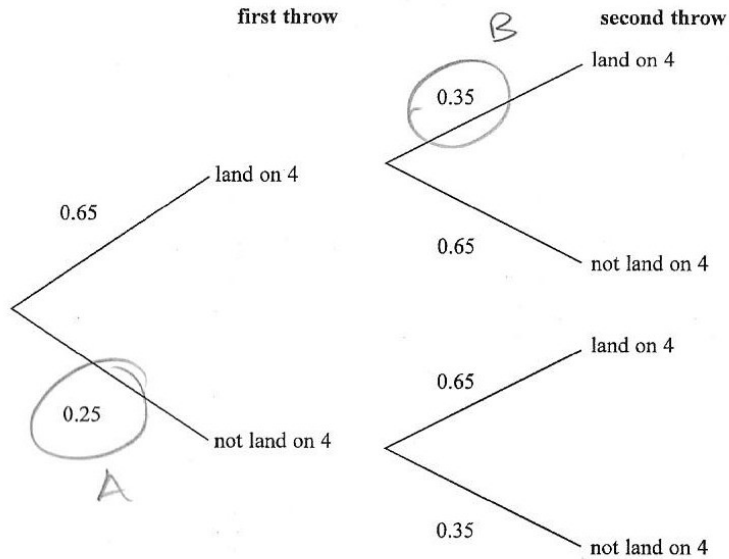$$= 18$$

Area of triangle $= \frac{1}{2}bh = 18$

$$\Rightarrow bh = 36$$

Different options, e.g. base = 6, height = 6
[or base = 9, height = 4 etc]

(Total for Question 3 is 2 marks)

4 When a biased 6-sided dice is thrown once, the probability that it will land on 4 is 0.65
The biased dice is thrown twice.

Amir draws this probability tree diagram.
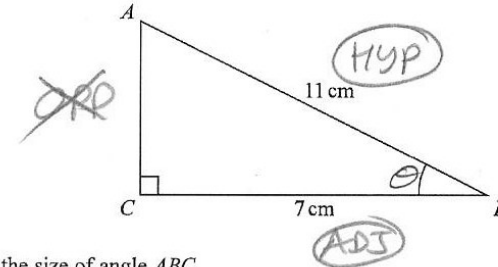The diagram is **not** correct.

| first throw | | second throw |

B

first throw:
0.65 — land on 4
0.25 (circled) — not land on 4 — *A*

second throw (top):
0.35 (circled) — land on 4
0.65 — not land on 4

second throw (bottom):
0.65 — land on 4
0.35 — not land on 4

Write down **two** things that are wrong with the probability tree diagram.

1 The probability marked Ⓐ should be 0.35

2 The probability marked Ⓑ should be 065

(Total for Question 4 is 2 marks)

---

5 *ABC* is a right-angled triangle.

(SOH CAH TOA)

(a) Work out the size of angle *ABC*.
Give your answer correct to 1 decimal place.

$$\cos\theta = \frac{ADJ}{HYP}$$

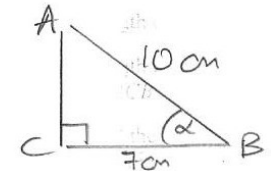$$\cos\theta = \frac{7}{11}$$

$$\theta = \cos^{-1}\left(\frac{7}{11}\right) = 50.4788\ldots$$

50.5°

(2)

The length of the side *AB* is reduced by 1 cm.

The length of the side *BC* is still 7 cm.
Angle *ACB* is still 90°

(b) Will the value of cos *ABC* increase or decrease?
You must give a reason for your answer.

Before, $\cos ABC = \frac{7}{11} = 0.636\ldots$

After, $\cos ABC = \frac{7}{10} = 0.7$ so it has increased

(1)

(Total for Question 5 is 3 marks)

6 There are some counters in a bag.
The counters are red or white or blue or yellow.

Bob is going to take at random a counter from the bag.

The table shows each of the probabilities that the counter will be blue or will be yellow.

