A Level Sample Paper 2 (Edexcel)
Worked Solutions

1) $f(x) = 2x^3 - 5x^2 + ax + a$

$f(-2) = 2(-8) - 5(4) + a(-2) + a$

$= -16 - 20 - 2a + a$

factor $\Rightarrow -36 - a = 0$

$-a = 36 \Rightarrow a = -36$

2)a) $\cos\theta = 2\sin\theta$

$1 = \dfrac{2\sin\theta}{\cos\theta} \Rightarrow 2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$

Student divided wrongly, should be $\tan\theta = \frac{1}{2}$ not $\tan\theta = 2$

b i) If $\theta = -26.6$; LHS $= \cos(-26.6) = 0.89\ldots$ } not the same
RHS $= 2\sin(-26.6) = -0.89\ldots$ } $\therefore$ not a solution

ii) the student squared both sides in their first step of working

3) $y = x \cdot (2x+1)^4$

$\dfrac{dy}{dx} = x \cdot 4(2x+1)^3 \cdot 2 + (2x+1)^4 \cdot 1$

$= 8x(2x+1)^3 + (2x+1)^4$

$= (2x+1)^3 [8x + (2x+1)]$

$= (2x+1)^3 (10x+1)$

4)a) $gf(x) = g(f(x))$

$= g(e^x) = 3\ln(e^x)$

$= 3x$

b) $fg(x) = f(g(x))$

$= f(3\ln x) = e^{3\ln x} = e^{\ln(x^3)} = x^3$

If $gf(x) = fg(x)$

$3x = x^3 \Rightarrow x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0$ or $x = \pm\sqrt{3}$

However $x > 0$ so $x = \sqrt{3}$ is the only solution

5)a) $\boxed{m = 25e^{-0.05t}}$ — ①

Note: t is in years (not months)

If $t = 0.5$, $m = 25e^{-0.05(0.5)}$

$= 24.3827\ldots$

$= 24.4\,g$ (3 s.f.)

b) $\frac{dm}{dt} = 25(-0.05)e^{-0.05t}$

$= 1.25e^{-0.05t}$ — ②

Rearrange ① $e^{-0.05t} = \frac{m}{25}$

Sub in ② $\frac{dm}{dt} = 1.25\frac{m}{25} \Rightarrow \frac{dm}{dt} = \frac{1}{20}m$

6i) Let $y = x^2 - 6x + 10$

$y = (x-3)^2 - 9 + 10$

$y = (x-3)^2 + 1$

Statement is ALWAYS TRUE because $x^2 - 6x + 10$ has minimum value of 1 (when $x = 3$).

ii) If $ax > b$ then $x > \frac{b}{a}$

Statement is SOMETIMES TRUE because, when $a > 0$ then $x > \frac{b}{a}$

however when $a < 0$ then $x < \frac{b}{a}$

iii) Examples: If numbers = 2 and 3 then $3^2 - 2^2 = 9 - 4 = 5$ (odd)

If numbers = 9 and 10 then $10^2 - 9^2 = 100 - 81 = 19$ (odd)

} seems to be true

Let numbers = n and $n+1$

then $(n+1)^2 - n^2 = \cancel{n^2} + 2n + 1 - \cancel{n^2} = 2n + 1$

$2n$ is always even, so $(2n+1)$ is always odd.

Statement is ALWAYS TRUE — see proof above.

7a) $\sqrt{4-x} = (4-x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1 + \left(-\frac{x}{4}\right)\right)^{\frac{1}{2}}$

$= 2\left\{1 + \frac{1}{2}\left(-\frac{x}{4}\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(-\frac{x}{4}\right)^2 + \ldots\right\}$

$= 2\left\{1 - \frac{x}{8} - \frac{1}{8}\cdot\frac{x^2}{16} + \ldots\right\}$

$= 2 - \frac{x}{2} - \frac{x^2}{64} + \ldots$

b) Valid for $|x| < 4$ ie. $-4 < x < 4$

Expansion IS VALID for $x = 1$

8)a) At A, $x=0$ so $5y + 2(0) = 10$

$y = 2 \quad \therefore A = (0,2)$

For $5y + 2x = 10$,

$5y = -2x + 10$

$y = -\frac{2x}{5} + 2 \quad \therefore$ Grad(given line) $= -\frac{2}{5}$

Line through $(0,2)$ with grad $= \frac{5}{2}$ ← note: perpendicular gradient

$$y - 2 = \frac{5}{2}(x - 0)$$

$$2y - 4 = 5x \implies 2y - 5x = 4$$

b) At B, $y=0$ so $\cancel{5(0)} + 2x = 10$

$x = 5 \quad \therefore B = (5,0)$

At D, $y=0$ so $\cancel{2(0)} - 5x = 4$

$x = -\frac{4}{5} \quad \therefore D = (-0.8, 0)$

$AB = \sqrt{(5-0)^2 + (0-2)^2}$

$= \sqrt{29}$

$AD = \sqrt{(-0.8-0)^2 + (0-2)^2}$

$= \frac{2\sqrt{29}}{5}$

Area of rectangle $= AB \times AD$

$= \sqrt{29} \times \frac{2\sqrt{29}}{5} = 11.6$

Alternative method for (b) Area of $\triangle ADB = \frac{1}{2} \times DB \times OA$

$= \frac{1}{2} \times (5.8) \times 2 = 5.8$

Area of rectangle $= 2 \times \triangle ADB = 2 \times 5.8 = 11.6$

9a) $\int_1^4 (3\sqrt{x} + A)\,dx = \int_1^4 (3x^{\frac{1}{2}} + A)\,dx$

$$= \left[\frac{\cancel{3}(2)x^{\frac{3}{2}}}{\cancel{3}} + Ax\right]_1^4 = \left[2(\sqrt{x})^3 + Ax\right]_1^4$$

$$= \left(2(\sqrt{4})^3 + A(4)\right) - \left(2(\sqrt{1})^3 + A\right)$$

$= 16 + 4A - 2 - A$

$= 14 + 3A$

If integral $= 2A^2$ then $14 + 3A = 2A^2$

$2A^2 - 3A - 14 = 0$

$(2A - 7)(A + 2) = 0$

$A = \frac{7}{2}$ or $A = -2$

10) Let GP $= a, ar, ar^2, \ldots$

then $S_6 = \dfrac{a(1-r^6)}{1-r}$ and $S_\infty = \dfrac{a}{1-r}$

If $S_\infty = \frac{8}{7} \times S_6$ then $\dfrac{\cancel{a}}{\cancel{1-r}} = \dfrac{8}{7} \cdot \dfrac{\cancel{a}(1-r^6)}{\cancel{(1-r)}}$

$7 = 8(1-r^6)$

$7 = 8 - 8r^6$

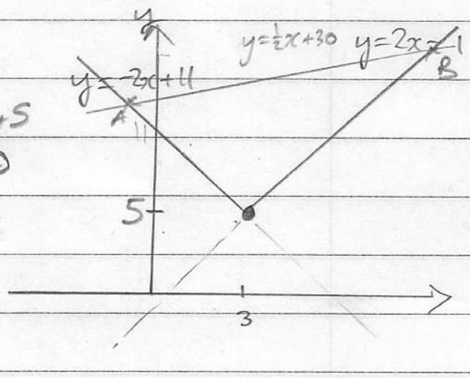
$8r^6 = 1$

$r^6 = \dfrac{1}{8} \Rightarrow r = \pm \dfrac{1}{\sqrt[6]{8}} = \pm \dfrac{1}{\sqrt{2}}$

11a) Branches are $y = 2(3-x)+5$ and $y = -2(3-x)+5$

$y = 6 - 2x + 5$ and $y = -6 + 2x + 5$

$y = -2x + 11$ ① and $y = 2x - 1$ ②



Find intersection by solving ① and ②

$-2x + 11 = 2x - 1$

$-2x - 2x = -1 - 11$

$-4x = -12 \Rightarrow x = 3$

Sub in ① $y = -2(3) + 11 = 5$ $\therefore$ Range $f(x) \geq 5$

b) Ⓐ $-2x + 11 = \frac{1}{2}x + 30$

$-2x - \frac{1}{2}x = 30 - 11$

$-\frac{5}{2}x = 19$

$x = -\dfrac{38}{5}$

(INVALID as $x \geq 0$)

Ⓑ $2x - 1 = \frac{1}{2}x + 30$

$2x - \frac{1}{2}x = 30 + 1$

$\frac{3}{2}x = 31$

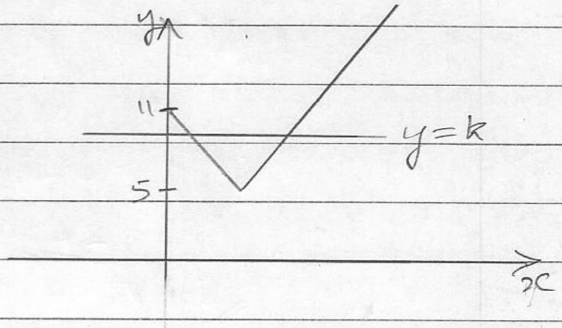
$x = \dfrac{62}{3}$

Ans: $x = \dfrac{62}{3}$ only

c) For two distinct roots,

must have $5 < k \leq 11$

(see sketch)



12a) $3\sin^2 x + \sin x + 8 = 9\cos^2 x$

$3\sin^2 x + \sin x + 8 = 9 - 9\sin^2 x$

$12\sin^2 x + \sin x - 1 = 0$

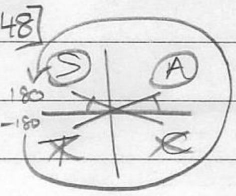
$(4\sin x - 1)(3\sin x + 1) = 0$

$\sin x = \frac{1}{4}$ or $\sin x = -\frac{1}{3}$ $\qquad -180 \leq x < 180$

$[\sin^{-1}(\tfrac{1}{4}) = 14.48]$

$x = 14.48$

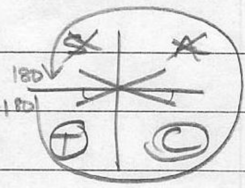
or $180 - 14.48$



$\sin^{-1}(-\tfrac{1}{3}) = -19.47$

RAA $= 19.47$

$x = -19.47, -180 + 19.47$



$x = 14.48°, 165.52°, -19.47°, -160.53°$

b) $3\sin^2(2\theta - 30) + \sin(2\theta - 30) + 8 = 9\cos^2(2\theta - 30)$

from (a) $\quad 2\theta - 30 = 14.48, 162.52, -19.47, -160.53$

$2\theta = 44.48, 192.52, 10.53, -130.53$

$\therefore$ Smallest positive solution $= \frac{10.53}{2} = 5.27°$

13a) $R\cos(\theta + \alpha) = 10\cos\theta - 3\sin\theta$

$R\cos\theta\cos\alpha - R\sin\theta\sin\alpha = 10\cos\theta - 3\sin\theta$

$\left.\begin{array}{l} R\cos\alpha = 10 \\ R\sin\alpha = 3 \end{array}\right\}$ $R = \sqrt{10^2 + 3^2} = \sqrt{109}$

$\sin\alpha = \frac{3}{\sqrt{109}} \Rightarrow \alpha = 16.699\ldots$

$\therefore$ Ans $\sqrt{109}\cos(\theta + 16.70)$

b) i) $H = a - 10\cos(80t) + 3\sin(80t)$

$= a - [10\cos(80t) - 3\sin(80t)]$

$\boxed{H = a - \sqrt{109}\cos(80t + 16.70)}$

If $t = 0$, $H = 1$ so $\quad 1 = a - \sqrt{109}\cos(16.70)$

$1 = a - 9.99996\ldots$

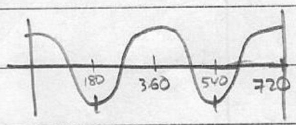
$a = 10.99996\ldots$

$a = 11.0$ (3sf) $\qquad H = 11.0 - \sqrt{109}\cos(80t + 16.70)$

ii) $\boxed{H = 11.0 - \sqrt{109}\cos(80t + 16.70)}$

$\begin{cases} H_{max} = 11.0 + \sqrt{109} \\ H_{min} = 11.0 - \sqrt{109} \end{cases}$ $\therefore$ max height $= 21.4\text{ m}$ //

c If $H = H_{max}$, $\cos(80t + 16.70) = -1$



$80t + 16.70 = 180$ or 540

$80t = 540 - 16.70$ look at <u>second</u> <u>cycle</u>

$t = 6.541$ mins

$t = 6$ min, 32 secs // (nearest sec)

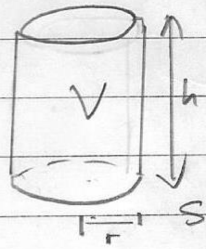
d) Period T (time to complete one cycle) is $80T = 360° \Rightarrow T = \frac{360}{80}$

If we increase the speed, we decrease the period.

To decrease the period, we need to increase the "80"

(i.e. the coefficient of t)

14a) $V = \pi r^2 h$ ①



$S = 2\pi r h + 2\pi r^2$ ②

S = surface Area

If $V = 500$, $\pi r^2 h = 500$

$h = \frac{500}{\pi r^2}$ ③

Sub ③ into ② $S = 2\cancel{\pi}\cancel{r}\left(\frac{500}{\cancel{\pi} \cancel{r^2}_{\,r}}\right) + 2\pi r^2$

$S = \frac{1000}{r} + 2\pi r^2$ //

b) $\boxed{S = 2\pi r^2 + \frac{1000}{r}} = 2\pi r^2 + 1000r^{-1}$

$\frac{dS}{dr} = 4\pi r - 1000r^{-2} = 4\pi r - \frac{1000}{r^2}$

If $\frac{dS}{dr} = 0$, $4\pi r - \frac{1000}{r^2} = 0$

$4\pi r^3 - 1000 = 0$

$r^3 = \frac{\overset{250}{\cancel{1000}}}{4\pi} \Rightarrow r^3 = \frac{250}{\pi} \Rightarrow r = 4.30127\ldots$

Sub in ③ $h = \frac{500}{\pi(4.30127)^2} = 8.6025\ldots$ $\therefore$ can has radius 4.30 m, height 8.60 m //

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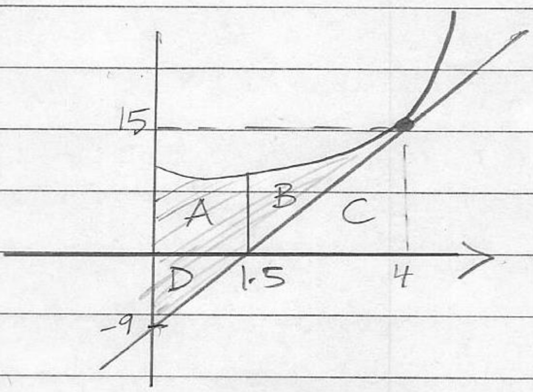
c) The shape of this can may not fit in standard vending machines

15) $y = 5x^{\frac{3}{2}} - 9x + 11$

$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9 = \frac{15}{2}\sqrt{x} - 9$

If $x=4$, $\frac{dy}{dx} = \frac{15}{2}\sqrt{4} - 9 = 6$



Tangent through $(4,15)$ grad$=6$

$y-15 = 6(x-4)$

$y-15 = 6x-24$

$y = 6x-9$

If $y=0$, $6x-9=0$

$x = \frac{9}{6} = \frac{3}{2} = 1.5$

Area under curve $= \int_0^4 (5x^{\frac{3}{2}} - 9x + 11)\,dx$ ← $A+B+C$

$= \left[\frac{2\cancel{5}x^{\frac{5}{2}}}{\cancel{5}} - \frac{9x^2}{2} + 11x\right]_0^4 = \left[2(\sqrt{x})^5 - \frac{9x^2}{2} + 11x\right]_0^4$

$= \left(2(32) - \frac{9(\cancel{16}^{8})}{\cancel{2}} + 11(4)\right) - (\cancel{0} - \cancel{0} + \cancel{0}) = 36$

Area of $\Delta C = \frac{1}{2}(4-1.5)(15) = 18.75$

Area of $\Delta D = \frac{1}{2}(1.5)(9) = 6.75$

$\therefore$ Area required = Area under curve − Area ΔC + Area ΔD

$= 36 - 18.75 + 6.75 = 24$

Alternative method

Use Area between curves $= \int_{x=0}^{4} (\text{upper} - \text{lower})\,dx$

$= \int \left((5x^{\frac{3}{2}} - 9x + 11) - (6x-9)\right) dx$

$= \int_0^4 (5x^{\frac{3}{2}} - 15x + 20)\,dx = 24$

16a) Let $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{11-2P}$

$$\boxed{1 \equiv A(11-2P) + BP}$$

If $P=0$, $\quad 1 = 11A + \cancel{B0} \Rightarrow A = \frac{1}{11}$

If $P=\frac{11}{2}$, $\quad 1 = \cancel{A0} + B\left(\frac{11}{2}\right) \Rightarrow B = \frac{2}{11}$

$$\text{Ans} = \frac{1}{11P} + \frac{2}{11(11-2P)}$$

b)
$$\frac{dP}{dt} = \frac{1}{22}P(11-2P)$$

$$\int \frac{1}{P(11-2P)}\,dP = \int \frac{1}{22}\,dt$$

$$\int \frac{1}{11P} + \frac{2}{11(11-2P)}\,dP = \int \frac{1}{22}\,dt$$

$$\frac{1}{11}\ln P + \frac{\cancel{2}^{1}}{11}\frac{\ln(11-2P)}{(-\cancel{2}_{1})} = \frac{1}{22}t + c$$

$(\times 11)$ $\quad \ln P - \ln(11-2P) = \frac{1}{2}t + c$

$$\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + c$$

GS $\quad \dfrac{P}{11-2P} = Ae^{\frac{1}{2}t}$ $\quad$ Note: P measured in thousands

If $t=0$, $P=1$ so $\quad \frac{1}{9} = Ae^{0} \Rightarrow A = \frac{1}{9}$

PS $\quad \dfrac{P}{11-2P} = \dfrac{1}{9}e^{\frac{1}{2}t}$

If $P=2$, $\quad \dfrac{2}{7} = \dfrac{1}{9}e^{\frac{1}{2}t}$

$$\frac{18}{7} = e^{\frac{1}{2}t}$$

$$\frac{1}{2}t = \ln\left(\frac{18}{7}\right) \Rightarrow t = 1.88892\ldots$$

$$t = 1.89 \text{ years}$$

c) Rearrange

$$9P = (11-2P)e^{\frac{1}{2}t}$$

$$9P = 11e^{\frac{1}{2}t} - 2Pe^{\frac{1}{2}t}$$

$$9P + 2Pe^{\frac{1}{2}t} = 11e^{\frac{1}{2}t}$$

$$P(9+2e^{\frac{1}{2}t}) = 11e^{\frac{1}{2}t}$$

$$P = \frac{(11e^{\frac{1}{2}t}) \times e^{-\frac{1}{2}t}}{(9+2e^{\frac{1}{2}t}) \times e^{-\frac{1}{2}t}}$$

$$P = \frac{11}{9e^{-\frac{1}{2}t}+2}$$

$$P = \frac{11}{2+9e^{-\frac{1}{2}t}} \quad //$$

GOAL

$$P = \frac{A}{B+Ce^{-\frac{1}{2}t}}$$