A Level Sample Paper 1 (Edexcel)
Worked solutions

1a) $y = 3x^4 - 8x^3 - 3$

i) $\dfrac{dy}{dx} = 12x^3 - 24x^2$ //

ii) $\dfrac{d^2y}{dx^2} = 36x^2 - 48x$ //

b) If $x=2$, $\dfrac{dy}{dx} = 12(8) - 24(4) = 0$ $\therefore$ stat. pt at $x=2$ //

c) If $x=2$, $\dfrac{d^2y}{dx^2} = 36(4) - 48(2) = 48 > 0$ $\therefore$ min. pt //

2a) Arc $= r\theta \Rightarrow 3 = r(0.4)$

$r = \dfrac{3}{0.4} = \dfrac{15}{2}$ $\therefore OD = 7.5$ cm //

b) $AO = 12 - 7.5 = 4.5$ cm

Angle $AOC = \pi - 0.4$

Angles on a straight line add to $180°$ (In radians, $180° = \pi$)

$\therefore$ Area of sector $= \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2}(4.5)^2(\pi - 0.4) = 27.8$ cm^2 // (3 s.f.)

3a) $x^2 + y^2 - 4x + 10y = k$

$x^2 - 4x + y^2 + 10y = k$

$(x-2)^2 - 4 + (y+5)^2 - 25 = k$

$(x-2)^2 + (y+5)^2 = k + 29$

Centre $= (2, -5)$

b) radius > 0 so $k + 29 > 0$

$k > -29$ //

4). $\displaystyle\int_a^{2a} \frac{t+1}{t}\,dt = \int_a^{2a}\left(1 + \frac{1}{t}\right)dt = \Big[t + \ln t\Big]_a^{2a}$

$= (2a + \ln(2a)) - (a + \ln a)$

$= 2a + \ln(2a) - a - \ln a$

$$= a + \ln\left(\frac{2a}{a}\right) = a + \ln 2$$

If Integral $= \ln 7$ then $a + \ln 2 = \ln 7$

$$a = \ln 7 - \ln 2$$

$$a = \ln\left(\frac{7}{2}\right)$$

5) $x = 2t - 1$ ① $\qquad y = 4t - 7 + \frac{3}{t}$ ②

Rearrange ① $t = \frac{x+1}{2}$ $\qquad$ Sub in ② $y = \cancel{4}^{2}\left(\frac{x+1}{\cancel{2}}\right) - 7 + \frac{3 \times 2}{\frac{(x+1)}{\cancel{2}} \times \cancel{2}}$

$$= 2x + 2 - 7 + \frac{6}{x+1}$$

$$= \frac{2x - 5}{1} + \frac{6}{x+1}$$

$$= \frac{(2x-5)(x+1) + 6}{x+1}$$

$$= \frac{2x^2 - 3x - 5 + 6}{x+1} = \frac{2x^2 - 3x + 1}{x+1}$$

6ai) $A(0, 16000)$ $\quad B(4, 9000)$

$$\text{Grad}(AB) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9000 - 16000}{4 - 0} = \frac{-7000}{4} = -1750$$

Line through $(0, 16000)$ grad $= -1750$

$$y - 16000 = -1750(x - 0)$$

$$y = -1750x + 16000$$

If $x = 3$, $\quad y = -1750(3) + 16000 = 10\,750$ barrels

ii) Model A indicates volume of oil extracted will fall below zero. This is not a realistic assumption

bi) Assume $V = Ae^{-kt}$

If $t = 0$, $V = 16\,000$ so $16\,000 = Ae^{0} \Rightarrow A = 16\,000$

If $t = 4$, $V = 9\,000$ so $9\,000 = Ae^{-k(4)} \Rightarrow e^{-4k} = \frac{9\,000}{16\,000}$

$$-4k = \ln\left(\frac{9}{16}\right)$$

$$k = -\frac{1}{4}\ln\left(\frac{9}{16}\right) = +0.14384.$$

$\therefore$ Model B is $V = 16000e^{-0.1438t}$

If $t=3$, $V = 16000e^{-0.1438(3)}$

$V = 10393.58\ldots$

$V = 10400$ barrels (3 s.f.)

7) $\vec{AC} = \vec{AB} + \vec{BC} = \begin{pmatrix}2\\3\\1\end{pmatrix} + \begin{pmatrix}1\\-9\\3\end{pmatrix} = \begin{pmatrix}3\\-6\\4\end{pmatrix}$

If $\theta = \angle BAC$ then $\cos\theta = \dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}||\vec{AC}|}$

$$\cos\theta = \frac{\begin{pmatrix}2\\3\\1\end{pmatrix}\cdot\begin{pmatrix}3\\-6\\4\end{pmatrix}}{\sqrt{2^2+3^2+1^2}\sqrt{3^2+6^2+4^2}}$$

$$\cos\theta = \frac{6-18+4}{\sqrt{14}\sqrt{61}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-8}{\sqrt{14}\sqrt{61}}\right) = 105.887\ldots$$

$$= 105.9^\circ \text{ (1dp)}$$

8a) $f(x) = \ln(2x-5) + 2x^2 - 30$

$f(3.5) = \ln(7-5) + 2(3.5)^2 - 30 = -4.8068\ldots$

$f(4) = \ln(8-5) + 2(4)^2 - 30 = +3.0986\ldots$

sign-change & continuous function

$\therefore 3.5 < \text{root} < 4$

b) $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$ ← From formula book

If $x_1 = 4$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 4 - \frac{3.099}{16.67} = 3.814\ldots = 3.81 \text{ (3 s.f.)}$$

c) $f'(x) = \dfrac{1}{2x-5}(2) + 4x = \dfrac{2}{2x-5} + 4x.$

If $f'(x) = 0$ then $\frac{2}{2x-5} + 4x = 0$

$2 + 4x(2x-5) = 0$

$2 + 8x^2 - 20x = 0$

$8x^2 - 20x + 2 = 0$

$(\div 2)$ $4x^2 - 10x + 1 = 0$

$x = \frac{10 \pm \sqrt{100 - 4(4)(1)}}{8} = \frac{10 \pm \sqrt{84}}{8}$

$x = 2.3956\ldots$ and $0.104\ldots$

Given that $x > 2.5$, this means no stationary points in domain.

$\therefore$ curve is increasing everywhere

$\therefore$ only one root of $f(x) = 0$

[Alternative method for (c): Sketch both $y = \ln(x-5)$ and $y = 30 - 2x^2$ and show one intersection only]

9a) LHS $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}$

$= \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta} = 2\operatorname{cosec}2\theta =$ RHS

RHS $= 2\operatorname{cosec}2\theta$

$= \frac{2}{\sin 2\theta}$

$= \frac{\cancel{2}}{\cancel{2}\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

b) $\tan\theta + \cot\theta = 1$

$2\operatorname{cosec}2\theta = 1$

$\frac{2}{\sin 2\theta} = 1$

$2 = \sin 2\theta$

$\sin 2\theta = 2$

No solutions, because $\sin 2\theta$ is always between $+1$ and -1

10) Let $f(\theta) = \sin\theta$

then $f'(\theta) = \lim_{h\to 0} \dfrac{f(\theta+h) - f(\theta)}{h}$ ← see formula book

$= \lim_{h\to 0} \dfrac{\sin(\theta+h) - \sin\theta}{h}$

$= \lim_{h\to 0} \dfrac{\sin\theta\cos h + \cos\theta\sin h - \sin\theta}{h}$

$= \lim_{h\to 0} \left(\dfrac{\sin\theta(\cos h - 1)}{h} + \dfrac{\cos\theta \sin h}{h} \right)$

$= \sin\theta \left(\lim_{h\to 0} \left(\dfrac{\cos h - 1}{h} \right) \right) + \cos\theta \left(\lim_{h\to 0} \left(\dfrac{\sin h}{h} \right) \right)$

$= \cancel{\sin\theta(0)} + \cos\theta(1)$

$= \cos\theta$

11) a) $H = 1.8 + 0.4d - 0.002d^2$

If $H=0$, $1.8 + 0.4d - 0.002d^2 = 0$

$(\times 1000)$ $2d^2 - 400d - 1800 = 0$

$d^2 - 200d - 900 = 0$

$d = \dfrac{200 \pm \sqrt{200^2 - 4(1)(900)}}{2}$

$= \dfrac{200 \pm \sqrt{43600}}{2} = 204.4\ldots \text{ or } -4.403\ldots$

$\therefore$ Distance travelled = 204.4 m

b) 1.8 represents the height (above ground) of the arrow when it is fired

c) $H = 1.8 + 0.4d - 0.002d^2$

$= -0.002d^2 + 0.4d + 1.8$

$= -0.002\left[d^2 - 200d\right] + 1.8$

$= -0.002\left[(d-100)^2 - 10000\right] + 1.8$

$= -0.002(d-100)^2 + 20 + 1.8$

$= -0.002(d-100)^2 + 21.8$ [OR $21.8 - 0.002(d-100)^2$]

d) If $H = 2.1 + 0.4d - 0.002d^2$

$= \ldots$ (as above

$= -0.002(d-100)^2 + 20 + 2.1$

$H = -0.002(d-100)^2 + 22.1$

i) If $d = 100$, $H = 22.1$

$\therefore$ vertex $= (100, 22.1)$

$H_{max} = 22.1 \text{ m}$ //

ii) If $H = H_{max}$, $d = 100 \text{ m}$ //

Alternative method for (d) Differentiate to get $\frac{dH}{dd}$

Solve $\frac{dH}{dd} = 0$ to find H_{max}

12a) $N = aT^b$

Taking logs, $\log N = \log(aT^b)$

$\log N = \log a + \log(T^b)$

$\log N = \log a + b\log T$

$= b\log T + \log a$

$\therefore m = b$ and $c = \log a$ //

b) From graph, $\text{grad} = \frac{\text{d. up}}{\text{d. ac}} = \frac{4.1 - 1.8}{1} = 2.3 \Rightarrow b = 2.3$

$y\text{-int} = 1.8 \Rightarrow \log a = 1.8 \Rightarrow a = 63.1$

$\boxed{N = 63.1 \times T^{2.3}}$

If $T = 3$, $N = 63.1 \times 3^{2.3}$

$= 789.602...$

$= 790$ // (3 s.f.) ← mark scheme allows any answer between 650 and 950

c) Given data goes up to $\log N \approx 4.5$

i.e. $N \approx 10^{4.5} = 31,600$

$N = 1\,000\,000$ is outside the range of given data.

The model may not hold true outside the range of given data //

d) If $T = 0$, $N = 63.1 \times 0^{2.3} = 0$

If $T = 1$, $N = 63.1 \times 1^{2.3} = 63.1$

$\therefore a = 63.1$ represents number of bacteria 1 day after start of experiment //

13a) $x = 2\cos t$ $y = \sqrt{3}\cos 2t$

$\frac{dx}{dt} = -2\sin t$ $\frac{dy}{dt} = -2\sqrt{3}\sin 2t$.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cancel{+}\cancel{2}\sqrt{3}\sin 2t \times \frac{1}{\cancel{+}\cancel{2}\sin t}$$

$$= \frac{\sqrt{3}\sin 2t}{\sin t} = \frac{\sqrt{3} \cdot 2\cancel{\sin t}\cos t}{\cancel{\sin t}}$$

$$= 2\sqrt{3}\cos t$$

b) If $t = \frac{2\pi}{3}$,

$$\begin{cases} x = 2\cos\left(\frac{2\pi}{3}\right) = -1 \\ y = \sqrt{3}\cos\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \\ \frac{dy}{dx} = 2\sqrt{3}\cos\left(\frac{2\pi}{3}\right) = -\sqrt{3} \end{cases}$$

Normal through $\left(-1, -\frac{\sqrt{3}}{2}\right)$ grad $= \frac{1}{\sqrt{3}}$ Note: Normal not tangent

$$y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$$

$$\sqrt{3}y + \frac{3}{2} = x + 1$$

$(\times 2)$ $\quad 2\sqrt{3}y + 3 = 2x + 2$

$$2x - 2\sqrt{3}y - 1 = 0$$

c) Solve

$$\begin{cases} 2x - 2\sqrt{3}y - 1 = 0 & (1) \\ x = 2\cos t & (2) \\ y = \sqrt{3}\cos 2t & (3) \end{cases}$$

Sub (2) and (3) into (1) $\quad 2(2\cos t) - 2\sqrt{3}(\sqrt{3}\cos 2t) - 1 = 0$

$$4\cos t - 6\cos 2t - 1 = 0$$

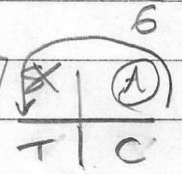
$$4\cos t - 6(2\cos^2 t - 1) - 1 = 0$$

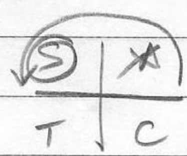
$$4\cos t - 12\cos^2 t + 6 - 1 = 0$$

$$12\cos^2 t - 4\cos t - 5 = 0$$

$$(6\cos t - 5)(2\cos t + 1) = 0$$

$\cos t = \frac{5}{6}$ or $\cos t = -\frac{1}{2}$ $\qquad 0 \leq t \leq \pi$

$\left[\cos^{-1}\left(\frac{5}{6}\right) = 0.5857\ldots\right]$ 

$\left[\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2}{3}\pi\right]$ 

$t = 0.5857\ldots$ $\qquad t = \frac{2\pi}{3}$

If $\cos t = \frac{5}{6}$, $\cos 2t = 2\left(\frac{5}{6}\right)^2 - 1 = \frac{7}{18}$

Sub in (2) $\quad x = 2\left(\frac{5}{6}\right) = \frac{5}{3}$

Sub in 3 $\quad y = \sqrt{3}\left(\frac{7}{18}\right) = \frac{7\sqrt{3}}{18} \qquad \therefore Q = \left(\frac{5}{3}, \frac{7\sqrt{3}}{18}\right)$

14a) $\text{Integral} \approx \frac{1}{2}(0.5)\left\{(3+2.2958) + 2(2.3041+1.9242+1.9089)\right\}$

$= 0.25\{5.2958 + 2(6.1372)\}$

$= 4.39255$

$= 4.39 \quad (3 \text{ s.f})$

b) For a more accurate estimate, use more strips

c) $\text{Area} = \int_1^3 \left(\frac{x^2 \ln x}{3} - 2x + 5\right) dx \quad (*)$

Now $\int \frac{x^2 \ln x}{3} dx = \int \ln x \left(\frac{x^2}{3}\right) dx$

$u = \ln x$	$\frac{dv}{dx} = \frac{x^2}{3}$
$\frac{du}{dx} = \frac{1}{x}$	$v = \frac{x^3}{9}$

$= (\ln x)\left(\frac{x^3}{9}\right) - \int \frac{1}{x} \cdot \frac{x^3}{9} dx$

$= \frac{x^3 \ln x}{9} - \int \frac{x^2}{9} dx$

$= \frac{x^3 \ln x}{9} - \frac{x^3}{27} + c$

$\therefore (*) = \left[\frac{x^3 \ln x}{9} - \frac{x^3}{27} - \frac{\cancel{2}x^2}{\cancel{2}} + 5x\right]_1^3$

$= \left(\frac{\cancel{27}^{3} \ln 3}{\cancel{9}} - \frac{\cancel{27}^{1}}{\cancel{27}_{1}} - 9 + 15\right) - \left(\frac{1 \cancel{\ln 1}}{9} - \frac{1}{27} - 1 + 5\right)$

$= 3\ln 3 - 1 - 9 + 15 + \frac{1}{27} + 1 - 5$

$= \frac{28}{27} + 3\ln 3 = \frac{28}{27} + \ln 27$ ← using $3\ln 3 = \ln 3^3 = \ln 27$

15) $f(x) = \frac{4\sin 2x}{e^{\sqrt{2}x - 1}}$

$f'(x) = \frac{e^{\sqrt{2}x-1} \cdot 8\cos 2x - 4\sin 2x \cdot \sqrt{2} e^{\sqrt{2}x - 1}}{(e^{\sqrt{2}x-1})^2}$

$$= \frac{8\cos 2x\, e^{\sqrt{2}x-1} - 4\sqrt{2}\sin 2x\, e^{\sqrt{2}x-1}}{\left(e^{\sqrt{2}x-1}\right)^2}$$

$$= \frac{4e^{\sqrt{2}x-1}\left[2\cos 2x - \sqrt{2}\sin 2x\right]}{\left(e^{\sqrt{2}x-1}\right)^2}$$

$$= \frac{4\left[2\cos 2x - \sqrt{2}\sin 2x\right]}{e^{\sqrt{2}x-1}}$$

If $f'(x) = 0$, then $\quad \frac{2\cos 2x}{\cos 2x} - \frac{\sqrt{2}\sin 2x}{\cos 2x} = 0$

$$2 - \sqrt{2}\tan 2x = 0$$

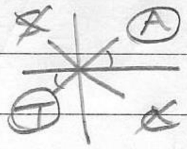
$$\tan 2x = \frac{2}{\sqrt{2}} \Rightarrow \tan 2x = \sqrt{2}$$

b) If $\tan 2x = \sqrt{2}$ $\quad$ ~~$0 \le x \le \pi$~~

$\left[\tan^{-1}\sqrt{2} = 0.9553\right]$ $\quad 0 \le 2x \le 2\pi$

$2x = 0.9553,\ \pi + 0.9553$

$x = 0.4777,\ 2.04845$



From sketch, $x = 2.04845$ at minimum.

i) $f(x) \to f(2x)$ $\quad$ H. stretch by $\times\frac{1}{2}$

$\therefore$ min. pt of $f(2x)$ will be at $x = 2.04845 \times \frac{1}{2}$

$= 1.0242\ldots$

$= 1.02$ (3 s.f.)

ii) $f(x) \to 2f(x)$ $\quad$ V. stretch $\times 2$

$\to -2f(x)$ $\quad$ V. reflection (in x-axis)

$\to -2f(x) + 3$ $\quad$ V. translation $+3$

Because of reflection, max pt of $f(x)$ becomes the min pt of $-2f(x)+3$.

$\therefore$ min pt of $-2f(x)+3$ will be at $x = 0.4777$

$x = 0.478$ (3 s.f.)