Colour	red	white	blue	yellow
Probability			0.45	0.25

There are 18 blue counters in the bag.

The probability that the counter Bob takes will be red is twice the probability that the counter will be white.

(a) Work out the number of red counters in the bag.

Let $P(\text{white}) = p$

so $P(\text{red}) = 2p$

then $2p + p + 0.45 + 0.25 = 1$

$3p = 0.3 \Rightarrow p = 0.1$

$P(\text{red}) = 2p = 0.2$

$(\text{blue}) = 0.45 \rightarrow 18$ counters $\Big) \div 0.45$

$1.0 \rightarrow 40$ counters $\Big) \times 0.2$

$(\text{red}) = 0.2 \rightarrow 8$ counters

8

(4)

A marble is going to be taken at random from a box of marbles.
The probability that the marble will be silver is 0.5

There must be an even number of marbles in the box.

(b) Explain why.

$P(\text{silver}) = 0.5 = \frac{1}{2}$. If the number of marbles was odd, the $P(\text{silver})$ could not equal to $\frac{1}{2}$.

(1)

(Total for Question 6 is 5 marks)

---

7 Solve $\dfrac{5 - x}{2} = 2x - 7$

$\left(\dfrac{5-x}{2}\right) \times 2 = (2x - 7) \times 2$

$5 - x = 4x - 14$

$-x - 4x = -14 - 5$

$-5x = -19$

$x = \dfrac{19}{5} = 3\frac{4}{5}$

$x = \underline{3\frac{4}{5}}$

(Total for Question 7 is 3 marks)

OR 3.8
OR $\dfrac{19}{5}$

8 *ABCDE* is a pentagon.

A pentagon *ABCDE* with angle at *A* = 125°, angle at *E* = 115°, angle at *B* = *x*, angle at *C* = 2*x*, and a right angle at *D*.

Angle *BCD* = 2 × angle *ABC*

Work out the size of angle *BCD*.
You must show all your working.

Let $\angle ABC = x$

then $\angle BCD = 2x$

Total of interior angles = 3 × 180
= 540

Pentagon = 3 triangles

∴ 115 + 125 + x + 2x + 90 = 540

3x + 330 = 540

3x = 210

x = 70°

∴ ∠BCD = 2 × 70
= 140°

140°

(Total for Question 8 is 5 marks)

---

9 $T = \sqrt{\dfrac{w}{d^3}}$

$w = 5.6 \times 10^{-5}$
$d = 1.4 \times 10^{-4}$

(a) Work out the value of *T*.
Give your answer in standard form correct to 3 significant figures.

$T = \sqrt{\dfrac{5.6 \times 10^{-5}}{(1.4 \times 10^{-4})^3}} = 4517.5395\ldots$

$= 4.5175395\ldots \times 10^3$

$= 4.52 \times 10^3$

$T = \underline{4.52 \times 10^3}$

(2)

w is increased by 10%
d is increased by 5%

Lottie says,

"The value of *T* will increase because both *w* and *d* are increased."

(b) Lottie is wrong.
Explain why.

new $w = 5.6 \times 10^{-5} \times 1.1$
$= 6.16 \times 10^{-5}$

new $d = 1.4 \times 10^{-4} \times 1.05$
$= 1.47 \times 10^{-4}$

new $T = 4403.665\ldots$

Lottie is wrong, as T has decreased from 4518 to 4404

(2)

(Total for Question 9 is 4 marks)

10 Here are three lamps.

lamp **A** lamp **B** lamp **C**

Lamp A flashes every 20 seconds.
Lamp B flashes every 45 seconds.
Lamp C flashes every 120 seconds.

The three lamps start flashing at the same time.

How many times in one hour will the three lamps flash at the same time?

A = 20, 40, 60, 80, 100, 120, 140, 160,
        180, 200, 220, 240...

B = 45, 90, 135, 180, 225, 270, 315,
        360, 405, 450, 495, 540,...

C = 120, 240, 360, 480, 600...

