Rayner Year 2 Textbook ANSWERS

Exercise 1A page 1

1 $\frac{4}{5}$ **2** $4a$ **3** $\frac{1}{3}$ **4** 4

5 $\frac{a}{2b}$ **6** 3 **7** $\frac{a}{2}$ **8** $2b$

9 $\frac{3}{4y}$ **10** $\frac{11y}{12x}$ **11** $\frac{2ya}{3}$ **12** $\frac{4m}{n}$

13 AG, BE, CH, DF

14 $\frac{a}{5b}$ **15** a **16** $\frac{7}{8}$ **17** $\frac{3}{4-x}$

18 $\frac{5+2x}{3}$ **19** $\frac{3x+1}{x}$ **20** $\frac{4+5a}{5}$ **21** $\frac{b}{3+2a}$

22 $\frac{3-x}{2}$ **23** $\frac{2x+1}{y}$ **24** $\frac{2x+5}{2}$ **25** $y + xy$

26 $\frac{5x+4}{3x-2}$ **27** $\frac{1+4a}{b+b^2}$ **28** $\frac{1+2x+2x^2}{x}$

29 $\frac{6n-3m}{2mn}$

30 a False **b** True **c** True **d** False

31 a $(x-3)(x+2)$ **b** $x-3$

32 a $x-4$ **b** $\frac{x-2}{x-1}$ **c** $\frac{x+5}{x+2}$

d $\frac{x}{x+1}$ **e** $\frac{x+4}{2(x-5)}$ **f** $\frac{x+5}{x-2}$

g $\frac{x}{2x-1}$ **h** $\frac{2x+1}{2x+3}$ **i** $\frac{x-1}{3x-4}$

33 $\frac{6x+3}{6x+2}$

34 a $\frac{x^2+1}{x^2}$ **b** $2x^2-1$ **c** $2x-1$

d $12x+1$ **e** $30x-2$ **f** $\frac{1-4x}{2}$

g $\frac{3x^2+1}{x^2+2}$ **h** $\frac{x+2}{x}$ **i** $\frac{x}{2x+3}$

j x^2-1 **k** $\frac{x+2}{x}$ **l** $\frac{x+3}{x+4}$

Exercise 1B page 3

1 a $\frac{3x}{5}$ **b** $\frac{3}{x}$ **c** $\frac{4x}{7}$ **d** $\frac{4}{7x}$

e $\frac{7x}{8}$ **f** $\frac{7}{8x}$ **g** $\frac{5x}{6}$ **h** $\frac{5}{6x}$

2 AF, BH, CD, EG

3 a $\frac{23x}{20}$ **b** $\frac{23}{20x}$ **c** $\frac{x}{12}$

d $\frac{1}{12x}$ **e** $\frac{5x+2}{6}$ **f** $\frac{7x+2}{12}$

4 a $\frac{1-2x}{12}$ **b** $\frac{2x-9}{15}$ **c** $\frac{3x+12}{14}$

5 a $\frac{9x+13}{10}$ **b** $\frac{3x+1}{x(x+1)}$

c $\frac{7x-8}{x(x-2)}$ **d** $\frac{8x+9}{(x-2)(x+3)}$

e $\frac{4x+11}{(x+1)(x+2)}$ **f** $\frac{x^2+4x+6}{2(x+2)}$

Exercise 1C page 4

1 a **2** $\frac{10m}{3}$ **3** $\frac{2y}{3x}$ **4** $\frac{15b}{4a}$

5 $\frac{4}{a^2}$ **6** $\frac{8}{3}$ **7** $\frac{x-1}{x+2}$ **8** $\frac{y^2}{5x}$

9 $\frac{x}{8}$ **10** $3(x+2)$

11 a 6 **b** 4 **c** $2a$

12 AF, BH, CE, DG

13 a $\frac{26}{3}$ **b** $\frac{6q}{p}$ **c** $\frac{2xy}{9}$ **d** $\frac{4}{aq}$

e 25 **f** $\frac{y}{2x}$ **g** $2a$ **h** 1

14 a 1 **b** $\frac{ab^3}{x}$ **c** $\frac{x}{x-1}$ **d** $\frac{z^2}{y}$

15 a $\frac{x}{10}$ **b** $3x$ **c** 11

16 $\frac{1}{4}$

17 $\frac{1}{5}$

Exercise 1D page 7

1 a one-one **b** many-one

c one-one **d** one-one

e many-one **f** many-one

g many-one **h** many-one

i one-one **j** one-one

2 a $f(x) \geqslant 3$ **b** $f(x) \geqslant 2$

c $f(x) \geqslant 1$ **d** $f(x) \geqslant 0$

e $0 \leqslant f(x) \leqslant 1$ **f** $-\frac{9}{4} \leqslant f(x) \leqslant 0$

g $0 \leqslant f(x) \leqslant \frac{25}{4}$ **h** $0 \leqslant f(x) \leqslant 5$

i $0 < f(x) \leqslant 1$ **j** $0 < f(x) \leqslant 1$

3 a $(x+3)^2 - 5$ **b** $f(x) \geqslant -5$

4 a $f(x) \geqslant 3$ **b** $f(x) \geqslant -50$

c $0 < f(x) \leqslant \frac{1}{2}$ **d** $0 < f(x) \leqslant 1$

Exercise 1E page 10

1 a $4(x+5)$ **b** $4x+5$

c $(4x)^2$ **d** $4x^2$

e x^2+5 **f** $x+10$

g $4(x^2+5)$ **h** $[4(x+5)]^2$

2 a -2.5 **b** $\pm\sqrt{\frac{5}{3}}$

3 **a** $x \mapsto 2(x-3)$ **b** $x \mapsto 2x-3$
c $x \mapsto x^2-3$ **d** $x \mapsto 4x^2$
e $x \mapsto (4x)$ **f** $x \mapsto (2x)^2-3$
g $x \mapsto (2x-3)^2$

4 **a** 2 **b** 11 **c** 6
d 2 **e** 1 **f** 64

5 **a** -3 **b** 2 **c** $1\frac{1}{2}$ **d** 5

6 **a** $2(3x-1)+1$ **b** $3(2x+1)-1$
c $2x^2+1$ **d** $(3x-1)^2$
e $2(3x-1)^2+1$ **f** $3(2x^2+1)-1$

7 **a** 11 **b** 9 **c** 11
d 14 **e** 81 **f** -1

8 **a** 2 **b** 0, 2 **c** $\pm\sqrt{2}$

9 $\frac{x+2}{5}$ **10** $\frac{x}{5}+2$

11 $\frac{x}{6}-2$ **12** $\frac{3x-1}{2}$

13 $\frac{4x}{3}+1$ **14** $\frac{(x+6)/2-4}{3}$

15 $x \mapsto \frac{2(x-10)-4}{5}$ **16** $x \mapsto \frac{2x-3}{-7}$

17 $x \mapsto \frac{12}{3x-5}$ **18** $x \mapsto \frac{3x-4}{5}$

19 $x \mapsto \frac{4(5x+3)+1}{2}$ **20** $x \mapsto \frac{12}{x}$

21 **a** $x \mapsto \frac{x}{3}$ **b** $x \mapsto x+5$ **c** $x \mapsto \frac{x-1}{2}$
d $x \mapsto 3(x-5)$ **e** $x \mapsto \frac{x}{3}+5$ **f** $x \mapsto \frac{x}{3}+5$

22 **a** $x \mapsto 6x+1$ **b** $x \mapsto \frac{2x}{3}+1$ **c** $x \mapsto \frac{x-1}{6}$
d $x \mapsto 2x-9$ **e** $x \mapsto \frac{x+9}{2}$ **f** $x \mapsto \frac{x+9}{2}$

23 **a** 7 **b** 21 **c** 5
d 5 **e** 2

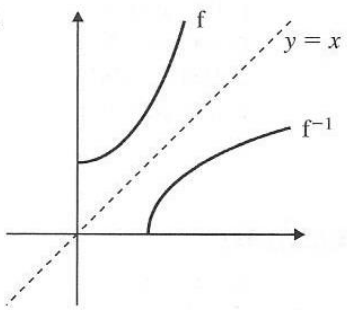
24 **a** 1 **b** **i** $x > -2\frac{1}{2}$ **ii** $x > 5$

25 **a** $x \mapsto \frac{3x-5}{2}$ **b** 2

Exercise 1F page 13

1 **a** $2-x$ **b** $\frac{13}{2x}$ **c** $\frac{3-x}{5}$
d $\frac{7x+5}{x-2}$ **e** $\frac{7x+5}{3-2x}$

2

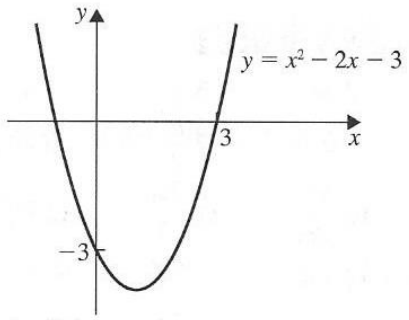


3 **a** $\frac{x+8}{3}, \frac{x+22}{3}$ **b** 4, 8

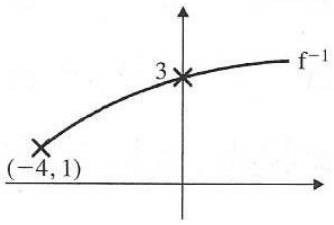
4 **a** $9x-4, \frac{x+1}{3}$ **b** Both are $\frac{x+4}{9}$

5 **a** $\frac{8x+10}{x-1}$ **b** $12\frac{1}{2}$
c 10 **d** 10

6 **a**



b **i** $f(x) \geqslant -4$
ii domain $x \geqslant -4$, range $f^{-1}(x) \geqslant 1$
iii



7 **a** $f^{-1}(x) = \frac{1}{2-x}, x \neq 2$
b $x = 1$

8 **a** domain $x \geqslant 0$, range $f^{-1}(x) \geqslant 2$
b $f^{-1}(x) = 2+\sqrt{x}$
c $x = 4$

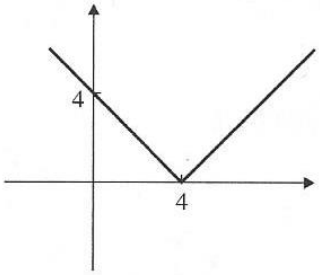
9 **a** $a = 2 \quad b = 0$
b $f^{-1}(x) = x^2+2, x \geqslant 0; g^{-1}(x) = \frac{1}{\sqrt{x}}, x \geqslant 0$

10 $f^{-1}(x) = \frac{x-1}{2}; g^{-1}(x) = \frac{5+3x}{x}, x \neq 0$

11 **a** $-\infty < x < \infty$ **b** $x \geqslant 0$
c $-90 < x \leqslant 90°$ **d** $x \geqslant 1$
e $x \leqslant -1$ **f** $x \geqslant -4$

Exercise 1G page 17

1 **a**



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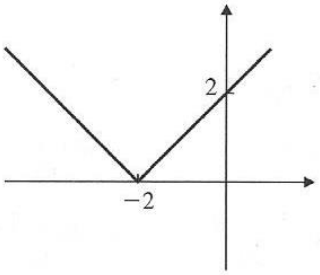

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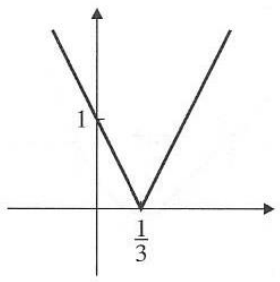

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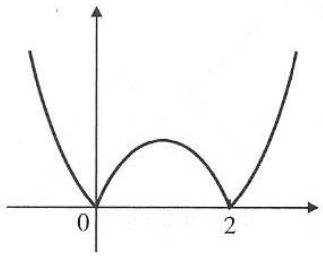

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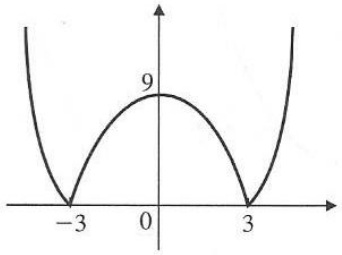

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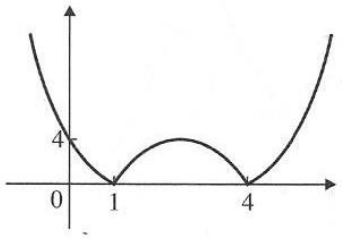

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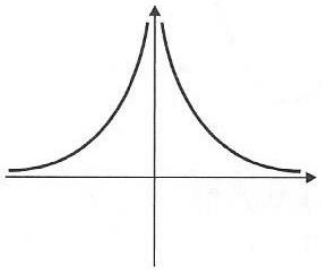

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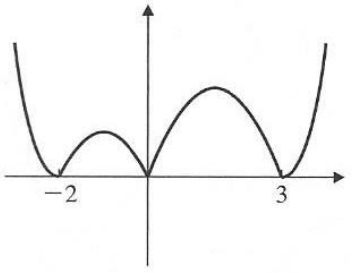
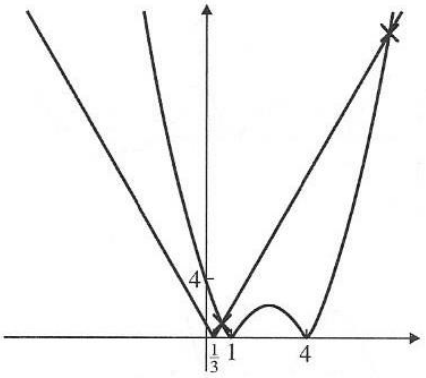


2 a $-3, 7$ **b** $0, 3$
c $-2, 3$ **d** $\pm 1, \pm 3$
e $\pm 5, \pm 7$ **f** $-7, 1$
g $-1, 2$ **h** $\pm \frac{1}{2}$

3 a $x < -1, x > 9$ **b** $-4 < x < 2$
c $x > 2, x < -5$ **d** all x
e $-\frac{14}{3} < x < 6$ **f** $-2 < x < 2$
g $\frac{1}{3} < x < 1$ **h** $x > -1$
i $0 < x < \frac{4}{3}$

4 $4 \pm \sqrt{11}$



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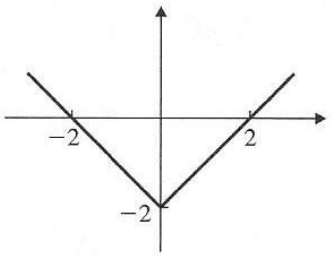

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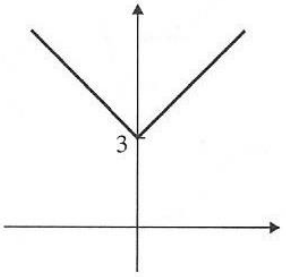

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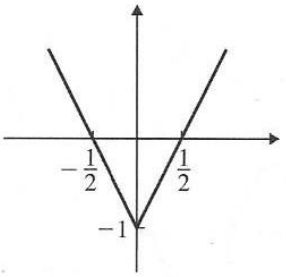

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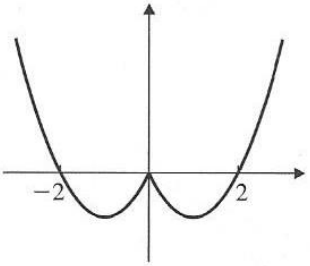

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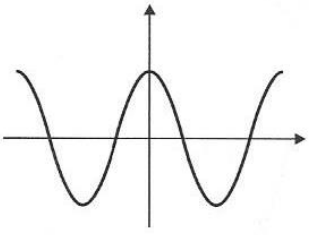

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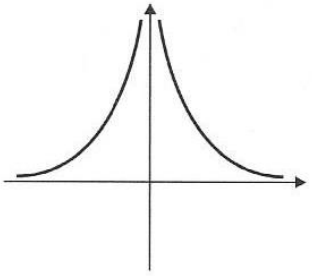

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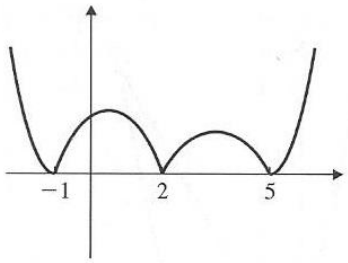

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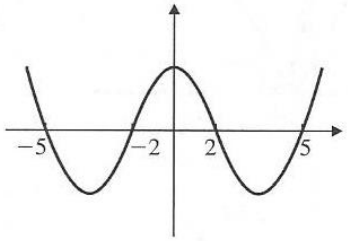

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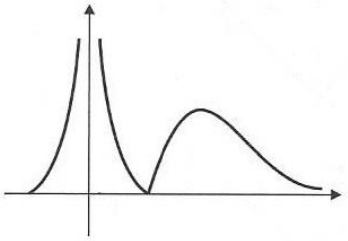

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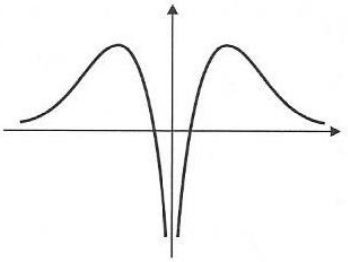

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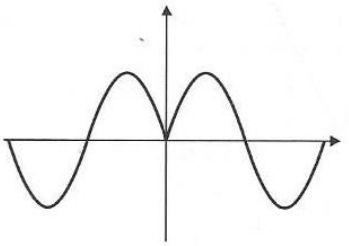
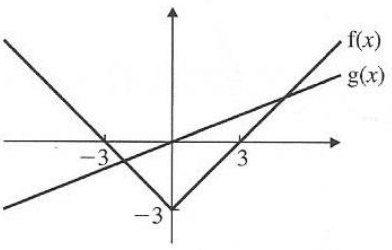


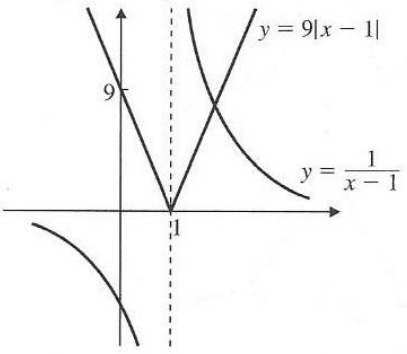
b 4

8 a



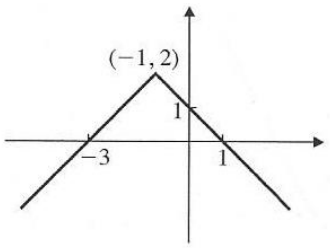
b $x > \frac{9}{2}, x < -\frac{9}{4}$

9 a



b $x > \frac{4}{3}, x < 1$

10 a i



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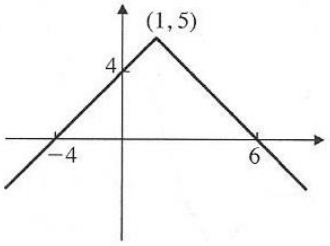
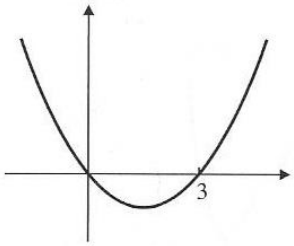


b i range $y \leqslant 2$ **ii** range $y \leqslant 5$

11 a



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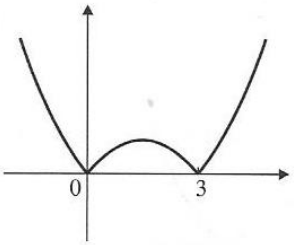

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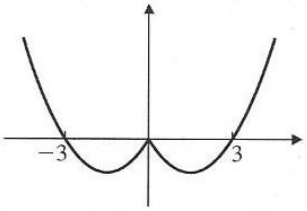

EXERCISE 1H PAGE 21

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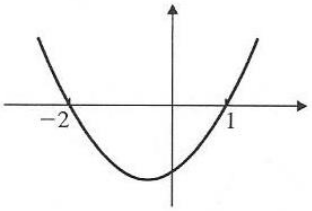

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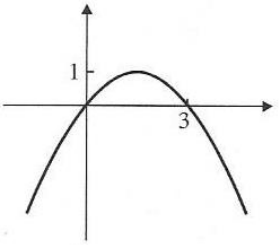

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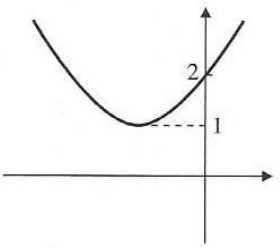

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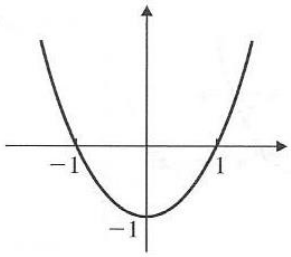

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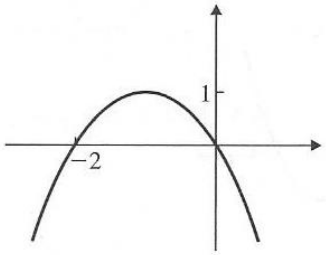

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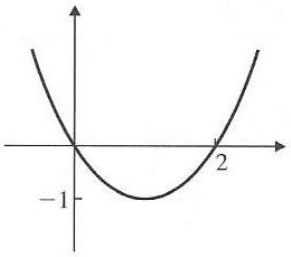

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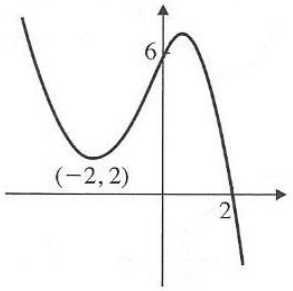

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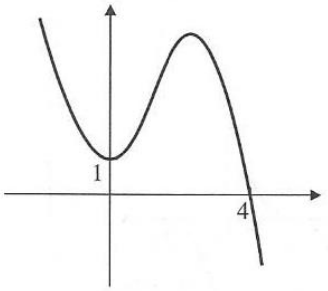
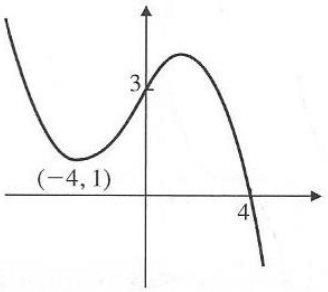


c



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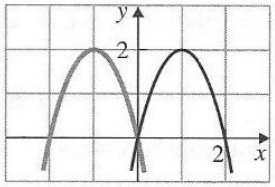
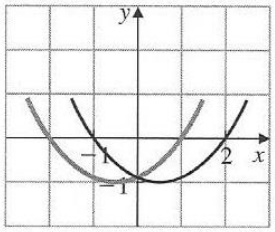


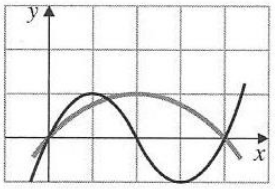
Sketch f(x + 2)

5



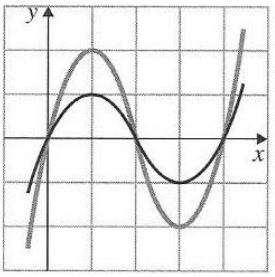
Sketch f($-x$)

6



Sketch f($\frac{1}{2}x$)

7



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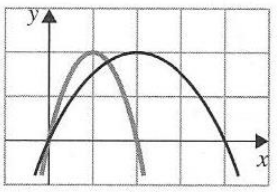

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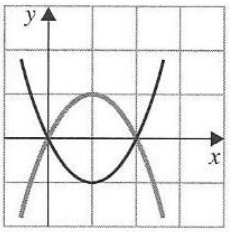
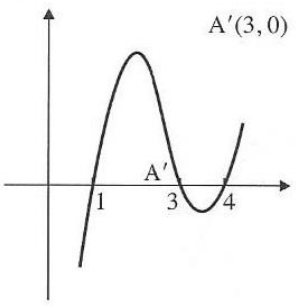


10 a



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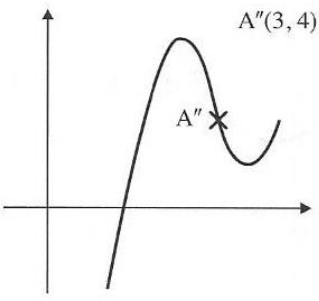
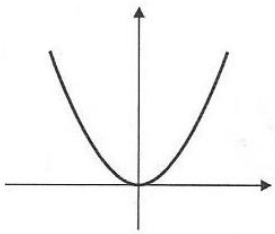


11 a



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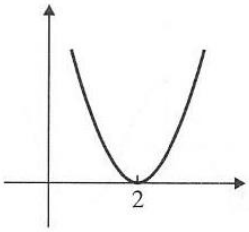

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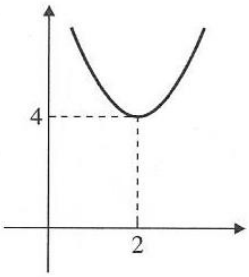
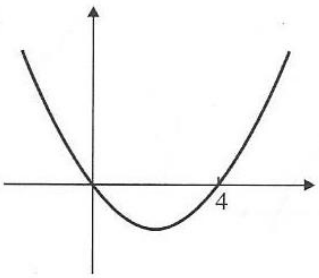


12 a



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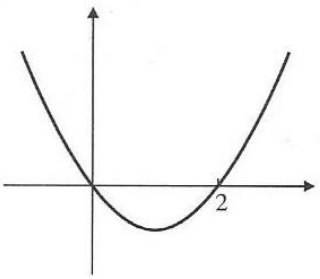

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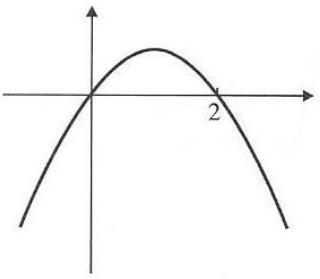
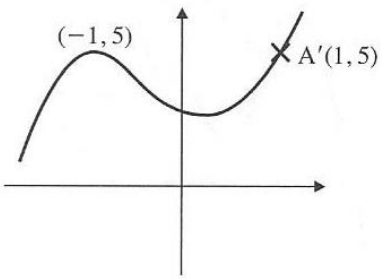


13 a



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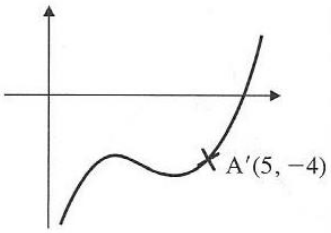

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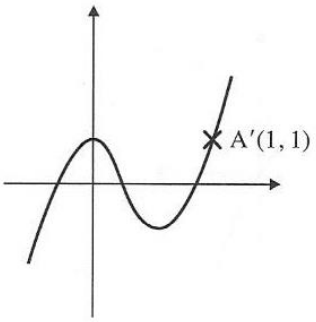
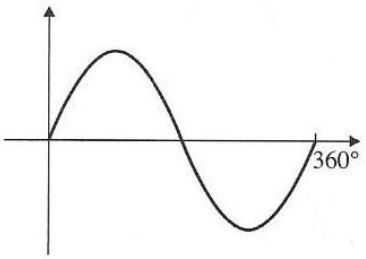


14 a $y = x^3 + 5$

b $y = (x - 2)^3$

c $y = (x - 2)^3 + 5$

15 a



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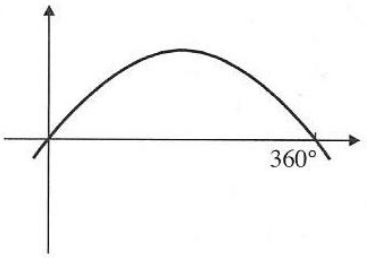
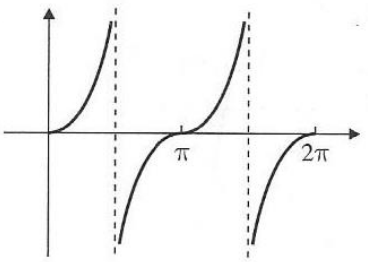


16 a



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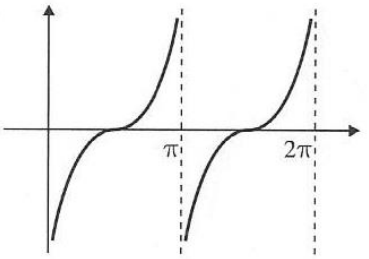


17 Stretch, scale factor 3 parallel to y-axis,

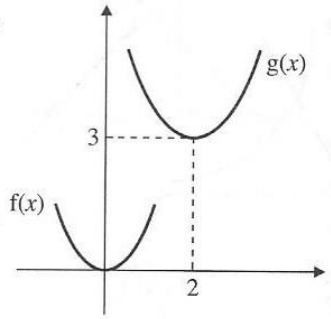
Translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

18 Stretch, scale factor 4 parallel to y-axis,

Translation $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$

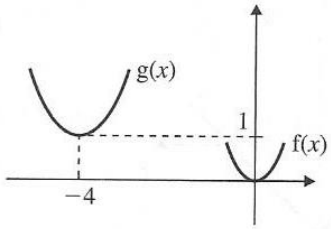
19 a $a = 2, b = 3$

b

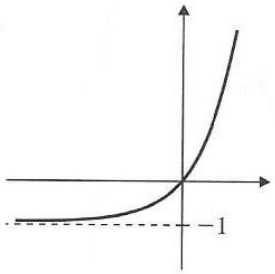


20 a $a = 4, b = 1$

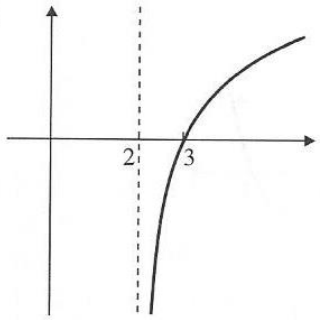
b



21



22 a



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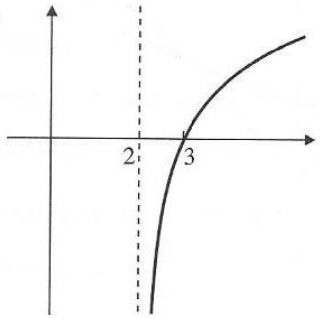


Review Exercise 1I page 24

1 a $\frac{5}{3-x}$ **b** $\frac{x^2}{2}$ **c** $\frac{3x+2}{x}$

d $1+x$ **e** $\frac{1+4x}{y+y^2}$ **f** $\frac{3x}{2}$

2 $4(x-1)(x+1), \frac{(x-4)}{4(x+1)}$

3 a $\frac{x+2}{x}$ **b** $\frac{x-3}{x+1}$ **c** $\frac{2x-1}{x-4}$

d $\frac{2x^2+1}{x^2}$ **e** $3x^2-1$ **f** $\frac{4x^2-1}{x}$

4 a $\frac{8x+5}{6}$ **b** $\frac{4x+1}{x(x+1)}$

c $\frac{x^2+3x-2}{(x+1)(x-1)}$ **d** $\frac{x+4}{12}$

e $-\frac{(3x+5)}{10}$ **f** $\frac{2(x-1)}{x+3}$

5 a $(2x-1)$ is common factor

b $\frac{3x+1}{x(2x+1)}$

6 a $f(x) \leqslant 1$

b $x = 0, \pm\sqrt{2}$

7 a $\frac{5x+2}{2x+1}$ **b** $\frac{1}{x-2}$

8 a $\frac{3}{4-x}$, not defined for $x = 4$

b 1, 3

9 a $0 \leqslant f(x) < 1$

b $\frac{3}{\sqrt{1-x^2}}$, range $f^{-1}(x) \geqslant 3$, domain $0 \leqslant x < 1$

10 a $g(x) \geqslant 3$ **b** $\frac{3}{2x^2+5}$

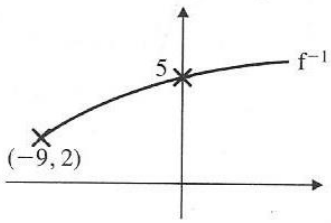
c $\frac{3+x}{2x}$ **d** $-1, \frac{3}{2}$

11 a $\frac{2}{2-x^2}$ **b** $\frac{2-x}{x}$ **c** $-2, 1$

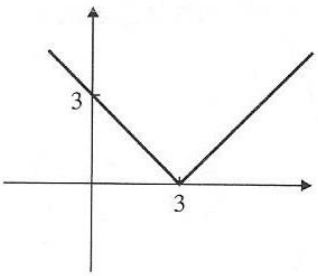
12 a $f(x) \geqslant -9$

b domain $x \geqslant -9$, range $f^{-1}(x) \geqslant 2$

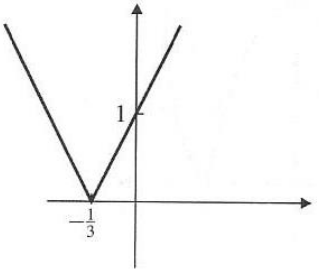
c



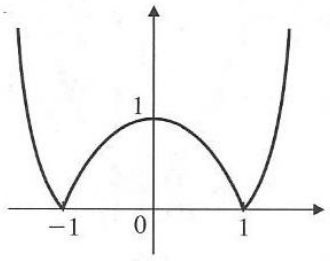
13 a



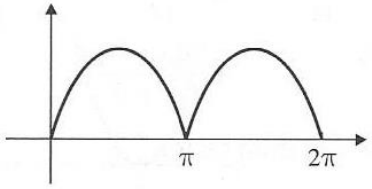
b



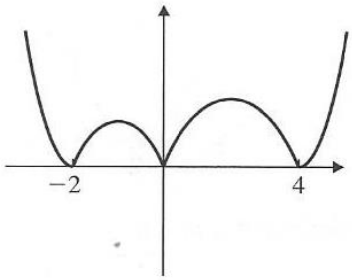
c



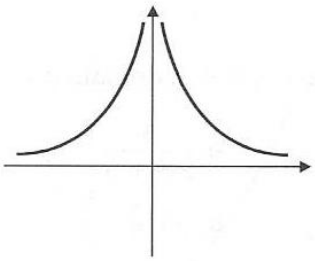
d



14 a



b



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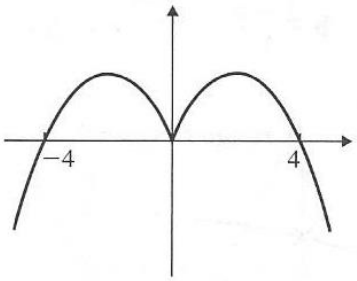
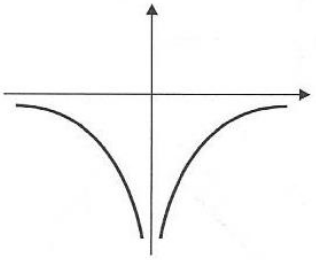


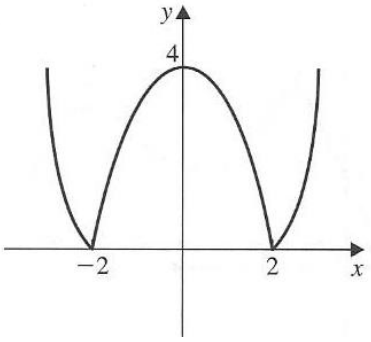
d



15 a $x > -\frac{1}{2}$ **b** $x < -2, x > 0$

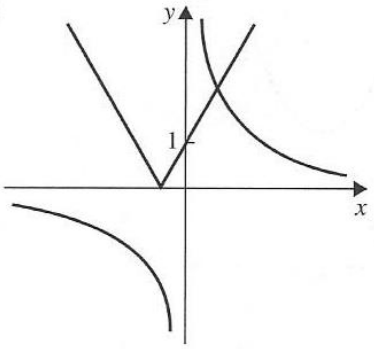
16 a $-4 < x < 6$ **b** $x < 1, x > 2$

17 a



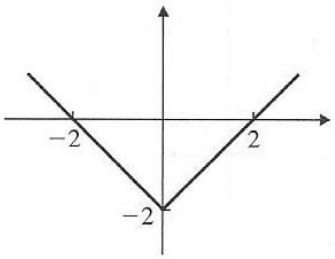
b $x > 3, x < -3$

18

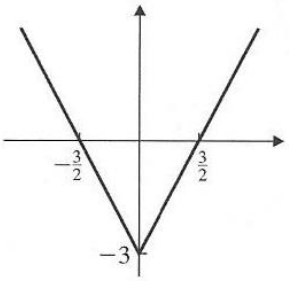


b one solution

19 a



b



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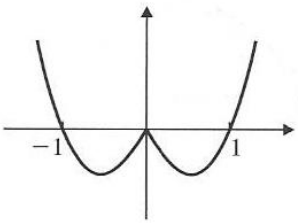
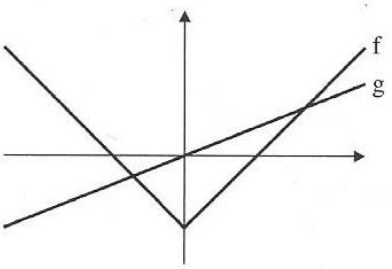
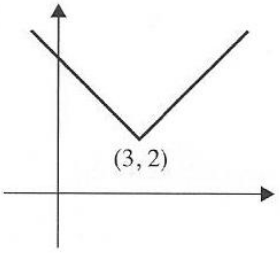


20 a



b $x > 10, x < -\frac{10}{3}$

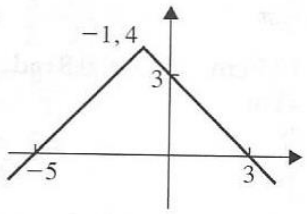
21 a



range: f(x) ⩾ 2

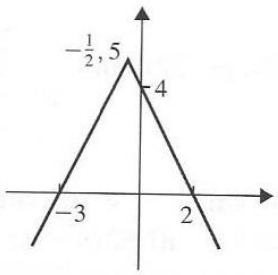
b $x = 0, 6$ **c** 3

22 a



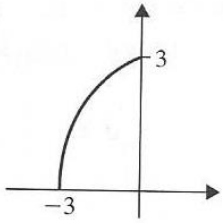
b $f(x) \leqslant 4$

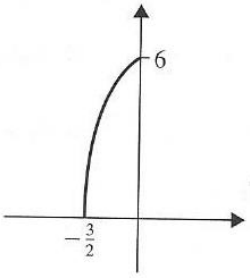
23 a

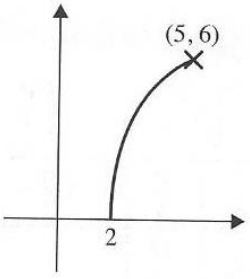


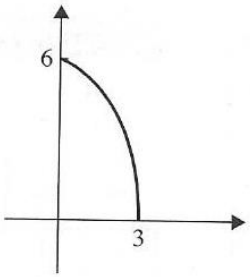
b 2 roots **c** $g(x) \leqslant 5$

24 a









25 a i (2, −9) **ii** (3, −4) **iii** (3, 9)
iv (1, −9) **v** (−3, −9)
b $f(x) = (x - 3)^2 - 9$

26 a i (−1, 5) **ii** (2, −1) **iii** (−2, 5)
b $1 + 4x - x^2$

27 translation $\begin{pmatrix}3\\0\end{pmatrix}$;
stretch scale factor 2 parallel to y-axis;
translation $\begin{pmatrix}0\\-4\end{pmatrix}$

28 stretch, scale factor $\frac{1}{2}$, parallel to x-axis;
translation $\begin{pmatrix}0\\-1\end{pmatrix}$

29 stretch, scale factor 2, parallel to x-axis;
translation $\begin{pmatrix}0\\3\end{pmatrix}$

30 f: translation $\begin{pmatrix}4\\0\end{pmatrix}$, g: translation $\begin{pmatrix}0\\-3\end{pmatrix}$

Examination Exercise 1 page 28

1 $\frac{1}{x+4}$

2 i $\frac{1}{2-x}$ **ii** $\frac{3}{(x-1)(x-4)}$

3 i $\frac{11-x}{(3-x)(1+x)}$ **ii** $\frac{-1}{1+x}$

4 i 26, 4 **ii** reflection in $y = x$

5 i −3

6 a $x = 6, x = -5$ **b** $a = 6$

7 a $f(x) < 5$

b i $\frac{1}{3}\ln(5 - x)$ **ii** 4

c $\frac{2x-3}{11-6x}$

8 a $0 \leqslant f(x) \leqslant 10$ **b** 3

c $\frac{5x-4}{3+x}$ **d** 6, 0.4

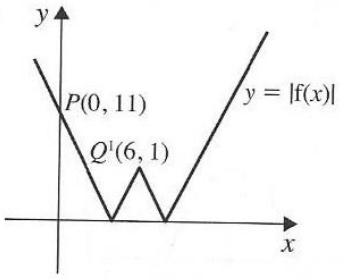
9 a $f(x) \geqslant -4$ **b** $3 + \sqrt{x+4}$

c i $|x^2 - 6x - 1|$ **ii** 5, 7

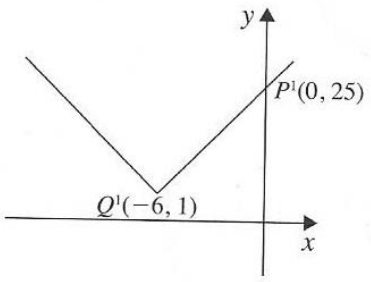
10 $x \leqslant -6, 1 \leqslant x \leqslant 2, x \geqslant 3$

11 $4a, -14a$

12 a



b



c $a = 2, b = 6$

13 i translation $\begin{pmatrix}-3\\0\end{pmatrix}$ and stretch s.f. 2 in the y direction

ii $-6 < x < -2$

Exercise 2A page 33

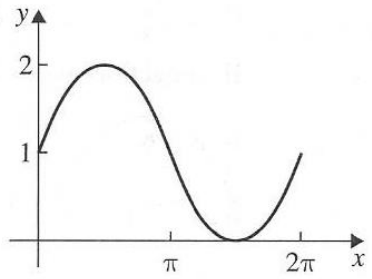
1 a $\frac{2}{3}\pi$ **b** $\frac{3}{2}\pi$ **c** $\frac{3}{4}\pi$
d $\frac{11}{6}\pi$ **e** $\frac{7}{4}\pi$ **f** 4π

2 a 0.698^c **b** 1.75^c **c** 0.873^c
d 3.80^c **e** 1.52^c **f** 0.0175^c

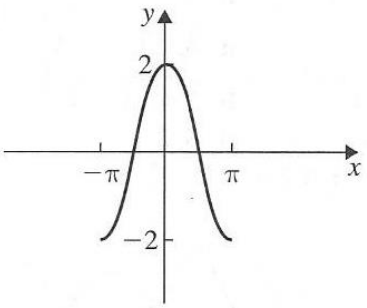
3 a 225° **b** 270° **c** 157.5°
d 115° **e** 74.5° **f** 28.6°

4 a $\frac{\sqrt{3}}{2}$ **b** 1 **c** $\frac{\sqrt{3}}{2}$
d −0.644 **e** −0.942 **f** 2.57

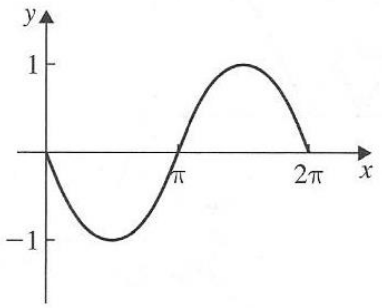
6 a



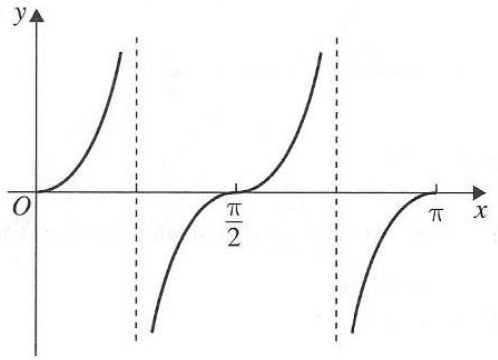
b



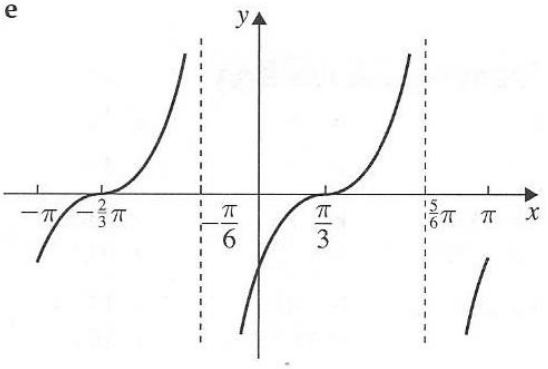
c



d



e



Exercise 2B page 36

1 a 8 cm **b** 40 cm²

2 a 4 cm **b** 60 cm² **c** 0.4 rad
d 6 cm **e** 0.072 m²

3 a $\frac{2}{9}\pi$ **b** 20.2 m **c** 1.04^c

4 a $\frac{\pi}{3}$ **b** $\frac{5}{18}\pi$

5 a 0.8 rad **b** 10.7 cm **c** 0.8 rad
d 7.98 cm **e** 4 cm

6 a 9 cm **b** $\frac{2}{3}^c$

7 a 20 cm **b** $\frac{3}{4}^c$

8 6.56 cm

9 5π cm (= 15.7 cm)

10 b $\frac{32\pi}{3}$ cm² **d** 18.5 cm², 37.0 cm²
e 3.44 cm²

11 $\frac{2}{3}$ rad

12 a 34.3 cm² **b** 24.1 cm² **c** 10.2 cm²

13 a i 50θ **ii** $50\sin\theta$ **iii** $50(\theta - \sin\theta)$

15 b 1.17^c

16 a $12r + 2\pi r + \frac{\pi}{3}r^2$ **c** 7.6 cm

17 a i 8θ **ii** $8\sin\theta$ **iii** $8\theta - 8\sin\theta$
b 1.1^c

18 b $\sqrt{72(1-\cos\theta)} + 60$ **d** 1.6^c

19 $16\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

Exercise 2C page 44

1 a 0 **b** 0 **c** 0 **d** 0
e 0 **f** −1 **g** −1 **h** 1

2

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

3 a $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{2}}$ **c** −1 **d** $\frac{1}{2}$

4 a 0.589 **b** 0.264 **c** −0.416
d 0.951 **e** 0.451 **f** 0.282

5 $\frac{\pi}{6}, \frac{5\pi}{6}$ **6** $\frac{\pi}{3}, \frac{4\pi}{3}$

7 a $\frac{\pi}{4}, \frac{7\pi}{4}$ **b** $\frac{\pi}{2}$ **c** $\frac{\pi}{6}, \frac{7\pi}{6}$

8 a 0.412, 2.730 **b** 0.708, 2.434
c 1.359, 4.924 **d** 1.107, 4.249

9 a $-\frac{\pi}{6}, \frac{\pi}{6}$ **b** 5.508, 3.917
c 1.039, −2.103 **d** $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

10 a 0.775 or 2.366

11 6.06 cm

12 a $\frac{\pi}{4}, \frac{5\pi}{4}$ **b** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
c 0.491, 2.062, 3.633, 5.204
d 2.319

13 a $\frac{7\pi}{12}, \frac{23\pi}{12}$ **b** $\frac{5\pi}{12}, \frac{11\pi}{12}$

c $\frac{\pi}{2}, \frac{3\pi}{2}$ **d** 1.620, 2.522

14 $\frac{\pi}{3}, \frac{2\pi}{3}$ **15** 0.983

16 $\frac{\pi}{4}, \frac{3\pi}{4}$ **17** $0, \pi, 1.107$

18 0.308, 2.834 **19** $0, \pi, \frac{\pi}{3}$

20 $\frac{\pi}{2}$ **21** 1.107, $\frac{3\pi}{4}$ (2.356)

22 $\frac{\pi}{8}, \frac{5\pi}{8}$ **23** $\frac{\pi}{6}, \frac{5\pi}{6}$

24 $\frac{\pi}{2}$ **25** $\frac{\pi}{2}$

Exercise 2D page 47

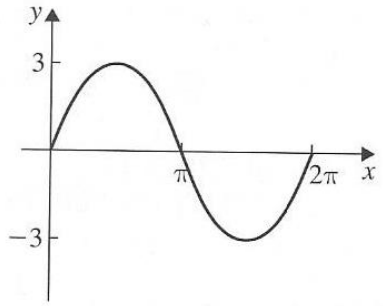
1 a 0.24740 **b** 0.25 **c** 1.05%
2 13.1%
3 0.294%
5 b 1
6 a 3 **b** 3 **c** 7 **d** 6
7 $x \approx 2, y \approx 9.8$
8 $a \approx 2.1, b \approx 7.7$

Review Exercise 2E page 48

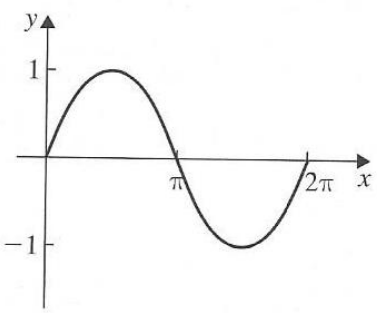
1 a 12.5 **b** 6.68 **c** 14.6
2 1.74 **3** 0.652 **4** 2.41
5 0.683
6 a 6.55 cm² **b** 26.6 cm²
7 $\frac{2\pi}{5}$

8 $\sin\frac{\pi}{4} = \cos\frac{\pi}{4}, \tan\left(-\frac{\pi}{6}\right) = \tan\frac{5\pi}{6}$,
$\cos\frac{\pi}{6} = \cos\frac{11\pi}{6} = -\sin\left(-\frac{\pi}{3}\right) = \sin\frac{2\pi}{3}$,
$\sin\frac{\pi}{6} = \sin\frac{5\pi}{6}, \tan\frac{\pi}{6} = \tan\left(-\frac{5\pi}{6}\right)$

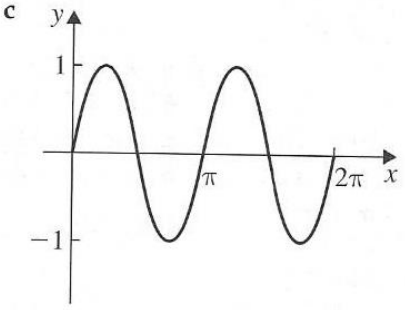
9 a



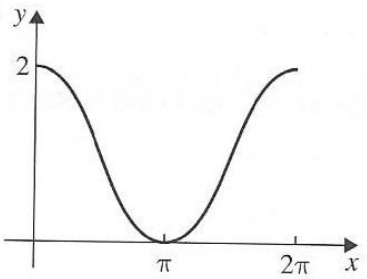
b



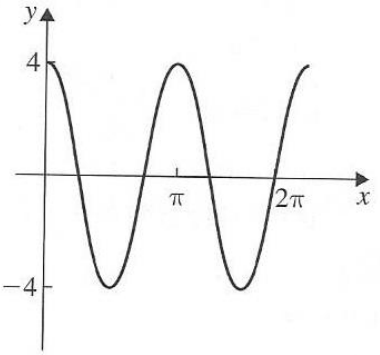
c



d



e



10 b i 2 **ii** 4
11 a 12 cm **b** 14.4 cm² **c** 12 cm
d 10ᶜ **e** 2.5ᶜ **f** $r = 2, \theta = 1.5$
g 4 cm
12 $50, 25\pi - 50$
13 a 4.2 cm **b** 1.0 cm²
14 a $20\sqrt{3}$ **b** 200π **c** $20\sqrt{3}\left(\frac{\pi}{3} + 2\right)$
15 a $r^2\left(\frac{5}{4} - \cos\theta\right)$ **b** 0.634
c 0.68 cm²
16 a 0.729 **b** 9.11 cm²
c 11.9 cm² **d** 15.6 cm
17 a $\frac{\pi}{3}, \frac{5\pi}{3}$ **b** $-\frac{3\pi}{4}, \frac{\pi}{4}$
c $\frac{\pi}{4}, \frac{7\pi}{4}$ **d** $\frac{\pi}{3}, \frac{2\pi}{3}$
e 3.9, 5.6 **f** 2.4, 3.9
g −0.6, 2.5 **h** $-\frac{\pi}{6}, -\frac{5\pi}{6}$
18 a $\frac{11\pi}{6}, \frac{\pi}{6}$ **b** $\frac{\pi}{2}, \frac{7\pi}{6}$
c $\frac{5\pi}{12}, \frac{13\pi}{12}$ **d** $\frac{\pi}{2}, -\frac{\pi}{2}$
19 $0.85, 2.3, \frac{7\pi}{6}, \frac{11\pi}{6}$
20 a $\frac{\pi}{2}$ **b** $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
c $\frac{\pi}{6}, \frac{5\pi}{6}, -0.3, -2.8$ **d** $-\frac{2\pi}{3}, \frac{2\pi}{3}, -0.8, 0.8$
e $-\frac{\pi}{6}, -\frac{5\pi}{6}$

21 a $-\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$

b $\frac{2\pi}{3}, \pi, \frac{5\pi}{3}$ **c** $\frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$

22 a $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$ **b** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c 0.8, 2.3, $\frac{7\pi}{6}, \frac{11\pi}{6}$ **d** 1.25, 4.39

e $0, \pi, 2\pi$, 0.46, 3.61

24 a -3 **b** 0

Examination Exercise 2 page 51

1 i $\frac{3\pi}{10}$ **ii** 20.4

2 b 40.9 **c** 96.7

3 i 8π **ii** $48\pi - 36\sqrt{3}$

4 6

5 a 17.5 **b** 0.943 **c** 38.9

6 ii 15.2 **iii** 9.76

7 a 18.8 cm^2 **b** 2 **c** 2π

8 b 19.9 cm^2 **c** 13.8 cm

9 $-\frac{2\pi}{15}, \frac{8\pi}{15}$

10 i 15 **ii** $(2x + 1)(2x - 3)(x + 1)$

iii $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$

11 $-\pi, -0.84, 0, 0.84, 0$

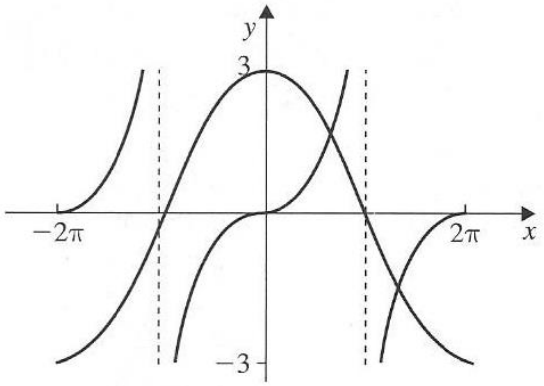
12 $\frac{\pi}{2}$, 0.85, 2.29

13 a $-0.056, 1.8$ **b i** $\frac{1}{2}, -\frac{1}{3}$

14 b 1.2, 4.4, 2.0, 5.2

15 $5 + 3\cos\theta$, least value is 2, occurs when $\theta = \pi$

16 i



ii 2.02, 4.26

Exercise 3A page 59

1 a $A = 1, B = 2$ **b** $A = 3, B = 2$

c $A = 2, B = 3$ **d** $A = 5, B = 2$

e $A = 7, B = -2$ **f** $A = 4, B = -3$

2 a $\frac{1}{x-3} + \frac{8}{x-2}$ **b** $\frac{1}{x-1} + \frac{2}{x-2}$

c $\frac{3}{x+1} + \frac{5}{x+2}$ **d** $\frac{1}{x+2} + \frac{6}{x+3}$

e $\frac{1}{x-1} + \frac{2}{x+2}$ **f** $\frac{2}{x-1} - \frac{1}{x+3}$

g $-\frac{4}{x} + \frac{5}{x-2}$ **h** $\frac{2}{x} + \frac{3}{x+1}$

3 a $A = 3, B = 1$ **b** $A = 4, B = -3$

c $A = 1, B = 3$ **d** $A = 2, B = 1$

e $A = -2, B = 5$ **f** $A = 7, B = -1$

4 a $\frac{3}{2x-3} + \frac{4}{x-3}$ **b** $\frac{3}{2x+1} + \frac{1}{x-1}$

c $\frac{5}{3x-2} + \frac{2}{3x+4}$ **d** $\frac{1}{2x-1} + \frac{1}{5x-1}$

e $\frac{4}{x+3} + \frac{3}{2x-1}$ **f** $\frac{2}{3x+5} + \frac{3}{2x-3}$

g $\frac{7}{2x-1} - \frac{1}{2x+1}$ **h** $\frac{2}{5x-2} + \frac{9}{5x+2}$

Exercise 3B page 61

1 a $A = 1, B = 3, C = 2$

b $A = 1, B = -1, C = 2$

c $A = 2, B = 3, C = 6$

d $A = 4, B = -1, C = 5$

e $A = 3, B = 2, C = 1$

2 a $\frac{1}{x-2} + \frac{3}{2x+1} + \frac{1}{x+2}$

b $\frac{5}{x-3} + \frac{1}{x+1} + \frac{1}{2x+3}$

c $\frac{2}{x-1} - \frac{4}{x} + \frac{1}{3x-4}$

d $\frac{3}{5x-2} + \frac{1}{4x-1} + \frac{2}{3x-2}$

e $\frac{5}{x-2} + \frac{5}{x-1} - \frac{3}{x+1}$

f $\frac{1}{5x+1} + \frac{2}{4x+3} + \frac{3}{4x-1}$

3 a $\frac{2}{x+3} + \frac{5}{x-2} + \frac{1}{(x-2)^2}$

b $\frac{2}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$

c $\frac{3}{2x-1} - \frac{7}{(3x+1)^2} - \frac{1}{(3x+1)}$

d $\frac{2}{3x-2} + \frac{4}{2x-1} + \frac{1}{(2x-1)^2}$

4 a $\frac{1}{x-2} + \frac{2}{(x-2)^2}$ **b** $\frac{1}{2x-1} - \frac{2}{(2x-1)^2}$

c $\frac{5}{x+1} + \frac{4}{(x-1)^2}$ **d** $\frac{3}{x-1} - \frac{1}{(x+3)^2}$

e $\frac{5}{(x+5)^2} + \frac{4}{3x-2}$ **f** $\frac{5}{x} + \frac{3}{x+1} - \frac{7}{x-1}$

g $\frac{7}{2x-1} - \frac{3}{x} - \frac{1}{2x+1}$

h $\frac{2}{x+4} + \frac{1}{(x-1)^2} - \frac{1}{(x-1)}$

Exercise 3C page 65

1 a $1 + \frac{1}{x+3} + \frac{1}{x-2}$

b $3 - \frac{1}{5x+1} + \frac{2}{2x-1}$

c $x + \frac{1}{x-3} + \frac{1}{x-1}$

d $2x + \frac{2}{x-2} + \frac{3}{x-1}$

e $x + 1 + \frac{3}{x+3} + \frac{4}{x+1}$

f $2x + 3 - \frac{2}{x-3} + \frac{5}{x+4}$

g $3x + 2 + \frac{2}{x+1} - \frac{1}{2x-1}$

h $2x - 1 + \frac{4}{2x-3} - \frac{3}{3x-5}$

2 a $3 + \frac{1}{x+1} + \frac{2}{(x-1)^2}$

b $2 - \frac{3}{2x-1} + \frac{4}{3x-2} - \frac{5}{(3x-2)^2}$

c $x + \frac{1}{x-2} + \frac{1}{x-1} + \frac{2}{(x-1)^2}$

d $x + \frac{2}{x+1} + \frac{3}{x-2} + \frac{4}{(x-2)^2}$

3 $\frac{1}{1-2x} - \frac{1}{1-x} + \frac{1}{(1-x)^2}$

Examination Exercise 3 page 66

1 $A = -\frac{5}{2}, B = \frac{9}{2}$

2 $A = 3, B = -1$

3 $A = 4, B = 3, C = 1$

4 $A = -2, B = 3$

5 $\frac{2}{x-2} - \frac{1}{x+1} - \frac{3}{(x-2)^2}$

6 $\frac{4}{x+2} - \frac{3}{x-1} + \frac{2}{(x-1)^2}$

7 $\frac{-50}{x} + \frac{25}{x^2} + \frac{100}{2x+1}$

Exercise 4A page 70

1 a 4, 7, 10, 13 **b** 3, 7, 11, 15

c 1, 4, 9, 16 **d** $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

e 2, 4, 8, 16 **f** 2, 6, 12, 20

g $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}$ **h** $-1, 1, -1, 1$

2 a $5n$ **b** $2n + 1$ **c** $4n - 3$

d n^2 **e** 3^n **f** 10^n

g $\frac{n}{n+1}$ **h** n^3 **i** $\frac{1}{n+1}$

3 a 2, 7, 12, 17, 22 **b** −1, 9, 19, 29, 39

c 1, 7, 1, 7, 1 **d** 2, 4, 16, 256, 65 536

e $2, \frac{1}{2}, 2, \frac{1}{2}, 2$ **f** 0, −2, −8, −26, −80

g 1000, 100, 10, 1, $\frac{1}{10}$ **h** $1, 2, 2\frac{1}{2}, \frac{29}{10}, \frac{941}{290}$

4 a $2n + 3$ **b** $14 - 4n$

c $7n - 15$ **d** $12 - 3n$

5 a $u_{n+1} = u_n + 5, u_1 = 7$

b $u_{n+1} = u_n + 3, u_1 = -2$

c $u_{n+1} = u_n - 2, u_1 = 2$

d $u_{n+1} = u_n + \frac{1}{2}, u_1 = 10\frac{1}{2}$

6 a $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ **b** convergent.

c 0

7 a $0, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}$ convergent. **b** 1

8 a $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$; convergent; limit 0

b 5, 9, 13, 17; divergent.

c 2, 5, 10, 17; divergent.

d $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$; convergent, limit 1

e −5, 5, −5, 5; periodic.

f 1, 5, 1, 5; periodic.

g $5, \frac{1}{5}, 5, \frac{1}{5}$; periodic.

h 1, 100, $\frac{1}{100}$, 1 000 000; divergent.

i 2, 12, 2, 12; periodic.

j −2, 4, −8, 16; divergent.

9 1, 2, 3, 5, 8, 13

10 1, 2, 5, 11, 26

11 3, 5.$\dot{3}$, 4.3125, 4.623 188…, 4.514 10…, 4.550 694, 4.538 226…, 4.542 452 7,

a $u = 4.54$

12 $1, 3, 2\frac{2}{3}$, 2.6875, 2.686 046 512, 2.686 147 186

a 2.69

13 a −2.79

14 a 3.30 **b** $x^2 - 3x - 1 = 0$

Exercise 4B page 73

1 a 24 **b** 14 **c** 10

d 38 **e** 30 **f** $2\frac{1}{12}$

g 21 **h** 28 **i** 63

j −6 **k** 35 **l** $\frac{23}{12}$

2 a $\sum_{r=1}^{4} r^2$ **b** $\sum_{r=1}^{4} 3^r$ **c** $\sum_{r=1}^{5} \frac{1}{r}$

d $\sum_{r=1}^{4} 2r$ **e** $\sum_{r=1}^{100} r$ **f** $\sum_{r=1}^{n} r(r+1)$

g $\sum_{r=1}^{100} \frac{r}{r+1}$ **h** $\sum_{r=1}^{6} (-1)^r$ **i** $\sum_{r=1}^{5} 4r + 1$

j $\sum_{r=1}^{5} 5r - 1$ **k** $\sum_{r=1}^{\infty} r^3$ **l** $\sum_{r=1}^{\infty} (-1)^{r+1} 2r$

3 10 **4** 10 000 **5** $\frac{1}{1000}$

Exercise 4C page 76

1 a Yes, 1 **b** Yes, −3 **c** No

d Yes, a **e** Yes, 3 **f** No

2 a $4n - 3$ **b** $2n + 8$ **c** $9n - 1$

d $22 - 3n$ **e** $29 - 4n$

3 **a** 106 **b** 107 **c** 4
d 7 **e** $\frac{11}{4}$
4 **a** $9n - 6$ **b** 16th
5 **a** $7n + 3$ **b** 23rd
6 **a** $35 - 6n$ **b** 15th
7 **a** 245 **b** 1170 **c** 48 **d** $13\frac{1}{2}$
8 **a** $4n - 3$ **b** 11th **c** 231
9 **a** $7n - 4$ **b** 6th **c** 123
10 **a** 1820 **b** 9 **c** 18 653
11 **a** 5 **b** 1130
12 **a** 11 **b** 182
13 **b** $a = 13, d = 7$
14 **b** $a = 17, d = -4$ **c** 42
15 **a** $a = 42, d = -3$ **c** 29
16 **a** $a = -18, d = 3$ **c** 14
17 $d = 5, a = -3$
18 15 500 **19** £317.20
20 **a** £1050 **b** £11 500
21 **a** 14 days **b** 104 days **c** 290 days
23 2600
24 **a** 15 050 **b** 10 150 **c** 9900
25 $n = 3$ **26** $x = 7$

Exercise 4D page 80

1 **b** $a = 7, d = 5$
2 **b** $a = 5, d = 2$
3 $a = 15, d = 7$
4 **a** $\frac{1}{2}(7n^2 + 3n)$ **b** 17
5 **a** 22 **b** 8, 14 **c** 6
6 **a** 2, 3 **b** 51
7 **a** $\frac{n}{2}(3n + 7)$ **b** 12
8 **a** $a = 11, d = 4$ **b** $r = 5$
9 **a** 7350 m
10 **a** 180 **b** 2940 **c** 7
d Unlikely to be able to increase sales month on month indefinitely.
11 **a** £67 500 **b** 4.3 km
12 $a = 9, d = 2$
13 1683 **14** 17
15 $a = 2, b = -3, S_{30} = 1365$

Exercise 4E page 85

1 **a** $3 \times 2^{n-1}$ **b** $2 \times 5^{n-1}$
c $20 \times (\frac{1}{2})^{n-1}$ **d** $200 \times (\frac{1}{5})^{n-1}$
e $\frac{1}{8} \times 2^{n-1} (= 2^{n-4})$ **f** $4 \times 3^{n-1}$
g $5 \times (-3)^{n-1}$
2 $a = 7, r = 3$
3 $a = \frac{35}{32}, r = 2$
4 $a = 5120, r = \frac{1}{2}$
5 **a** ± 2 **b** ± 6
6 192, $\frac{3}{4}$
7 2
8 2
9 **a** $a = 1, d = \frac{1}{2}$ **b** $r = 2$
10 **a** 1023 **b** 2186
c 195 312 **d** 615
11 **a** 79.9 **b** 80.0
c 607.5 **d** 1706.7
e 66.7
12 **a** $7r$ **b** 5
13 **b** $-4, 3$
14 **a** £2048 **b** £4095
c No. The amounts are increasing too much by the end of the year.
15 **a** 94.3 m **b** 637.5 m
16 **a** 6th week **b** 15 weeks
17 **a** £29 458 **b** 18th year
c Unlikely to stay in the same role for 17 years, rate of inflation likely to change.
18 £376 000 (3 sf). No, the market can fluctuate significantly over this time.
19 **c** $r = 3, a = 7$
20 **b** 15
21 **a** −177 147 **c** 11
22 5
23 **a** $\frac{5}{2}(3^n - 1)$ **b** 1 328 600 **c** 16
24 7
25 4, −12
26 **a** 88 572 **b** 12 582 900 **c** 850

Exercise 4F page 89

1 **a** 10 **b** 6 **c** 20
d 4 **e** 10 **f** $\frac{14}{5}$
2 $\frac{2}{3}$ **3** 6 **4** 21 **5** $-\frac{2}{7}$
6 **a** 40 **b** 3 **c** 5 **d** 9
8 $r = \frac{1}{3}, a = 18$ or $r = \frac{2}{3}, a = 9$
9 3 **10** $\frac{x-1}{x+1}$ **11** $\frac{1}{10}$
12 $\frac{3}{5}$ **13** 18 m
14 **a** $a = \frac{10}{3}, r = \frac{2}{3}$ **b** $10[1 - (\frac{2}{3})^n]$
c 3 **d** 10
15 **a** 192 **b** $3 \times 4^{n-1}$ **c** $a = 3, r = 4$
16 $r = \frac{1}{2}, a = 10$
17 $a = 2, r = \sqrt{2}, S_{10} = 62(\sqrt{2} + 1)$
18 **a** $6n + 7$ **b** Arithmetic
19 **a** $\frac{3}{2}(3^{20} - 1)$ **b** $\frac{4}{3}(4^{30} - 1)$ **c** $14(2^{10} - 1)$
20 **a** 5 **b** $-2\frac{1}{3}$

Review Exercise 4G page 91

1 **a** 3, 5, 7, 9 **b** 1, 4, 7, 10
c 1, 4, 9, 16 **d** 10, 100, 1000, 10 000
e $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ **f** 3, 8, 15, 24
g $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ **h** −1, 1, −1, 1
2 **a** $4n$ **b** n^2 **c** $3n + 1$
d $\frac{n}{n+2}$ **e** $\frac{1}{n}$

3 a 4, 7, 10, 13 **b** 50, 45, 40, 35
c $3, \frac{1}{3}, 3, \frac{1}{3}$ **d** 64, 32, 16, 8
e 10, 100, 10 000, 10^8 **f** 1,1, 1, 1

4 a 34 **b** 30 **c** 25
d 20 **e** 31 **f** 12
g −6 **h** $\frac{23}{12}$ **i** $\frac{13}{12}$

5 a $\sum_{r=1}^{4} 3n$ **b** $\sum_{r=3}^{6} r^2$ **c** $\sum_{r=1}^{4} 10^r$
d $\sum_{r=1}^{50} 2n$ **e** $\sum_{r=1}^{10} r^{-2}$ **f** $\sum_{r=3}^{6} \frac{1}{r}$

6 a 39 **b** 151 **c** 13 **d** 21
7 a 820 **b** 8200 **c** 1 101 100 **d** $n(5n-3)$
8 $a = -3, d = 5$ **9** $a = 3, d = 5$
10 $a = 121, d = -6$ **11** 15 500
12 £923 **13** 30
14 2 **15** 4234
16 $a = 6, d = 2$
17 a 17 **b** 7 **c** −330
18 a −25 **b** 345
19 a $5 \times 2^{n-1}$ **b** $2 \times 3^{n-1}$ **c** $100 \times (\frac{1}{2})^{n-1}$
20 765 **21** $a = 2\frac{1}{2}, r = 2$
22 $r = 4$ **23** 4, −5
24 a $x + 1$ **b** $x(x+1)^{99}$
25 10 **26** 48 **27** 34.9 cm
28 a $(x^2 + x - 2)$ **b** 192

Examination Exercise 4 page 94

1 b −4 **c** 1953
2 b i 6 **ii** 90
3 a 11 **b** £557 500
4 b 3010 **c** 2009
5 a i $\frac{1}{2}$, 4 **ii** periodic/oscillating
b $a = -2, d = \frac{5}{2}$
6 b 180 **c** £1710
e $n = 18$, aged 27
7 i −2 **ii** 3072 **iii** −1 048 575
8 b 256 **c** 1.602
9 a 17.28 **b** 120 **c** 25.92
10 i $k = 28$ **b** $r = -\frac{1}{2}, |r| < 1, a = 6$
11 i a $\frac{8}{9}$ **b** 18 **ii** 28
12 a 0.75 **b** 256 **c** 1024 **d** 14
13 a $p = \frac{4}{5}, q = 4$ **b** 195
14 i 7.5 **ii** 64 **iii** $-\frac{2}{3}$
15 i 80, 75, 70 **ii** 650 **iii** 8 **iv** 320
16 i 91 **ii** 978 **iii** 38
17 a 2 **b** 198
19 a $5k + 3$ **b** $156k + 114 = 6(26k + 19)$
20 a $a_2 = 5 - 4k$
$a_3 = 5 - k(5 - 4k)$
b $17 - 9k + 4k^2$
c 500
21 i a 4 **b** 80
ii a $3k, 4k$ **b** 11

Exercise 5A page 101

1 a $1 - 2x + 3x^2$ **b** $1 - 3x + 6x^2$
c $1 - 6x + 21x^2$ **d** $1 - 10x + 55x^2$
e $1 + \frac{1}{2}x - \frac{1}{8}x^2$ **f** $1 + \frac{3}{2}x + \frac{3}{8}x^2$
g $1 - \frac{1}{2}x + \frac{3}{8}x^2$ **h** $1 - \frac{3}{4}x + \frac{21}{32}x^2$
i $1 - \frac{2}{3}x - \frac{1}{9}x^2$ **j** $1 + \frac{2}{5}x + \frac{7}{25}x^2$
k $1 - \frac{1}{2}x - \frac{1}{8}x^2$ **l** $1 + \frac{1}{3}x + \frac{2}{9}x^2$

2 0.980
3 1.010

4 a $-\frac{1}{2} < x < \frac{1}{2}$ **b** $-1 < x < 1$
c $-\frac{1}{5} < x < \frac{1}{5}$ **d** $-\frac{1}{2} < x < \frac{1}{2}$
e $-\frac{1}{2} < x < \frac{1}{2}$ **f** $-\frac{1}{4} < x < \frac{1}{4}$
g $-\frac{1}{3} < x < \frac{1}{3}$ **h** $-\frac{1}{2} < x < \frac{1}{2}$
i $-3 < x < 3$ **j** $-2 < x < 2$

5 a $1 - 6x + 24x^2 - 80x^3$
b $1 + 2x + 3x^2 + 4x^3$
c $1 - 20x + 250x^2 - 2500x^3$
d $1 - 14x + 112x^2 - 672x^3$
e $1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$
f $1 + 10x + 30x^2 + 20x^3$
g $1 + \frac{3}{2}x + \frac{27}{8}x^2 + \frac{135}{16}x^3$
h $1 - \frac{1}{2}x - \frac{3}{8}x^2 - \frac{7}{16}x^3$
i $1 - \frac{x}{6} + \frac{x^2}{72} - \frac{x^3}{432}$
j $1 + \frac{x}{6} + \frac{x^2}{18} + \frac{7x^3}{324}$

6 $-\frac{77}{128}x^4$ **7** $\frac{4389}{256}x^3$ **8** $-\frac{1}{125}x^3$
9 a $x^3 + 6x^2 + 12x + 8$
b $15\sqrt{3} + 26$
10 b $264\sqrt{2}$
11 $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$, 1.0392
12 −20
13 $1 - \frac{1}{2x} - \frac{1}{8x^2} - \frac{1}{16x^3}$, 9.94987
14 a $1 - \frac{x}{3} - \frac{x^2}{9}$ **b** 3.332222

Exercise 5B page 103

1 a $\frac{1}{2} - \frac{x}{4} + \frac{x^2}{8}$, $|x| < 2$

b $\frac{1}{64}\left(1 + \frac{3x}{4} + \frac{3}{8}x^2\right)$, $|x| < 4$

c $\frac{1}{9}\left(1 - \frac{2}{3}x + \frac{x^2}{3}\right)$, $|x| < 3$

d $\frac{1}{625}\left(1 - \frac{4x}{5} + \frac{2x^2}{5}\right)$, $|x| < 5$

e $2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2\right)$, $|x| < 4$

f $\frac{1}{3}\left(1 + \frac{1}{18}x + \frac{1}{216}x^2\right)$, $|x| < 9$

g $2\left(1-\frac{1}{24}x-\frac{1}{576}x^2\right)$, $|x|<8$

h $\frac{1}{4}\left(1-\frac{1}{12}x+\frac{5}{576}x^2\right)$, $|x|<8$

i $5^5\left(1+\frac{x}{10}+\frac{3x^2}{1000}\right)$, $|x|<25$

2 a $2\left(1+\frac{3x}{8}-\frac{9x^2}{128}\right)$, $|x|<\frac{4}{3}$

b $\frac{1}{3}\left(1+\frac{x}{9}+\frac{x^2}{54}\right)$, $|x|<\frac{9}{2}$

c $2\left(1-\frac{5x}{24}-\frac{25x^2}{576}\right)$, $|x|<\frac{8}{5}$

d $125\left(1-\frac{21x}{50}+\frac{147}{5000}x^2\right)$, $|x|<\frac{25}{7}$

3 a $a=\frac{1}{2}, b=-\frac{3}{16}$ **b** $a=2, b=\frac{1}{4}$

c $a=\frac{1}{\sqrt{2}}, b=\frac{7}{4\sqrt{2}}$ **d** $a=\frac{1}{3}, b=-\frac{2}{243}$

4 $3+\frac{2x}{3}-\frac{2x^2}{27}+\frac{4x^3}{243}$, $|x|<\frac{9}{4}$

5 a $1-x-\frac{x^2}{2}$, $|x|<\frac{1}{2}$

b 0.990

6 a $1-\frac{3x}{5}-\frac{18}{25}x^2$ **b** 1.961

Exercise 5C page 104

1 a $-\left(\frac{13}{3}+\frac{19}{9}x+\frac{28}{27}x^2\right)$, $|x|<2$

b $-\left(2+\frac{3x}{2}+\frac{5x^2}{4}\right)$, $|x|<1$

c $\frac{11}{2}-\frac{17x}{4}+\frac{29x^2}{8}$, $|x|<1$

d $\frac{5}{2}-\frac{11x}{12}+\frac{25x^2}{72}$, $|x|<2$

e $-\frac{3}{2}x-\frac{3}{4}x^2$, $|x|<1$

f $-\frac{7}{3}-\frac{17x}{9}-\frac{55x^2}{27}$, $|x|<1$

2 a $1+\frac{1}{x-2}+\frac{1}{x-1}$

b $-\left(\frac{1}{2}+\frac{5x}{4}+\frac{9x^2}{8}+\frac{17x^3}{16}\right)$

c $|x|<1$

3 a $1-x-4x^2-6x^3$ **b** $|x|<\frac{1}{4}$

4 $\lambda=2, n=-3$

5 Both equal -2

6 $1+\frac{3x}{2}+\frac{7x^2}{8}$, $|x|<1$

7 a $1+x+\frac{1}{2}x^2$, $|x|<1$

8 a $a=\frac{3}{2}, b=\frac{1}{2}$ **b** $\frac{1970}{1393}$ or $\frac{1393}{985}$

9 a $q=3$

b $-x^3$

c $-1<x<\frac{1}{3}$

10 a $\frac{[1-(-x)^n]}{1+x}$ **b** $-1<x<1$ **c** $\frac{1}{1+x}$

11 b $1+2x+3x^2+4x^3+\ldots$

c $1+3x+6x^2+10x^3+\ldots$

d $1+4x+10x^2+\ldots$

13 a $a_0=1, a_1=-\frac{1}{2}, a_2=\frac{3}{8}, a_3=-\frac{5}{16}$

Review Exercise 5D page 106

1 a $|x|<\frac{2}{3}$ **b** $|x|<\frac{5}{3}$ **c** $|x|<\frac{2}{7}$

d $|x|<\frac{9}{2}$ **e** $|x|<\frac{3}{2}$ **f** $|x|<\frac{5}{3}$

2 $2-\frac{x}{4}-\frac{x^2}{32}$, $|x|<\frac{8}{3}$

3 a $a=\frac{1}{9}, b=\frac{4}{729}$ **b** $a=\frac{1}{8}, b=\frac{9}{64}$

4 a $\frac{1}{3(x+1)}+\frac{2}{3(1-2x)}$ **b** $1+x+3x^2$

c $|x|<\frac{1}{2}$

5 a $\frac{1}{2x+1}+\frac{3}{x-1}$ **b** $-2-5x+x^2$

c $|x|<\frac{1}{2}$

6 a $\frac{4}{3x-1}+\frac{8}{x+2}$ **b** $-108\frac{1}{2}$

c $|x|<\frac{1}{3}$

Examination Exercise 5 page 107

1 a $1+x-2x^2$

b i $\frac{1}{4}-\frac{1}{16}x+\frac{5}{256}x^2$ **ii** 0.2313

2 a $2+\frac{5}{4}x-\frac{25}{64}x^2$ **b** $\frac{3}{2}\sqrt{2}$ **c** $\frac{181}{128}$

3 a $2-\frac{3}{4}x-\frac{9}{32}x^2-\frac{45}{256}x^3$

b $x=0.1, 19.2201$

4 i $1-2x+7x^2-22x^3$

ii $|x|<\frac{1}{4}$

5 ii 0.136 **iii** $-\frac{1}{x^2}-\frac{3}{x^3}-\frac{6}{x^4}$

6 $n=-1.5, k=4$

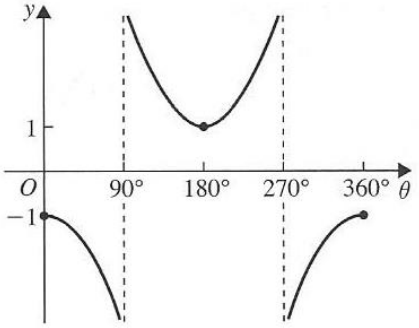
7 $p=2, q=-2, |x|<2$

Exercise 6A page 114

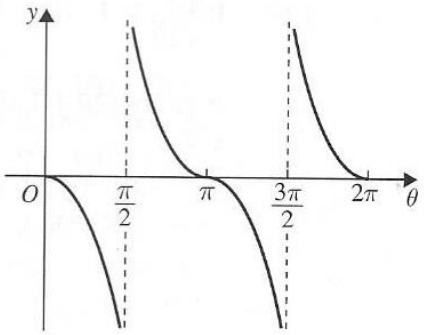
1 a $\sqrt{3}$ **b** $\frac{2}{\sqrt{3}}$ **c** 1.015

d $\sqrt{2}$ **e** 0.325 **f** $\frac{4}{3}$

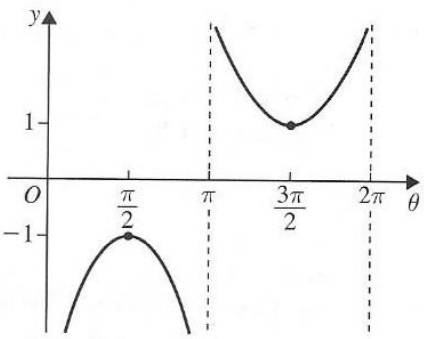
2 a



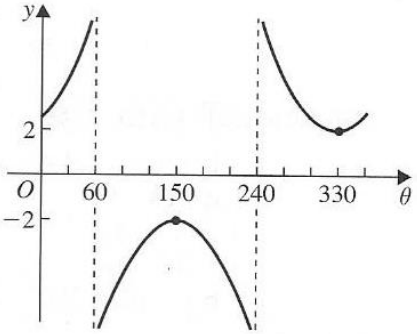
b



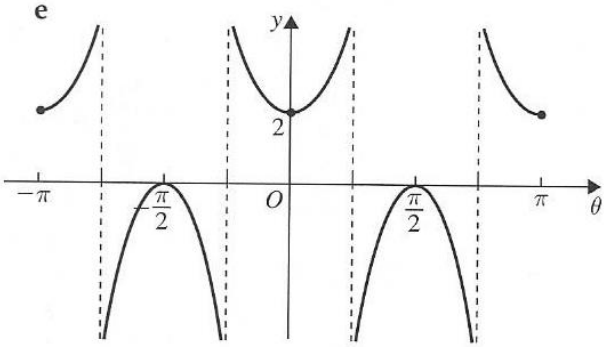
c



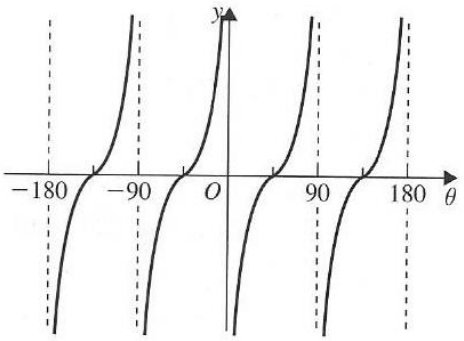
d



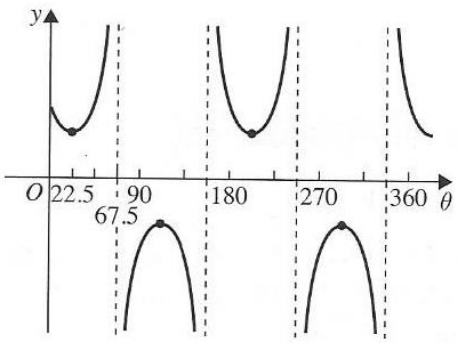
e



f



g



3 **a** $\sec^2 x$ **b** $\sec^2 2x$ **c** $\sin^2 x$
d $\sec x$ **e** $\sin\theta\cos\theta$ **f** $\operatorname{cosec}^2\theta$
5 **a** 60°, 300° **b** 30°, 210° **c** 45°, 135°
d 33.6°, 326.4° **e** 18.4°, 198.4° **f** 90°
7 **a** 45°, 225°, 63.4°, 243.4°
b −180°, 0, 180°, −135°, 45°
c 90°, 30°, 150°
9 $30° < \theta < 150°$
10 **a** $\cos\theta$ **b** $-60° < \theta < 60°$
11 **a** 35.3°, 144.7°, 215.3°, 324.7°
b 30°, 60°, 120°, 150°
12 (22.5, 7.5), (67.5, 142.5)

Exercise 6B page 117

1 **a** 30° **b** 90° **c** 45°
d 120° **e** −30° **f** 60°
2 **a** $-\frac{\pi}{4}$ **b** $\frac{\pi}{3}$ **c** $\frac{\pi}{4}$
d 0 **e** $\frac{\pi}{3}$ **f** $-\frac{\pi}{2}$
3/4 See text
5 **a** 60°, 120° **b** 45°, 225° **c** 0, 360°
6 **a** $\frac{1}{2}$ **b** 0 **c** θ **d** θ

Exercise 6C page 120

1 **b** 1
2 **b** 0
3 **a** False **b** True
4 **a** $\sin 3\theta$ **b** $\cos 2A$ **c** $\sin 3x$ **d** 1
5 **a** $\frac{1+\sqrt{3}}{2\sqrt{2}}$ **b** $\frac{1+\sqrt{3}}{2\sqrt{2}}$ **c** $\frac{\sqrt{3}-1}{2\sqrt{2}}$
d $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **e** $\sqrt{3}-2$
6 **a** $\frac{63}{65}$ **b** $\frac{56}{65}$ **c** $\frac{63}{16}$
7 73.2°, 286.8°
8 **a** $-\sin x$ **b** $-\cos x$ **c** $-\sin x$ **d** $-\cos x$
9 $\frac{1}{3}$
10 1
11 45°, 225°
12 **a** 1, $\theta = 50°$ **b** 1, $\theta = 105°$
14 $\tan(x + 45°)$
15 **a** $\frac{2\tan A}{1-\tan^2 A}$ **b** $\frac{1}{2}$
17 **a** $2+\sqrt{3}$ **b** $\sqrt{6}+\sqrt{2}$ **c** $\sqrt{6}-\sqrt{2}$

23 $\frac{1}{3}$

24 b 0.625, 2.195

Exercise 6D page 124

1 a 1 **b** $\frac{\sqrt{3}}{2}$ **c** $\frac{1}{2}$ **d** 1

2 a $\sin 20°$ **b** $\cos 34°$ **c** $\cos 70°$ **d** $\tan 22°$
e $\frac{1}{2}\sin 2\theta$ **f** $\frac{1}{2}\tan 4\theta$ **g** $2\sin A$ **h** 1

3 $\sin 2\theta = \frac{24}{25}$, $\cos 2\theta = \frac{7}{25}$

4 $\sin 2\theta = \frac{120}{169}$, $\cos 2\theta = -\frac{119}{169}$

5 a $\frac{4}{3}$ **b** $\frac{24}{7}$

6 $\frac{\pm\sqrt{7}}{4}$

7 $\pm\frac{4}{5}$

8 $\frac{1}{3}$ or -3

9 0, 180°, 360°, 60°, 300°

10 90°, 270°, 194.5°, 345.5°

11 60°, 300°, 75.5°, 284.5°

12 31.7°, 121.7°, 211.7°, 301.7°

13 0°, 180°, 360°, 30°, 150°

14 60°, 300°, 109.5°, 250.5°

15 0°, 60°, 120°, 180°, 240°, 300°, 360°

16 0°, 360°, 240°

17 0°, 120°, 360°

18 14.5°, 165.5°

37 $2\sin\frac{x}{2}\cos\frac{x}{2}$

38 $\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

41 b $2+\sqrt{3}$

42 18.4°, 161,6°, 198.4°, 341.6°

43 $A = 1, B = 4, C = 3$

Exercise 6E page 128

1 a $5\cos(\theta - 36.9)°$ **b** $13\cos(\theta - 67.4)°$
c $\sqrt{5}\cos(\theta - 63.4)°$

2 a $5\sin(\theta + 53.1)°$ **b** $5\cos(\theta + 36.9)°$
c $\sqrt{5}\cos(\theta - 26.6)°$ **d** $17\sin(\theta - 61.9)°$
e $10\sin(\theta + 36.9)°$ **f** $\sqrt{2}\cos(\theta + 45)°$

3 a $2\sin(\theta + 60)°$
b Maximum value = 2 at $\theta = 30°$

4 a 5, 53.1° **b** 13, 157.4°
c $\sqrt{2}$, 45° **d** 17, 298.1°

5 a $5\cos(\theta - 36.9)°$ **b** 103.3°, 330.5°

6 a $3\sin(\theta + 70.53)°$ **b** 67.7°, 331.3°

7 a 1.9°, 121.9° **b** 80.0°, 325.2°
c 90°, 330° **d** 60.4°, 193.3°
e 257.6°, 349.8° **f** 40.8°, 201.1°
g 78.4°, 244.8°
h 39.0°, 162.8°, 219.0°, 342.8°

8 a 2.25, 0.15 **b** 0.38, 1.97

9 a $\sqrt{7}\cos(x - 0.714)$ **b** 1.43

10 a 0.36, 2.14 **b** 0, 4.07, 6.28
c 0, 4.71, 6.28 **d** 1.70, 3.29

11 a $\sqrt{10}\sin(\theta - 71.6)°$ **b** $\frac{4}{\sqrt{10}} > 1$

12 $-\sqrt{3} \leqslant k \leqslant \sqrt{3}$

13 a $c = 12$ **b** $13\cos(2\theta + 1.176)$

14 a $5\cos(\theta + 0.93)°$ **b** 2

15 a $13\cos(\theta - 1.176)$ **b** 2

16 a $5\sin(2\theta + 36.9)° - 3$
b 8.1°, 45.0°

17 a $5\cos(x - 0.6435)$
b $\frac{1}{2}, \frac{1}{12}$
c 1.429, 3.000

18 a i 15.6°C **ii** 4.45
b 2126 and 1128

19 a i max: $h = 68.6$ m, $t = 1.30$
min: $h = 51.4$ m, $t = 0.30$
b 2 hours
c 41.5 minutes

20 a i $L = 2\cos\theta + 4\sin\theta$
ii $L = 2\sqrt{5}\sin(\theta + 0.464)$
b i $2\sqrt{5}$ **ii** 1.11

21 a $0 \leqslant \theta \leqslant 360$ and $0 \leqslant t \leqslant 20 \Rightarrow 0 \leqslant 18t \leqslant 360$
b 5.4 minutes

Review Exercise 6F page 131

1 a 41.8°, 138.2° **b** 138.6°, 221.4°
c 145.0°, 325.0° **d** 210°, 330°
e $\frac{\pi}{3}, \frac{5\pi}{3}$ **f** $\frac{\pi}{4}, -\frac{3\pi}{4}$
g 60°, 300° **h** 0°, 180°, 360°

2 a 0°, 132°, 228°, 360°
b 58°, 238°, 148°, 328°

3 $\frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}$

4 a 45° **b** 0° **c** 60°

5 a $\frac{\pi}{6}$ **b** $\frac{\pi}{6}$ **c** $\frac{\pi}{6}$

7 a 18.4°, 198.4°, 26.6°, 206.6°
b 60°, 300°
c 90°, 270°, 199.5°, 340.5°

8 $-\frac{16}{65}$

9 a $-\frac{3}{5}$
b $\sin 2\theta = -\frac{24}{25}$, $\cos 2\theta = -\frac{7}{25}$

11 a 60°, 300°, 180°
b 0, 180°, 360°, 80.4°, 279.6°
c 25.2°, 154.8°
d 90°, 270°, 30°, 150°
e 0, 360°, 120°, 240°
f 0, 180°, 360°, 30°, 330°, 150°, 210°

12 a $-\frac{7}{25}$ **b** $\frac{24}{25}$ **c** $-\frac{24}{7}$

13 a i $\frac{\sqrt{6}-\sqrt{2}}{4}\left(=\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

ii $2+\sqrt{3}\left(=\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$

iii $\frac{\sqrt{2}-\sqrt{6}}{4}\left(=\frac{1-\sqrt{3}}{2\sqrt{2}}\right)$

14 a $-\frac{119}{169}$ **b** $\frac{2}{13}\sqrt{13}$ **c** $-\frac{120}{119}$

15 $\frac{1}{3}$

16 a 30°, 150°, 270°

b 0, 180°, 360°, 30°, 150°, 210°, 330°

c 90°, 120°, 240°, 270°

d 0, 120°, 240°, 360°

e 0, 60°, 300°, 360°

17 $3\sin\theta - 4\sin^3\theta$

18 a $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)$ **b** $0, \frac{3\pi}{2}, 2\pi$

c $\sqrt{2}$

19 a $\sqrt{3}\sin(\theta + 54.74)°$ **b** 90°, 340.5°

20 a $2\cos(\theta + 60)°$ **b** $\frac{1}{4}$

21 a $5\sin(\theta + 36.9)°$ **b** 120°, 347°

22 $d = 0.838$ m, 0823 and 2023

23 a $x = 0.3\cos\theta, y = 0.9\sin\theta$

b 6.5°

Examination Exercise 6 page 134

1 6.5°, 53.5°, 126.5°, 173.5°

2 i $\frac{5}{3}$ **ii** $\sqrt{17}$

3 i $\tan\alpha = 2, \tan\beta = 5$

ii $-\frac{7}{9}$

4 a ii $-\frac{5}{13}$ **b** −21.3°

5 37.9°, 142°

6 i $\lambda = \frac{1}{2}$ **ii** $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$

7 b 2.820, 5.961

8 ii a $4 + 2\sqrt{2}$ **b** 0.659 **c** $32\sin\theta$

9 99.6°, 260.4°

10 i a $1 + \frac{\sqrt{2}}{2}$ **ii a** $k = 2$

b 0°, 30°, 150°, 180°

11 b ii 22.5°, 112.5°, 202.5°, 292.5°

12 ii 70.5°, −70.5°

iii $0 < k < \frac{3}{2}$

13 a $R = \sqrt{29}, \alpha = 1.19$

b i 5.09 **ii** 0.567, 3.34

14 a $R = \sqrt{5}, \alpha = 26.57°$

b 33.0°, 273.9° **c** 86.1°

15 a $R = 6.5, \alpha = 0.395$

b (0, 6), (1.97, 0), (5.11, 0)

c $H_{max} = 18.5, H_{min} = 5.5$

d 11, 48

16 i $R = 5, \alpha = 53.1$

ii a −64.7°, 138°

b $k = 8, c = 3$

17 a i $R = 5, \alpha = 53.1$

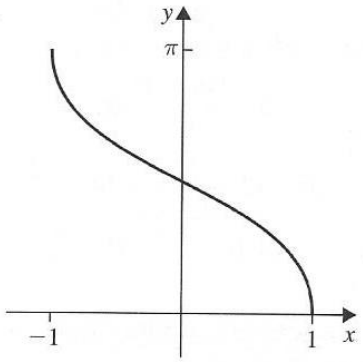
ii 18.4°, 198.4°

b ii 35.3°, 144.7°

c iii $\cos 72° = \frac{\sqrt{5} - 1}{4}$

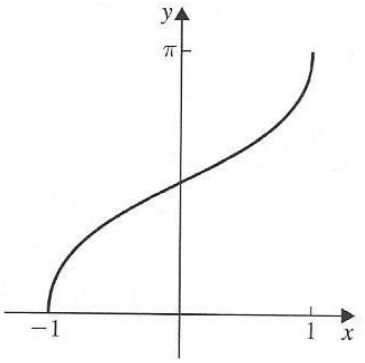
18 i 0.5 **ii** $\frac{\sqrt{2}}{2}$

19 a



(−1, π), (1, 0)

b



(−1, 0), (1, π)

Exercise 7A page 140

1 a $6(2x + 5)^2$ **b** $8x(x^2 + 7)^3$

2 a $12(3x - 4)^3$ **b** $40(8x + 11)^4$

c $4x(x^2 - 3)$ **d** $27x^2(3x^3 + 1)^2$

e $-12(1 - 3x)^3$ **f** $-10(3 - x)^9$

3 a $-3(1 + 3x)^{-2}$ **b** $-4(2x + 1)^{-3}$

c $-15(5x + 2)^{-4}$ **d** $-24(4x - 1)^{-3}$

e $-10x(x^2 + 1)^{-2}$ **f** $40(1 - x)^{-5}$

4 b $\frac{4}{7}\sqrt{7}$

5 a $\frac{5x}{\sqrt{5x^2 + 3}}$ **b** $(3x + 1)^{-\frac{2}{3}}$

c $-x(x^2 - 3)^{-\frac{3}{2}}$ **d** $-\frac{3x^2}{(x^3 + 1)^2}$

e $-4(8x + 7)^{-\frac{3}{2}}$ **f** $-\frac{1}{2\sqrt{x}(\sqrt{x} + 2)^2}$

6 $y = 64x - 48$ **7** $x + 18y - 1157 = 0$

8 (0, 8) **9** 48

10 $2y = x + 3$

11 a $3(2\sqrt{x} - 3x)^2\left(\frac{1}{\sqrt{x}} - 3\right)$ **b** $\frac{1}{2x^2}\left(1 - \frac{1}{x}\right)^{-\frac{1}{2}}$

c $-3\sqrt{x}(x^{\frac{3}{2}} + 2)^{-2}$ **d** $-\frac{1}{\sqrt{x}}(1 + \sqrt{x})^{-3}$

e $\frac{1}{2}\left(x^2 - \frac{1}{x}\right)^{-\frac{1}{2}}\left(2x + \frac{1}{x^2}\right)$ **f** $20x + (2x - 3)^{-\frac{1}{2}}$

12 a (1, 7) minimum and (−3, −9) maximum

13 a (1, 2)

Exercise 7B page 143

1 $(2x + 1)(6x + 1)$ **2** $2x(3x - 1)(6x - 1)$

3 $x^2(x - 1)(5x - 3)$ **4** $2(x + 2)(x + 3)(2x + 5)$

5 $2(1-x)^2(1-4x)$ **6** $2(x-1)(10x^3-6x^2+1)$

7 $5(1+2x)^3(10x+1)$ **8** $5(3x+1)^3(3x-7)$

9 $\frac{\sqrt{x}}{2}(5x+6)$ **10** $\frac{(4x+1)}{2\sqrt{x}}(20x+1)$

11 $\frac{x}{\sqrt{2x+1}}(5x+2)$ **12** $\frac{x^2}{\sqrt{4x-1}}(14x-3)$

13 $y = 19x - 1$ **14** 16 **15** $2, \frac{2}{5}$

16 a $-3, -\frac{1}{3}$

b maximum at $x = -3$, minimum at $x = -\frac{1}{3}$

17 a i $(-2, 0), (4, 0), (0, 32)$

ii $(0, 32)$ maximum; $(4, 0)$ minimum

18 a $(2, -4)$ **b** $(-1, -4)$

19 i $k = 3$ **ii** $\frac{16}{3}$

Exercise 7C page 147

1 $\frac{2}{(x+1)^2}$ **2** $-\frac{15}{(2x-1)^2}$

3 $\frac{3x^2+2x}{(3x+1)^2}$ **4** $\frac{4-10x-4x^2}{(x^2+1)^2}$

5 $\frac{-2(x+1)}{x^3}$ **6** $\frac{-4x-1}{(x+1)^4}$

7 $\frac{-6(3x+2)(2x+3)}{(4x+1)^4}$ **8** $\frac{-x^3-9x^2}{(x-3)^5}$

9 $\frac{(3x-4)^2(6x+25)}{(2x+1)^3}$ **10** $\frac{-x-2}{2x^2\sqrt{x+1}}$

11 $\frac{2x+3}{(4x+3)^{\frac{3}{2}}}$ **12** $\frac{3x-4}{(2x-1)^{\frac{3}{2}}}$

13 $16y = x + 1$ **14** $y = -3x - 1$

15 $y = 4 - x$ **16** $\frac{3}{2}$

17 $(3, -\frac{1}{6})$ minimum, $(-1, \frac{1}{2})$ maximum

Exercise 7D page 151

1 a e^x **b** $3e^{3x}$ **c** $2e^x$

d $2x\,e^{x^2}$ **e** $2e^{2x+1}$ **f** $-e^{-x}$

g $20e^{4x}$ **h** $e^x + 2e^{2x}$ **i** $2x + e^{-x}$

j $-2e^{-2x}$ **k** $-4e^{-x}$ **l** $3x^2e^{x^3} + 3x^2$

m $(4\ln 5)5^{4x}$ **n** $(300\ln 2)2^{10x}$ **o** $(-14\ln 3)3^{-7x}$

2 a $xe^x + e^x$ **b** $x^2e^x + 2xe^x$

c $2e^{2x}(2x+1)$ **d** $e^x(x^2+2x+1)$

e $4x^2e^{2x}(2x+3)$ **f** $3x^2e^{3x} + 2xe^{3x} - 3x^2$

g $\frac{e^{3x}}{x^2}(3x-1)$ **h** $\frac{xe^x - 2e^x - 2}{x^3}$

i $\frac{2x-x^2}{e^x}$ **j** $e^x(1+x)^2(4+x)$

k $e^{-x}(3x^2-x^3-1)$ **l** $\frac{2e^{2x}-e^{3x}}{(1-e^x)^2}$

3 a $3e^2$ **b** 3

c 1 **d** $-\frac{4e^2}{(e^2-1)^2}$

4 $y = e^2(x-1)$

5 $y = x$

6 b $y + e^2x = \frac{1}{e^2}$ **c** $\left(\frac{1}{e^4}, 0\right)$

7 $\left(-1, -\frac{1}{e}\right)$ minimum

8 $(1, e)$ minimum

9 a $(0, 2)$

10 a $(0, 0)$ minimum, $(-2, 4e^{-2})$ maximum

16 $(\frac{1}{3}\ln 2, 8)$ minimum

Exercise 7E page 155

1 a $\frac{1}{x}$ **b** $\frac{3}{x}$ **c** $\frac{6}{x}$

d $\frac{3}{3x-1}$ **e** $\frac{-2}{1-2x}$ **f** $\frac{3x^2+1}{x^3+x}$

g $\frac{1}{x+1}$ **h** $\frac{2}{x}$ **i** $\frac{3}{x+2}$

j $-\frac{1}{x}$ **k** $\frac{1}{2x}$ **l** $\frac{2x+1}{x^2+x-2}$

2 a $\ln(x+4) - \ln(x-2)$ **b** $\frac{1}{x+4} - \frac{1}{x-2}$

3 a $\frac{1}{x} - \frac{1}{x+1}$ **b** $\frac{2}{2x+3} - \frac{4}{4x-1}$

4 a $1 + \ln x$ **b** $x + 2x\ln x$

c $\frac{x}{1+x} + \ln(1+x)$ **d** $\frac{1-\ln x}{x^2}$

e $\frac{(x+1)-2x\ln x}{x(x+1)^3}$ **f** $\frac{\ln x - 1}{(\ln x)^2}$

g $2x + \frac{1}{x}$ **h** $\frac{3}{x} - \frac{1}{x^2}$

i $\frac{1}{2x} + \frac{1}{2\sqrt{x}}$ **j** $-\frac{\ln x}{x^2}$

k $\frac{2x^3}{1+x^2} + 2x\ln(1+x^2)$ **l** $\frac{1}{x+1} - \frac{1}{x+2}$

5 $x + 2x\ln x, 3 + 2\ln x$

7 $\frac{e^x}{1+e^x}$

8 b $3y = x + 3\ln 3 - 2$

9 $y = -x + \ln 2 + 1$

10 a $y = x - 2$

b 2 square units

11 A$(1, 0)$ B$\left(e, \frac{1}{e}\right)$

12 $\left(\frac{1}{e}, -\frac{1}{e}\right)$

13 $\frac{1}{2e}$

14 a $(3.67, -4.57)$

15 a $\frac{f'(x)}{f(x)}$ **b** $\ln(x^3+1)$

16 $y = e(x-1)$

17 $[1, \ln(e+1)]$

Exercise 7F page 159

1 a $\cos x$ **b** $-3\sin 3x$
c $4\sec^2 x$ **d** $6\cos 6x$
e $-\frac{3}{2}\sin\frac{3}{2}x$ **f** $10\cos 2x$
g $\frac{1}{2}\sec^2\frac{1}{2}x$ **h** $-10\sin x + \cos x$
i $\sec^2 x + \sin x$ **j** $\cos(x+1)$
k $2\sec^2(2x-1)$ **l** $\frac{1}{2}\sin\frac{1}{2}x$

2 a $2\sin x\cos x$ **b** $-6\cos x\sin x$
c $3\sin^2 x\cos x$ **d** $-6\cos^2 x\sin x$
e $-\frac{1}{2}(\cos x)^{-\frac{1}{2}}\sin x$ **f** $-8\cos 4x\sin 4x$
g $18\tan 9x\sec^2 9x$ **h** $1 + \dfrac{\cos x}{2\sqrt{\sin x}}$
i $3(\sin 2x)^{\frac{1}{2}}\cos 2x$

3 a $\sin x + x\cos x$ **b** $\cos 2x - 2x\sin 2x$
c $x^2\cos x + 2x\sin x$ **d** $\cos^2 x - \sin^2 x$
e $-\left(\dfrac{x\sin x + \cos x}{x^2}\right)$ **f** $\dfrac{2x\cos 2x - 2\sin 2x}{x^3}$
g $\dfrac{\sin x - x\cos x}{\sin^2 x}$ **h** $\dfrac{3x^2\cos x + x^3\sin x}{\cos^2 x}$
i $-\dfrac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$

6 a -6 **b** 4 **c** -2 **d** $\frac{\pi}{2} + 1$

9 $y = 2x + 1 - \frac{\pi}{2}$

10 a $\pi\cos\pi x$ **b** $2\pi\sec^2 2\pi x$
c $-\sin(x-\pi)$ **d** $-\frac{\pi}{2}\sin\frac{\pi}{2}x$
e $2x\cos x^2$ **f** $3x^2\sec^2 x^3$
g $2\cos(2x-\pi)$ **h** $\frac{1}{\pi}\cos\frac{x}{\pi}$
i $4x\sin(x^2)\cos(x^2)$

11 $0, \sqrt{\frac{\pi}{2}}$ **13** $2y + x = \frac{3\pi}{2}$

15 $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ **16** $-3\sqrt{3}$

18 b -4 **19** $\dfrac{\pi\sqrt{3}+3}{6}$

20 -4

21 $\left(\frac{\pi}{4}, \sqrt{2}\right)$ maximum $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$ minimum

22 a $(\cos x)^3 - 2(\sin x)^2\cos x$
b 0.62 1.57 2.53

23 a $2y + \sqrt{3} = -x + \frac{4\pi}{3}$
b $\left(0, \dfrac{4\pi - 3\sqrt{3}}{6}\right)$

24 a $-\pi$ **b** $y\pi = x - 1$

Exercise 7G page 162

1 a $-\operatorname{cosec} x\cot x$ **b** $-\operatorname{cosec}^2 x$
c $2\sec 2x\tan 2x$ **d** $-3\operatorname{cosec}^2 3x$
e $-4\operatorname{cosec} 4x\cot 4x$ **f** $-2\operatorname{cosec}^2 x$
g $2\sec(2x+1)\tan(2x+1)$
h $2x + \cos x + \sec x\tan x$

2 a $-2\cot x\operatorname{cosec}^2 x$ **b** $2\sec^2 x\tan x$
c $\sec x\tan^2 x + \sec^3 x$ **d** $6\sec^2 3x\tan 3x$
e $\sec x(x\tan x + 1)$
f $2x\cot 2x - 2x^2\operatorname{cosec}^2 2x$
g $\dfrac{\sec x(x\tan x - 1)}{x^2}$
h $\dfrac{2x\cot x + x^2\operatorname{cosec}^2 x}{\cot^2 x}$
i $2\sec x\tan x(1 + \sec x)$

6 $-2\cot x - \operatorname{cosec} x + c$

Exercise 7H page 163

1 a $\dfrac{2}{\sqrt{1-(2x)^2}}$ **b** $\dfrac{-1}{\sqrt{1-(x+1)^2}}$
c $\dfrac{1}{3\left(1+\left(\frac{x}{3}\right)^2\right)}$ **d** $\dfrac{1}{2\sqrt{1-\left(\frac{x+1}{2}\right)^2}}$
e $\dfrac{2x}{1+x^4}$ **f** $\dfrac{-1}{x\sqrt{x^2-1}}$
g $\dfrac{-1}{1+x^2}$ **h** $\dfrac{1}{\sqrt{1-(3-x)^2}}$
i $3x^2\arcsin(x) + \dfrac{x^3}{\sqrt{1-x^2}}$ **j** $\dfrac{-e^{\arccos(x)}}{\sqrt{1-x^2}}$

Exercise 7I page 165

1 a $\frac{1}{x}$ **b** $\frac{1}{x}$ **c** $\frac{2}{x}$
d $1 + \frac{1}{x}$ **e** $5e^x + \frac{1}{x}$ **f** $2e^x + \frac{5}{x}$

2 a $(\frac{1}{3}, \ln\frac{1}{3})$

3 a $(1, \ln 4 - 3)$

4 b $4x + 12y = 5$

5 $y = 6x - 2$

6 a $5e^x$ **b** $7e^x$ **c** $20e^{4x}$
d $\dfrac{1}{x+1}$ **e** $\dfrac{-4}{e^x}$ **f** $6e^{3x}(e^{3x}+1)$
g $2e^x + \frac{1}{x}$ **h** $5e^x$ **i** $\dfrac{-2e^x}{(e^x-1)^2}$

7 e^x

8 a $\frac{2}{x}$ **b** $\frac{3}{x}$ **c** $\frac{1}{2x}$
d $-\frac{1}{x}$ **e** $\frac{1}{2x}$ **f** $-\frac{1}{3x}$

9 a $2(x+1)$ **b** $14(2x+1)^6$
c $24(3x-5)^7$ **d** $45(3x-7)^2$
e $-12(4x+3)^{-4}$ **f** $-20(5x-2)^{-5}$
g $2(4x+11)^{-\frac{1}{2}}$ **h** $5(15x-17)^{-\frac{2}{3}}$
i $na(ax+b)^{n-1}$

11 $(1, 1)$

12 $(0, -1)$

13 a $1 + \ln x$ **b** $x^2e^x + 2xe^x$
c $\cos^2 x - \sin^2 x$ **d** $e^x\left(\sqrt{x} + \dfrac{1}{2\sqrt{x}}\right)$
e $e^x(\sec^2 x + \tan x)$ **f** $\cot x - x\operatorname{cosec}^2 x$
g $e^x\operatorname{cosec} x(1 - \cot x)$ **h** $x^2\sec x(x\tan x + 3)$
i $x^2(3\cos x - x\sin x)$

15 a $\dfrac{1-\ln x}{x^2}$ **b** $\dfrac{e^x}{x^3}(x-2)$

c $\dfrac{\sec x}{e^x}(\tan x - 1)$ **d** $\sec^2 x$

e $-\operatorname{cosec}^2 x$ **f** $\dfrac{x}{e^x}(2-x)$

g $\dfrac{x-1}{2x\sqrt{x}}$ **h** $\dfrac{x^2(3\cos x + x\sin x)}{\cos^2 x}$

i $-\dfrac{(x\operatorname{cosec}^2 x + 2\cot x)}{x^3}$

17 $a = 1, b = -1, c = -1$

18 a $2\cos 2x$ **b** $5e^{5x}$

c $\cos x\, e^{\sin x}$ **d** $-2x\sin(x^2)$

e $\sec x \tan x\, e^{\sec x}$

f $-3\operatorname{cosec}\left(3x + \frac{\pi}{3}\right)\cot\left(3x + \frac{\pi}{3}\right)$

g $-3\operatorname{cosec}^2 3x$ **h** $\dfrac{3x^2}{x^3+3}$

i $2\cot 2x$ **j** $-\tan x$

k $3x^2 e^{x^3}$ **l** $\tan x$

m $6x(x^2+1)^2$ **n** $3\sin^2 x\cos x$

o $6\tan^2(2x)\sec^2 2x$

19 $-\dfrac{1}{\sqrt{1-x^2}}$ **20** $\dfrac{1}{2\sqrt{1-x^2}}$

21 a $\dfrac{1}{1+x^2}$ **b** $\dfrac{1}{3(1+x^2)}$

c $\dfrac{1}{2x}$ **d** $-\dfrac{1}{x}$

e $\dfrac{e^x}{5}$ **f** $\dfrac{e^{\frac{x}{2}}}{2}$

g $\dfrac{1}{2\sqrt{1-x^2}}$

22 a $\dfrac{1}{2\sqrt{x}}$ **b** $\dfrac{1}{2\sqrt{x}}$

23 a $nf(x)^{n-1}f'(x)$ **b** $f'(x)e^{f(x)}$

c $f'(x)\cos[f(x)]$

24 a $x = a^p, a = e^q, x = e^r$

c $\log_a x = \dfrac{\log_e x}{\log_e a}$

Review Exercise 7J page 168

1 a $12(3x-1)^3$ **b** $45(3x-4)^2$

c $-8(4x+3)^{-3}$ **d** $-15(5x-1)^{-4}$

e $12(2-3x)^{-5}$ **f** $\frac{3}{2}(3x+1)^{-\frac{1}{2}}$

2 a $8x(x^2+3)^3$ **b** $-15x^2(2-x^3)^4$

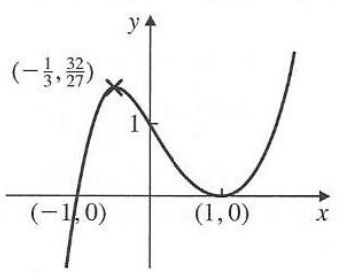
c $30x(3x^2+1)^4$ **d** $4(2x+1)(x^2+x)^3$

e $-\dfrac{3}{x^2}\left(1+\dfrac{1}{x}\right)^2$ **f** $-3\left(x^2+\dfrac{1}{x}\right)^{-4}\left(2x-\dfrac{1}{x^2}\right)$

3 a $(x+1)^2(4x+1)$ **b** $5(2x-3)^3(2x+1)$

c $(x+1)^2(x-1)(5x-1)$ **d** $4x(3x-2)(3x-1)$

4



5 a $(-3, 0)$ maximum, $(\frac{1}{3}, -\frac{500}{27})$ minimum

6 a $\dfrac{x^2+2x}{(x+1)^2}$ **b** $\dfrac{-(x+2)}{2x^2\sqrt{x+1}}$

c $-\dfrac{1}{(x+1)^2}$ **d** $\dfrac{e^{3x}}{x^3}(3x-2)$

e $\dfrac{2x-6}{x^3}$ **f** $\dfrac{x\cos x - \sin x}{x^2}$

8 a e^x **b** $-3e^x$

c $\frac{1}{2}e^x$ **d** $6x^2 - 4e^x$

e $3e^x - \frac{3}{2}x^{-\frac{1}{2}}$ **f** $\frac{1}{6}x^{-\frac{2}{3}} - \frac{1}{4}e^x$

9 a $\dfrac{1}{x}$ **b** $\dfrac{4}{x}$ **c** $\dfrac{1}{x}$ **d** $\dfrac{2}{x}$

e $\dfrac{1}{2x}$ **f** $\dfrac{1}{x}$ **g** $-\dfrac{1}{x}$

10 a $e^2 - 1$ **b** 1 **c** 0 **d** 9

11 a $y = x + 1$ **b** $y = x - 1$

c $y = e^2 + 1 - e^2 x$

12 a $ey = 1 + e^2 - x$ **b** $y = \frac{9}{2} + \ln 9 - \frac{3}{2}x$

13 a $(0, 1)$ minimum

b $(0, 2)$ minimum

c $(1, -1)$ maximum

d $(\frac{1}{2}, \frac{1}{2} + \ln 2)$ minimum

e $(\frac{1}{8}, \ln\frac{1}{8})$ maximum

14 A$(1, 0)$ B$\left(e, \dfrac{1}{e}\right)$

16 a $\cos x$ **b** $-2\sin x$

c $\cos x - \sin x$ **d** $2\cos x + 3\sin x$

e $\sec^2 x + 2x$ **f** $3\cos x - \sec^2 x$

17 a $\frac{1}{2}$ **b** -1 **c** 0 **d** $\frac{1}{2}$ **e** 1

18 $y = 2x - \dfrac{\pi}{2}$ **19** $y = x + 2 - \dfrac{\pi}{2}$

20 a $3\cos 3x$ **b** $-4\sin 4x$

c $5\sec^2 5x$ **d** $2\sin x\cos x$

e $4\tan^3 x\sec^2 x$ **f** $2\cos\left(2x + \dfrac{\pi}{4}\right)$

g $\dfrac{1}{2}\dfrac{\cos x}{\sqrt{\sin x}}$ **h** $6\sin 3x\cos 3x$

i $-6\cos^2 2x\sin 2x$

21 a $2e^{2x}$ **b** $-3e^{-3x}$

c $2x\,e^{x^2}$ **d** $-e^{-x}$

e $\cos x\, e^{\sin x}$ **f** $\dfrac{2x}{x^2+1}$

g $\dfrac{3x^2}{x^3-2}$ **h** $\dfrac{3x^2+2}{x^3+2x-1}$

i $\dfrac{2x}{x^2+1} - \dfrac{1}{x-1}$ **j** $-\left(\dfrac{2x}{x^2+3}\right)$

k 1 **l** $\dfrac{1}{2\sqrt{x}}e^{\sqrt{x}}$

23 $\dfrac{e^x\cos x}{(1+\sin x)} + e^x\ln(1+\sin x)$

24 a $e^x(x+1)$ **b** $1 + \ln x$

c $e^x\left(\dfrac{1}{x} + \ln x\right)$ **d** $x\cos x + \sin x$

e $-x^2\sin x + 2x\cos x$ **f** $e^x\sec^2 x + e^x\tan x$

25 $\frac{2\pi\ell^3\sqrt{3}}{27}$ at $\theta = \tan^{-1}\sqrt{2}$

26 $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{16}\right)$ maximum $\left(\frac{\pi}{2}, 0\right)$ shoulder

$\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{16}\right)$ minimum

27 a $\sin x\,(2\cos^2 x - \sin^2 x)$

b $\frac{2}{2x+5}$

c $2\cos 2x \cos 3x - 3\sin 2x \sin 3x$

d $2x\,e^{x^2+2}$

e $\frac{2}{x}\ln x$

f $2x\cos x^2$

28 a $\sec x \tan x$ **b** $-\text{cosec}\, x \cot x$

c $-\text{cosec}^2 x$

29 $6\sqrt{3}$

30 $(2, 4 - \ln 16)$ minimum

31 a $\frac{1}{\sqrt{1-x^2}}$ **b** $\frac{1}{3x}$

c e^x **d** $\frac{1}{2}e^x$

32 a $y^2 + 3y^2 \ln 2y$ **b** $\frac{1}{(4e^2 + 3e^2 \ln 2)}$

33 -1 and 3

Examination Exercise 7 page 171

1 a i $5x(x^2+1)^{\frac{3}{2}}$ **ii** 2

b $2, -\frac{1}{2}$

2 i a $3x^2 \ln 2x + x^2$

b $3(x + \sin 2x)^2(1 + 2\cos 2x)$

3 a $2(4x+1)^3 \cos 2x + 12\,(4x+1)^2 \sin 2x$

b $p = -2$

c $\frac{-2x}{(2x^2+3)(3x^2+4)}$

4 a $4x^3 \tan 2x + 2x^4 \sec^2 2x$

b $\frac{3}{4}$

5 i 0.12 **ii** 1.6

6 i $\ln(2y+3) + \frac{2(y+4)}{2y+3}$

ii A: 0.27, B: 0.17

7 28

8 b $(5, 7)$

9 a $3e^{3x} + \frac{1}{x}$

10 a $x^2 + 3x^2 \ln x$

b i $y - e^3 = 4e^2(x - e)$

ii $x = \frac{3}{4}e$

11 $y = 12x + 15$

12 a i $2xe^{2x} + e^{2x}$

ii $y - e^2 = 3e^2(x-1)$

b $k = 6$

13 $27x + 3y - 32 = 0$

14 a $\frac{-2}{2e - x}$

b $y = \frac{e}{2}x - \frac{e^2}{2} + 2$

15 $y - \frac{\pi}{4} = -8(x - 2\sqrt{3})$

16 a $\frac{2\pi}{9}$ **b** $y = -\frac{1}{3}x$

17 d i $x = 1, 2$

ii $\frac{-(2x-2)}{(x^2-2x+2)^2} + \frac{1}{x^2}$

iii at $x = 1$, $\frac{d^2y}{dx^2} > 0$ $\therefore$ min, and $y = 0$

Exercise 8A page 177

1 b 3.4

2 b 4.0

3 a $-1, -2$ **b** -1.2 **c** 1.4

4 c 1.8 **d** $(-3, -2)$

5 b $a = 9$ **c** 0.9

6 a 5.85, -173, -8030

b $(1, 2)$ **c** 1.52

7 b 0.6, 8.4

Exercise 8B page 180

1 a 2.571 **b** -2.714 **c** 0.143

d 8.602 **e** 2.714 **f** 1.935

2 a $x^3 - 7x + 1 = 0$ **b** $x^3 - 7x + 1 = 0$

c $x^3 - 7x + 1 = 0$ **d** $x^2 - 74 = 0$

e $x^3 - 20 = 0$ **f** $x - \sin x - 1 = 0$

3 a i 0.562 **ii** -7.140 **iii** 0.372

b i $x^2 + 3x - 2 = 0$

ii $x^2 + 7x - 1 = 0$

iii $x^2 + 5x - 2 = 0$

c i $\frac{\sqrt{17}-3}{2}$ **ii** $\frac{-7-\sqrt{53}}{2}$ **iii** $\frac{-5+\sqrt{33}}{2}$

4 a $f(-2) = 2$, $f(-3) = -14$ **c** -2.196

5 c 1.7

6 c 5.82

7 $p = 7.2\%$

8 c 6.61

9 b 0.572 hrs, 201 km/h

Exercise 8C page 184

1 a $3x^2 + 5$ **b** 1.42

2 6.662 **3** 1.378 **4** 2.88

5 2.285 **6** 1.856

7 a $(2, 3)$ **b** 2.807

8 2.646 **9** 4.123 **10** 1.360

11 1.252

Exercise 8D page 187

1 a 21.5 **b** above **c** 21

2 5.83 **3** 58.0

4 a 0.977 **b** 0.994

5 a 0.076 **b** 0.720 **c** 0.785

6 a 3.92 **b** 2.78 **c** 3.39

7 0.937

Review Exercise 8E page 188

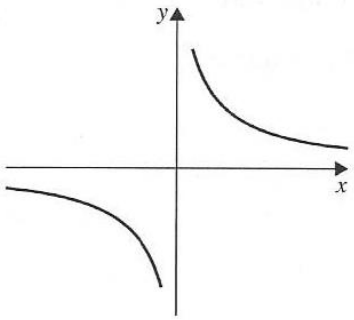
2 b 0.629
3 1.78
4 c 2.21
5 −1.67 **6** 0.697 **7** 1.70
8 1.31 **9** 0.821 **10** −1.84
11 1.87 **12** −2.279 **13** −3.080
14 a (2, 3) **b** 2.15 **15** 1.824
16 a 4.5×10^6 **b** 5.3 **c** 0.95

Examination Exercise 8 page 189

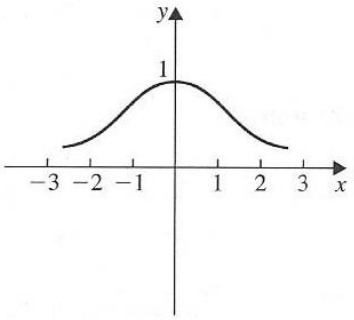
1 iii 5.828, 5.557
2 b 0.485, 0.492, 0.489
c 0.49
3 a 3 and 4 **b** 3.24
4 c 2.236, 2.054
5 b 0.80219, 0.80133, 0.80167
6 i $y = 9 - x^2$
ii b $\alpha = 2.156$
7 b 1.41,1.20,1.31
8 iii 1.917 **iv** (3.92, 2.60)
9 b 3.880, 3.918
10 a −1.439, 0.268
b 1.384
11 ii 39.59
12 b 2.219
13 a at a stationary point $f'(x) = 0$
b 0.622
14 i a 4.146
b staircase diagram will always move to upper root
ii b 1.159
15 i $x_2 = -1.5$
16 ii values alternate
17 a $y = 0.6595$ **b** 1.083
18 6.39
19 i 21.4
ii more/narrower strips
20 ii curve is above tops of trapezia
iii $69\frac{1}{3}$
21 i 6.97
ii tops of trapezia are below the curve

Exercise 9A page 198

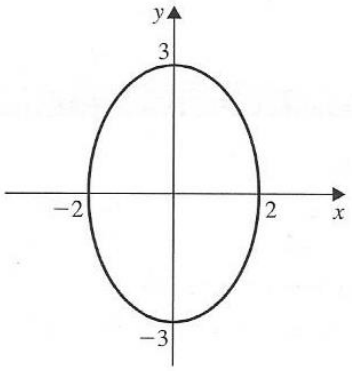
1 a $y = (x-1)^2,\ x, y \in \mathbb{R},\ y \geqslant 0$
b $y^2 = x^3,\ x, y \in \mathbb{R},\ x \geqslant 0$
c $y = \frac{9}{x},\ x, y \in \mathbb{R},\ x \neq 0,\ y \neq 0$
d $y = x^3 - 4x,\ x, y \in \mathbb{R}$
e $y = x^2 + 2x + 2,\ x, y \in \mathbb{R},\ y \geqslant 1$
f $y^2 = \frac{1}{x},\ x, y \in \mathbb{R},\ x > 0,\ y \neq 0$
g $y = \frac{18}{x^2} + 1,\ x, y \in \mathbb{R},\ x > 0,\ y > 1$
h $y = \frac{3x^4}{16} + 4,\ x, y \in \mathbb{R},\ x > 0,\ y > 4$
i $y = \frac{x}{2x-1},\ x, y \in \mathbb{R},\ x \neq \frac{1}{2},\ y \neq \frac{1}{2}$
j $y = 3x^2 - 10x + 9,\ x, y \in \mathbb{R},\ y \geqslant \frac{2}{3}$
2 a $y = t^2 + 1$ **b** $y = \sin t$
c $y = \frac{2}{t}$ **d** $y = 9t^3 - t$
3 $y = 2x - 3$
4 a



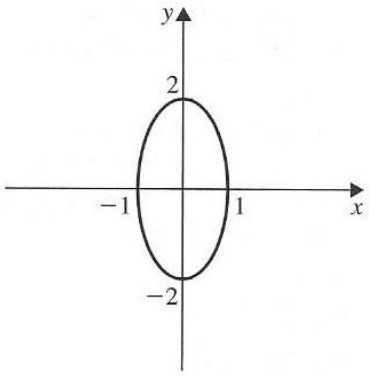
b



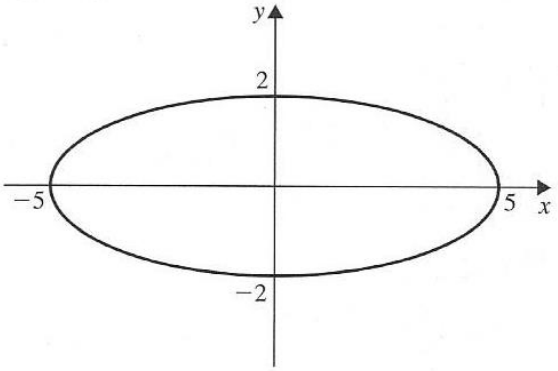
c



5 a $x^2 + \frac{y^2}{4} = 1$

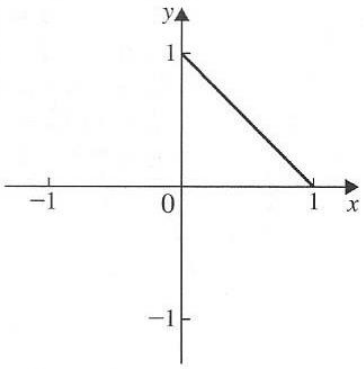


6 a $\frac{x^2}{25} + \frac{y^2}{4} = 1$



7 a $x + y = 1$

b $0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant 1$



8 $x = 2y^2 - 1$

9 a $y = 1 - 2x^2$

10 $(x + y)(x - y)^2 = 8$

Exercise 9B page 201

1 a $\frac{t}{2}$ **b** $-\frac{3}{2}\tan t$

2 a $\frac{1}{t}$ **b** $-\frac{3}{5t^2}$ **c** $-\frac{4}{7}\cot t$

d $\frac{1}{2\sin t}$ **e** $\frac{e^t + e^{-t}}{e^t}$

3 b $3y = x + 1$

4 a $t = -1$ **b** $\frac{2}{3}$ **c** $2y + 3x = 10$

5 b $y = x$ **c** $16y = -9x + \frac{9}{4}$

6 a $\left(-\frac{20}{9}, -\frac{40}{27}\right)$, $(-3, -9)$ **b** $(-4, -8)$

7 a $\frac{t^2 - 1}{t}$ **b** $(8, -4)$ and $(8, 4)$

8 $(1, 0)$ and $(-1, -4)$

9 $y + x = \sqrt{2}$

11 $4y = 4x + a$

12 $\frac{2y}{\sin\theta} - \frac{x}{\cos\theta} = 3$

13 a $-\frac{\cos 2t}{\sin t}$ **b** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

14 $(0, 0)$, $(\pi, 2)$, $(2\pi, 0)$

15 a $(x - 2)^2 + (y - 2)^2 = 9$; circle, centre $(2, 2)$, radius 3

b $x + y = 4 + 3\sqrt{2}$

Exercise 9C page 203

1 a $\frac{26}{3}$ **b** 818.4 **c** $25\ln 3$

2 a $\frac{28}{3}$ **b** 9 **c** 8

3 a $9\ln 4$ **b** 174 **c** $7\frac{1}{2}$ **d** 261.2

4 $A = 3\pi$

Exercise 9D page 205

1 a $(18, 0)$ **b** $(-0.73, 0)$, $(2.73, 0)$

c $(\pm 3, 0)$

2 a $(0, 2)$ **b** $(0, 3)$ **c** $(0, 0.37)$

3 a $(2.2, 0)$, $(0, -4)$ **b** $(-0.33, 0)$

c $(1, 0)$, $(0, -1)$ **d** $(4, 0)$, $(0, -4)$

e $(\pm 2, 0)$, $(0, \pm 4)$ **f** $(\pm 3, 0)$, $(0, \pm 2)$

4 $(4, 16)$

5 $\left(1, \frac{\pm 5\sqrt{3}}{2}\right)$

6 $(1, 4)$, $(6, 9)$

7 $(0.317, 5.32)$, $(-6.32, -1.32)$

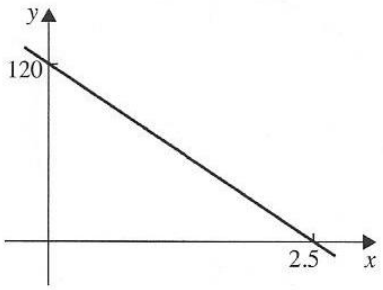
9 a $2y + x - 3 = 0$ **b** $(-6, 4.5)$

10 $\left(\frac{729}{32}, -\frac{97}{8}\right)$

Exercise 9E page 208

1 a 120 km **b** 2.5 hrs

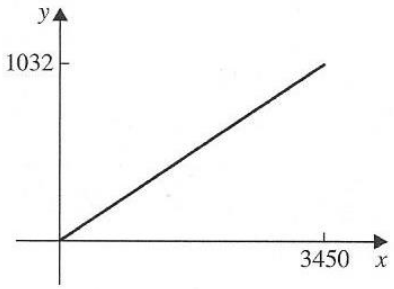
c



d 48 km/h

e No – constant speed is unlikely throughout the whole journey.

2 a



b 59.8 km

c The plane cannot keep climbing indefinitely.

3 a 20 m **b** 2.0 sec **c** 7.07 m

4 a 2.96 sec **b** 12.7 m **c** 10.7 m

5 a $r = 40$ m **b** $(40, 80)$

c $t = 6\pi = 18.8$ mins **d** 0.2 m/s

6 a $(3, -2.5)$ **b** $(0, 2.5)$

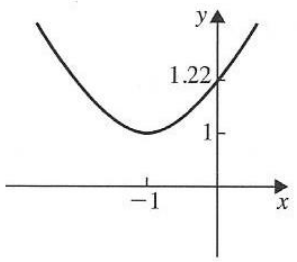
c $4\pi = 12.6$ mins

Review Exercise 9F page 210

1 a $y = (x - 1)^2$ **b** $y = \frac{3}{x + 1}$ **c** $xy = 8$

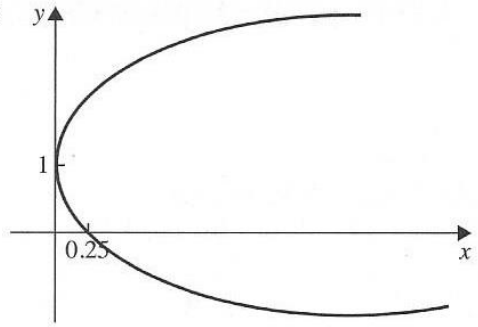
d $y = 1 - x$ **e** $\frac{x^2}{25} + y^2 = 1$ **f** $y = 4x^2 - 2$

3 a



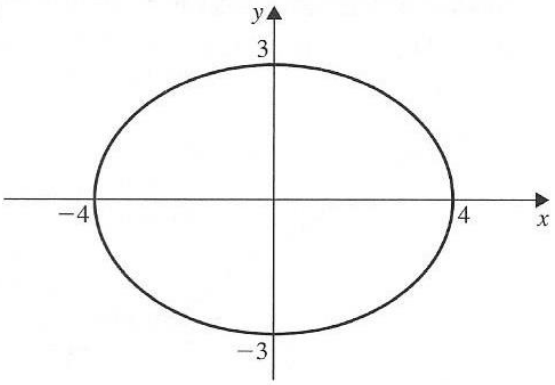
$x, y \in \mathbb{R}, y \geqslant 1$

b



$x, y \in \mathbb{R}, x \geqslant 0$

c



$x, y \in \mathbb{R}, -4 \leqslant x \leqslant 4$
$-3 \leqslant y \leqslant 3$

4 a $\frac{1}{2t}$ **b** $-\tan t$ **c** $\frac{2}{3t}$

5 $(0, -1)$ **6** $(1, -2), (-1, 2)$

7 a $y = -2x - 1$ **b** $y + x = \sqrt{2}$
c $x + y = 2$ **d** $9y = 24\sqrt{3} - 4\sqrt{3}x$
e $2y = 9 - 3x$ **f** $y = x(1 + \sqrt{2}) + 3$

8 $(\sqrt{3}, 1 + \sqrt{3}), (-\sqrt{3}, 1 - \sqrt{3})$

9 $(-3, 8)$

10 a 1500 m **b** 187.5 sec **c** 12.2 km

11 b $\frac{\pi}{4} = 0.79$ sec **c** 2.6 m/s

12 a $(8, -5\sqrt{3})$ **b** $\pi = 3.14$ mins

Examination Exercise 9 page 212

1 b $a = 3, b = 12$

2 ii $\left(\frac{4}{9}, \frac{64}{27}\right)$ **iv** $y^2 = x(4 - x)^2$

3 i $\frac{2}{t} - 2t^2$

ii $(0, 3)$ minimum

iii $y = \frac{2}{x + 1} + (x + 1)^2$

4 b $x = 1$ **c** $y = 3 + \frac{1}{8 \ln 2}x$

5 a $\frac{\pi}{3}$ **b** $k = \frac{17}{16}$

6 a $k = -\frac{2}{3}$ **b** $y = 2x - 2$

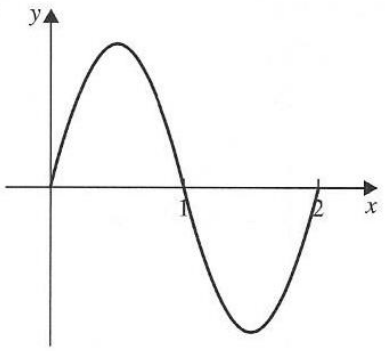
c $x^2 = 3y\left(1 - \frac{y}{4}\right)$ OR $\frac{x^2}{3} + \frac{(y - 2)^2}{4} = 1$

7 i $(0, 0), (1, 0), (2, 0)$

ii $\left(1 + \frac{1}{\sqrt{3}}, \frac{-4}{3\sqrt{3}}\right), \left(1 - \frac{1}{\sqrt{3}}, \frac{4}{3\sqrt{3}}\right)$

iii $y = 2x^3 - 6x^2 + 4x$

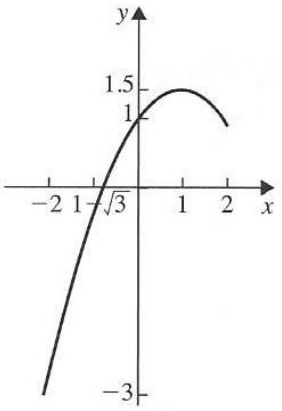
iv



8 i $\left(1, 1\frac{1}{2}\right)$

ii $y = 1 + x - \frac{1}{2}x^2$

iii $-2 \leqslant x \leqslant 2$



9 a $\frac{1}{2}e^4$ **b** $y = \frac{e}{2\sqrt{1 - x}\,(1 - \ln 2\sqrt{1 - x})}$

Exercise 10A page 217

1 a A, B positive, C, D, E negative
b B, E positive, A, C negative, D zero
c A, C, D, E positive, B negative

2 a all $x \in \mathbb{R}$ **b** $x \geqslant 1$
c $x \leqslant -\frac{1}{2}, x \geqslant 1$ **d** all $x \in \mathbb{R}$

3 a $x \leqslant -\frac{5}{3}$
b $\frac{-1-\sqrt{3}}{2} \leqslant x \leqslant \frac{-1+\sqrt{3}}{2}$
c $0 \leqslant x \leqslant \frac{\pi}{2}, \frac{3\pi}{2} \leqslant x \leqslant 2\pi$
d $x \leqslant -\frac{3}{2}$

4 a $\frac{2}{3}$ **b** $e^{-\frac{5}{6}}$ **c** $\frac{-5 \pm \sqrt{5}}{2}$

6 a (1.25, 3.16) max
(4.39, −3.16) min
b (2.82, 0), (5.96, 0)

EXERCISE 10A
(alternative answers for OCR & MEI)

2 a all $x \in \mathbb{R}$ **b** $x > 1$
c $x < -\frac{1}{2}, x > 1$ **d** all $x \in \mathbb{R}$

3 a $x < -\frac{5}{3}$
b $\frac{-1-\sqrt{3}}{2} < x < \frac{-1+\sqrt{3}}{2}$
c $0 < x < \frac{\pi}{2}, \frac{3\pi}{2} < x < 2\pi$
d $x < -\frac{3}{2}$

Exercise 10B page 219

1 a $-\frac{x}{y}$ **b** $-\frac{y}{x}$ **c** $-\left(\frac{2x+y}{x+2y}\right)$
d $\frac{3-x}{y+4}$ **e** $-\frac{x}{3y-2}$ **f** $\frac{2x}{3y^2}$
g $-\left(\frac{9x^2+4y^2}{8xy+3y^2}\right)$

2 a -3 b -1 c $-\frac{3}{2}$ d 3

3 $x + y = 4$ **4** $y = x$ **5** $x + y = 4$

6 $y = x$ **7** $4y + 3x = 20$ **8** $y = x$

10 a -1 b 6 c -4 d -8

11 b 2, -2 c (2, 4), $(-2, -4)$

12 $(-4, 2), (4, -2)$

13 a $\dfrac{2 - x + y}{3y - x}$ c (5, 3)

14 $(-7, 7)$

15 a $-\frac{17}{8}$ b $\frac{1}{7}$ c $-\frac{4}{3}$ d $-\frac{2}{5}$ e $-\frac{1}{5}$

16 a i $\dfrac{-1}{\sin y}$ ii $-\dfrac{1}{\sqrt{1 - x^2}}$ b $-\dfrac{1}{\sqrt{1 - x^2}}$

17 a i $\cos^2 y$ ii $\dfrac{1}{1 + x^2}$ b $\dfrac{1}{1 + x^2}$

18 $x^{x^2}(x + 2x \ln x)$

19 a $\frac{1}{x}$ or $\frac{1}{e^y}$ b $-\frac{y}{x}$ c $-\frac{y^2}{x^2}$

d $\tan x \tan y$ e $\dfrac{1}{\cos y - \sin y}$

Exercise 10C page 222

1 a $3^x \ln 3$ b $5^{x-1} \ln 5$ c $3(4^x) \ln 4$

d $2x(\ln 2)2^{x^2}$ e $-\frac{2x}{3}e^{-\frac{x^2}{3}}$

2 a $7^t(\ln 7)$ b $4^t(\ln 4)$ c $-3^{-t} \ln 3$

3 a $-15e^{-\frac{t}{10}}$ b $2e^{-2t}$ c $6^t(10 \ln 6)$

4 372°C/min

5 a 220 b 1.36 g/year

6 a 2000 b 12.8 min

c $\frac{180}{e}$ (= 66.2°C/min)

7 a 190°C b 22.0 min c 11.2°C/min

Exercise 10D page 224

1 0.6 cm s^{-1} **2** 10π cm^2 s^{-1}

3 0.2 cm s^{-1}

4 a 1.92 cm^3 s^{-1} b 0.96 cm^2 s^{-1}

5 300π cm^3 s^{-1} **6** 0.0637 cm s^{-1}

7 800π cm^3 s^{-1} **8** 225 cm^3 s^{-1}

9 2 cm s^{-1} **10** 5 cm s^{-1}

11 $\frac{4}{\pi}$ cm s^{-1} **12** b 1500 cm^3 s^{-1}

13 $\frac{3}{2}$ units s^{-1} **14** $\frac{1}{3\pi}$ cm s^{-1} ($\approx$0.106)

15 $\frac{2}{\pi}$ cm s^{-1}

Review Exercise 10E page 226

1 a $\left(\frac{2}{9}, -\frac{205}{243}\right)$ b (0.908, 0), (2.48, 0)

2 a i $x \leqslant -\dfrac{\sqrt{30}}{6}, x \geqslant \dfrac{\sqrt{30}}{6}$

ii $-\dfrac{\sqrt{30}}{6} \leqslant x \leqslant \dfrac{\sqrt{30}}{6}$

b i $x \geqslant -1.6$ ii $x \leqslant -1.6$

3 a $\dfrac{1}{2y + 1}$ b $-\left(\dfrac{y}{1 + x + 2y}\right)$ c $-\dfrac{x}{y}$

4 a $-\dfrac{x}{3y}$ b $\dfrac{2 + 8x}{3y^2 - 3}$

c $\dfrac{3x + 2y}{5y - 2x}$ d $\dfrac{1 - y^3 - 3x^2y}{1 + 3xy^2 + x^3}$

e $\dfrac{\cos x}{2 \sin y}$ f $\dfrac{2xy^2}{1 - 2x^2y}$

5 $\frac{3}{4}$

6 a $-\left(\dfrac{2x + 7y}{7x + 6y}\right)$ b $-\dfrac{16}{19}$

7 $7x + 11y - 32 = 0$

8 $16x - 10y - 33 = 0$

9 a $\dfrac{2 - x}{4y - 4}$

b At (2, 2) gradient = 0. At (0, 1) gradient of curve is undefined.

12 b 3

13 a $-\frac{3}{2}$ b $-\frac{4}{3}$

14 a $8^x \ln 8$ b $(2 \ln 3)3^x$ c $(2x \ln 4)4^{x^2}$

d $-12e^{-3x}$ e $x^x(1 + \ln x)$

16 0.121°C/min

17 100π cm^3 s^{-1}

18 216 cm^3 s^{-1}

19 64π cm^2 s^{-1}

20 3 cm s^{-1}

21 0.25 cm s^{-1}

22 b $\frac{45}{2\pi}$ cm s^{-1}

23 a 3.08×10^{-7} cm s^{-1} b 7.4×10^{-6} cm^2 s^{-1}

24 2 cm s^{-1}

25 12 cm s^{-1}

Examination Exercise 10 page 228

1 $\dfrac{1 - 2x^2y \cos 2x}{(\sin 2x + 2y)x^2}$

2 $-\frac{3}{8}$

3 a $\frac{1}{4} - \ln 2$ b $\frac{11}{8}$

4 $\dfrac{1 - x^{-2}}{2y}$, $(1, \sqrt{2}), (1, -\sqrt{2})$

5 a $-\frac{4}{9}$ b $9x - 4y + 13 = 0$

6 a i $\dfrac{-3ye^{3x}}{e^{3x} - 2 \sin 2y}$ ii $-\pi$

b $y = \frac{\pi}{4} - \frac{\ln 2}{\pi}$

7 i $\dfrac{2x - y - 4}{x - 2y + 3}$

ii Denominator is zero $\Rightarrow$ tangents are parallel to y-axis.

iii $8x - 9y = 48$

8 a $\dfrac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)}$

b $\dfrac{3\pi + 62}{\pi + 22}$

Alternative answer for Q4:

$$\frac{dy}{dx} = \frac{2x - y^2}{2xy}$$

9 ii 0.28 cm/min
10 a $0.0149\ \text{cm s}^{-1}$ b $1.5\ \text{cm}^2\text{s}^{-1}$
11 0.06 m/h
12 $1.25\ \text{cm s}^{-1}$
13 i 12.17 ii $0.18\ \text{m}^3/\text{h}$
14 (10, 2), (−2, 2)

Exercise 11A page 233

1 a $\frac{(1+x)^4}{4}+c$ b $\frac{1}{7}(x+4)^7+c$
c $\frac{1}{10}(2x+1)^5+c$ d $-\frac{1}{5}(1-x)^5+c$
e $-\frac{1}{(x+3)}+c$ f $\frac{2}{9}(3x-1)^3+c$
g $-2(x+1)^{-2}+c$ h $-\frac{1}{5}(5x+3)^{-1}+c$
i $7e^x+c$ j $\frac{x^2}{2}+5e^x+c$
k $\frac{1}{5}e^{5x}+c$ l $\frac{e^{6x}}{6}-x+c$
m $\frac{3}{2}e^{2x}+c$ n $-9e^{-x}+c$
o $-\frac{4}{e^x}+c$ p $-\frac{1}{e^x}+x^2+c$

2 a $-\cos x+c$ b $\frac{1}{4}\sin 4x+c$
c $-\frac{1}{10}\cos 10x+c$ e $\frac{2}{5}\sin 5x+c$
e $-\frac{1}{12}\cos 6x+c$ f $\frac{1}{2}\sin 2x+x^2+c$
g $-\frac{1}{4}\cos(4x-1)+c$ h $\frac{x^2}{2}-\sin(x+1)+c$
i $x^2+\ln x+c$ j $\sin x+2\ln x+c$
k $\frac{1}{4}\ln x+c$ l $\ln(x+3)+c$
m $6\ln(1+x)+c$ n $\frac{1}{3}\ln(3x+2)+c$
o $\frac{1}{2}e^{2x}+2\ln x+c$

3 a $6(4x+1)^{\frac{1}{2}}$ b $\frac{1}{6}(4x+1)^{\frac{1}{2}}+c$
4 a $\frac{5}{2}(5x-2)^{-\frac{1}{2}}$ b $\frac{2}{5}(5x-2)^{\frac{1}{2}}+c$
5 a $\frac{1}{8}(6x+1)^{\frac{4}{3}}+c$ b $2(x+2)^{\frac{1}{2}}+c$
c $\frac{1}{5}(1+2x)^{\frac{5}{2}}+c$
6 e^2-1
7 a $\frac{1}{3}e^{3x+1}+c$ b $2e^{2x-3}+c$
c $-\frac{5}{e^x}+c$ d $\frac{1}{8}(2x+1)^4+c$
e $-\frac{1}{4}(4x-3)^{-1}+c$ f $e^x-e^{-x}+c$
g $\frac{1}{3}(2x+1)^{\frac{3}{2}}+c$ h $\frac{1}{2}(4x+5)^{\frac{1}{2}}+c$
i $\frac{1}{2}e^{2x}+2e^x+x+c$
8 a $2\ln(2x+7)+c$ b $2x+e^x-e^{-x}+c$
c $-\frac{2}{3}(3x-2)^{-1}+c$ d $\frac{1}{6}(4x+3)^{\frac{3}{2}}+c$
e $\frac{1}{4}(8x-1)^{\frac{1}{2}}+c$ f $\frac{1}{8}(6x+5)^{\frac{4}{3}}+c$
9 $\frac{x^2}{2}+5\ln x-\frac{1}{x}+c$
10 a $x-\frac{4}{x}-4\ln x+c$ b $x+5\ln x-\frac{3}{x}+c$
c $\frac{1}{2}(e^x+x)+c$ d $x-5\ln x-\frac{6}{x}+c$
e $\frac{x^2}{2}-25\ln x+c$ f $e^{x+2}-\frac{1}{2}e^{-2x}+c$
11 a $\frac{\pi}{4}$ b $\frac{1}{2}$
12 $\frac{1}{2}$ **13** $\frac{1}{3}(e^3-1)$ **14** $\ln\frac{5}{2}$
15 $-\frac{1}{12}$ **16** $\frac{1}{4}$ **17** $\frac{1}{2}e^3-\frac{1}{2}e^{-1}$
18 $10\ln 2$ **19** $\ln\frac{3}{2}$ **20** $12\frac{2}{3}$
21 $0.0339=\frac{1}{2}(\sin 2^c-\sin 1^c)$ **22** $8+\frac{1}{2}\ln 3$
23 1 **24** $3-\sqrt{3}$ **25** $4\frac{5}{6}$
26 $\frac{1}{2}\ln 5$ **27** $\frac{1}{2}+\frac{3\pi}{4}$ **28** $\frac{7}{12}$
30 a 2, 3 b $6\ln\frac{3}{2}-\frac{1}{2}$
31 $4-3\ln 3$
32 b 0, 5 c $\frac{1}{6}$
34 a $\ln 2, \ln 6$ b $16-12\ln 3$

Exercise 11B page 236

1 $\ln(x+3)+c$ **2** $\ln(2x+1)+c$ **3** $\ln(x^2+5)+c$
4 $\ln(x^3+2)+c$ **5** $\frac{1}{4}\ln(4x+1)+c$ **6** $\frac{1}{7}\ln(7x-1)+c$
7 $\ln(e^x+3)+c$ **8** $\frac{1}{2}\ln(x^2+3)+c$ **9** $\ln(\sin x)+c$
10 $\ln(x+1)(x-2)+c$
11 $\ln(2x+1)(5x+2)+c$
12 $3\ln(2x+1)-\ln(x+1)+c$
13 $\frac{1}{3}\ln\frac{5}{2}$
14 a $2\ln 4$ b $\ln 2$ c $6+\ln 2$
d $12-\ln 2$ e $\ln 4$ f $2\ln 3$
g $\ln\frac{27}{10}$ h $-\ln 2$ i $10+\ln 3$
j $\ln 30$ k $10+3\ln 2$ l $4-4\ln 3$
15 $\ln\left(\frac{3}{2}\right)$
16 a −1.10 b −0.24 c −0.32
17 a $-\ln\cos x+c$ b $\frac{1}{2}\ln\sin 2x+c$
c $\ln(\tan x)+c$ d $-\ln(\cos x+3)+c$
e $-\frac{1}{2}\ln\cos 2x+c$ f $\ln(\ln x)+c$

Exercise 11C page 240

Note: Instructions do not say answers must be factorised.

1 a $\frac{(x+2)^4}{4}-\frac{2(x+2)^3}{3}+c$
b $\frac{(x-3)^4}{4}+(x-3)^3+c$
c $\frac{2(x+4)^5}{5}-2(x+4)^4+c$
d $\frac{(x-1)^4}{4}+\frac{(x-1)^3}{3}+c$
e $\frac{2(x+3)^{\frac{5}{2}}}{5}-2(x+3)^{\frac{3}{2}}+c$
f $\frac{2(x-2)^{\frac{5}{2}}}{5}+\frac{8(x-2)^{\frac{3}{2}}}{3}+8(x-2)^{\frac{1}{2}}+c$

2 a $\frac{(x+1)^5}{5} - \frac{(x+1)^4}{4} + c$

b $\frac{(x-1)^7}{7} + \frac{(x-1)^6}{6} + c$

c $\frac{(2x+1)^5}{20} - \frac{(2x+1)^4}{16} + c$

d $\frac{(4x-1)^{\frac{5}{2}}}{40} + \frac{(4x-1)^{\frac{3}{2}}}{24} + c$

e $\frac{2(5x+1)^{\frac{3}{2}}}{75} - \frac{2(5x+1)^{\frac{1}{2}}}{25} + c$

f $\frac{(3x-2)^6}{54} + \frac{(3x-2)^5}{9} + c$

3 a $\frac{3}{2}(x^2+1)^4 + c$ **b** $2(x^3-3)^3 + c$

c $\frac{1}{4}(e^x-1)^4 + c$ **d** $2(e^x+2)^{\frac{1}{2}} + c$

e $\frac{1}{3}\sin^3 x + c$ **f** $-\frac{1}{4}\cos^4 x + c$

g $\frac{1}{4}\tan^4 x + c$ **h** $\sin^{-1} x + c$

4 a $\frac{49}{20}$, $u = x+1$ **b** $\frac{23}{30}$, $u = x-3$

c $21\frac{11}{15}$, $u = x-2$ **d** $3\frac{1}{3}$, $u = 2x+1$

5 a $e - \frac{1}{e}$ **b** $2\sqrt{3} - \frac{2}{3}\sqrt{5}$

c $\frac{1}{2}\ln 2$ **d** $\frac{2}{3}$

7 0.18

Exercise 11D page 243

1 a $\frac{x}{3}(1+x)^3 - \frac{(1+x)^4}{12} + c$

b $x\sin x + \cos x + c$

2 a $xe^x - e^x + c$ **b** $\frac{x}{4}(1+x)^4 - \frac{(1+x)^5}{20} + c$

c $-xe^{-x} - e^{-x} + c$ **d** $\frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + c$

e $-x\cos x + \sin x + c$ **f** $\frac{3x}{2}\sin 2x + \frac{3}{4}\cos 2x + c$

g $\frac{x}{2}(x-1)^4 - \frac{1}{10}(x-1)^5 + c$

h $\frac{2x}{3}(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}} + c$

i $\frac{x^2}{2}\ln 2x - \frac{x^2}{4} + c$ **j** $\frac{x^3}{3}\ln x - \frac{x^3}{9} + c$

k $-\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + c$ **l** $e^x(x^2-2x+2) + c$

3 $1 - \frac{2}{e} (= 0.264)$

4 a $\frac{1}{9} - \frac{4}{9e^3}$ **b** $\frac{1}{4}$ **c** $\frac{17}{6}$

d $71\frac{11}{15}$ **e** $\frac{2e^3}{9}$ **f** $\pi - 2$

5 a $7\frac{1}{10}$

Misprint in Q6b:

Should say $-\left(\frac{2x+1}{4e^{2x}}\right) + c$

[Not $\left(\frac{2x+1}{4e^{2x}}\right) + c$ **]**

7 a (1, 0),(0, 1) **b** $e - 2$

8 a $2xe^{x^2}$ **b** $\frac{e^{x^2}}{2}(x^2-1) + c$

9 b $-e^x\cos x + \int e^x\cos x\,dx$

c $\frac{e^x}{2}(\sin x + \cos x) + c$

10 a $\frac{1}{34}e^{5x}[-3\cos 3x + 5\sin 3x] + c$

b $\frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx] + c$

Exercise 11E page 245

1 a $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$ **b** $\frac{1}{2}x + \frac{1}{4}\sin 2x + c$

c $\tan x - x + c$ **d** $\frac{1}{2}x - \frac{1}{12}\sin 6x + c$

e $\frac{1}{2}x + \frac{1}{8}\sin 4x + c$ **f** $3x + 4\cos x - \sin 2x + c$

g $\frac{5}{2}x + \frac{1}{4}\sin 2x + \tan x + c$

h $\frac{1}{2}\tan 2x - x + c$ **i** $\tan x - 2\ln\cos x + c$

2 a $\frac{\pi}{8} - \frac{1}{4}$ **b** $\frac{1}{2} + \frac{\pi}{4}$ **c** $\sqrt{3} - \frac{\pi}{3}$

d $\frac{3\pi}{4} + 2$ **e** $\frac{\pi}{12}$

Exercise 11F page 246

1 a $\frac{1}{1+x} - \frac{1}{1+2x}$

b $\ln(1+x) - \frac{1}{2}\ln(1+2x) + c$

2 a $\ln(x-1) - \ln(x+1) + c$

b $2\ln(x+4) - \ln(x+5) + c$

c $5\ln(x-7) - 4\ln(x-2) + c$

4 0.235

5 a $\frac{4}{2x+1} + \frac{2}{x+1} + \frac{1}{(x+1)^2}$

6 a $x - \frac{1}{3} - \frac{2}{2x-1} + \frac{26}{3(3x-1)}$

9 a $(x+1)(x+2)(x+3)$

b $-\frac{2}{x+2} + \frac{1}{x+3} + \frac{1}{x+1}$

c $\ln\frac{32}{27}$

Exercise 11G page 249

Section A

1 $\frac{1}{4}(x+2)^4 + c$ **2** $\frac{1}{3}\sin 3x + c$

3 $4e^x + c$ **4** $\ln x + c$

5 $-\frac{1}{x} + c$ **6** $\ln\left(\frac{x-1}{x+1}\right) + c$

7 $x\sin x + \cos x + c$ **8** $\ln(5x-1) + c$

9 $\frac{1}{2}\tan 2x + c$ **10** $\ln(x^2+a) + c$

11 $\frac{(x+4)^3}{12}(3x-4) + c$

or $\frac{(x+4)^4}{4} - \frac{4(x+4)^3}{3} + c$

or $\frac{x(x+4)^3}{3} - \frac{(x+4)^4}{12} + c$

12 $\frac{x}{5}e^{5x} - \frac{1}{25}e^{5x} + c$

13 $\frac{1}{3}\ln\sin 3x + c$ **14** $\frac{1}{3}e^{3x+2} + c$

15 $\frac{1}{6}(x^4-1)^{\frac{3}{2}} + c$ **16** $5\ln(x-7) + c$

17 $2\ln(x+1) + 3\ln(x+2) + c$

18 $\frac{1}{2}x + \frac{1}{4}\sin 2x + c$ **19** $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$

20 $x + \ln x + c$ **21** $\ln(1+x) + c$

22 $e^x(x^2-2x+2) + c$ **23** $\frac{1}{4}\cos(3-4x) + c$

24 $4\tan x + c$ **25** $\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + c$

26 $\frac{x^2}{2} + 3x + \ln x + c$ **27** $-\ln\cos x + c$

28 $\tan x - x + c$ **29** $\frac{1}{8}(x^2+1)^4 + c$

30 $-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + c$

31 **a** $e^{x^3} + c$ **b** $e^{\sin x} + c$ **d** $\frac{34}{3}$

32 **a** $e - 1$ **b** $\frac{1}{2}(e^2-1)$

c $\ln 3$ **d** $\frac{1}{2}\ln 4$

33 **a** $2 + \ln 3$ **b** $3 + 4\ln 2$

c $6\ln 2 + 27\frac{1}{6}$ **d** $2e^2 + 4e - 2$

34 $y = \frac{1}{2}\ln x + 3$

35 $y = \frac{1}{3}(5\ln x + 1)$

36 $y = \frac{1}{6}(x^2 + 5 + 2\ln x)$

37 $y = e^x + x - 1$

38 a $\cos 2x = 1 - 2\sin^2 x$ **b** $\frac{\pi}{8} + \frac{1}{4}$

39 b $\sqrt{3} - 1 - \frac{\pi}{12}$

40 $2x\sin x + (2 - x^2)\cos x + c$

Section B

1 a 2 **b** 0 **c** $\frac{1}{3} - \frac{\sqrt{2}}{6}$

d $\sqrt{3}$ **e** $\frac{e}{2}(e^2 - 1)$ **f** $\frac{1}{3}$

g $44\frac{1}{3}$ **h** $\frac{98}{3}$ **i** 4

2 $\frac{3}{2}\ln(2x + 5) + c$ **3** $e^{x^2} + c$

4 $\ln(4 + \sin x) + c$ **5** $2\ln x + \ln(x - 3) + c$

6 $\frac{3}{2}x^2\ln x - \frac{3}{4}x^2 + c$ **7** $\frac{(1 + x)^{11}}{132}(11x - 1) + c$

8 $x\ln x - x + c$ **9** $-e^{-3x} + c$

10 $-2\cos\left(\frac{x}{2}\right) + c$ **11** $x - 3\ln(x + 3) + c$

12 $\sin x + \ln\cos x + c$ **13** $\frac{1}{2}x - \frac{1}{8}\sin 4x + c$

14 $\frac{2}{15}(5x + 1)^{\frac{3}{2}} + c$ **15** $2(x - 4)^{\frac{1}{2}} + c$

16 $2x^3 - \frac{3x^2}{2} + c$ **17** $e - \frac{1}{e}$

18 $\frac{1}{2}e^4 + \frac{3}{2}e^2 - 2e + 1$ **19** $2 + \frac{1}{e} - \frac{1}{e^3}$

20 38 **21** $64\frac{3}{4}$

22 11005 **23** $\frac{8}{9}$

24 $\frac{2}{9}$ **25** $\frac{4}{5}\ln 4$

26 $\frac{2e^3 + 1}{9}$ **27** -2

28 $2\ln 2 - \frac{3}{4}$ **29** $29.6 = \frac{148}{5}$

30 $2\ln 4$ **31** $\ln 2$

32 b $\frac{8}{3}$ **33** $\frac{1}{2}$

34 b $1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$

35 a $-\frac{1}{3}(1 - x^2)^{\frac{3}{2}}$

b $-\frac{x^2}{3}(1 - x^2)^{\frac{3}{2}} - \frac{2}{15}(1 - x^2)^{\frac{5}{2}}$

38 3 **39** $\frac{1}{2}\ln(\sec 2x + \tan 2x) + c$

Exercise 11H page 253

1 $\ln 8 - \ln 3 = 0.981$ **2** $\frac{9}{4}$

3 0.591 **4** 1.87

5 0.390 **6** 1.93

7 9 **8** 1.15

9 a 2, 3 **b** $\frac{5}{2} + 6\ln\frac{2}{3}$

10 $4 - 3\ln 3$

11 a 0, 5 **b** $\frac{1}{6}u^2$

13 a $\ln 2, \ln 6$ **b** $12\ln 3 - 16$

Exercise 11I page 256

1 a $y = x + \ln x + c$ **b** $y = x^3 - x + c$

c $y = e^x - x + c$ **d** $y = \sin x + c$

e $y = x - 3e^{-x} + c$ **f** $y = \ln(\sin x) + c$

2 a $y = \frac{1}{6}(2x + 1)^3 + \frac{5}{6}$ **b** $y = \frac{2}{3}(x - 1)^{\frac{3}{2}} + \frac{4}{3}$

c $y = \frac{3}{2} - \frac{1}{2}\cos 2x$ **d** $y = \frac{1}{2}\ln(2x - 1) + 1$

Note: Equivalent answers acceptable.

Exercise 11J page 259

1 a $\ln y = \frac{x^3}{3} + c$ **b** $y^2 = x^2 + c$

c $y = kx$ **d** $y = x + \ln x + c$

e $y^2 = x^3 + c$ **f** $y^2 = \frac{1}{2}e^{4x} + c$

g $y = -\frac{1}{(e^x + c)}$ **h** $y^2 = 2\sin x + c$

i $y = \ln(1 + x) + c$ **j** $y = e^{2x + c}$

k $y = e^{x^3 + c} - 3$ **l** $\sin y = \ln x + c$

m $y = \ln(2x^2 + c)$ **n** $y = e^{x^2 + c}$

o $y = -\frac{1}{(x + c)}$

2 $y = 4x^2$

3 a $x = 10e^{3t}$ **b** $y = 1000e^{-10t}$

c $x = 15e^t + 5$ **d** $y = 18e^{-t} - 8$

4 a $x = 100e^{5t}$ **b** $y = 1000e^{-2t}$

c $x = x_0 e^{3t}$ **d** $y = y_0 e^{-4t}$

e $x = x_0 e^{kt}$ **f** $y = y_0 e^{-kt}$

5 a $-e^{-y} = \frac{x^2}{2} - 3$ or $y = \ln\left(\frac{2}{6 - x^2}\right)$

b $y = e^{-\cos x}$

c $y = \frac{3e^{x^2} - 5}{2}$

d $y = \sqrt{(1 - \ln 2 - 2\ln\cos x)}$

e $\tan y = 2 - \cos x$

f $y = 2e^{\left(\frac{x^2 - 1}{2}\right)} - 1$

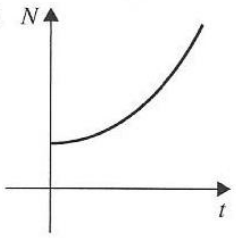
6 a $x = 20 - 15e^{-\frac{t}{10}}$ **b** 4.05

7 $y = \frac{2}{1 + e^{-2x}}$

8 a $\frac{1}{1 + x} - \frac{1}{2 + x}$ **b** $y = \frac{4}{3}\left(\frac{1 + x}{2 + x}\right)$

Exercise 11K page 262

2



4 $P = 50\,000\,000\,e^{0.00912t}$

5 b 46.1 min **c** $100e^3 \approx 2008$

6 b 12 min **c** 45°C

7 b £7746 **c** 9 years

8 10 hours

9 20 minutes ($c = 2, k = \frac{1}{10}$)

10 $V = 300\,e^{-\frac{t}{10}} + 200$ **c** 200 cm³

Review Exercise 11L page 264

1 a $-\cos x + c$ **b** $\frac{1}{2}\sin 2x + c$

c $-4\cos\frac{x}{4} + c$ **d** $\tan x + c$

e $\frac{1}{7}e^{7x} + c$ **f** $-\frac{1}{3}e^{-3x} + c$

g $\frac{1}{5}\sin(5x + 4) + c$ **h** $\frac{1}{8}(2x + 3)^4 + c$

i $-\frac{1}{4}(1 + 4x)^{-1} + c$ **j** $\ln x + c$

k $5\ln x + c$ **l** $\frac{3}{2}\ln(2x + 9) + c$

m $5\ln(x - 7) + c$ **n** $\sin x - \cos x + c$

o $\frac{1}{9}(6x - 1)^{\frac{3}{2}} + c$

3 a $\frac{1}{2} - \frac{\sqrt{3}}{4}$ **b** $\ln 2$ **c** 14

4 a $2\ln 2 - 2$ **b** maximum

5 a $2\ln(x + 1) + 3\ln(x + 2) + c$

b $4\ln(x + 1) + 6\ln(x + 2) + c$

6 a $\ln(x^3 + 5) + c$ **b** $\frac{1}{4}\ln(4x^2 - 7) + c$

c $\frac{1}{2}\ln(1 + e^{2x}) + c$ **d** $-\ln\cos x + c$

e $\ln f(x) + c$ **f** $\frac{1}{2}\ln\sin 2x + c$

7 a $\frac{71}{10}$ **b** $\frac{41}{288}$

c $\frac{19}{108}$ **d** $1 - e$

e 18, $u^2 = 2x - 1$ **f** $-\frac{2048}{45}$, $u = x - 1$

g $e^8 - 1$, $u = x^3$ **h** $\frac{3}{16}$, $u = x^2 + 1$

8 $\frac{7}{9}$

9 $\frac{46}{15}$

11 a $\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + c$ **b** $\sin x - x\cos x + c$

c $\frac{3}{2}x^2\ln x - \frac{3}{4}x^2 + c$ **d** $\frac{x^3}{9}(3\ln x - 1) + c$

12 $e^2 + 1$ **13** $\frac{e^4 - 1}{2}$

15 c $\frac{\pi}{4}$

16 a $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$ **b** $\tan x - x + c$

17 a $y = \left[\frac{3}{2}(x + 14)\right]^{\frac{2}{3}}$ **b** $y = \sqrt{40x + 4}$

c $y = e^{x^2 - 4}$ **d** $y = 2e^{\frac{1}{2}(x^2 - 1)} - 1$

e $y = x + \ln x + 2$

f $y = \sqrt{5e^{2\tan x} - 1}$ or $\ln\left(\frac{y^2 + 1}{5}\right) = 2\tan x$

19 $y = e^{2x^2 - 2}$

20 c $\frac{1}{10}\ln\left(\frac{3}{2}\right)$ **d** 17.1 hours

21 $\frac{75}{2} - 50\ln 2$

23 a $\sec x + c$ **b** $\frac{1}{3}\cot 3x + c$ **c** $x\ln 2x - x + c$

d $x^2\sin x + 2x\cos x - 2\sin x + c$

e $\ln(x + 2) + \frac{3}{x + 2} + c$

Examination Exercise 11 page 267

2 i $-\frac{1}{24}(4 - 3x)^8 + c$

ii $-\frac{1}{3}\ln(4 - 3x) + c$

3 i $4x - 4\ln x - \frac{1}{x} + c$

ii $\frac{3}{16}(4x + 1)^{\frac{4}{3}} + c$

4 b $\frac{1}{2}e^2 + 4e - \frac{1}{2}$

5 a $A = 2, B = 3$

b $y = x^2 + 3\ln(2x^2 - x + 2) + 1 - 3\ln 5$

7 $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

8 $\frac{3}{4}e^2 - \frac{3}{4}$

9 a $-\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + c$

b $\frac{3}{16} - \frac{1}{8}\ln 2$

10 a $a = 5, b = 3$

11 $\frac{13}{3}\sqrt{5} - \frac{16}{3}\sqrt{2}$

12 $2(x^2 - 2)^{\frac{3}{2}} + 16(x^2 - 2)^{\frac{1}{2}} + c$

13 $-\frac{23}{384}$

14 $\frac{47}{480}$

15 a $A = 1, B = 1, C = -4$

b $-\frac{1}{2}e^{-2y} = -\ln(1 - 3x) + \ln(1 + x) + \frac{4}{1 + x} - \frac{9}{2}$

16 $y = \frac{1}{2}\ln(1 - 2\ln|\sec x|)$

17 a $\frac{dx}{dt}$ is the rate of increase of the mass of waste products. M is the initial mass of unburned fuel.

b $x = M(1 - e^{-kt})$

c $x = \frac{2}{3}M$

18 ii $h = \sqrt[3]{\frac{729}{8} - \frac{27t}{400\pi}}$

iii 71 minutes

19 b 161

20 a $\frac{1}{P - 2} - \frac{1}{P}$

c 0.473

Exercise 12A page 274

1 a $\sqrt{29}, \frac{1}{\sqrt{29}}(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$

b $\sqrt{178}, \frac{1}{\sqrt{178}}(-5\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$

c $5\sqrt{26}, \frac{1}{5\sqrt{26}}(7\mathbf{i} - 24\mathbf{j} - 5\mathbf{k})$

2 a $3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$ **b** $15\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$

c $5\mathbf{i} - 6\mathbf{j}$ **d** $-5\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}$

3 a 7 **b** 9 **c** $3\sqrt{5}$ **d** $3\sqrt{3}$

4 a $\frac{1}{3}(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ **b** $\frac{1}{6}(4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})$

c $\frac{1}{7}(3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$ **d** $\frac{1}{3}(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

5 $a = 5, b = 2$

6 $8\mathbf{i} + 24\mathbf{j} + 12\mathbf{k}$

7 $12\mathbf{i} - 3\mathbf{j} - 24\mathbf{k}$

8 a $\begin{pmatrix}2\\3\\1\end{pmatrix}$ **b** $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ **c** $\begin{pmatrix}0\\2\\1\end{pmatrix}$

d $\begin{pmatrix}-1\\1\\4\end{pmatrix}$ **e** $\begin{pmatrix}0\\2\\0\end{pmatrix}$ **f** $\begin{pmatrix}2\\0\\3\end{pmatrix}$

9 $n = -1$

10 a $3\mathbf{i} + 9\mathbf{j} + 4\mathbf{k}$ **b** $\sqrt{106}$

c $\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ **d** $\mathbf{i} + 8\mathbf{j} - 7\mathbf{k}$

11 a $\begin{pmatrix}3\\3\\5\end{pmatrix}$ **b** $\sqrt{43}$ **c** $\begin{pmatrix}-5\\1\\1\end{pmatrix}$

12 a $\begin{pmatrix}-6\\-9\\9\end{pmatrix}$ **b** $\begin{pmatrix}-12\\-13\\-9\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-4\\18\end{pmatrix}$

d 13 **e** $\begin{pmatrix}-0.8\\0\\0.6\end{pmatrix}$

13 a $-2\mathbf{i} - 8\mathbf{j} - 3\mathbf{k}$ **b** $2\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$

14 a $2\mathbf{i} + 6\mathbf{j}$ **b** $\mathbf{i} + 3\mathbf{j}$

c $2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$

15 a $3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ **b** 13

c (7, 6, 11) **d** (31, 12, 19)

e 2:1

16 a 143° **b** 65.9° **c** 40.2°

17 a (10, 1, 3) **b** 84.5°

Exercise 12B page 278

1 $AB = AC = \sqrt{30}$

2 Right-angle at P

3 $2\overrightarrow{AB} = \overrightarrow{CD}$

4 $\overrightarrow{EF} = \overrightarrow{GH}$ and $\overrightarrow{EG} = \overrightarrow{FH}$

5 $AB = DA = 3$, $BC = CD = \sqrt{14}$

6 $\overrightarrow{AC} = \overrightarrow{BD}$, $\overrightarrow{AB} = \overrightarrow{CD}$, $\overrightarrow{AD} = \overrightarrow{BC} = \sqrt{581}$

7 (12, 2, 1)

8 D(6, 0, 5), Area = 25

9 V = 125, SA = 150

Exercise 12C page 282

1 a $8\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ **b** $3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$

c $5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}$ **d** $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

2 a $5\mathbf{i}$, 5

b $0.5\mathbf{i} - 3\mathbf{j} - \mathbf{k}$, $\frac{\sqrt{161}}{4}$

c $2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\sqrt{6}$

d $0.5\mathbf{i} + 2.5\mathbf{j} + 4\mathbf{k}$, $\frac{3\sqrt{10}}{2}$

3 a $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ **b** $-2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

c $\mathbf{i} + 3\mathbf{j} + 1.75\mathbf{k}$ **d** $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

4 a $8\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ **b** $8\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$

c $5\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ **d** $10\mathbf{i} - 4\mathbf{j} + 17\mathbf{k}$

5 a $4\mathbf{k}$ **b** $30\mathbf{i} - 10\mathbf{j} - 15\mathbf{k}$

c $-4\mathbf{i} + 16\mathbf{j} + 32\mathbf{k}$

6 $4\mathbf{i} - 6\mathbf{j} - \mathbf{k}$

7 $8\mathbf{i} + 2\mathbf{j} - 11\mathbf{k}$

8 a 5 seconds **b** $y = -11, z = 2$

9 $x = -\sqrt{651}$

10 a $13\mathbf{i} + 4\mathbf{k}$ **b** No

11 $-4\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$

12 $5\sqrt{5}$

13 16.1 km

14 a $2\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$ **b** $\mathbf{i} + \mathbf{j} + 4.5\mathbf{k}$

c 59.0 m

15 a $a = -3, b = -3, c = 1$

b $0.75\mathbf{i} + 0.75\mathbf{j} - 0.25\mathbf{k}$

c $7.63\,\mathrm{m\,s^{-1}}$

16 $t = 4$ s, $-39\mathbf{i} + 10\mathbf{j} - 6\mathbf{k}$

Review Exercise 12D page 284

1 a $\frac{1}{\sqrt{35}}(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$

b $\frac{\sqrt{6}}{18}(-2\mathbf{i} - 7\mathbf{j} + \mathbf{k})$

c $\frac{\sqrt{26}}{26}(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$

2 $2.5\,(3\mathbf{i} - 12\mathbf{j} - 4\mathbf{k})$

3 a i $6\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$ **ii** $\sqrt{77}$

b i $2\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}$ **ii** $\sqrt{89}$

c i $-13\mathbf{i} - 12\mathbf{j} + 4\mathbf{k}$ **ii** $\sqrt{329}$

4 a $\sqrt{34}$ **b** $\sqrt{34} + \sqrt{6} = 8.28$

c 6 **d** $-5\mathbf{i} + 20\mathbf{j} + 5\mathbf{k}$

e $s = 4$

5 a $3\mathbf{i} - 4\mathbf{j} - 8\mathbf{k}$ **b** $1.5\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$

c $-0.5\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

6 (−0.5, 1.5, 1)

7 a 24.1° **b** 64.8° **c** 150°

8 (10, −9, −16)

9 scalene

10 504

11 a rectangle **b** trapezium

c parallelogram

12 $8(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})\,\mathrm{m\,s^{-1}}$

13 $(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})\,\mathrm{m\,s^{-1}}$

14 $(13\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ m

15 $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})\,\mathrm{m\,s^{-2}}$

16 $(27\mathbf{i} + 3\mathbf{j} - 15\mathbf{k})\,\mathrm{m\,s^{-1}}$

17 a $-\mathbf{i} - \frac{10}{3}\mathbf{j}$ **b** 111 m

18 7 seconds, $(10\mathbf{i} - 9\mathbf{j} - 3\mathbf{k})$ m

Exercise 13A page 288

1 Assume $\exists\, m, n \in \mathbb{Z}$ s.t. $28m + 21n = 1$.
Then $4m + 3n = \frac{1}{7}$.
But $m, n \in \mathbb{Z} \Rightarrow 4m + 3n \in \mathbb{Z}$, and $\frac{1}{7} \notin \mathbb{Z}$.
This contradicts our assumption that $m, n \in \mathbb{Z}$, and proves that there exist no integers m and n such that $28m + 21n = 1$.

2 a Assume $\exists\, m, n \in \mathbb{Z}$ s.t. $27a + 36b = 1$.
Then $3a + 4b = \frac{1}{9}$.
But $a, b \in \mathbb{Z} \Rightarrow 3a + 4b \in \mathbb{Z}$, and $\frac{1}{9} \notin \mathbb{Z}$.
This contradicts our assumption that $a, b \in \mathbb{Z}$, and proves that there exist no integers a and b such that $27a + 36b = 1$.

b e.g. $x = 7, y = -5$

3 Assume $ab \in \mathbb{Q}$, i.e. $ab = \frac{m}{n}$, $m, n \in \mathbb{Z}$.

Also, $b = \frac{c}{d}$ as $b \in \mathbb{Q}$.

Then $a\frac{c}{d} = \frac{m}{n}$

$a = \frac{dm}{cn} \Rightarrow a \in \mathbb{Q}$ as $dm, cn \in \mathbb{Z}$.

This contradicts the statement that a is irrational. Therefore the original statement that the product ab is **irrational** must be true.

4 Assume that n is odd, i.e. $n = 2m + 1$.

Then $n^2 = (2m + 1)^2 = 4m^2 + 4m + 1$
$= 2(2m^2 + 2m) + 1$

which is odd.

This contradicts the statement that n^2 is even. Therefore the original statement that if n^2 is even n must also be even must be true.

5 Assume that $\sqrt{2}$ is rational, i.e. $\sqrt{2} = \frac{a}{b}$, $a, b \in \mathbb{Z}$, and that this fraction is written in its simplest terms.

Then $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2$ i.e. a^2 is even.

If a^2 is even then a must also be even, i.e. $a = 2c$, $c \in \mathbb{Z}$.

Then $a^2 = 4c^2$, $2b^2 = 4c^2$, $b^2 = 2c^2$ i.e. b^2 is even, and therefore b is also even.

But this contradicts the assumption that $\frac{a}{b}$ was a fraction written in its simplest terms.

Therefore the original statement that $\sqrt{2}$ is irrational is true.

6 Assume there does exist a greatest even number $2n$. Now consider $2n + 2$.

$2n + 2 > 2n$ and also $2n + 2 = 2(n + 1)$ so it is also even. This contradicts our assumption that $2n$ was the greatest even number, and we have therefore proved that there is no greatest even number.

7 Assume that there does exist a greatest odd number $2n + 1$. Now consider $2n + 3$.

$2n + 3 > 2n + 1$ and also $2n + 3 = 2(n + 1) + 1$ so it is also odd. This contradicts our assumption that $2n + 1$ was the greatest odd number, and proves that there is no greatest odd number.

8 Assume that there does exist a greatest rational number $q = \frac{a}{b}$, $a, b \in \mathbb{Z}$.

Now consider $q + 1 = \frac{a}{b} + 1 = \frac{a + b}{b}$.

$a, b \in \mathbb{Z} \Rightarrow a + b \in \mathbb{Z}$. $\therefore \frac{a + b}{b} \in \mathbb{Q}$ and

$\frac{a + b}{b} > q$. This contradicts the assumption that q was the greatest rational number, and proves that there is no greatest rational number.

9 Assume that there does exist a smallest positive rational number $q = \frac{a}{b}$, $a + b \in \mathbb{Z}^+$.

Now consider $\frac{a}{b + 1}$. $b \in \mathbb{Z}^+ \Rightarrow b + 1 \in \mathbb{Z}^+$, so $\frac{a}{b + 1} \in \mathbb{Q}^+$, and $\frac{a}{b + 1} < \frac{a}{b}$.

This contradicts the assumption that q was the smallest positive rational number, and proves that there is no smallest positive rational number.

10 Let a^2 and b^2 be odd, and assume that $a^2 + b^2 = c^2$ for some $c \in \mathbb{Z}$.

Now a^2, b^2 odd $\Rightarrow a, b$ also odd

i.e. $a = 2m + 1$ and $b = 2n + 1$

$a^2 = 4m^2 + 4m + 1$ $b^2 = 4n^2 + 4n + 1$

Then $a^2 + b^2 = 4m^2 + 4m + 1 + 4n^2 + 4n + 1$
$= 2(2m^2 + 2m + 2n^2 + 2n + 1)$
$= c^2$, so c^2 is even.

Now, if c^2 is even then c must also be even, i.e. $c = 2k$ and $c^2 = 4k^2$, so c^2 is in fact a multiple of 4.

But $c^2 = 4m^2 + 4m + 4n^2 + 4n + 2$
$= 4(m^2 + m + n^2 + n) + 2$

which is **not** a multiple of 4.

This contradiction proves that the original statement that the sum of any two odd square numbers cannot itself be a square number must be true.

11 Let $x, y \in \mathbb{Z}^+$. Then $x + y > 0$ and $2\sqrt{xy} > 0$.

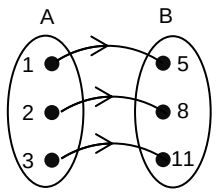
Now assume $x + y < 2\sqrt{xy}$. Since both sides of the inequality are positive, we can square each side to get

$(x + y)^2 < 4xy$
$x^2 + 2xy + y^2 < 4xy$
$x^2 - 2xy + y^2 < 0$
$(x - y)^2 < 0$

But no square can be negative.

This contradiction shows that $x + y \geqslant 2\sqrt{xy}$ $\forall$ $x, y \in \mathbb{Z}^+$.

ERRORS IN TEXT

Page	Exercise	Question	Should be	Not
5	Functions & Mappings	Diagram	Type 1 	(Current diagram very confused!)
244	Ex 11D	6b	$-\left(\frac{2x+1}{4e^{2x}}\right)+c$	$\left(\frac{2x+1}{4e^{2x}}\right)+c$