A Level Sample Paper 3 (Edexcel)
Worked Solutions

1a)

Hours	$0 \le y < 5$	...	$8 \le y < 11$
Freq	12		8
CW	5		3
FD	2.4		$\frac{8}{3}$

(width) (8-11 group) $CW = 3 \to 1.5$ cm

(0-5 group) $CW = 2.4 \to 1.2$ cm

(height) (8-11 group) $FD = 8/3 \to 8$ cm

(0-5 group) $FD = 2.4 \to 7.2$ cm

$\therefore$ Ans: width = 1.2 cm, height = 7.2 cm

b) $\Sigma fx = 12 \times 2.5 + 6 \times 6.5 + \cdots + 2 \times 13 = 205.5$

$\Sigma fx^2 = 12 \times 2.5^2 + 6 \times 6.5^2 + \cdots + 2 \times 13^2 = 1785.25$

$$\bar{x} = \frac{\Sigma fx}{n} = \frac{205.5}{31} = \frac{411}{62} = 6.6290\ldots$$

$$Var = \frac{\Sigma fx^2}{n} - \bar{x}^2 = \frac{1785.25}{31} - \left(\frac{411}{62}\right)^2 = 13.6446\ldots$$

$$s.d. = \sqrt{13.6446\ldots} = 3.6938\ldots$$

$\therefore$ mean = 6.63 hours, s.d. = 3.69 hours

c) Hurn is further South than Heathrow, but has fewer hours of daily sunshine.

$\therefore$ Calculations do not support Thomas' belief.

Note: knowledge of large data set needed — you should know which place is further south

d) 1 sd. above mean = 6.63 + 3.69

= 10.32 hours.

Days with more than 10.32 hours sunshine = (fraction of 8-11 group) + (all of 11-12, 12-14 groups)

$$= \left(\frac{0.68}{3}\right) \times 8 + 3 + 2$$

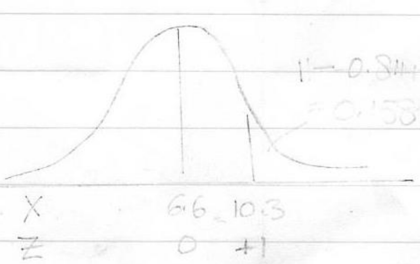
$$= 1.8133 + 5 = 6.81 \text{ days}$$

e) $X \sim N(6.6, 3.7^2)$

$P(X > (6.6 + 3.7))$

$= P(X > 10.3)$

$= 0.15865\ldots$



$\therefore$ No of days $= 0.15865\ldots \times 31 = 4.92$ days

f) Answers to (d) and (e) do not agree.
So Helen's model does <u>not</u> seem suitable//

2a) $t = 24°C$ is outside range of given data $\therefore$ unreliable//

b) PMCC measures how closely the data fits to a straight line relationship//

c) Test statistic = PMCC

H_0: no underlying correlation ($\rho = 0$)
H_1: positive underlying correlation ($\rho > 0$) (one-tailed test, $\alpha = 5\%$)

$n = 9$, critical value $= 0.5822$

$0.609 > 0.5822$ so significant.

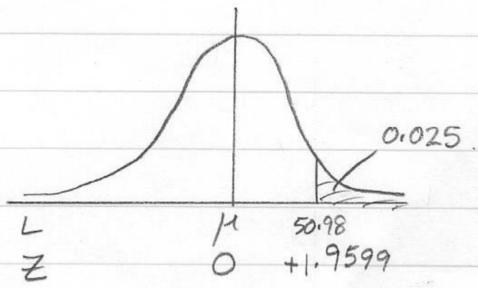
Strong evidence to say underlying correlation is positive.

d) Average temperature $= 27.2°C$, so either Beijing or Jacksonville//

3a) Let L = length of strip (in cm).

$L \sim N(\mu, 0.5^2)$

$P(L > 50.98) = 0.025$.



$$\frac{50.98 - \mu}{0.5} = 1.9599$$

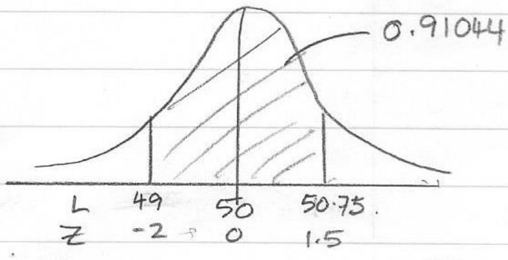
$$\mu = 50.00$$

$P(49 < L < 50.75)$

$$= P\left(\frac{49-50}{0.5} < Z < \frac{50.75-50}{0.5}\right)$$

$$= P(-2 < Z < 1.5)$$

$$= 0.91044 //$$



b) Let N = number of unusable strips (out of 10)

$N \sim B(10, 0.08956)$

$P(N < 4) = P(N \leq 3)$

$= 0.9913 //$

$\ldots 3 | 4 \; 5 \; 6 \ldots$

c) Let $\bar{X}$ = sample mean of 15 strips

$$\bar{X} \sim N\left(\mu, \frac{0.6^2}{15}\right)$$

H_0: $\mu = 50.1$
H_1: $\mu > 50.1$ (one-tailed test, $\alpha = 1\%$)

Assuming H_0, $E(\bar{X}) = 50.1$

$P(\bar{X} \geq 50.4)$

$= 0.0264037\ldots$

$= 2.64\% > 1\%$

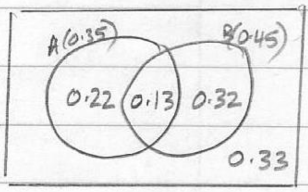
use calculator with $\sigma = 0.6 \div \sqrt{15}$

Not significant. Not enough evidence to reject H_0.
Not enough evidence to say that the mean length is greater than 50.1 cm

4a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cup B) = 0.35 + 0.45 - 0.13$

$= 0.67$



$P(A' \cap B') = 1 - 0.67 = 0.33.$

$$P(A' | B') = \frac{P(A' \cap B')}{P(B')}$$

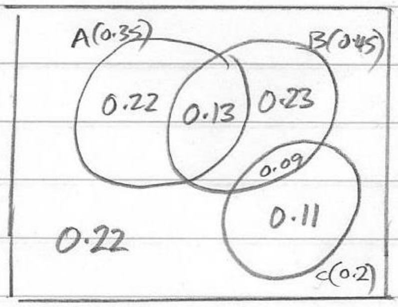
$$= \frac{0.33}{0.55} = 0.6$$

b) $P(A) \times P(B) = 0.35 \times 0.45 = 0.1575$ } Not the same

$P(A \cap B) = 0.13$ } $\therefore$ not independent

c) $P(B \cap C) = P(B) \times P(C)$

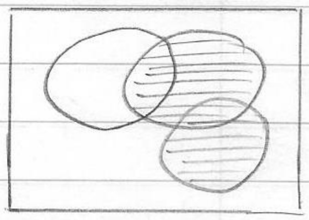
$= 0.45 \times 0.2 = 0.09$



d) $P([B \cup C]') = 0.22 + 0.22$

$= 0.44$

$B \cup C$ is shaded region, we want areas outside this



5a) If they tested all the seeds, they'd have no seeds left to sell.

b) Let X = no. of seeds germinating (out of 24)

$X \sim B(24, 0.55)$ 14 | (15) 16

$P(X \geqslant 15) = 1 - P(X \leqslant 14)$

$= 1 - 0.70087 = 0.2991$

Let U = no. of trays with ≥15 seeds germinating (out of 10)

$U \sim B(10, 0.2991)$

$P(U \geqslant 5) = 1 - P(U \leqslant 4)$

$= 1 - 0.85127 = 0.1487$

c) Binomial can approximate to normal provided n is large, and p is middling (close to 0.5).

d) Let V = no of seeds germinating (out of 240)

$V \sim B(240, 0.55)$

$V \approx N(240 \times 0.55,\ 240 \times 0.55 \times 0.45)$

$\approx N(132, 59.4)$ 149 | (150) 151

$P(V \geqslant 150) \approx$

continuity correction! $\approx P(V > 149.5)$

use calculator with $\sigma = \sqrt{59.4}$

$= 0.01158\ldots$

$= 0.0116$ (3 sf)

e) Let $W \sim B(240, p)$

$H_0: p = 0.55$

$H_1: p \neq 0.55$ (two-tailed test, take $\alpha = 5\%$ or 2.5% each tail)

Assuming H_0, $P(W \geqslant 150) = 0.0113$

$= 1.13\% < 2.5\%$.

Significant. Strong evidence (at the 5% level) to reject H_0.
Strong evidence to say that the company's claim is wrong.

Mechanics

$\underline{s} \xrightarrow{diff} \underline{v} \xrightarrow{diff} \underline{a}$

$\xleftarrow{integ}$

6) $\underline{a} = 5t\,\underline{i} - 15t^{\frac{1}{2}}\,\underline{j}$

Integrating,

$\underline{v} = \frac{5t^2}{2}\underline{i} - (2)\frac{\cancel{15}^{5}\,t^{\frac{3}{2}}}{\cancel{3}}\underline{j} + \underline{c}$

$\underline{v} = \frac{5t^2}{2}\underline{i} - 10t^{\frac{3}{2}}\underline{j} + \underline{c}$

when $t=0$, $\underline{v} = 20\underline{i}$ so $20\underline{i} = \frac{5(0)^2}{2}\underline{i} - 10(0)^{\frac{3}{2}}\underline{j} + \underline{c}$

$\underline{c} = 20\underline{i}$

$\underline{v} = \frac{5t^2}{2}\underline{i} - 10t^{\frac{3}{2}}\underline{j} + 20\underline{i}$

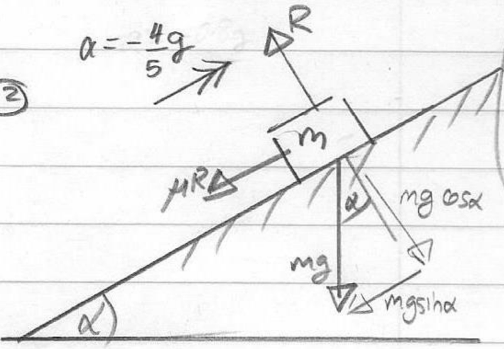
If $t=4$, $\underline{v} = \frac{5(\cancel{16}^{8})}{\cancel{2}}\underline{i} - 10(4^{\frac{3}{2}})\underline{j} + 20\underline{i}$

$= 60\underline{i} - 80\underline{j}$

$\therefore$ speed $= \sqrt{60^2+80^2} = 100\ ms^{-1}$

7a) (Res ↖) $R = mg\cos\alpha$ ①

(F=ma ↗) $-\mu R - mg\sin\alpha = m\left(-\frac{4}{5}g\right)$ ②



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

Sub ① into ②

$-\mu \cancel{mg}\cos\alpha - \cancel{mg}\sin\alpha = -\frac{4}{5}\cancel{mg}$

$-\mu.\frac{4}{5} - \frac{3}{5} = -\frac{4}{5}$

(×5) $-4\mu - 3 = -4$

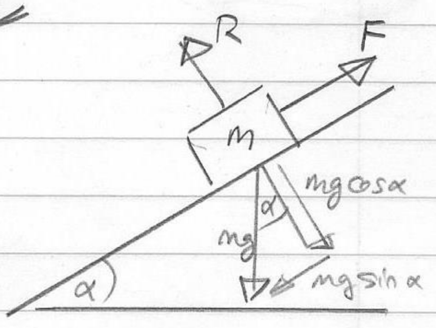
$-4\mu = -1 \Rightarrow \mu = 0.25$

b) Assume equilibrium

(Res ↖) $R = mg\cos\alpha$ ①

(Res ↗) $F = mg\sin\alpha$ ②

From ① $R = m \times 9.8 \times \frac{4}{5} = 7.84m$



$F_{max} = \mu R$

$= 0.25 \times 7.84m = 1.96m$

From ② $\quad F = m \times 9.8 \times \frac{3}{5} = 5.88m$.

$F > F_{max}$ which is impossible

$\therefore$ particle would not remain in equilibrium at A

8a) $\underline{s} =$

$\underline{u} = 0.6\underline{j}$

$\underline{v} = 10.5\underline{i} - 0.9\underline{j}$

$\underline{a} = ?$

$t = 15$

$$\underline{v} = \underline{u} + \underline{a}t$$

$$10.5\underline{i} - 0.9\underline{j} = 0.6\underline{j} + (\underline{a})(15)$$

$$15\underline{a} = 10.5\underline{i} - 1.5\underline{j}$$

$$\underline{a} = \frac{1}{15}(10.5\underline{i} - 1.5\underline{j})$$

$$= 0.7\underline{i} - 0.1\underline{j}$$

b) $\underline{s} = \underline{r} - \underline{r_0}$

$\underline{u} = 0.6\underline{j}$

$\underline{v} =$

$a = 0.7\underline{i} - 0.1\underline{j}$

$t = t$

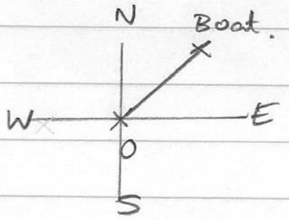
$$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$$

$$\underline{r} - \underline{0} = (0.6\underline{j})t + \frac{1}{2}(0.7\underline{i} - 0.1\underline{j})t^2$$

$$\underline{r} = 0.6t\underline{j} + 0.35t^2\underline{i} - 0.05t^2\underline{j}$$

$$\underline{r} = (0.35t^2)\underline{i} + (0.6t - 0.05t^2)\underline{j} \quad - ⊛$$

c)



dist. east = dist. north

$$0.35t^2 = 0.6t - 0.05t^2$$

$$0.4t^2 - 0.6t = 0$$

$$(\times 10) \quad 4t^2 - 6t = 0$$

$$2t(2t - 3) = 0$$

$$t = 0 \text{ or } t = 1.5.$$

If $t = 0$, $\underline{r} = 0\underline{i} + 0\underline{j}$

If $t = 1.5$, $\underline{r} = 0.7875\underline{i} + 0.7875\underline{j}$

Ans: $t = 1.5\text{ s}$

d) Differentiating ⊛

$$\underline{v} = 0.7t\underline{i} + (0.6 - 0.1t)\underline{j}$$

moving to NE $\Rightarrow$ component east = component north

$$0.7t = 0.6 - 0.1t$$

$$0.8t = 0.6$$

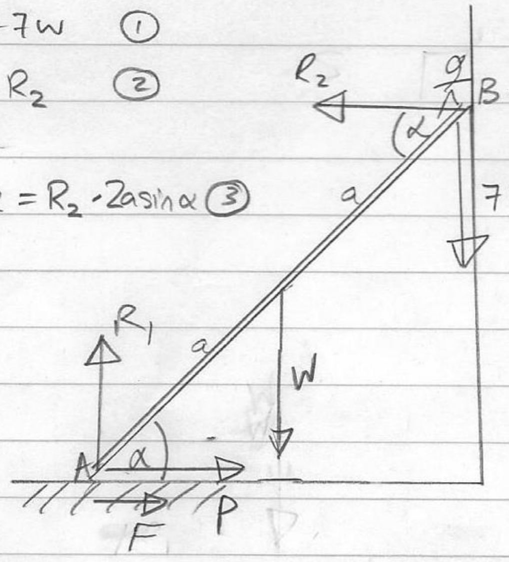
$$t = 0.75\text{ s}$$

9a) (Res ↑) $R_1 = W + 7W$ ①

(Res →) $F + P = R_2$ ②

(Mom about A)

$W \cdot a\cos\alpha + 7W \cdot 2a\cos\alpha = R_2 \cdot 2a\sin\alpha$ ③



$\mu = \frac{1}{4}$

$\tan\alpha = \frac{5}{2}$

From ③ $\quad \frac{W\cancel{a}\cancel{\cos\alpha}}{\cancel{\cos\alpha}} + \frac{14W\cancel{a}\cancel{\cos\alpha}}{\cancel{\cos\alpha}} = 2\cancel{a}R_2\frac{\sin\alpha}{\cos\alpha}$

$W + 14W = \cancel{2}R_2\left(\frac{5}{\cancel{2}}\right)$

$15W = 5R_2 \Rightarrow R_2 = 3W$

b) From ① $\quad R_1 = 8W$

$\therefore F_{max} = \mu R_1 = \mu \times 8W$

$= \frac{1}{4} \times 8W = 2W.$

From ② $\quad F = R_2 - P$

$F = 3W - P.$

If P is small then Friction acts to right.

must have $F \leq F_{max} \Rightarrow 3W - P \leq 2W$

$W \leq P \Rightarrow P \geq W.$

If P is large then Friction acts to left

must have $|F| \leq F_{max} \Rightarrow P - 3W \leq 2W$

$P \leq 5W.$

Range of possible values for equilibrium $W \leq P \leq 5W$

c) (Res ↑) $R_1 = 8W + W_A$ ①

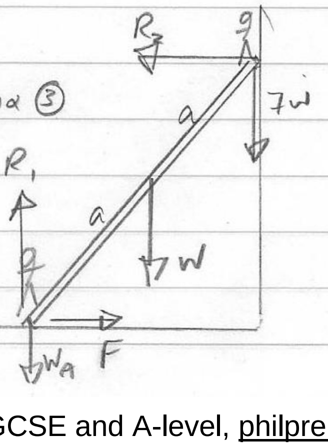
(Res →) $R_2 = F$ ②

(Mom about A) $Wa\cos\alpha + 14Wa\cos\alpha = 2R_2 a\sin\alpha$ ③

From ③ $\quad R_2 = 3W$ (as before)

From ② $\quad F = 3W$

From ①, Reaction force at A will INCREASE, so max friction force at A also INCREASES.



10a)

	H.	V.
s	x	$y-18$
u	$U\cos\alpha$	$U\sin\alpha$
v	v_x	v_y
a	0	-9.8
t	t	t

$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\underline{v = u + at}$

(H) $v_x = U\cos\alpha = \frac{4u}{5}$ ①

(V) $v_y = U\sin\alpha - 9.8t$

$v_y = \frac{3U}{5} - 9.8t$ ②

$\underline{s = ut + \frac{1}{2}at^2}$

(H) $x = U\cos\alpha\, t = \frac{4Ut}{5}$ ③

(V) $y - 18 = U\sin\alpha\, t - 4.9t^2$

$y = 18 + \frac{3Ut}{5} - 4.9t^2$ ④

Hits ground when $x = 36$, $y = 0$

From ③ $36 = \frac{4Ut}{5} \Rightarrow t = \frac{45}{U}$

Sub in ④ $0 = 18 + \frac{3\cancel{U}}{\cancel{5}}\left(\frac{\overset{9}{\cancel{45}}}{\cancel{U}}\right) - 4.9\left(\frac{45}{U}\right)^2$

$0 = 18 + 27 - \frac{9922.5}{U^2}$

$\frac{9922.5}{U^2} = 45$

$9922.5 = 45U^2 \Rightarrow U = 14.8 \text{ ms}^{-1}$ (3sf.)

b) If $y = 10.8$,

Sub in ④ $10.8 = 18 + \frac{3(14.8)t}{5} - 4.9t^2$

$-7.2 = 8.88t - 4.9t^2$

$4.9t^2 - 8.88t - 7.2 = 0$

$t = \frac{8.88 \pm \sqrt{8.88^2 - 4(4.9)(-7.2)}}{2(4.9)}$

$t = 2.4195$ or -0.6073

If $t = 2.4195$, $v_y = \frac{3(14.8)}{5} - 9.8(2.4195) = -14.8311$

$v_x = \frac{4}{5}(14.8) = 11.84$

$\therefore$ speed of stone $= \sqrt{14.8311^2 + 11.84^2}$

$= 18.9775\ldots$

$= 19\ ms^{-1}$ (2 sf)

c) Possible improvements : might include air resistance

might include air speed/wind.