LCM = 360 secs = 6 mins

Lamps flash at the same time every 6 mins,
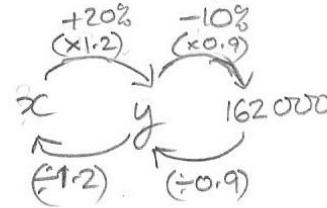   ie. 10 times per hour

10

(Total for Question 10 is 3 marks)

11 In 2003, Jerry bought a house.

In 2007, Jerry sold the house to Mia.
He made a profit of 20%

In 2012, Mia sold the house for £162 000
She made a loss of 10%

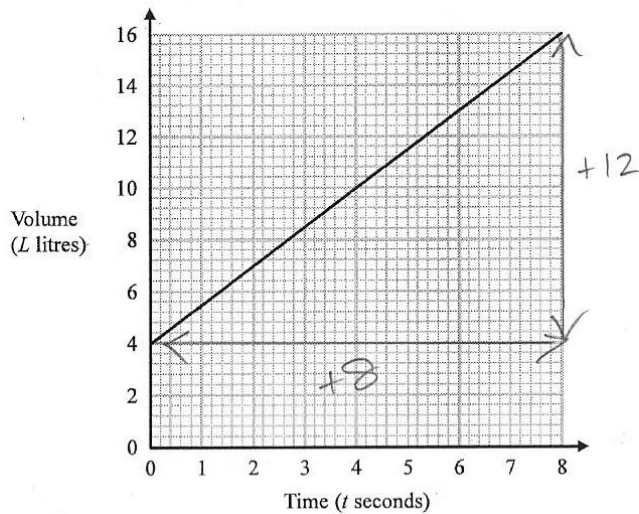Work out how much Jerry paid for the house in 2003

$+20\%$ -10%
$(\times 1.2)$ $(\times 0.9)$
x y 162 000
$(\div 1.2)$ $(\div 0.9)$

y = price in 2007 = 162 000 ÷ 0.9
                    = 180 000

x = price in 2003 = 180 000 ÷ 1.2
                    = 150 000

£ 150 000

(Total for Question 11 is 3 marks)

12 The graph shows the volume of liquid (L litres) in a container at time t seconds.

Volume
(L litres)

+12

+8

Time (t seconds)

(a) Find the gradient of the graph.

$$Grad = \frac{d \cdot up}{d \cdot ac} = \frac{12}{8} = \frac{3}{2}$$

$\frac{3}{2}$

(2)

(b) Explain what this gradient represents.

$\frac{3}{2}$ = 1.5 litres per second. This represents the increase of liquid in the tank per second
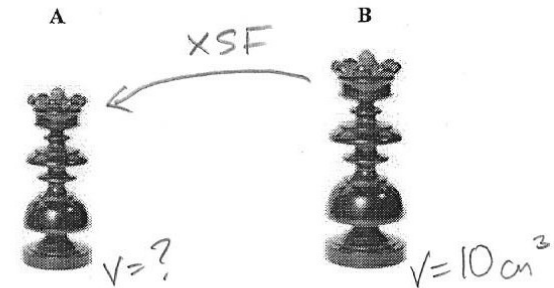
(1)

The graph intersects the volume axis at $L = 4$

(c) Explain what this intercept represents.

$L = 4$ litres. This represents the amount of liquid in the tank at the start

(1)

(Total for Question 12 is 4 marks)

---

13 Here are two similar solid shapes.

A   B

×SF

V=?   V = 10 cm³

surface area of shape **A** : surface area of shape **B** = 3 : 4

The volume of shape **B** is 10 cm³

Work out the volume of shape **A**.
Give your answer correct to 3 significant figures.

Area scale factor = ASF = $(SF)^2$

Ratio of areas = 3:4

$\therefore$ ASF = $\frac{3}{4}$ so $(SF)^2 = \frac{3}{4}$

$SF = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ (=0.866)

New vol = old vol × SF^3

$= 10 \times \left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{15\sqrt{3}}{4} = 6.49519....$

6.50 cm³

(Total for Question 13 is 3 marks)

14 There are 16 hockey teams in a league.
Each team played two matches against each of the other teams.

Work out the total number of matches played.

Each team plays $2 \times 15 = 30$ matches.

If 16 teams plays 30 matches,

then $16 \times 30 = 480$ matches

HOWEVER each match counted twice,

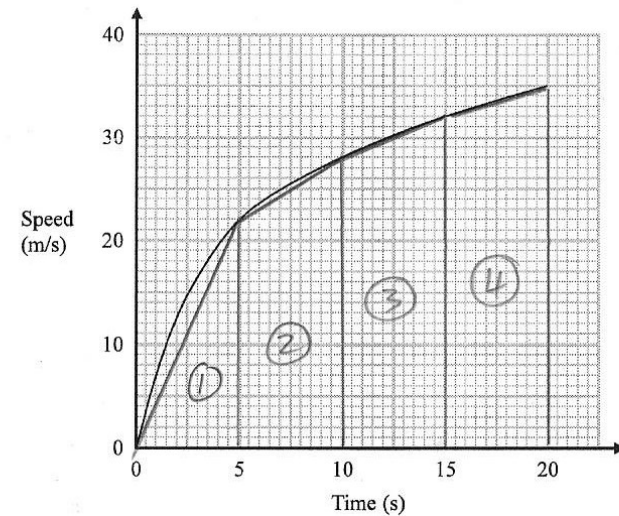so actual number of matches $= 480 \div 2$
$= 240$

240

(Total for Question 14 is 2 marks)

Alternative method

• If teams are A, B, C, D, ...

• Imagine a table showing AvB and BvA.

	A	B	C	D
A	✗			
B		✗		
C			✗	
D				✗

• Cross out AvA (team cannot play itself)

• Number of matches = (columns) × (rows) − (crosses)

$= 16 \times 16 - 16$

$= 240$

15 The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.

(a) Work out an estimate for the distance the car travelled in the first 20 seconds.
Use 4 strips of equal width.

Distance covered = Area under graph

Area of ① $= \frac{1}{2}bh = \frac{1}{2}(5)(22) = 55$

Area of ② $= \frac{1}{2}(a+b)h = \frac{1}{2}(22+28)(5) = 125$

Area of ③ $= \frac{1}{2}(28+32)(5) = 150$

Area of ④ $= \frac{1}{2}(32+35)(5) = 167.5$

Total $= 55 + 125 + 150 + 167.5$

$= 497.5$

497.5 metres

(3)

(b) Is your answer to part (a) an underestimate or an overestimate of the actual distance the car travelled in the first 20 seconds?
Give a reason for your answer.

Answer is an underestimate, as tops of trapeziums are below the actual curve

(1)

(Total for Question 15 is 4 marks)

16 The nth term of a sequence is given by $an^2 + bn$ where a and b are integers.

The 2nd term of the sequence is -2
The 4th term of the sequence is 12

(a) Find the 6th term of the sequence.

If $n=2$, 2nd term $= a(2)^2 + b(2) = -2$

$\Rightarrow 4a + 2b = -2$

$2a + b = -1$ —①

If $n=4$, 4th term $= a(4)^2 + b(4) = 12$

$16a + 4b = 12$

$4a + b = 3$ —②

② −①

$2a = 4$

$a = 2$

Sub in ① $4 + b = -1$

$b = -5$

6th term $= 2(6)^2 - 5(6) =$

42

(4)

Here are the first five terms of a different quadratic sequence.

$$0 \qquad 2 \qquad 6 \qquad 12 \qquad 20$$

(b) Find an expression, in terms of n, for the nth term of this sequence.

seq $= 0 \quad 2 \quad 6 \quad 12 \quad 20$

$\begin{array}{ccccc} & +2 & +4 & +6 & +8 \\ & & +2 & +2 & +2 \end{array}$

$n^2 = 1 \quad 4 \quad 9 \quad 16 \quad 25$

$-1 \quad -2 \quad -3 \quad -4 \quad -5$

$-n = -1 \quad -2 \quad -3 \quad -4 \quad -5$

$0 \quad 0 \quad 0 \quad 0 \quad 0$

nth term $= n^2 - n$

$n^2 - n$
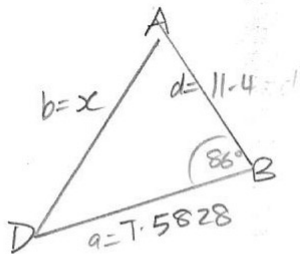
(2)

(Total for Question 16 is 6 marks)

17

Work out the length of AD.
Give your answer correct to 3 significant figures.

$$\frac{y}{\sin 35} = \frac{12.5}{\sin 109}$$

$$y = \frac{12.5 \sin 35}{\sin 109}$$

$$y = 7.5828$$

$$b^2 = a^2 + d^2 - 2ad \cos B$$

$$x^2 = 11.4^2 + 7.5828^2$$
$$- 2(11.4)(7.5828)\cos 86$$

$$x^2 = 175.3988$$

$$x = 13.2438\ldots$$

13.2 _____ cm

(Total for Question 17 is 5 marks)

---

18 (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

If $x = 1$, $\quad x^3 + x = 1^3 + 1 = 2$ (too small)

If $x = 2$, $\quad x^3 + x = 2^3 + 2 = 10$ (too big)

So must be a solution between $x = 1$ and $x = 2$

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$$x^3 = 7 - x$$
$$x = \sqrt[3]{7 - x}$$

(1)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$$x_0 = 2$$
$$x_1 = \sqrt[3]{7 - 2} = 1.709975\ldots$$
$$x_2 = \sqrt[3]{1.709975 - 2} = 1.742418\ldots$$

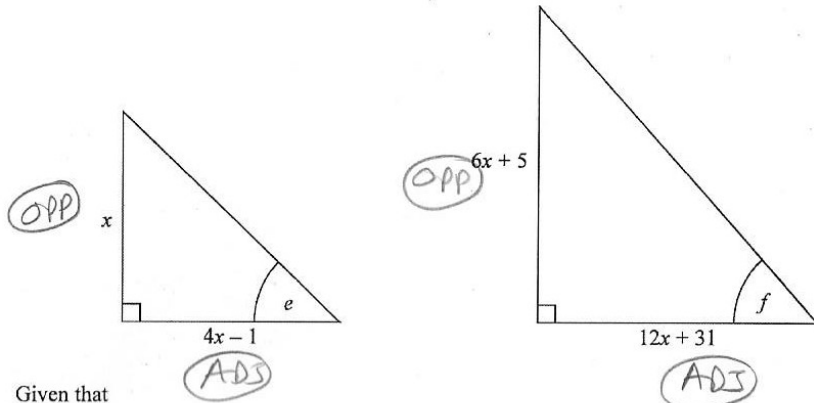
$$x_2 = \sqrt[3]{1.709975 - 2} = 1.742418\ldots$$
$$x_3 = 1.7388495\ldots$$

Solution is approx 1.74 (3 sf)

1.74

(3)

(Total for Question 18 is 6 marks)

19 Here are two right-angled triangles.

OPP

x

OPP $6x + 5$

e

f

$4x - 1$

ADJ

$12x + 31$

ADJ

Given that

$\tan e = \tan f$ ——— ⓧ

find the value of x.

You must show all your working.

$\tan e = \dfrac{OPP}{adj} = \dfrac{x}{4x-1}$

$\tan f = \dfrac{6x+5}{12x+31}$

Using ⓧ

$\dfrac{x}{4x-1} = \dfrac{6x+5}{12x+31}$

$x(12x+31) = (6x+5)(4x-1)$

$12x^2 + 31x = 24x^2 - 6x + 20x - 5$

$12x^2 + 31x - 24x^2 + 6x - 20x + 5 = 0$

$-12x^2 + 17x + 5 = 0$

(×-1) $12x^2 - 17x - 5 = 0$

$(3x - 5)(4x + 1) = 0$ (x cannot be negative!)

$x = \dfrac{5}{3}$ or $x \neq -\dfrac{1}{4}$

$x = \dfrac{5}{3}$

(Total for Question 19 is 5 marks)

---

20 50 people were asked if they speak French or German or Spanish.

Of these people,
   31 speak French
   2 speak French, German and Spanish
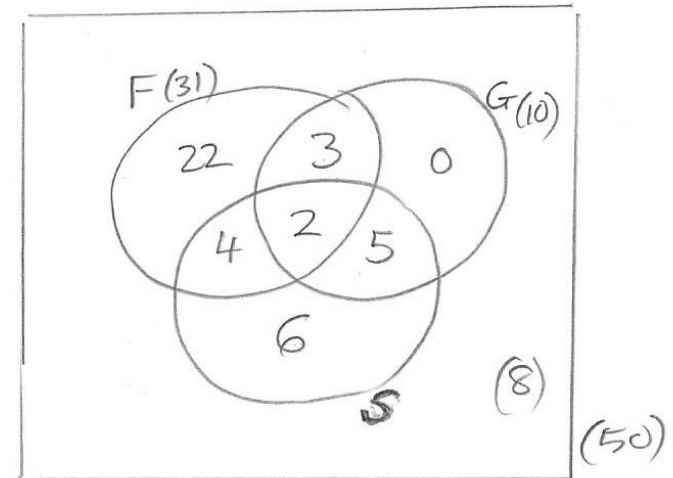   4 speak French and Spanish but not German
   7 speak German and Spanish
   8 do not speak any of the languages
   all 10 people who speak German speak at least one other language

Two of the 50 people are chosen at random.

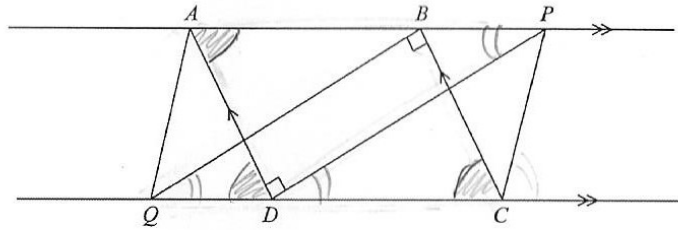Work out the probability that they both only speak Spanish.

F (31)   G (10)

22   3   0

4   2   5

6

S   (8)

(50)

P(both people speak only spanish) = $\dfrac{6}{50} \times \dfrac{5}{49}$

$= \dfrac{3}{245}$
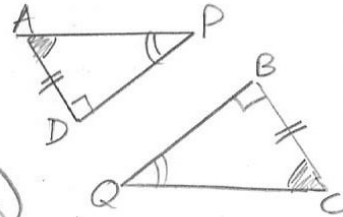
$\dfrac{3}{245}$

(Total for Question 20 is 5 marks)

21

ABCD is a parallelogram.
ABP and QDC are straight lines.
Angle ADP = angle CBQ = 90°

(a) Prove that triangle ADP is congruent to triangle CBQ.

∠PAD = ∠ADQ (alternate angles)
and ∠ADQ = ∠BCQ (corresponding angles)

So   ∠PAD = ∠BCQ
Similarly ∠APD = ∠BQC
Also   length AD = length BC (opposite sides of a parallelogram)

∴ congruent triangles (all three angles match, and one side)

(3)

(b) Explain why AQ is parallel to PC.

∠BQD = ∠BPD
So BQ and PD are parallel
Also ABP and QDC are parallel
∴ BPDQ is a parallelogram.
From this BP = QD and AB = DC
So AP = QC thus APCQ is a parallelogram.
Therefore AQ is parallel to PC

(2)

(Total for Question 21 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS