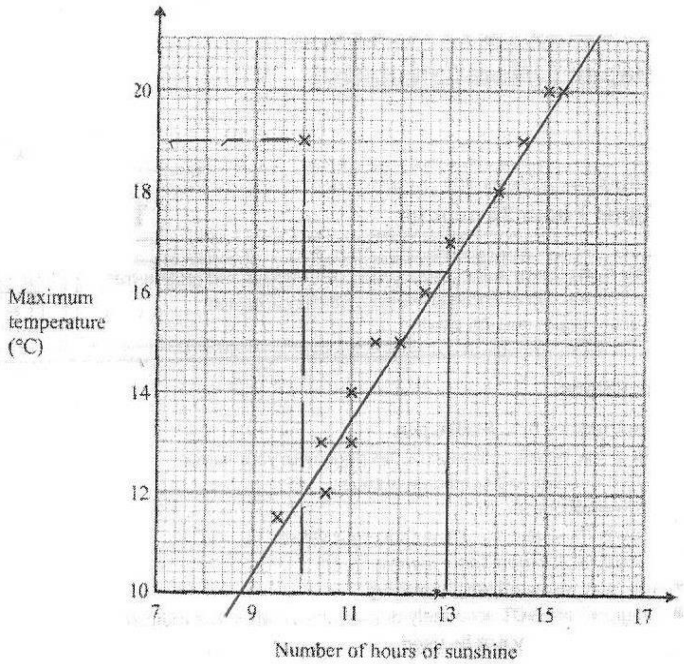
2017 paper 1 (Edexcel GCSE) – solutions

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The scatter graph shows the maximum temperature and the number of hours of sunshine in fourteen British towns on one day.



One of the points is an outlier.

(a) Write down the coordinates of this point.

(10, 19)

(1)

(b) For all the other points write down the type of correlation.

positive

(1)

On the same day, in another British town, the maximum temperature was 16.4 °C.

(c) Estimate the number of hours of sunshine in this town on this day.

13 hours

(2)

A weatherman says,

"Temperatures are higher on days when there is more sunshine."

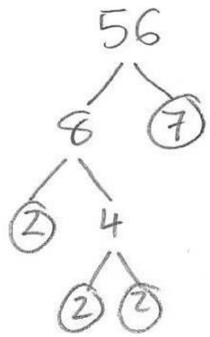
(d) Does the scatter graph support what the weatherman says? Give a reason for your answer.

Yes, overall there is a positive correlation on the graph – as the hours of sunshine increase, the temperature increases.

(1)

(Total for Question 1 is 5 marks)

2 Express 56 as the product of its prime factors.



$2 \times 2 \times 2 \times 7$

(Total for Question 2 is 2 marks)

OR $2^3 \times 7$

3 Work out 54.6×4.3

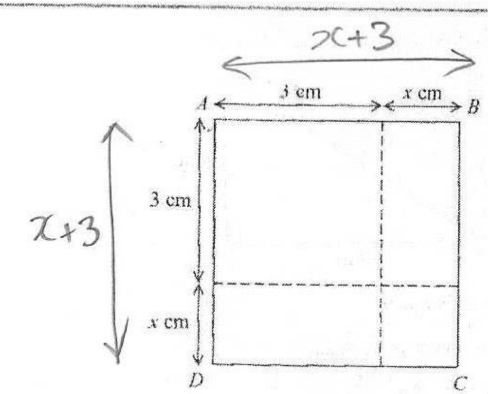
$= 234.78$

$$\begin{array}{r} 546 \\ \times\ 43 \\ \hline 1638 \\ 21840 \\ \hline 23478 \end{array}$$

234.78

(Total for Question 3 is 3 marks)

4



$A = 10$.

The area of square *ABCD* is 10 cm².

Show that $x^2 + 6x = 1$

$(x+3)(x+3) = 10$

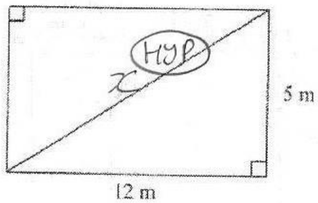
$x^2 + 3x + 3x + 9 = 10$

$x^2 + 6x = 10 - 9$

$x^2 + 6x = 1$

(Total for Question 4 is 3 marks)

5 This rectangular frame is made from 5 straight pieces of metal.



The weight of the metal is 1.5 kg per metre.

Work out the total weight of the metal in the frame.

$12^2 + 5^2 = x^2$

$144 + 25 = x^2$

$169 = x^2$

$x^2 = 169$

$x = \sqrt{169} = 13$

Total length of frame $= 12 + 5 + 12 + 5 + 13$

$= 24 + 10 + 13 = 47$

Total weight $= 47 \times 1.5$

$= 70.5$

$$\begin{array}{r} 47 \\ \times\ 15 \\ \hline 235 \\ 470 \\ \hline 705 \end{array}$$

70.5 kg

(Total for Question 5 is 5 marks)

6 The equation of the line L_1 is $y = 3x - 2$
The equation of the line L_2 is $3y - 9x + 5 = 0$

Show that these two lines are parallel.

L_2 $3y - 9x + 5 = 0$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{\cancel{9}^{3}x}{\cancel{3}_{1}} - \frac{5}{3}$

$y = 3x - \frac{5}{3}$

Grad of $L_2 = 3$

Grad of $L_1 = 3$

$\Big\}$ same gradient $\therefore$ parallel

(Total for Question 6 is 2 marks)

7 There are 10 boys and 20 girls in a class.
The class has a test.

The mean mark for all the class is 60
The mean mark for the girls is 54

Work out the mean mark for the boys.

mean = $\frac{\text{total}}{\text{no. of nos}}$

	Number	mean mark	total marks
Boys	10	?	
Girls	20	54	1080
Class	30	60	1800

total marks for Boys = 1800 − 1080 = 720

mean mark for Boys = $\frac{720}{10}$ = 72

72

(Total for Question 7 is 3 marks)

8 (a) Write 7.97×10^{-6} as an ordinary number.

0.0000797

0.00000797

(1)

(b) Work out the value of $(2.52 \times 10^5) \div (4 \times 10^{-3})$
Give your answer in standard form.

$$\frac{2.52 \times 10^5}{4 \times 10^{-3}} = 0.63 \times 10^8$$

$$= 6.3 \times 10^{-1} \times 10^8$$

$$4\overline{)2.52} = 0.63$$

6.3×10^7

(2)

(Total for Question 8 is 3 marks)

9 Jules buys a washing machine.

20% VAT is added to the price of the washing machine.
Jules then has to pay a total of £600

What is the price of the washing machine with **no** VAT added?

$$? \xrightarrow{+20\%} £600$$

Let original price = x

$$x \times 1.2 = 600$$

$$x = \frac{600 \times 10}{1.2 \times 10}$$

$$= \frac{\cancel{6000}^{1000\ 500}}{\cancel{12}_{2\ 1}}$$

£ 500

(Total for Question 9 is 2 marks)

10 Show that $(x + 1)(x + 2)(x + 3)$ can be written in the form $ax^3 + bx^2 + cx + d$
where a, b, c and d are positive integers.

$$(x+1)[(x+2)(x+3)] = (x+1)[x^2 + 3x + 2x + 6]$$

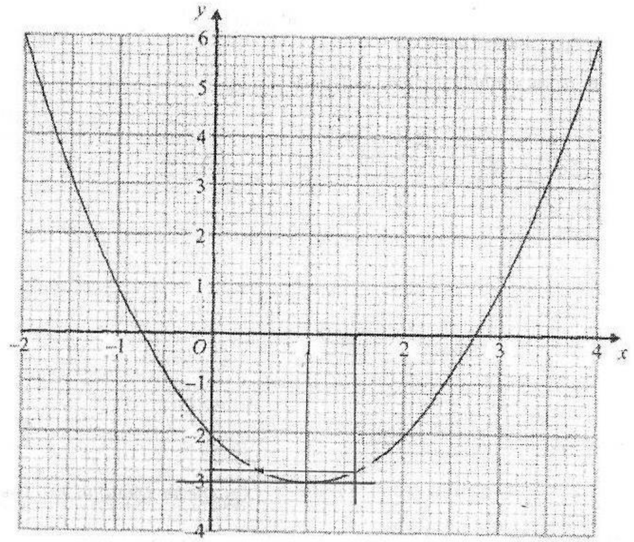
$$= (x+1)(x^2 + 5x + 6)$$

$$= x^3 + 5x^2 + 6x$$
$$+ x^2 + 5x + 6$$

$$= x^3 + 6x^2 + 11x + 6$$

(Total for Question 10 is 3 marks)

11 The graph of $y = f(x)$ is drawn on the grid.



(a) Write down the coordinates of the turning point of the graph.

(1, −3)

(1)

(b) Write down estimates for the roots of $f(x) = 0$

−0.7, 2.7

(1)

(c) Use the graph to find an estimate for $f(1.5)$

−2.8

(1)

(Total for Question 11 is 3 marks)

12 (a) Find the value of $81^{-\frac{1}{2}}$ $= \frac{1}{81^{\frac{1}{2}}} = \frac{1}{\sqrt{81}} = \frac{1}{9}$

$\frac{1}{9}$

(2)

(b) Find the value of $\left(\frac{64}{125}\right)^{\frac{2}{3}}$ $= \left(\sqrt[3]{\frac{64}{125}}\right)^2$

$= \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

$\frac{16}{25}$

(2)

(Total for Question 12 is 4 marks)

13 The table shows a set of values for x and y.

x	1	2	3	4
y	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

y is inversely proportional to the square of x.

(a) Find an equation for y in terms of x

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

If $x=1, y=9, \quad 9 = \frac{k}{1^2} \Rightarrow k = 9$

$y = \frac{9}{x^2}$

(2)

(b) Find the positive value of x when $y = 16$

If $y=16$, $\quad 16 = \frac{9}{x^2}$

$x^2 = \frac{9}{16}$

$x = \pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$

$\frac{3}{4}$

(2)

(Total for Question 13 is 4 marks)

14 White shapes and black shapes are used in a game.
Some of the shapes are circles.
All the other shapes are squares.

W   B

The ratio of the number of white shapes to the number of black shapes is 3 : 7

WC   BC

The ratio of the number of white circles to the number of white squares is 4 : 5

The ratio of the number of black circles to the number of black squares is 2 : 5

BC   BS

Work out what fraction of all the shapes are circles.

	White	Black	TOTAL
circles	$13\frac{1}{3}\%$	20%	
squares			
TOTAL	30%	70%	100%

$$\text{white} = \frac{3}{10} \text{ of } 100\% = 30\%$$

$$\text{white circles} = \frac{4}{9} \text{ of } 30\% = \frac{4}{\cancel{9}_3} \times \frac{\cancel{30}^{10}}{1} = \frac{40}{3} = 13\tfrac{1}{3}\%$$

$$\text{Black circles} = \frac{2}{7} \text{ of } 70\% = \frac{2}{\cancel{7}_1} \times \frac{\cancel{70}^{10}}{1} = 20\%$$

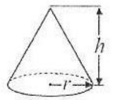
$$\text{circles} = 13\tfrac{1}{3}\% + 20\% = 33\tfrac{1}{3}\% = \frac{1}{3}$$

Ans $= \frac{1}{3}$

(Total for Question 14 is 4 marks)

15 A cone has a volume of 98 cm³.
The radius of the cone is 5.13 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$



(a) Work out an estimate for the height of the cone.

$$V = \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

$$h = \frac{3V}{\pi r^2} = \frac{3 \times 98}{\pi \times (5.13)^2} \approx \frac{\cancel{3} \times 100}{\cancel{3} \times 5^2} = \frac{100}{25} = 4$$

4 cm
(3)

John uses a calculator to work out the height of the cone to 2 decimal places.

(b) Will your estimate be more than John's answer or less than John's answer?
Give reasons for your answer.

Rounding 98 to 100 increases the estimate.
Rounding π to 3 and 5.13 to 5 will decrease the denominator, but increase the overall estimate. So estimate more than John's answer

(1)

(Total for Question 15 is 4 marks)

16 n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n-2)^2$ is always an even number.

$$n^2 - 2 - [(n-2)(n-2)] \leftarrow \text{use 'safety brackets'}$$

$$= n^2 - 2 - [n^2 - 2n - 2n + 4]$$

$$= \cancel{n^2} - 2 - \cancel{n^2} + 4n - 4$$

$$= 4n - 6$$

$$= 2(2n - 3)$$

Therefore this is always an even number

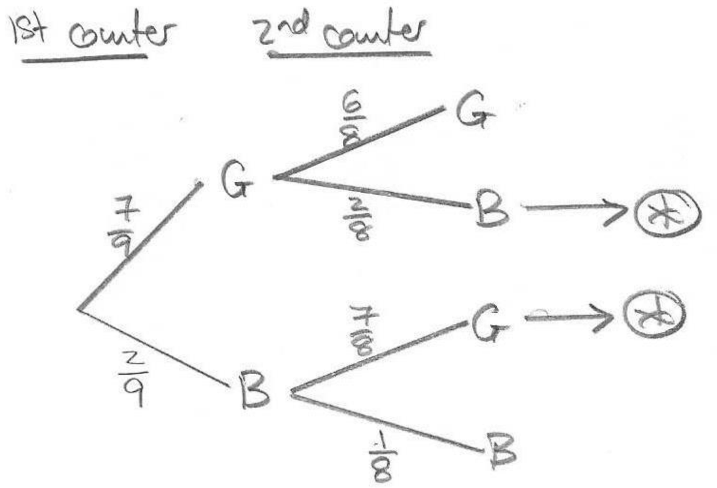
(Total for Question 16 is 4 marks)

17 There are 9 counters in a bag.

7 of the counters are green.
2 of the counters are blue.

Ria takes at random two counters from the bag.

Work out the probability that Ria takes one counter of each colour.
You must show your working.



$$P(\text{one of each}) = P(GB) + P(BG)$$

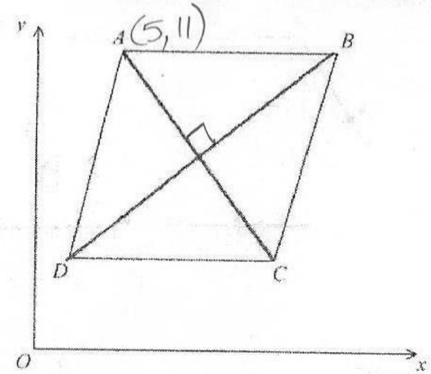
$$= \frac{7}{9} \times \frac{\cancel{2}^{1}}{\cancel{8}_{4}} + \frac{\cancel{2}^{1}}{9} \times \frac{7}{\cancel{8}_{4}}$$

$$= \frac{7}{36} + \frac{7}{36} = \frac{\cancel{14}^{7}}{\cancel{36}_{18}}$$

Ans $= \frac{7}{18}$

(Total for Question 17 is 4 marks)

18



ABCD is a rhombus.
The coordinates of *A* are (5, 11)
The equation of the diagonal *DB* is $y = \frac{1}{2}x + 6$

Find an equation of the diagonal *AC*.

Diagonals of a rhombus are perpendicular.

For DB, $y = \frac{1}{2}x + 6$

$\therefore$ Gradient of DB $= \frac{1}{2}$

Gradient of AC $= -2$

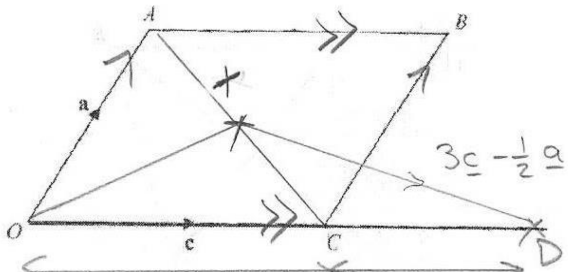
AC goes through (5, 11) with gradient $= -2$

$$y = -2x + c$$
$$11 = -2(5) + c$$
$$11 = -10 + c$$
$$11 + 10 = c$$
$$c = 21$$

$y = -2x + 21$

(Total for Question 18 is 4 marks)

19



$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of k.

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$
$$= \overrightarrow{OA} + \tfrac{1}{2}(\overrightarrow{AC})$$
$$= \underline{a} + \tfrac{1}{2}(\underline{c} - \underline{a}) = \underline{a} + \tfrac{1}{2}\underline{c} - \tfrac{1}{2}\underline{a} = \tfrac{1}{2}\underline{a} + \tfrac{1}{2}\underline{c}$$

$$\overrightarrow{OD} = \overrightarrow{OX} + \overrightarrow{XD}$$
$$= \cancel{\tfrac{1}{2}\underline{a}} + \tfrac{1}{2}\underline{c} + 3\underline{c} - \cancel{\tfrac{1}{2}\underline{a}} = 3.5\underline{c}$$

$$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$$
$$= -\underline{c} + 3.5\underline{c} = 2.5\underline{c}$$

$$\therefore OC : CD = \underline{c} : 2.5\underline{c}$$
$$= 1^{\times 2} : \tfrac{5}{2}^{\times 2}$$
$$= \tfrac{2}{5} : \tfrac{5}{5}$$
$$= \tfrac{2}{5} : 1$$

OR 0.4

$k = \frac{2}{5}$

(Total for Question 19 is 4 marks)

20 Solve algebraically the simultaneous equations

$$x^2 + y^2 = 25 \quad — ①$$
$$y - 3x = 13 \quad — ②$$

Rearrange ② $\quad y = 3x + 13 \quad — ③$

Sub in ① $\quad x^2 + (3x+13)^2 = 25$

$$x^2 + (3x+13)(3x+13) = 25$$
$$x^2 + 9x^2 + 39x + 39x + 169 = 25$$
$$10x^2 + 78x + 144 = 0$$
$$(\div 2) \quad 5x^2 + 39x + 72 = 0$$
$$(5x + 24)(x + 3) = 0$$

72: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12; 8, 9

$5x + 24 = 0 \quad$ or $\quad x + 3 = 0$

$5x = -24 \qquad x = -3$

$x = -\frac{24}{5}$

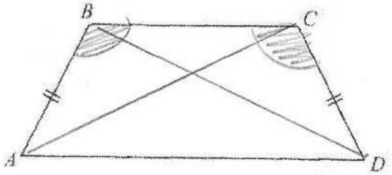
Sub in ③ $\quad$ If $x = -3$, $y = 3(-3) + 13$
$$= -9 + 13 = 4$$

If $x = -\frac{24}{5}$, $y = 3(-\frac{24}{5}) + 13$
$$= -\frac{72}{5} + 13 = -\frac{72}{5} + \frac{65}{5} = \frac{-7}{5}$$

$x = -3, y = 4$ or $x = -\frac{24}{5}, y = -\frac{7}{5}$

(Total for Question 20 is 5 marks)

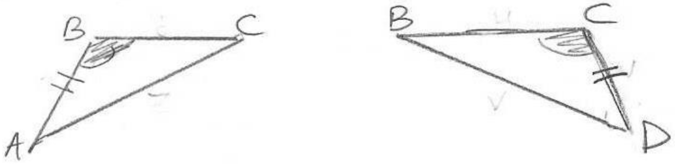
21 $ABCD$ is a quadrilateral.



$AB = CD$.
Angle ABC = angle BCD.

Prove that $AC = BD$.

Compare triangles ABC and BCD:



We are told that $AB = CD$

We are told that $\angle ABC = \angle BCD$

We know that $BC = BC$ as this length is common to both triangles.

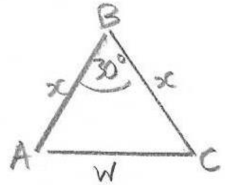
$\therefore$ ABC and BCD are congruent, because of SAS

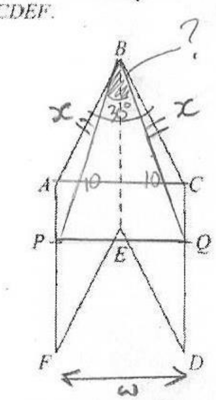
Side, included angle, side

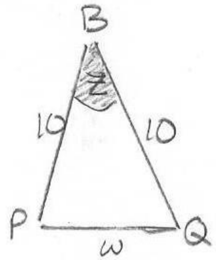
Thus AC must equal BD

(Total for Question 21 is 4 marks)

22 The diagram shows a hexagon $ABCDEF$.







$ABEF$ and $CBED$ are congruent parallelograms where $AB = BC = x$ cm.
P is the point on AF and Q is the point on CD such that $BP = BQ = 10$ cm.

Given that angle $ABC = 30°$,

prove that $\cos PBQ = 1 - \frac{(2-\sqrt{3})}{200}x^2$

Let width = w (same as AC or PQ or FD)
and let required angle = PBQ = Z

(cosine rule, $\triangle ABC$) $w^2 = x^2 + x^2 - 2(x)(x)\cos 30$

$w^2 = 2x^2 - \cancel{2}x^2\left(\frac{\sqrt{3}}{\cancel{2}}\right)$

$w^2 = 2x^2 - \sqrt{3}x^2 = (2\sqrt{3})x^2$ — ①

(cosine rule, $\triangle PBQ$) $w^2 = 10^2 + 10^2 - 2(10)(10)\cos Z$

$w^2 = 200 - 200\cos Z$ — ②

From ① & ② $(2-\sqrt{3})x^2 = 200 - 200\cos Z$

$\frac{\cancel{200}\cos Z}{\cancel{200}} = \frac{200}{200} - \frac{(2-\sqrt{3})x^2}{200}$

$\cos PBQ = 1 - \frac{(2-\sqrt{3})x^2}{200}$

(Total for Question 22 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS