

Rayner Year 2 Textbook ANSWERS

EXERCISE 1A PAGE 1

- 1 $\frac{4}{5}$ 2 $4a$ 3 $\frac{1}{3}$ 4 4
 5 $\frac{a}{2b}$ 6 3 7 $\frac{a}{2}$ 8 $2b$
 9 $\frac{3}{4y}$ 10 $\frac{11y}{12x}$ 11 $\frac{2ya}{3}$ 12 $\frac{4m}{n}$
 13 AG, BE, CH, DF
 14 $\frac{a}{5b}$ 15 a 16 $\frac{7}{8}$ 17 $\frac{3}{4-x}$
 18 $\frac{5+2x}{3}$ 19 $\frac{3x+1}{x}$ 20 $\frac{4+5a}{5}$ 21 $\frac{b}{3+2a}$
 22 $\frac{3-x}{2}$ 23 $\frac{2x+1}{y}$ 24 $\frac{2x+5}{2}$ 25 $y+xy$
 26 $\frac{5x+4}{3x-2}$ 27 $\frac{1+4a}{b+b^2}$ 28 $\frac{1+2x+2x^2}{x}$
 29 $\frac{6n-3m}{2mn}$
 30 a False b True c True d False
 31 a $(x-3)(x+2)$ b $x-3$
 32 a $x-4$ b $\frac{x-2}{x-1}$ c $\frac{x+5}{x+2}$
 d $\frac{x}{x+1}$ e $\frac{x+4}{2(x-5)}$ f $\frac{x+5}{x-2}$
 g $\frac{x}{2x-1}$ h $\frac{2x+1}{2x+3}$ i $\frac{x-1}{3x-4}$
 33 $\frac{6x+3}{6x+2}$
 34 a $\frac{x^2+1}{x^2}$ b $2x^2-1$ c $2x-1$
 d $12x+1$ e $30x-2$ f $\frac{1-4x}{2}$
 g $\frac{3x^2+1}{x^2+2}$ h $\frac{x+2}{x}$ i $\frac{x}{2x+3}$
 j x^2-1 k $\frac{x+2}{x}$ l $\frac{x+3}{x+4}$

EXERCISE 1B PAGE 3

- 1 a $\frac{3x}{5}$ b $\frac{3}{x}$ c $\frac{4x}{7}$ d $\frac{4}{7x}$
 e $\frac{7x}{8}$ f $\frac{7}{8x}$ g $\frac{5x}{6}$ h $\frac{5}{6x}$
 2 AF, BH, CD, EG
 3 a $\frac{23x}{20}$ b $\frac{23}{20x}$ c $\frac{x}{12}$
 d $\frac{1}{12x}$ e $\frac{5x+2}{6}$ f $\frac{7x+2}{12}$
 4 a $\frac{1-2x}{12}$ b $\frac{2x-9}{15}$ c $\frac{3x+12}{14}$

5 a $\frac{9x+13}{10}$

c $\frac{7x-8}{x(x-2)}$

e $\frac{4x+11}{(x+1)(x+2)}$

b $\frac{3x+1}{x(x+1)}$

d $\frac{8x+9}{(x-2)(x+3)}$

f $\frac{x^2+4x+6}{2(x+2)}$

EXERCISE 1C PAGE 4

- 1 a 2 $\frac{10m}{3}$ 3 $\frac{2y}{3x}$ 4 $\frac{15b}{4a}$
 5 $\frac{4}{a^2}$ 6 $\frac{8}{3}$ 7 $\frac{x-1}{x+2}$ 8 $\frac{y^2}{5x}$
 9 $\frac{x}{8}$ 10 $3(x+2)$
 11 a 6 b 4 c $2a$
 12 AF, BH, CE, DG
 13 a $\frac{26}{3}$ b $\frac{6q}{p}$ c $\frac{2xy}{9}$ d $\frac{4}{aq}$
 e 25 f $\frac{y}{2x}$ g $2a$ h 1
 14 a 1 b $\frac{ab^3}{x}$ c $\frac{x}{x-1}$ d $\frac{z^2}{y}$
 15 a $\frac{x}{10}$ b $3x$ c 11
 16 $\frac{1}{4}$
 17 $\frac{1}{5}$

EXERCISE 1D PAGE 7

- 1 a one-one b many-one
 c one-one d one-one
 e many-one f many-one
 g many-one h many-one
 i one-one j one-one
 2 a $f(x) \geq 3$ b $f(x) \geq 2$
 c $f(x) \geq 1$ d $f(x) \geq 0$
 e $0 \leq f(x) \leq 1$ f $-\frac{9}{4} \leq f(x) \leq 0$
 g $0 \leq f(x) \leq \frac{25}{4}$ h $0 \leq f(x) \leq 5$
 i $0 < f(x) \leq 1$ j $0 < f(x) \leq 1$
 3 a $(x+3)^2-5$ b $f(x) \geq -5$
 4 a $f(x) \geq 3$ b $f(x) \geq -50$
 c $0 < f(x) \leq \frac{1}{2}$ d $0 < f(x) \leq 1$

EXERCISE 1E PAGE 10

- 1 a $4(x+5)$ b $4x+5$
 c $(4x)^2$ d $4x^2$
 e x^2+5 f $x+10$
 g $4(x^2+5)$ h $[4(x+5)]^2$
 2 a -2.5 b $\pm\sqrt{\frac{5}{3}}$

3 a $x \mapsto 2(x-3)$ b $x \mapsto 2x-3$
 c $x \mapsto x^2-3$ d $x \mapsto 4x^2$
 e $x \mapsto (4x)$ f $x \mapsto (2x)^2-3$
 g $x \mapsto (2x-3)^2$

4 a 2 b 11 c 6
 d 2 e 1 f 64

5 a -3 b 2 c $1\frac{1}{2}$ d 5

6 a $2(3x-1)+1$ b $3(2x+1)-1$
 c $2x^2+1$ d $(3x-1)^2$
 e $2(3x-1)^2+1$ f $3(2x^2+1)-1$

7 a 11 b 9 c 11
 d 14 e 81 f -1

8 a 2 b 0, 2 c $\pm\sqrt{2}$

9 $\frac{x+2}{5}$ 10 $\frac{x}{5}+2$

11 $\frac{x}{6}-2$ 12 $\frac{3x-1}{2}$

13 $\frac{4x}{3}+1$ 14 $\frac{(x+6)/2-4}{3}$

15 $x \mapsto \frac{2(x-10)-4}{5}$ 16 $x \mapsto \frac{2x-3}{-7}$

17 $x \mapsto \frac{12}{3x-5}$ 18 $x \mapsto \frac{3x-4}{5}$

19 $x \mapsto \frac{4(5x+3)+1}{2}$ 20 $x \mapsto \frac{12}{x}$

21 a $x \mapsto \frac{x}{3}$ b $x \mapsto x+5$ c $x \mapsto \frac{x-1}{2}$

d $x \mapsto 3(x-5)$ e $x \mapsto \frac{x}{3}+5$ f $x \mapsto \frac{x}{3}+5$

22 a $x \mapsto 6x+1$ b $x \mapsto \frac{2x}{3}+1$ c $x \mapsto \frac{x-1}{6}$

d $x \mapsto 2x-9$ e $x \mapsto \frac{x+9}{2}$ f $x \mapsto \frac{x+9}{2}$

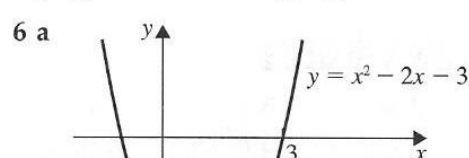
23 a 7 b 21 c 5
 d 5 e 2

24 a 1 b i $x > -2\frac{1}{2}$ ii $x > 5$

25 a $x \mapsto \frac{3x-5}{2}$ b 2

5 a $\frac{8x+10}{x-1}$ b $12\frac{1}{2}$

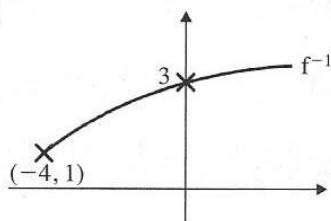
c 10 d 10



b i $f(x) \geq -4$

ii domain $x \geq -4$, range $f^{-1}(x) \geq 1$

iii



7 a $f^{-1}(x) = \frac{1}{2-x}, x \neq 2$

b $x = 1$

8 a domain $x \geq 0$, range $f^{-1}(x) \geq 2$

b $f^{-1}(x) = 2 + \sqrt{x}$

c $x = 4$

9 a $a = 2$ b $b = 0$

b $f^{-1}(x) = x^2 + 2, x \geq 0; g^{-1}(x) = \frac{1}{\sqrt{x}}, x \geq 0$

10 $f^{-1}(x) = \frac{x-1}{2}; g^{-1}(x) = \frac{5+3x}{x}, x \neq 0$

11 a $-\infty < x < \infty$ b $x \geq 0$

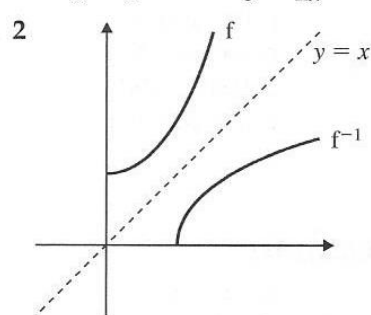
c $-90 < x \leq 90^\circ$ d $x \geq 1$

e $x \leq -1$ f $x \geq -4$

EXERCISE 1F PAGE 13

1 a $2-x$ b $\frac{13}{2x}$ c $\frac{3-x}{5}$

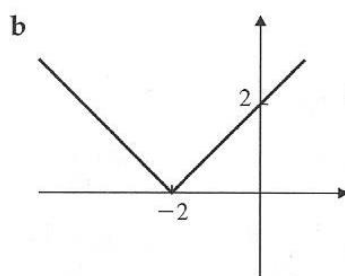
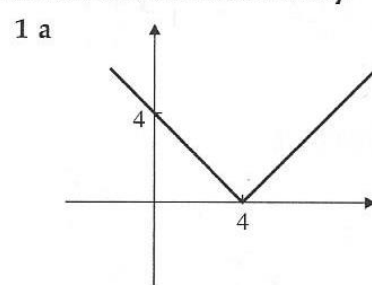
d $\frac{7x+5}{x-2}$ e $\frac{7x+5}{3-2x}$

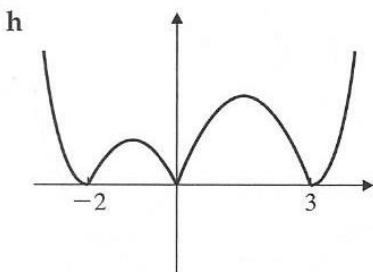
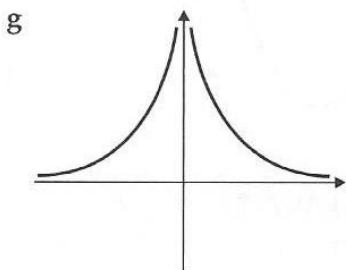
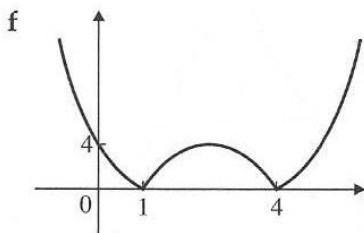
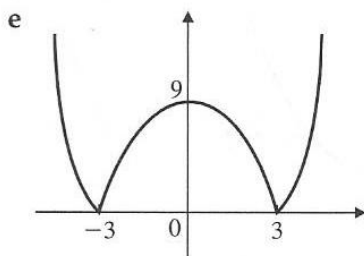
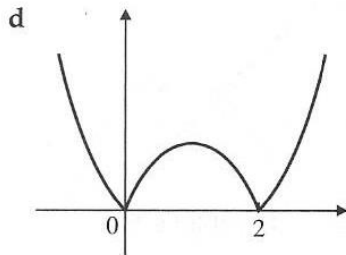
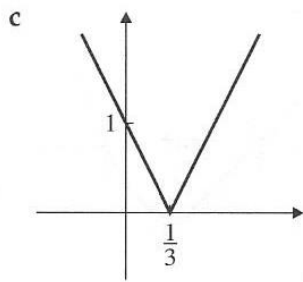


3 a $\frac{x+8}{3}, \frac{x+22}{3}$ b 4, 8

4 a $9x-4, \frac{x+1}{3}$ b Both are $\frac{x+4}{9}$

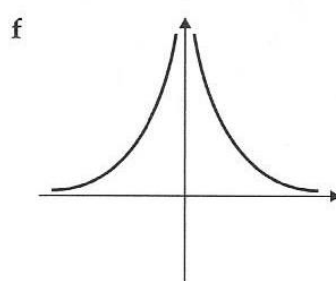
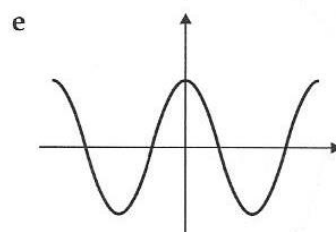
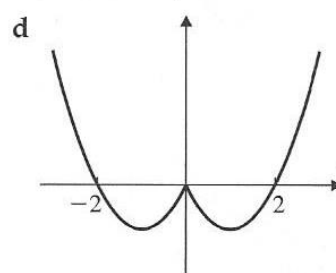
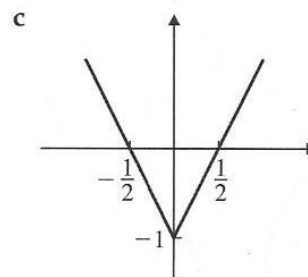
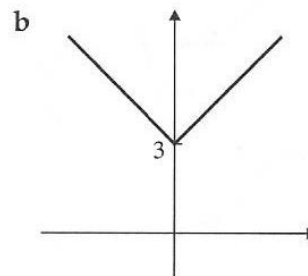
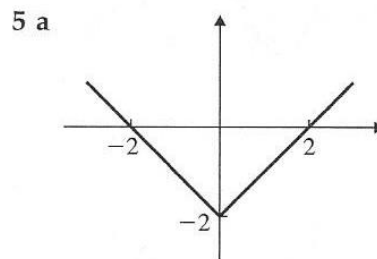
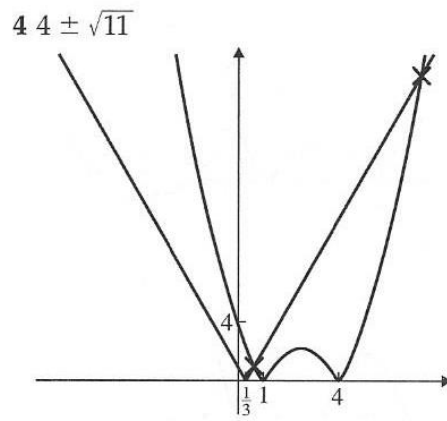
EXERCISE 1G PAGE 17



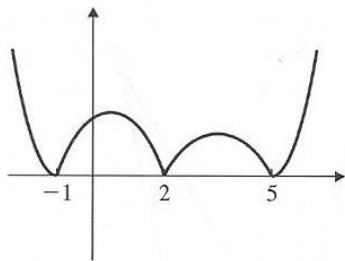


- 2 a $-3, 7$
 c $-2, 3$
 e $\pm 5, \pm 7$
 g $-1, 2$
 3 a $x < -1, x > 9$
 c $x > 2, x < -5$
 e $-\frac{14}{3} < x < 6$
 g $\frac{1}{3} < x < 1$
 i $0 < x < \frac{4}{3}$

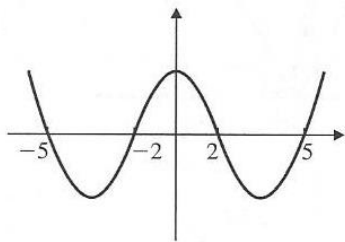
- b $0, 3$
 d $\pm 1, \pm 3$
 f $-7, 1$
 h $\pm \frac{1}{2}$
 b $-4 < x < 2$
 d all x
 f $-2 < x < 2$
 h $x > -1$



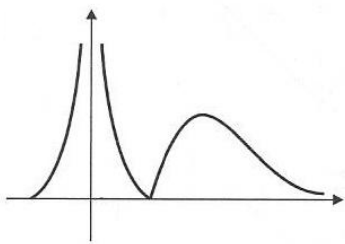
6 a



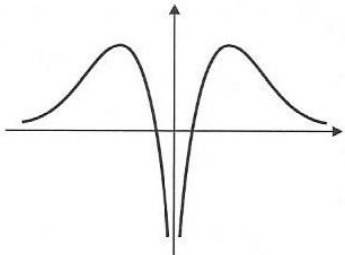
b



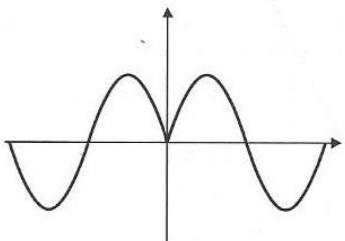
c



d

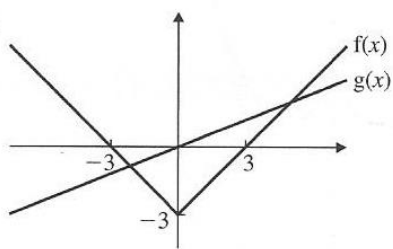


7 a



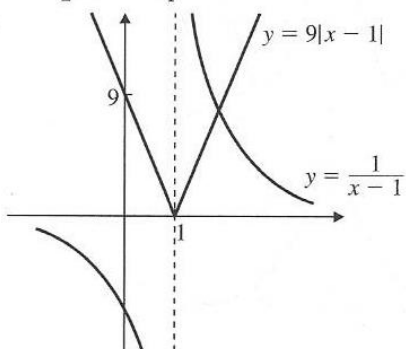
b 4

8 a



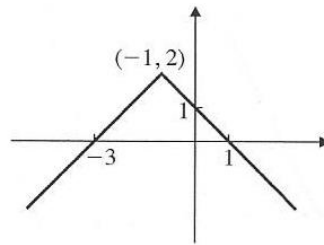
b $x > \frac{9}{2}, x < -\frac{9}{4}$

9 a

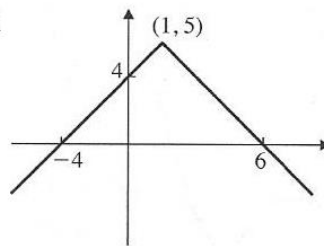


b $x > \frac{4}{3}, x < 1$

10 a i

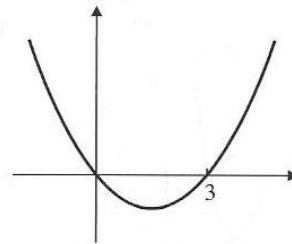


ii

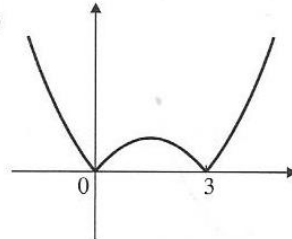


b i range $y \leq 2$ ii range $y \leq 5$

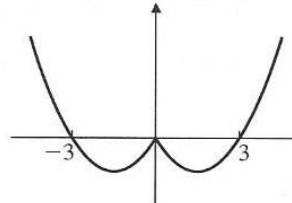
11 a



b

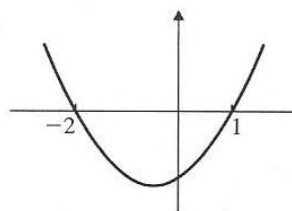


c

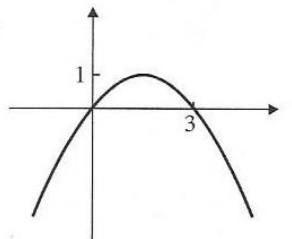


EXERCISE 1H PAGE 21

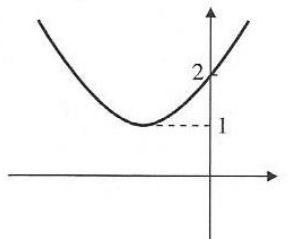
1 a

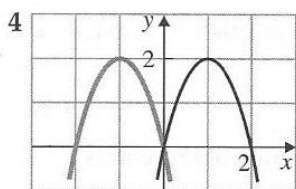
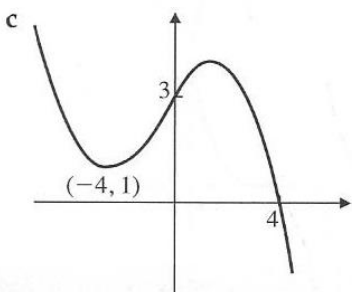
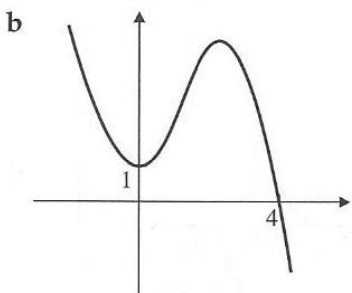
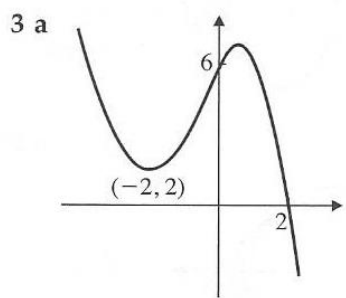
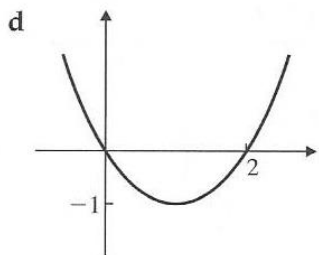
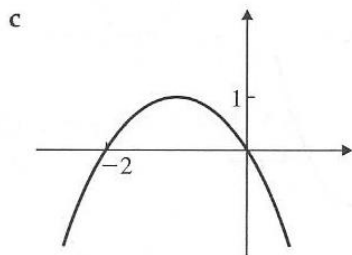
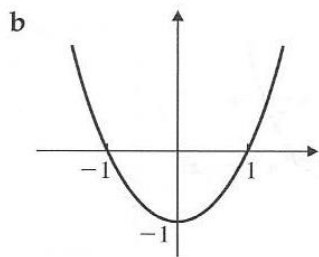


b

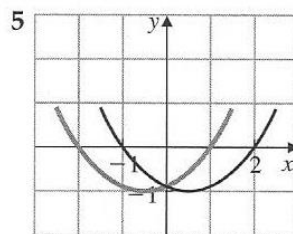


2 a

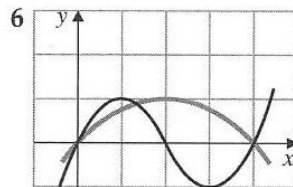




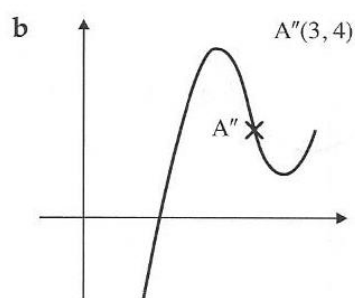
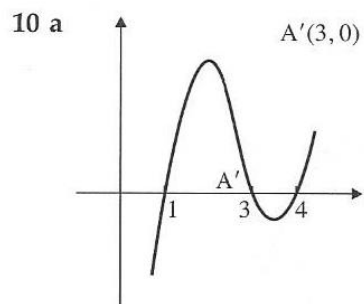
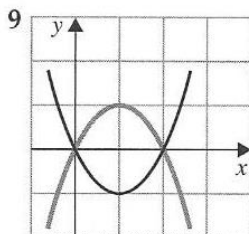
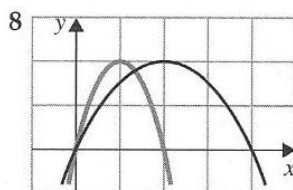
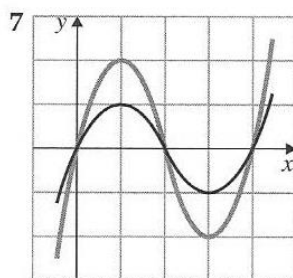
Sketch $f(x + 2)$



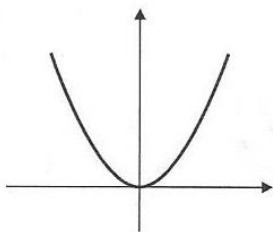
Sketch $f(-x)$



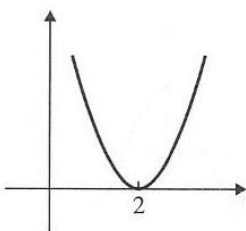
Sketch $f(\frac{1}{2}x)$



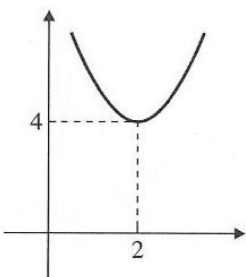
11 a



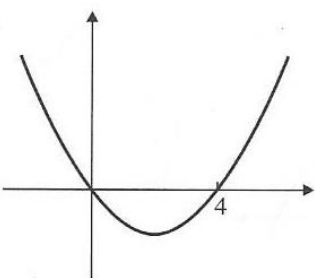
b



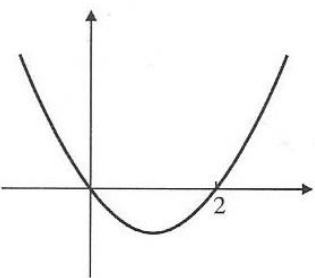
c



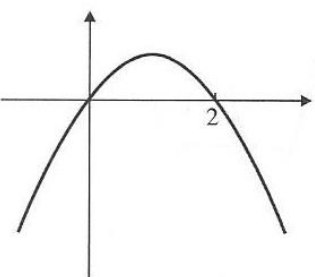
12 a



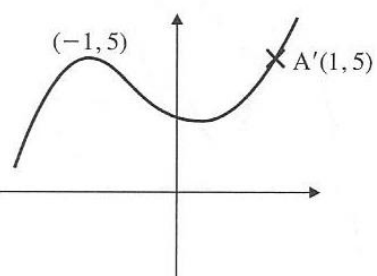
b



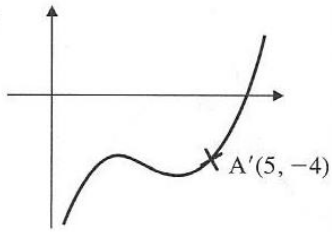
c



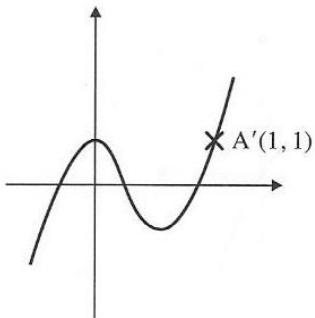
13 a



b



c

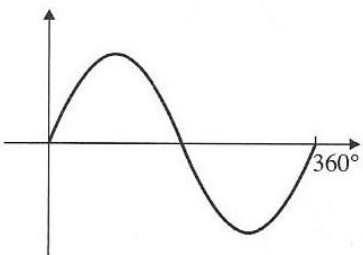


14 a $y = x^3 + 5$

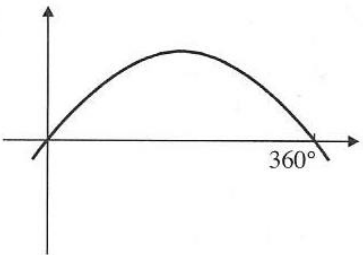
b $y = (x - 2)^3$

c $y = (x - 2)^3 + 5$

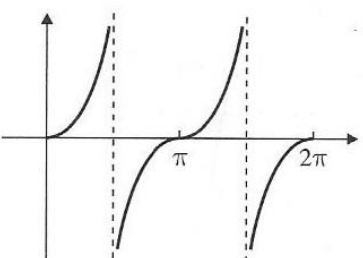
15 a



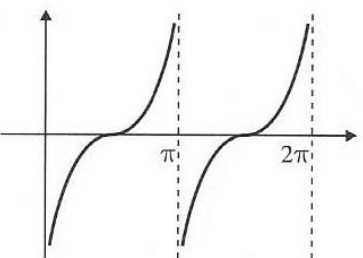
b



16 a



b



17 Stretch, scale factor 3 parallel to y -axis,

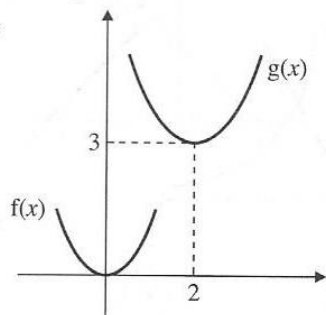
Translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

18 Stretch, scale factor 4 parallel to y -axis,

Translation $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$

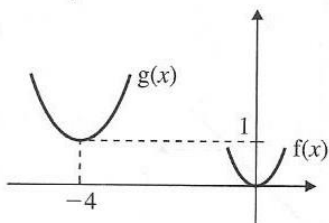
19 a $a = 2, b = 3$

b

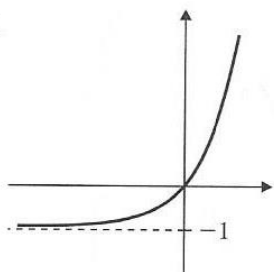


20 a $a = 4, b = 1$

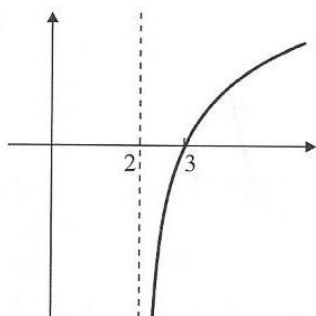
b



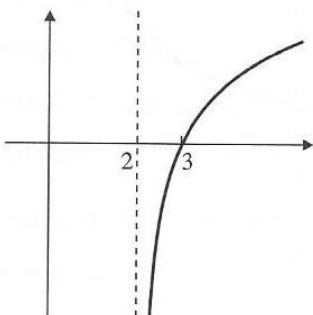
21



22 a



b



3 a $\frac{x+2}{x}$

b $\frac{x-3}{x+1}$

c $\frac{2x-1}{x-4}$

d $\frac{2x^2+1}{x^2}$

e $3x^2 - 1$

f $\frac{4x^2-1}{x}$

4 a $\frac{8x+5}{6}$

b $\frac{4x+1}{x(x+1)}$

c $\frac{x^2+3x-2}{(x+1)(x-1)}$

d $\frac{x+4}{12}$

e $-\frac{(3x+5)}{10}$

f $\frac{2(x-1)}{x+3}$

5 a $(2x-1)$ is common factor

b $\frac{3x+1}{x(2x+1)}$

6 a $f(x) \leq 1$

b $x = 0, \pm\sqrt{2}$

7 a $\frac{5x+2}{2x+1}$

b $\frac{1}{x-2}$

8 a $\frac{3}{4-x}$, not defined for $x = 4$

b 1, 3

9 a $0 \leq f(x) < 1$

b $\frac{3}{\sqrt{1-x^2}}$, range $f^{-1}(x) \geq 3$, domain $0 \leq x < 1$

10 a $g(x) \geq 3$

b $\frac{3}{2x^2+5}$

c $\frac{3+x}{2x}$

d $-1, \frac{3}{2}$

11 a $\frac{2}{2-x^2}$

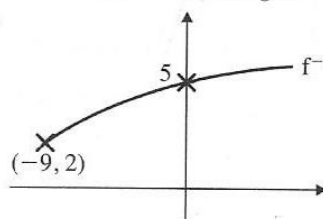
b $\frac{2-x}{x}$

c -2, 1

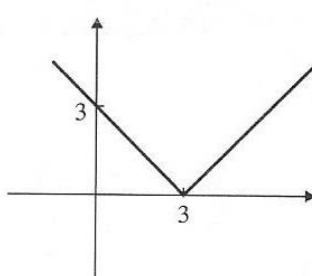
12 a $f(x) \geq -9$

b domain $x \geq -9$, range $f^{-1}(x) \geq 2$

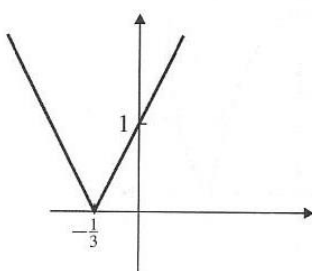
c



13 a



b



REVIEW EXERCISE 1I PAGE 24

1 a $\frac{5}{3-x}$

b $\frac{x^2}{2}$

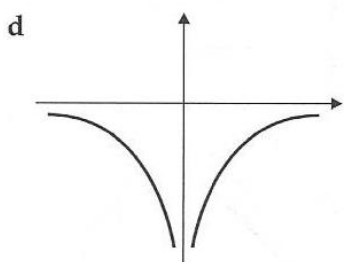
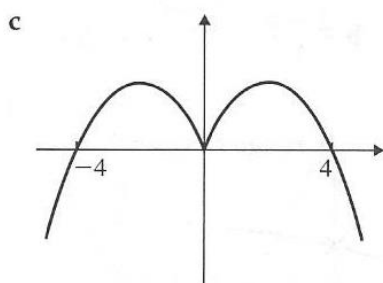
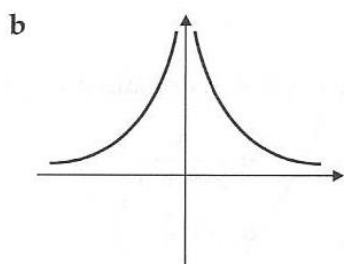
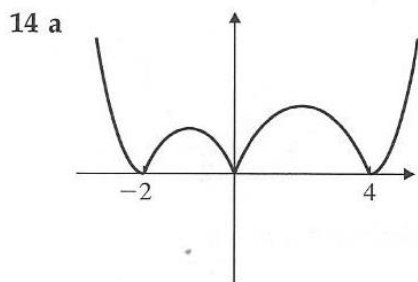
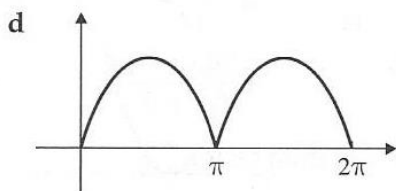
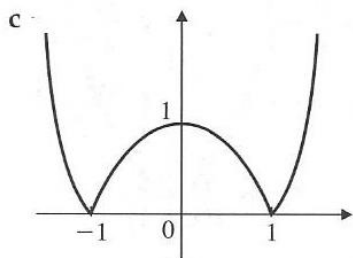
c $\frac{3x+2}{x}$

d $1+x$

e $\frac{1+4x}{y+y^2}$

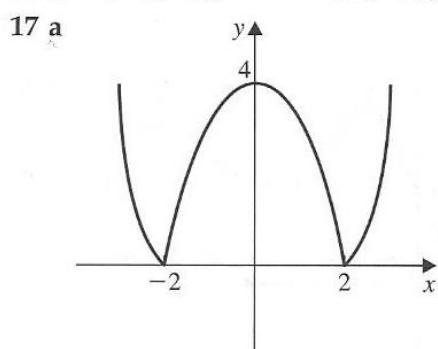
f $\frac{3x}{2}$

2 $4(x-1)(x+1), \frac{(x-4)}{4(x+1)}$

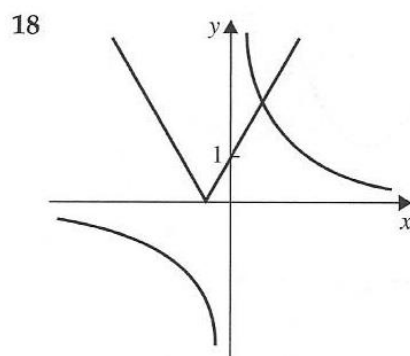


15 a $x > -\frac{1}{2}$ b $x < -2, x > 0$

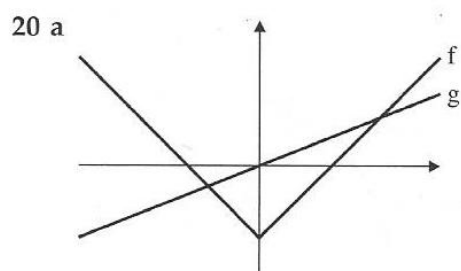
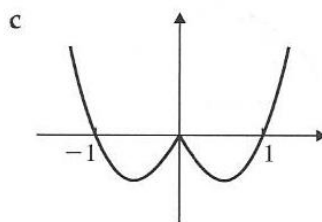
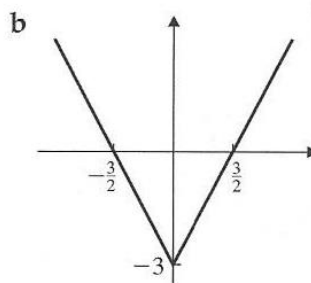
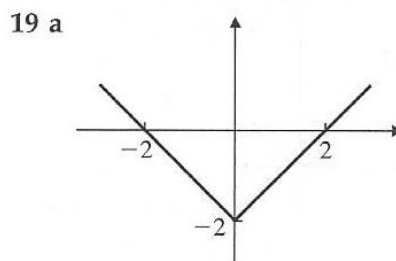
16 a $-4 < x < 6$ b $x < 1, x > 2$



b $x > 3, x < -3$

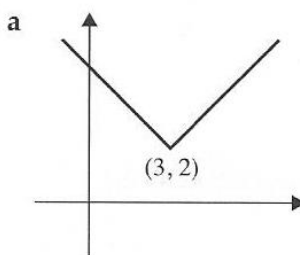


b one solution



b $x > 10, x < -\frac{10}{3}$

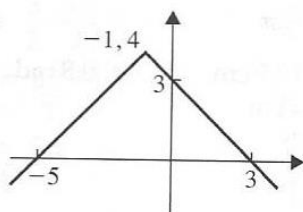
21 a range: $f(x) \geq 2$



b $x = 0, 6$

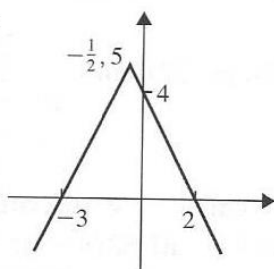
c 3

22 a



b $f(x) \leq 4$

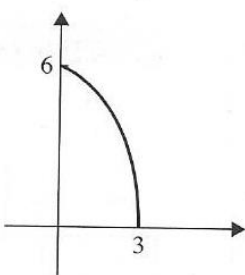
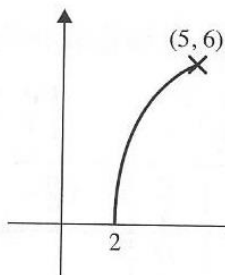
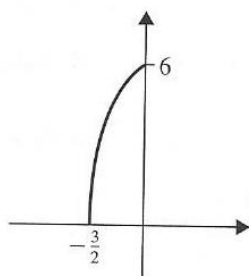
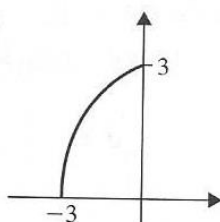
23 a



b 2 roots

c $g(x) \leq 5$

24 a



- 25 a i (2, -9) ii (3, -4) iii (3, 9)
iv (1, -9) v (-3, -9)

b $f(x) = (x - 3)^2 - 9$

- 26 a i (-1, 5) ii (2, -1) iii (-2, 5)
b $1 + 4x - x^2$

27 translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$;stretch scale factor 2 parallel to y -axis;

translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

28 stretch, scale factor $\frac{1}{2}$, parallel to x -axis;

translation $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

29 stretch, scale factor 2, parallel to x -axis;

translation $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

30 f: translation $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$, g: translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

EXAMINATION EXERCISE 1 PAGE 28

1 $\frac{1}{x+4}$

2 i $\frac{1}{2-x}$

ii $\frac{3}{(x-1)(x-4)}$

3 i $\frac{11-x}{(3-x)(1+x)}$

ii $\frac{-1}{1+x}$

4 i 26, 4

ii reflection in $y = x$

5 i -3

6 a $x = 6, x = -5$

b $a = 6$

7 a $f(x) < 5$

b i $\frac{1}{3} \ln(5-x)$

ii 4

c $\frac{2x-3}{11-6x}$

8 a $0 \leq f(x) \leq 10$

b 3

c $\frac{5x-4}{3+x}$

d 6, 0.4

9 a $f(x) \geq -4$

b $3 + \sqrt{x+4}$

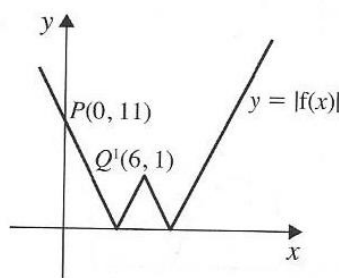
c i $|x^2 - 6x - 1|$

ii 5, 7

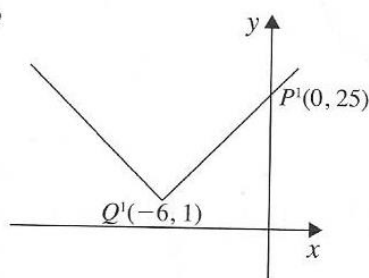
10 $x \leq -6, 1 \leq x \leq 2, x \geq 3$

11 $4a, -14a$

12 a



b



c $a = 2, b = 6$

13 i translation $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and stretch s.f. 2 in the y direction

ii $-6 < x < -2$

EXERCISE 2A PAGE 33

1 a $\frac{2}{3}\pi$

b $\frac{3}{2}\pi$

c $\frac{3}{4}\pi$

d $\frac{11}{6}\pi$

e $\frac{7}{4}\pi$

f 4π

2 a 0.698°

b 1.75°

c 0.873°

d 3.80°

e 1.52°

f 0.0175°

3 a 225°

b 270°

c 157.5°

d 115°

e 74.5°

f 28.6°

4 a $\frac{\sqrt{3}}{2}$

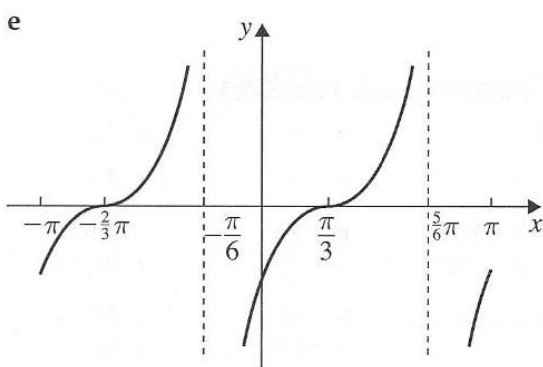
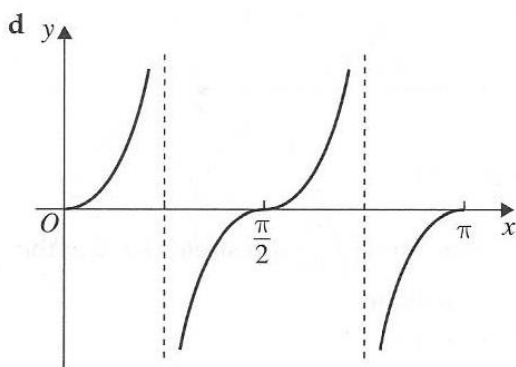
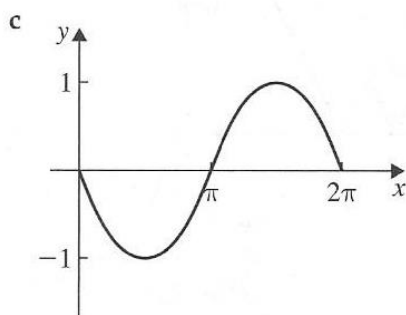
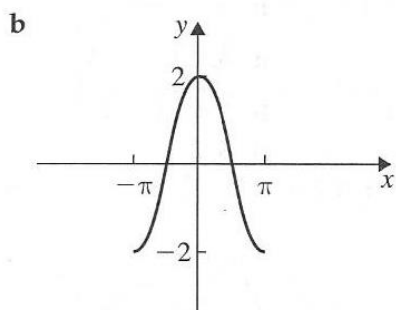
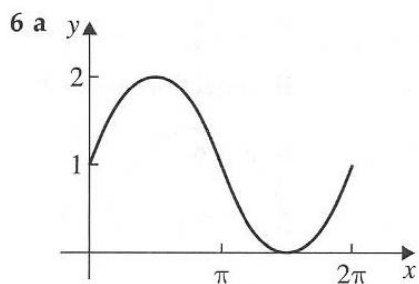
b 1

c $\frac{\sqrt{3}}{2}$

d -0.644

e -0.942

f 2.57



EXERCISE 2B PAGE 36

- 1 a 8 cm b 40 cm²
 2 a 4 cm b 60 cm² c 0.4 rad
 d 6 cm e 0.072 m²
 3 a $\frac{2}{9}\pi$ b 20.2 m c 1.04°

- 4 a $\frac{\pi}{3}$ b $\frac{5}{18}\pi$
 5 a 0.8 rad b 10.7 cm c 0.8 rad
 d 7.98 cm e 4 cm
 6 a 9 cm b $\frac{2}{3}c$
 7 a 20 cm b $\frac{3}{4}c$
 8 6.56 cm
 9 5π cm (= 15.7 cm)
 10 b $\frac{32\pi}{3}$ cm² d 18.5 cm², 37.0 cm²
 e 3.44 cm²
 11 $\frac{2}{3}$ rad
 12 a 34.3 cm² b 24.1 cm² c 10.2 cm²
 13 a i 50θ ii 50 sin θ iii 50(θ - sin θ)
 15 b 1.17°
 16 a $12r + 2\pi r + \frac{\pi}{3}r^2$ c 7.6 cm
 17 a i 8θ ii 8 sin θ iii 8θ - 8 sin θ
 b 1.1°
 18 b $\sqrt{72(1 - \cos \theta)} + 60$ d 1.6°
 19 $16\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

EXERCISE 2C PAGE 44

- 1 a 0 b 0 c 0 d 0
 e 0 f -1 g -1 h 1

2		0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
	sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
	cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
	tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

- 3 a $\frac{\sqrt{3}}{2}$ b $-\frac{1}{\sqrt{2}}$ c -1 d $\frac{1}{2}$
 4 a 0.589 b 0.264 c -0.416
 d 0.951 e 0.451 f 0.282
 5 $\frac{\pi}{6}, \frac{5\pi}{6}$ 6 $\frac{\pi}{3}, \frac{4\pi}{3}$
 7 a $\frac{\pi}{4}, \frac{7\pi}{4}$ b $\frac{\pi}{2}$ c $\frac{\pi}{6}, \frac{7\pi}{6}$
 8 a 0.412, 2.730 b 0.708, 2.434
 c 1.359, 4.924 d 1.107, 4.249
 9 a $-\frac{\pi}{6}, \frac{\pi}{6}$ b 5.508, 3.917
 c 1.039, -2.103 d $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$
 10 a 0.775 or 2.366
 11 6.06 cm
 12 a $\frac{\pi}{4}, \frac{5\pi}{4}$ b $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
 c 0.491, 2.062, 3.633, 5.204
 d 2.319
 13 a $\frac{7\pi}{12}, \frac{23\pi}{12}$ b $\frac{5\pi}{12}, \frac{11\pi}{12}$

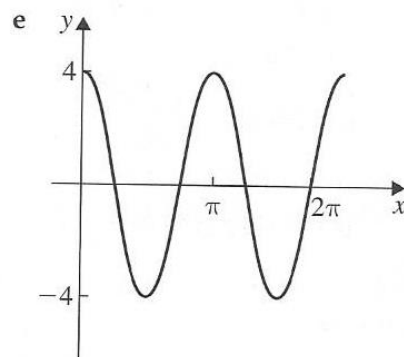
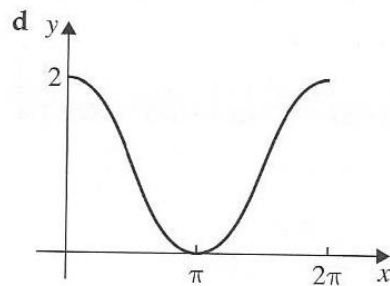
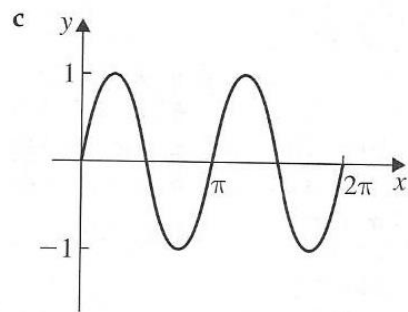
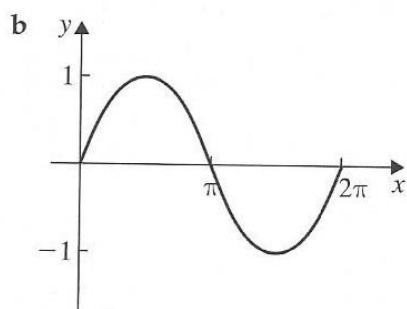
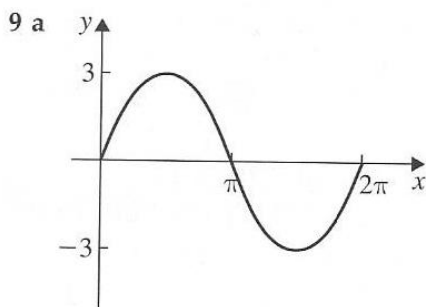
- c $\frac{\pi}{2}, \frac{3\pi}{2}$ d 1.620, 2.522
- 14 $\frac{\pi}{3}, \frac{2\pi}{3}$ 15 0.983
- 16 $\frac{\pi}{4}, \frac{3\pi}{4}$ 17 0, π , 1.107
- 18 0.308, 2.834 19 0, π , $\frac{\pi}{3}$
- 20 $\frac{\pi}{2}$ 21 1.107, $\frac{3\pi}{4}$ (2.356)
- 22 $\frac{\pi}{8}, \frac{5\pi}{8}$ 23 $\frac{\pi}{6}, \frac{5\pi}{6}$
- 24 $\frac{\pi}{2}$ 25 $\frac{\pi}{2}$

EXERCISE 2D PAGE 47

- 1 a 0.24740 b 0.25 c 1.05%
- 2 13.1%
- 3 0.294%
- 5 b 1
- 6 a 3 b 3 c 7 d 6
- 7 $x \approx 2, y \approx 9.8$
- 8 $a \approx 2.1, b \approx 7.7$

REVIEW EXERCISE 2E PAGE 48

- 1 a 12.5 b 6.68 c 14.6
- 2 1.74 3 0.652 4 2.41
- 5 0.683
- 6 a 6.55 cm^2 b 26.6 cm^2
- 7 $\frac{2\pi}{5}$
- 8 $\sin \frac{\pi}{4} = \cos \frac{\pi}{4}, \tan \left(-\frac{\pi}{6}\right) = \tan \frac{5\pi}{6},$
 $\cos \frac{\pi}{6} = \cos \frac{11\pi}{6} = -\sin \left(-\frac{\pi}{3}\right) = \sin \frac{2\pi}{3},$
 $\sin \frac{\pi}{6} = \sin \frac{5\pi}{6}, \tan \frac{\pi}{6} = \tan \left(-\frac{5\pi}{6}\right)$

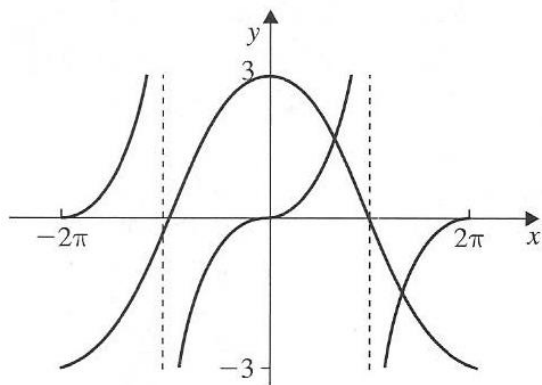


- 10 b i 2 ii 4
- 11 a 12 cm b 14.4 cm^2 c 12 cm
d 10° e 2.5° f $r = 2, \theta = 1.5$
g 4 cm
- 12 50, $25\pi - 50$
- 13 a 4.2 cm b 1.0 cm^2
- 14 a $20\sqrt{3}$ b 200π c $20\sqrt{3} \left(\frac{\pi}{3} + 2\right)$
- 15 a $r^2 \left(\frac{5}{4} - \cos \theta\right)$ b 0.634
c 0.68 cm^2
- 16 a 0.729 b 9.11 cm^2
c 11.9 cm^2 d 15.6 cm
- 17 a $\frac{\pi}{3}, \frac{5\pi}{3}$ b $-\frac{3\pi}{4}, \frac{\pi}{4}$
c $\frac{\pi}{4}, \frac{7\pi}{4}$ d $\frac{\pi}{3}, \frac{2\pi}{3}$
e 3.9, 5.6 f 2.4, 3.9
g -0.6, 2.5 h $-\frac{\pi}{6}, -\frac{5\pi}{6}$
- 18 a $\frac{11\pi}{6}, \frac{\pi}{6}$ b $\frac{\pi}{2}, \frac{7\pi}{6}$
c $\frac{5\pi}{12}, \frac{13\pi}{12}$ d $\frac{\pi}{2}, -\frac{\pi}{2}$
- 19 0.85, 2.3, $\frac{7\pi}{6}, \frac{11\pi}{6}$
- 20 a $\frac{\pi}{2}$ b $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
c $\frac{\pi}{6}, \frac{5\pi}{6}, -0.3, -2.8$ d $-\frac{2\pi}{3}, \frac{2\pi}{3}, -0.8, 0.8$
e $-\frac{\pi}{6}, -\frac{5\pi}{6}$

- 21 a $-\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$
 b $\frac{2\pi}{3}, \pi, \frac{5\pi}{3}$ c $\frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$
 22 a $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$ b $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
 c $0.8, 2.3, \frac{7\pi}{6}, \frac{11\pi}{6}$ d $1.25, 4.39$
 e $0, \pi, 2\pi, 0.46, 3.61$
 24 a -3 b 0

EXAMINATION EXERCISE 2 PAGE 51

- 1 i $\frac{3\pi}{10}$ ii 20.4
 2 b 40.9 c 96.7
 3 i 8π ii $48\pi - 36\sqrt{3}$
 4 6
 5 a 17.5 b 0.943 c 38.9
 6 ii 15.2 iii 9.76
 7 a 18.8 cm^2 b 2 c 2π
 8 b 19.9 cm^2 c 13.8 cm
 9 $-\frac{2\pi}{15}, \frac{8\pi}{15}$
 10 i 15 ii $(2x+1)(2x-3)(x+1)$
 iii $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$
 11 $-\pi, -0.84, 0, 0.84, 0$
 12 $\frac{\pi}{2}, 0.85, 2.29$
 13 a $-0.056, 1.8$ b i $\frac{1}{2}, -\frac{1}{3}$
 14 b $1.2, 4.4, 2.0, 5.2$
 15 $5 + 3 \cos \theta$, least value is 2, occurs when $\theta = \pi$
 16 i



ii 2.02, 4.26

EXERCISE 3A PAGE 59

- 1 a $A = 1, B = 2$ b $A = 3, B = 2$
 c $A = 2, B = 3$ d $A = 5, B = 2$
 e $A = 7, B = -2$ f $A = 4, B = -3$
 2 a $\frac{1}{x-3} + \frac{8}{x-2}$ b $\frac{1}{x-1} + \frac{2}{x-2}$
 c $\frac{3}{x+1} + \frac{5}{x+2}$ d $\frac{1}{x+2} + \frac{6}{x+3}$

- e $\frac{1}{x-1} + \frac{2}{x+2}$ f $\frac{2}{x-1} - \frac{1}{x+3}$
 g $-\frac{4}{x} + \frac{5}{x-2}$ h $\frac{2}{x} + \frac{3}{x+1}$
 3 a $A = 3, B = 1$ b $A = 4, B = -3$
 c $A = 1, B = 3$ d $A = 2, B = 1$
 e $A = -2, B = 5$ f $A = 7, B = -1$
 4 a $\frac{3}{2x-3} + \frac{4}{x-3}$ b $\frac{3}{2x+1} + \frac{1}{x-1}$
 c $\frac{5}{3x-2} + \frac{2}{3x+4}$ d $\frac{1}{2x-1} + \frac{1}{5x-1}$
 e $\frac{4}{x+3} + \frac{3}{2x-1}$ f $\frac{2}{3x+5} + \frac{3}{2x-3}$
 g $\frac{7}{2x-1} - \frac{1}{2x+1}$ h $\frac{2}{5x-2} + \frac{9}{5x+2}$

EXERCISE 3B PAGE 61

- 1 a $A = 1, B = 3, C = 2$
 b $A = 1, B = -1, C = 2$
 c $A = 2, B = 3, C = 6$
 d $A = 4, B = -1, C = 5$
 e $A = 3, B = 2, C = 1$
 2 a $\frac{1}{x-2} + \frac{3}{2x+1} + \frac{1}{x+2}$
 b $\frac{5}{x-3} + \frac{1}{x+1} + \frac{1}{2x+3}$
 c $\frac{2}{x-1} - \frac{4}{x} + \frac{1}{3x-4}$
 d $\frac{3}{5x-2} + \frac{1}{4x-1} + \frac{2}{3x-2}$
 e $\frac{5}{x-2} + \frac{5}{x-1} - \frac{3}{x+1}$
 f $\frac{1}{5x+1} + \frac{2}{4x+3} + \frac{3}{4x-1}$
 3 a $\frac{2}{x+3} + \frac{5}{x-2} + \frac{1}{(x-2)^2}$
 b $\frac{2}{x-1} - \frac{1}{x+2} - \frac{3}{(x+2)^2}$
 c $\frac{3}{2x-1} - \frac{7}{(3x+1)^2} - \frac{1}{(3x+1)}$
 d $\frac{2}{3x-2} + \frac{4}{2x-1} + \frac{1}{(2x-1)^2}$
 4 a $\frac{1}{x-2} + \frac{2}{(x-2)^2}$ b $\frac{1}{2x-1} - \frac{2}{(2x-1)^2}$
 c $\frac{5}{x+1} + \frac{4}{(x-1)^2}$ d $\frac{3}{x-1} - \frac{1}{(x+3)^2}$
 e $\frac{5}{(x+5)^2} + \frac{4}{3x-2}$ f $\frac{5}{x} + \frac{3}{x+1} - \frac{7}{x-1}$
 g $\frac{7}{2x-1} - \frac{3}{x} - \frac{1}{2x+1}$
 h $\frac{2}{x+4} + \frac{1}{(x-1)^2} - \frac{1}{(x-1)}$

EXERCISE 3C PAGE 65

- 1 a $1 + \frac{1}{x+3} + \frac{1}{x-2}$
 b $3 - \frac{1}{5x+1} + \frac{2}{2x-1}$
 c $x + \frac{1}{x-3} + \frac{1}{x-1}$
 d $2x + \frac{2}{x-2} + \frac{3}{x-1}$
 e $x + 1 + \frac{3}{x+3} + \frac{4}{x+1}$
 f $2x + 3 - \frac{2}{x-3} + \frac{5}{x+4}$
 g $3x + 2 + \frac{2}{x+1} - \frac{1}{2x-1}$
 h $2x - 1 + \frac{4}{2x-3} - \frac{3}{3x-5}$
- 2 a $3 + \frac{1}{x+1} + \frac{2}{(x-1)^2}$
 b $2 - \frac{3}{2x-1} + \frac{4}{3x-2} - \frac{5}{(3x-2)^2}$
 c $x + \frac{1}{x-2} + \frac{1}{x-1} + \frac{2}{(x-1)^2}$
 d $x + \frac{2}{x+1} + \frac{3}{x-2} + \frac{4}{(x-2)^2}$
- 3 $\frac{1}{1-2x} - \frac{1}{1-x} + \frac{1}{(1-x)^2}$

EXAMINATION EXERCISE 3 PAGE 66

- 1 $A = -\frac{5}{2}, B = \frac{9}{2}$
 2 $A = 3, B = -1$
 3 $A = 4, B = 3, C = 1$
 4 $A = -2, B = 3$
 5 $\frac{2}{x-2} - \frac{1}{x+1} - \frac{3}{(x-2)^2}$
 6 $\frac{4}{x+2} - \frac{3}{x-1} + \frac{2}{(x-1)^2}$
 7 $\frac{-50}{x} + \frac{25}{x^2} + \frac{100}{2x+1}$

EXERCISE 4A PAGE 70

- 1 a 4, 7, 10, 13
 c 1, 4, 9, 16
 e 2, 4, 8, 16
 g $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}$
- b 3, 7, 11, 15
 d $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
 f 2, 6, 12, 20
 h -1, 1, -1, 1
- 2 a $5n$
 d n^2
 g $\frac{n}{n+1}$
- b $2n+1$
 e 3^n
 h n^3
- c $4n-3$
 f 10^n
 i $\frac{1}{n+1}$
- 3 a 2, 7, 12, 17, 22
 c 1, 7, 1, 7, 1
 e $2, \frac{1}{2}, 2, \frac{1}{2}, 2$
 g 1000, 100, 10, 1, $\frac{1}{10}$
- b -1, 9, 19, 29, 39
 d 2, 4, 16, 256, 65 536
 f 0, -2, -8, -26, -80
 h $1, 2, 2\frac{1}{2}, \frac{29}{10}, \frac{941}{290}$

- 4 a $2n+3$
 c $7n-15$
- b $14-4n$
 d $12-3n$
- 5 a $u_{n+1} = u_n + 5, u_1 = 7$
 b $u_{n+1} = u_n + 3, u_1 = -2$
 c $u_{n+1} = u_n - 2, u_1 = 2$
 d $u_{n+1} = u_n + \frac{1}{2}, u_1 = 10\frac{1}{2}$
- 6 a $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
 c 0
 b convergent.
- 7 a $0, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}$ convergent.
 b 1
- 8 a $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$; convergent; limit 0
 b 5, 9, 13, 17; divergent.
 c 2, 5, 10, 17; divergent.
 d $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$; convergent, limit 1
 e -5, 5, -5, 5; periodic.
 f 1, 5, 1, 5; periodic.
 g $5, \frac{1}{5}, 5, \frac{1}{5}$; periodic.
 h 1, 100, $\frac{1}{100}$, 1 000 000; divergent.
 i 2, 12, 2, 12; periodic.
 j -2, 4, -8, 16; divergent.
- 9 1, 2, 3, 5, 8, 13
- 10 1, 2, 5, 11, 26
- 11 $3, 5.\dot{3}, 4.3125, 4.623\ 188\dots, 4.514\ 10\dots,$
 $4.550\ 694, 4.538\ 226\dots, 4.542\ 452\ 7,$
 a $u = 4.54$
- 12 $1, 3, 2\frac{2}{3}, 2.6875, 2.686\ 046\ 512, 2.686\ 147\ 186$
 a 2.69
- 13 a -2.79
- 14 a 3.30
 b $x^2 - 3x - 1 = 0$

EXERCISE 4B PAGE 73

- 1 a 24
 d 38
 g 21
 j -6
- b 14
 e 30
 h 28
 k 35
- c 10
 f $2\frac{1}{12}$
 i 63
 l $\frac{23}{12\frac{5}{5}}$
- 2 a $\sum_{r=1}^4 r^2$
 d $\sum_{r=1}^4 2r$
 g $\sum_{r=1}^{100} \frac{r}{r+1}$
- b $\sum_{r=1}^4 3^r$
 e $\sum_{r=1}^{100} r$
 h $\sum_{r=1}^6 (-1)^r$
- c $\sum_{r=1}^4 \frac{1}{r}$
 f $\sum_{r=1}^n r(r+1)$
 i $\sum_{r=1}^5 4r+1$
- j $\sum_{r=1}^5 5r-1$
 k $\sum_{r=1}^{\infty} r^3$
 l $\sum_{r=1}^{\infty} (-1)^{r+1} 2r$
- 3 10
 4 10 000
 5 $\frac{1}{1000}$

EXERCISE 4C PAGE 76

- 1 a Yes, 1
 d Yes, a
 2 a $4n-3$
 d $22-3n$
- b Yes, -3
 e Yes, 3
 b $2n+8$
 e $29-4n$
- c No
 f No
 c $9n-1$

- 3 a 106 b 107 c 4
 d 7 e $\frac{11}{4}$
- 4 a $9n - 6$ b 16th
 5 a $7n + 3$ b 23rd
 6 a $35 - 6n$ b 15th
- 7 a 245 b 1170 c 48 d $13\frac{1}{2}$
 8 a $4n - 3$ b 11th c 231
 9 a $7n - 4$ b 6th c 123
 10 a 1820 b 9 c 18 653
 11 a 5 b 1130
 12 a 11 b 182
 13 b $a = 13, d = 7$
 14 b $a = 17, d = -4$ c 42
 15 a $a = 42, d = -3$ c 29
 16 a $a = -18, d = 3$ c 14
 17 $d = 5, a = -3$
 18 15 500 19 £317.20
 20 a £1050 b £11 500
 21 a 14 days b 104 days c 290 days
 23 2600
 24 a 15 050 b 10 150 c 9900
 25 $n = 3$ 26 $x = 7$

EXERCISE 4D PAGE 80

- 1 b $a = 7, d = 5$
 2 b $a = 5, d = 2$
 3 $a = 15, d = 7$
 4 a $\frac{1}{2}(7n^2 + 3n)$ b 17
 5 a 22 b 8, 14 c 6
 6 a 2, 3 b 51
 7 a $\frac{n}{2}(3n + 7)$ b 12
 8 a $a = 11, d = 4$ b $r = 5$
 9 a 7350 m
 10 a 180 b 2940 c 7
 d Unlikely to be able to increase sales month on month indefinitely.
 11 a £67 500 b 4.3 km
 12 $a = 9, d = 2$
 13 1683 14 17
 15 $a = 2, b = -3, S_{30} = 1365$

EXERCISE 4E PAGE 85

- 1 a $3 \times 2^{n-1}$ b $2 \times 5^{n-1}$
 c $20 \times (\frac{1}{2})^{n-1}$ d $200 \times (\frac{1}{5})^{n-1}$
 e $\frac{1}{8} \times 2^{n-1} (= 2^{n-4})$ f $4 \times 3^{n-1}$
 g $5 \times (-3)^{n-1}$
 2 $a = 7, r = 3$
 3 $a = \frac{35}{32}, r = 2$
 4 $a = 5120, r = \frac{1}{2}$
 5 a ± 2 b ± 6
 6 $192, \frac{3}{4}$
 7 2
 8 2
 9 a $a = 1, d = \frac{1}{2}$ b $r = 2$

- 10 a 1023 b 2186
 c 195 312 d 615
 11 a 79.9 b 80.0
 c 607.5 d 1706.7
 e 66.7
 12 a $7r$ b 5
 13 b $-4, 3$
 14 a £2048 b £4095
 c No. The amounts are increasing too much by the end of the year.
 15 a 94.3 m b 637.5 m
 16 a 6th week b 15 weeks
 17 a £29 458 b 18th year
 c Unlikely to stay in the same role for 17 years, rate of inflation likely to change.
 18 £376 000 (3 sf). No, the market can fluctuate significantly over this time.
 19 c $r = 3, a = 7$
 20 b 15
 21 a $-177\ 147$ c 11
 22 5
 23 a $\frac{5}{2}(3^n - 1)$ b 1 328 600 c 16
 24 7
 25 4, -12
 26 a 88 572 b 12 582 900 c 850

EXERCISE 4F PAGE 89

- 1 a 10 b 6 c 20
 d 4 e 10 f $\frac{14}{5}$
 2 $\frac{2}{3}$ 3 6 4 21 5 $-\frac{2}{7}$
 6 a 40 b 3 c 5 d 9
 8 $r = \frac{1}{3}, a = 18$ or $r = \frac{2}{3}, a = 9$
 9 3 10 $\frac{x-1}{x+1}$ 11 $\frac{1}{10}$
 12 $\frac{3}{5}$ 13 18 m
 14 a $a = \frac{10}{3}, r = \frac{2}{3}$ b $10[1 - (\frac{2}{3})^n]$
 c 3 d 10
 15 a 192 b $3 \times 4^{n-1}$ c $a = 3, r = 4$
 16 $r = \frac{1}{2}, a = 10$
 17 $a = 2, r = \sqrt{2}, S_{10} = 62(\sqrt{2} + 1)$
 18 a $6n + 7$ b Arithmetic
 19 a $\frac{3}{2}(3^{20} - 1)$ b $\frac{4}{3}(4^{30} - 1)$ c $14(2^{10} - 1)$
 20 a 5 b $-2\frac{1}{3}$

REVIEW EXERCISE 4G PAGE 91

- 1 a 3, 5, 7, 9 b 1, 4, 7, 10
 c 1, 4, 9, 16 d 10, 100, 1000, 10 000
 e $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ f 3, 8, 15, 24
 g $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ h -1, 1, -1, 1
 2 a $4n$ b n^2 c $3n + 1$
 d $\frac{n}{n+2}$ e $\frac{1}{n}$

3 a 4, 7, 10, 13

c $3, \frac{1}{3}, 3, \frac{1}{3}$

e 10, 100, 10 000, 10^8

4 a 34

d 20

g -6

b 30

e 31

h $\frac{23}{12}$

b 50, 45, 40, 35

d 64, 32, 16, 8

f 1, 1, 1, 1

c 25

f 12

i $\frac{13}{12}$

5 a $\sum_{r=1}^4 3n$

b $\sum_{r=3}^6 r^2$

c $\sum_{r=1}^4 10^r$

d $\sum_{r=1}^{50} 2n$

e $\sum_{r=1}^{10} r^{-2}$

f $\sum_{r=3}^6 \frac{1}{r}$

6 a 39

b 151

c 13

d 21

7 a 820

b 8200

c 1 101 100

d $n(5n - 3)$

8 a = -3, d = 5

9 a = 3, d = 5

10 a = 121, d = -6

11 15 500

12 £923

13 30

14 2

15 4234

16 a = 6, d = 2

17 a 17

b 7

c -330

18 a -25

b 345

19 a $5 \times 2^{n-1}$

b $2 \times 3^{n-1}$

c $100 \times (\frac{1}{2})^{n-1}$

20 765

21 a = $2\frac{1}{2}$, r = 2

22 r = 4

23 4, -5

24 a $x + 1$

b $x(x + 1)^{99}$

25 10

26 48

27 34.9 cm

28 a $(x^2 + x - 2)$

b 192

EXAMINATION EXERCISE 4 PAGE 94

1 b -4

c 1953

2 b i 6

ii 90

3 a 11

b £557 500

4 b 3010

c 2009

5 a i $\frac{1}{2}, 4$

ii periodic/oscillating

b a = -2, d = $\frac{5}{2}$

6 b 180

c £1710

e n = 18, aged 27

7 i -2

ii 3072

iii -1 048 575

8 b 256

c 1.602

9 a 17.28

b 120

c 25.92

10 i k = 28

b $r = -\frac{1}{2}, |r| < 1, a = 6$

11 i a $\frac{8}{9}$

b 18

ii 28

12 a 0.75

b 256

c 1024

d 14

13 a $p = \frac{4}{5}, q = 4$

b 195

14 i 7.5

ii 64

iii $-\frac{2}{3}$

15 i 80, 75, 70

ii 650

iii 8

iv 320

16 i 91

ii 978

iii 38

17 a 2

b 198

19 a $5k + 3$

b $156k + 114 = 6(26k + 19)$

20 a $a_2 = 5 - 4k$

$a_3 = 5 - k(5 - 4k)$

b $17 - 9k + 4k^2$

c 500

21 i a 4

b 80

ii a $3k, 4k$

b 11

EXERCISE 5A PAGE 101

1 a $1 - 2x + 3x^2$

c $1 - 6x + 21x^2$

e $1 + \frac{1}{2}x - \frac{1}{8}x^2$

g $1 - \frac{1}{2}x + \frac{3}{8}x^2$

i $1 - \frac{2}{3}x - \frac{1}{9}x^2$

k $1 - \frac{1}{2}x - \frac{1}{8}x^2$

b $1 - 3x + 6x^2$

d $1 - 10x + 55x^2$

f $1 + \frac{3}{2}x + \frac{3}{8}x^2$

h $1 - \frac{3}{4}x + \frac{21}{32}x^2$

j $1 + \frac{2}{5}x + \frac{7}{25}x^2$

l $1 + \frac{1}{3}x + \frac{2}{9}x^2$

2 0.980

3 1.010

4 a $-\frac{1}{2} < x < \frac{1}{2}$

b $-1 < x < 1$

c $-\frac{1}{5} < x < \frac{1}{5}$

d $-\frac{1}{2} < x < \frac{1}{2}$

e $-\frac{1}{2} < x < \frac{1}{2}$

f $-\frac{1}{4} < x < \frac{1}{4}$

g $-\frac{1}{3} < x < \frac{1}{3}$

h $-\frac{1}{2} < x < \frac{1}{2}$

i $-3 < x < 3$

j $-2 < x < 2$

5 a $1 - 6x + 24x^2 - 80x^3$

b $1 + 2x + 3x^2 + 4x^3$

c $1 - 20x + 250x^2 - 2500x^3$

d $1 - 14x + 112x^2 - 672x^3$

e $1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3$

f $1 + 10x + 30x^2 + 20x^3$

g $1 + \frac{3}{2}x + \frac{27}{8}x^2 + \frac{135}{16}x^3$

h $1 - \frac{1}{2}x - \frac{3}{8}x^2 - \frac{7}{16}x^3$

i $1 - \frac{x}{6} + \frac{x^2}{72} - \frac{x^3}{432}$

j $1 + \frac{x}{6} + \frac{x^2}{18} + \frac{7x^3}{324}$

6 $-\frac{77}{128}x^4$

7 $\frac{4389}{256}x^3$

8 $-\frac{1}{125}x^3$

9 a $x^3 + 6x^2 + 12x + 8$

b $15\sqrt{3} + 26$

10 b $264\sqrt{2}$

11 $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3, 1.0392$

12 -20

13 $1 - \frac{1}{2x} - \frac{1}{8x^2} - \frac{1}{16x^3}, 9.94987$

14 a $1 - \frac{x}{3} - \frac{x^2}{9}$

b 3.332222

EXERCISE 5B PAGE 103

1 a $\frac{1}{2} - \frac{x}{4} + \frac{x^2}{8}, |x| < 2$

b $\frac{1}{64}\left(1 + \frac{3x}{4} + \frac{3}{8}x^2\right), |x| < 4$

c $\frac{1}{9}\left(1 - \frac{2}{3}x + \frac{x^2}{3}\right), |x| < 3$

d $\frac{1}{625}\left(1 - \frac{4x}{5} + \frac{2x^2}{5}\right), |x| < 5$

e $2\left(1 - \frac{1}{8}x - \frac{1}{128}x^2\right), |x| < 4$

f $\frac{1}{3}\left(1 + \frac{1}{18}x + \frac{1}{216}x^2\right), |x| < 9$

$$g \ 2\left(1 - \frac{1}{24}x - \frac{1}{576}x^2\right), |x| < 8$$

$$h \ \frac{1}{4}\left(1 - \frac{1}{12}x + \frac{5}{576}x^2\right), |x| < 8$$

$$i \ 5^5\left(1 + \frac{x}{10} + \frac{3x^2}{1000}\right), |x| < 25$$

$$2 \ a \ 2\left(1 + \frac{3x}{8} - \frac{9x^2}{128}\right), |x| < \frac{4}{3}$$

$$b \ \frac{1}{3}\left(1 + \frac{x}{9} + \frac{x^2}{54}\right), |x| < \frac{9}{2}$$

$$c \ 2\left(1 - \frac{5x}{24} - \frac{25x^2}{576}\right), |x| < \frac{8}{5}$$

$$d \ 125\left(1 - \frac{21x}{50} + \frac{147}{5000}x^2\right), |x| < \frac{25}{7}$$

$$3 \ a \ a = \frac{1}{2}, b = -\frac{3}{16} \quad b \ a = 2, b = \frac{1}{4}$$

$$c \ a = \frac{1}{\sqrt{2}}, b = \frac{7}{4\sqrt{2}} \quad d \ a = \frac{1}{3}, b = -\frac{2}{243}$$

$$4 \ 3 + \frac{2x}{3} - \frac{2x^2}{27} + \frac{4x^3}{243}, |x| < \frac{9}{4}$$

$$5 \ a \ 1 - x - \frac{x^2}{2}, |x| < \frac{1}{2}$$

$$b \ 0.990$$

$$6 \ a \ 1 - \frac{3x}{5} - \frac{18}{25}x^2 \quad b \ 1.961$$

EXERCISE 5C PAGE 104

$$1 \ a \ -\left(\frac{13}{3} + \frac{19}{9}x + \frac{28}{27}x^2\right), |x| < 2$$

$$b \ -\left(2 + \frac{3x}{2} + \frac{5x^2}{4}\right), |x| < 1$$

$$c \ \frac{11}{2} - \frac{17x}{4} + \frac{29x^2}{8}, |x| < 1$$

$$d \ \frac{5}{2} - \frac{11x}{12} + \frac{25x^2}{72}, |x| < 2$$

$$e \ -\frac{3}{2}x - \frac{3}{4}x^2, |x| < 1$$

$$f \ -\frac{7}{3} - \frac{17x}{9} - \frac{55x^2}{27}, |x| < 1$$

$$2 \ a \ 1 + \frac{1}{x-2} + \frac{1}{x-1}$$

$$b \ -\left(\frac{1}{2} + \frac{5x}{4} + \frac{9x^2}{8} + \frac{17x^3}{16}\right)$$

$$c \ |x| < 1$$

$$3 \ a \ 1 - x - 4x^2 - 6x^3 \quad b \ |x| < \frac{1}{4}$$

$$4 \ \lambda = 2, n = -3$$

$$5 \ \text{Both equal } -2$$

$$6 \ 1 + \frac{3x}{2} + \frac{7x^2}{8}, |x| < 1$$

$$7 \ a \ 1 + x + \frac{1}{2}x^2, |x| < 1$$

$$8 \ a \ a = \frac{3}{2}, b = \frac{1}{2} \quad b \ \frac{1970}{1393} \text{ or } \frac{1393}{985}$$

$$9 \ a \ q = 3$$

$$b \ -x^3$$

$$c \ -1 < x < \frac{1}{3}$$

$$10 \ a \ \frac{[1 - (-x)^n]}{1 + x} \quad b \ -1 < x < 1 \quad c \ \frac{1}{1 + x}$$

$$11 \ b \ 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$c \ 1 + 3x + 6x^2 + 10x^3 + \dots$$

$$d \ 1 + 4x + 10x^2 + \dots$$

$$13 \ a \ a_0 = 1, a_1 = -\frac{1}{2}, a_2 = \frac{3}{8}, a_3 = -\frac{5}{16}$$

REVIEW EXERCISE 5D PAGE 106

$$1 \ a \ |x| < \frac{2}{3} \quad b \ |x| < \frac{5}{3} \quad c \ |x| < \frac{2}{7}$$

$$d \ |x| < \frac{9}{2} \quad e \ |x| < \frac{3}{2} \quad f \ |x| < \frac{5}{3}$$

$$2 \ 2 - \frac{x}{4} - \frac{x^2}{32}, |x| < \frac{8}{3}$$

$$3 \ a \ a = \frac{1}{9}, b = \frac{4}{729} \quad b \ a = \frac{1}{8}, b = \frac{9}{64}$$

$$4 \ a \ \frac{1}{3(x+1)} + \frac{2}{3(1-2x)} \quad b \ 1 + x + 3x^2$$

$$c \ |x| < \frac{1}{2}$$

$$5 \ a \ \frac{1}{2x+1} + \frac{3}{x-1} \quad b \ -2 - 5x + x^2$$

$$c \ |x| < \frac{1}{2}$$

$$6 \ a \ \frac{4}{3x-1} + \frac{8}{x+2} \quad b \ -108\frac{1}{2}$$

$$c \ |x| < \frac{1}{3}$$

EXAMINATION EXERCISE 5 PAGE 107

$$1 \ a \ 1 + x - 2x^2$$

$$b \ i \ \frac{1}{4} - \frac{1}{16}x + \frac{5}{256}x^2 \quad ii \ 0.2313$$

$$2 \ a \ 2 + \frac{5}{4}x - \frac{25}{64}x^2 \quad b \ \frac{3}{2}\sqrt{2} \quad c \ \frac{181}{128}$$

$$3 \ a \ 2 - \frac{3}{4}x - \frac{9}{32}x^2 - \frac{45}{256}x^3$$

$$b \ x = 0.1, 19.2201$$

$$4 \ i \ 1 - 2x + 7x^2 - 22x^3$$

$$ii \ |x| < \frac{1}{4}$$

$$5 \ ii \ 0.136 \quad iii \ -\frac{1}{x^2} - \frac{3}{x^3} - \frac{6}{x^4}$$

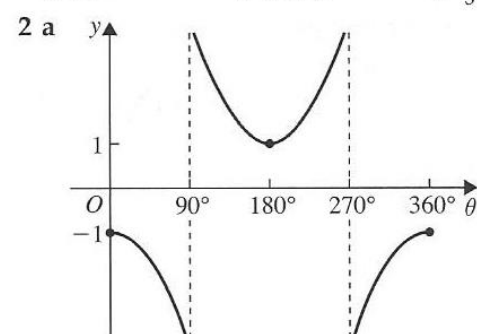
$$6 \ n = -1.5, k = 4$$

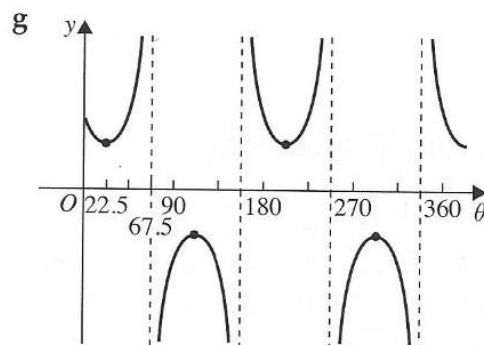
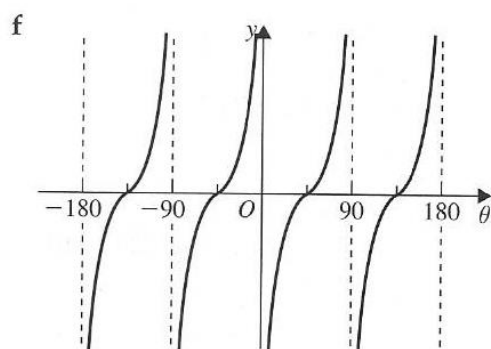
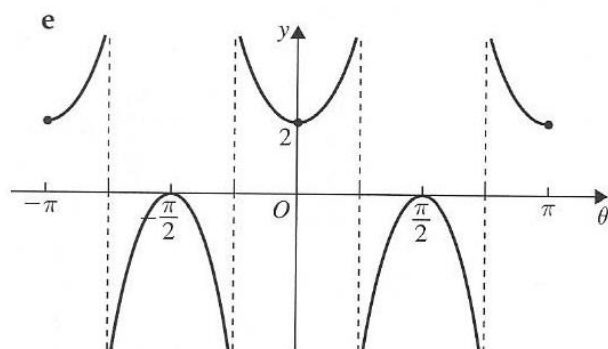
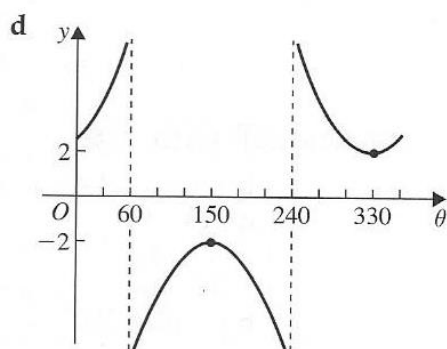
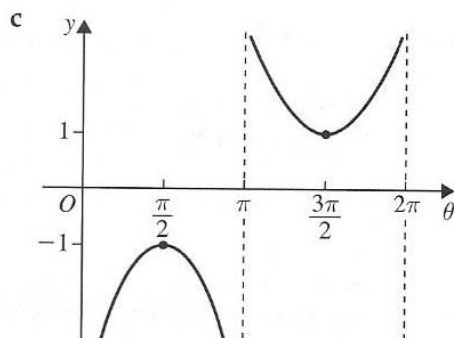
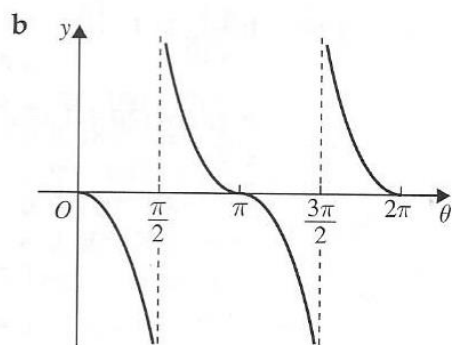
$$7 \ p = 2, q = -2, |x| < 2$$

EXERCISE 6A PAGE 114

$$1 \ a \ \sqrt{3} \quad b \ \frac{2}{\sqrt{3}} \quad c \ 1.015$$

$$d \ \sqrt{2} \quad e \ 0.325 \quad f \ \frac{4}{3}$$





- 3 a $\sec^2 x$ b $\sec^2 2x$ c $\sin^2 x$
 d $\sec x$ e $\sin \theta \cos \theta$ f $\operatorname{cosec}^2 \theta$
 5 a $60^\circ, 300^\circ$ b $30^\circ, 210^\circ$ c $45^\circ, 135^\circ$
 d $33.6^\circ, 326.4^\circ$ e $18.4^\circ, 198.4^\circ$ f 90°
 7 a $45^\circ, 225^\circ, 63.4^\circ, 243.4^\circ$
 b $-180^\circ, 0, 180^\circ, -135^\circ, 45^\circ$
 c $90^\circ, 30^\circ, 150^\circ$
 9 $30^\circ < \theta < 150^\circ$
 10 a $\cos \theta$ b $-60^\circ < \theta < 60^\circ$
 11 a $35.3^\circ, 144.7^\circ, 215.3^\circ, 324.7^\circ$
 b $30^\circ, 60^\circ, 120^\circ, 150^\circ$
 12 $(22.5, 7.5), (67.5, 142.5)$

EXERCISE 6B PAGE 117

- 1 a 30° b 90° c 45°
 d 120° e -30° f 60°
 2 a $-\frac{\pi}{4}$ b $\frac{\pi}{3}$ c $\frac{\pi}{4}$
 d 0 e $\frac{\pi}{3}$ f $-\frac{\pi}{2}$
 3/4 See text
 5 a $60^\circ, 120^\circ$ b $45^\circ, 225^\circ$ c 0, 360°
 6 a $\frac{1}{2}$ b 0 c θ d θ

EXERCISE 6C PAGE 120

- 1 b 1
 2 b 0
 3 a False b True
 4 a $\sin 3\theta$ b $\cos 2A$ c $\sin 3x$ d 1
 5 a $\frac{1+\sqrt{3}}{2\sqrt{2}}$ b $\frac{1+\sqrt{3}}{2\sqrt{2}}$ c $\frac{\sqrt{3}-1}{2\sqrt{2}}$
 d $\frac{\sqrt{3}+1}{2\sqrt{2}}$ e $\sqrt{3}-2$
 6 a $\frac{63}{65}$ b $\frac{56}{65}$ c $\frac{63}{16}$
 7 $73.2^\circ, 286.8^\circ$
 8 a $-\sin x$ b $-\cos x$ c $-\sin x$ d $-\cos x$
 9 $\frac{1}{3}$
 10 1
 11 $45^\circ, 225^\circ$
 12 a $1, \theta = 50^\circ$ b $1, \theta = 105^\circ$
 14 $\tan(x + 45^\circ)$
 15 a $\frac{2 \tan A}{1 - \tan^2 A}$ b $\frac{1}{2}$
 17 a $2 + \sqrt{3}$ b $\sqrt{6} + \sqrt{2}$ c $\sqrt{6} - \sqrt{2}$

- 23 $\frac{1}{3}$
 24 b 0.625, 2.195

EXERCISE 6D PAGE 124

- 1 a 1 b $\frac{\sqrt{3}}{2}$ c $\frac{1}{2}$ d 1
 2 a $\sin 20^\circ$ b $\cos 34^\circ$ c $\cos 70^\circ$ d $\tan 22^\circ$
 e $\frac{1}{2}\sin 2\theta$ f $\frac{1}{2}\tan 4\theta$ g $2\sin A$ h 1
 3 $\sin 2\theta = \frac{24}{25}$, $\cos 2\theta = \frac{7}{25}$
 4 $\sin 2\theta = \frac{120}{169}$, $\cos 2\theta = -\frac{119}{169}$
 5 a $\frac{4}{3}$ b $\frac{24}{7}$
 6 $\frac{\pm\sqrt{7}}{4}$
 7 $\pm\frac{4}{5}$
 8 $\frac{1}{3}$ or -3
 9 $0^\circ, 180^\circ, 360^\circ, 60^\circ, 300^\circ$
 10 $90^\circ, 270^\circ, 194.5^\circ, 345.5^\circ$
 11 $60^\circ, 300^\circ, 75.5^\circ, 284.5^\circ$
 12 $31.7^\circ, 121.7^\circ, 211.7^\circ, 301.7^\circ$
 13 $0^\circ, 180^\circ, 360^\circ, 30^\circ, 150^\circ$
 14 $60^\circ, 300^\circ, 109.5^\circ, 250.5^\circ$
 15 $0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ, 360^\circ$
 16 $0^\circ, 360^\circ, 240^\circ$
 17 $0^\circ, 120^\circ, 360^\circ$
 18 $14.5^\circ, 165.5^\circ$
 37 $2\sin\frac{x}{2}\cos\frac{x}{2}$
 38 $\frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}}$
 41 b $2 + \sqrt{3}$
 42 $18.4^\circ, 161.6^\circ, 198.4^\circ, 341.6^\circ$
 43 $A = 1, B = 4, C = 3$

EXERCISE 6E PAGE 128

- 1 a $5\cos(\theta - 36.9^\circ)$ b $13\cos(\theta - 67.4^\circ)$
 c $\sqrt{5}\cos(\theta - 63.4^\circ)$
 2 a $5\sin(\theta + 53.1^\circ)$ b $5\cos(\theta + 36.9^\circ)$
 c $\sqrt{5}\cos(\theta - 26.6^\circ)$ d $17\sin(\theta - 61.9^\circ)$
 e $10\sin(\theta + 36.9^\circ)$ f $\sqrt{2}\cos(\theta + 45^\circ)$
 3 a $2\sin(\theta + 60^\circ)$
 b Maximum value = 2 at $\theta = 30^\circ$
 4 a 5, 53.1° b 13, 157.4°
 c $\sqrt{2}$, 45° d 17, 298.1°
 5 a $5\cos(\theta - 36.9^\circ)$ b $103.3^\circ, 330.5^\circ$
 6 a $3\sin(\theta + 70.53^\circ)$ b $67.7^\circ, 331.3^\circ$
 7 a $1.9^\circ, 121.9^\circ$ b $80.0^\circ, 325.2^\circ$
 c $90^\circ, 330^\circ$ d $60.4^\circ, 193.3^\circ$
 e $257.6^\circ, 349.8^\circ$ f $40.8^\circ, 201.1^\circ$
 g $78.4^\circ, 244.8^\circ$
 h $39.0^\circ, 162.8^\circ, 219.0^\circ, 342.8^\circ$
 8 a 2.25, 0.15 b 0.38, 1.97
 9 a $\sqrt{7}\cos(x - 0.714)$ b 1.43
 10 a 0.36, 2.14 b 0, 4.07, 6.28
 c 0, 4.71, 6.28 d 1.70, 3.29

11 a $\sqrt{10}\sin(\theta - 71.6^\circ)$ b $\frac{4}{\sqrt{10}} > 1$

12 $-\sqrt{3} \leq k \leq \sqrt{3}$

13 a $c = 12$ b $13\cos(2\theta + 1.176)$

14 a $5\cos(\theta + 0.93)^\circ$ b 2

15 a $13\cos(\theta - 1.176)$ b 2

16 a $5\sin(2\theta + 36.9^\circ) - 3$

b $8.1^\circ, 45.0^\circ$

17 a $5\cos(x - 0.6435)$

b $\frac{1}{2}, \frac{1}{12}$

c 1.429, 3.000

18 a i 15.6°C ii 4.45

b 2126 and 1128

19 a i max: $h = 68.6\text{ m}$, $t = 1.30$

min: $h = 51.4\text{ m}$, $t = 0.30$

b 2 hours

c 41.5 minutes

20 a i $L = 2\cos\theta + 4\sin\theta$

ii $L = 2\sqrt{5}\sin(\theta + 0.464)$

b i $2\sqrt{5}$ ii 1.11

21 a $0 \leq \theta \leq 360$ and $0 \leq t \leq 20 \Rightarrow 0 \leq 18t \leq 360$

b 5.4 minutes

REVIEW EXERCISE 6F PAGE 131

- 1 a $41.8^\circ, 138.2^\circ$ b $138.6^\circ, 221.4^\circ$
 c $145.0^\circ, 325.0^\circ$ d $210^\circ, 330^\circ$
 e $\frac{\pi}{3}, \frac{5\pi}{3}$ f $\frac{\pi}{4}, -\frac{3\pi}{4}$
 g $60^\circ, 300^\circ$ h $0^\circ, 180^\circ, 360^\circ$
 2 a $0^\circ, 132^\circ, 228^\circ, 360^\circ$
 b $58^\circ, 238^\circ, 148^\circ, 328^\circ$
 3 $\frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}$
 4 a 45° b 0° c 60°
 5 a $\frac{\pi}{6}$ b $\frac{\pi}{6}$ c $\frac{\pi}{6}$
 7 a $18.4^\circ, 198.4^\circ, 26.6^\circ, 206.6^\circ$
 b $60^\circ, 300^\circ$
 c $90^\circ, 270^\circ, 199.5^\circ, 340.5^\circ$
 8 $-\frac{16}{65}$
 9 a $-\frac{3}{5}$
 b $\sin 2\theta = -\frac{24}{25}$, $\cos 2\theta = -\frac{7}{25}$
 11 a $60^\circ, 300^\circ, 180^\circ$
 b $0^\circ, 180^\circ, 360^\circ, 80.4^\circ, 279.6^\circ$
 c $25.2^\circ, 154.8^\circ$
 d $90^\circ, 270^\circ, 30^\circ, 150^\circ$
 e $0^\circ, 360^\circ, 120^\circ, 240^\circ$
 f $0^\circ, 180^\circ, 360^\circ, 30^\circ, 330^\circ, 150^\circ, 210^\circ$
 12 a $-\frac{7}{25}$ b $\frac{24}{25}$ c $-\frac{24}{7}$
 13 a i $\frac{\sqrt{6} - \sqrt{2}}{4} \left(= \frac{\sqrt{3} - 1}{2\sqrt{2}} \right)$
 ii $2 + \sqrt{3} \left(= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \right)$
 iii $\frac{\sqrt{2} - \sqrt{6}}{4} \left(= \frac{1 - \sqrt{3}}{2\sqrt{2}} \right)$

14 a $-\frac{119}{169}$ b $\frac{2}{13}\sqrt{13}$ c $-\frac{120}{119}$

15 $\frac{1}{3}$

16 a $30^\circ, 150^\circ, 270^\circ$

b $0, 180^\circ, 360^\circ, 30^\circ, 150^\circ, 210^\circ, 330^\circ$

c $90^\circ, 120^\circ, 240^\circ, 270^\circ$

d $0, 120^\circ, 240^\circ, 360^\circ$

e $0, 60^\circ, 300^\circ, 360^\circ$

17 $3 \sin \theta - 4 \sin^3 \theta$

18 a $\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)$ b $0, \frac{3\pi}{2}, 2\pi$

c $\sqrt{2}$

19 a $\sqrt{3} \sin(\theta + 54.74^\circ)$ b $90^\circ, 340.5^\circ$

20 a $2 \cos(\theta + 60^\circ)$ b $\frac{1}{4}$

21 a $5 \sin(\theta + 36.9^\circ)$ b $120^\circ, 347^\circ$

22 d = 0.838 m, 0823 and 2023

23 a $x = 0.3 \cos \theta, y = 0.9 \sin \theta$

b 6.5°

EXAMINATION EXERCISE 6 PAGE 134

1 $6.5^\circ, 53.5^\circ, 126.5^\circ, 173.5^\circ$

2 i $\frac{5}{3}$ ii $\sqrt{17}$

3 i $\tan \alpha = 2, \tan \beta = 5$

ii $-\frac{7}{9}$

4 a ii $-\frac{5}{13}$ b -21.3°

5 $37.9^\circ, 142^\circ$

6 i $\lambda = \frac{1}{2}$ ii $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$

7 b 2.820, 5.961

8 ii a $4 + 2\sqrt{2}$ b 0.659 c $32 \sin \theta$

9 $99.6^\circ, 260.4^\circ$

10 i a $1 + \frac{\sqrt{2}}{2}$ ii a $k = 2$

b $0^\circ, 30^\circ, 150^\circ, 180^\circ$

11 b ii $22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$

12 ii $70.5^\circ, -70.5^\circ$

iii $0 < k < \frac{3}{2}$

13 a $R = \sqrt{29}, \alpha = 1.19$

b i 5.09 ii 0.567, 3.34

14 a $R = \sqrt{5}, \alpha = 26.57^\circ$

b $33.0^\circ, 273.9^\circ$ c 86.1°

15 a $R = 6.5, \alpha = 0.395$

b $(0, 6), (1.97, 0), (5.11, 0)$

c $H_{\max} = 18.5, H_{\min} = 5.5$

d 11, 48

16 i $R = 5, \alpha = 53.1$

ii a $-64.7^\circ, 138^\circ$

b $k = 8, c = 3$

17 a i $R = 5, \alpha = 53.1$

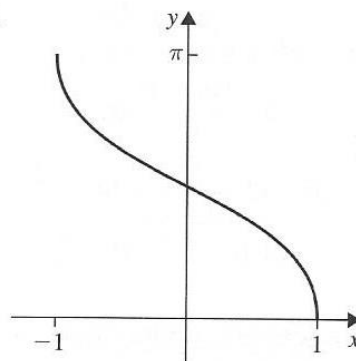
ii $18.4^\circ, 198.4^\circ$

b ii $35.3^\circ, 144.7^\circ$

c iii $\cos 72^\circ = \frac{\sqrt{5} - 1}{4}$

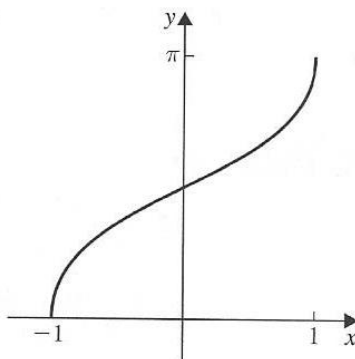
18 i 0.5 ii $\frac{\sqrt{2}}{2}$

19 a



$(-1, \pi), (1, 0)$

b



$(-1, 0), (1, \pi)$

EXERCISE 7A PAGE 140

1 a $6(2x + 5)^2$

b $8x(x^2 + 7)^3$

2 a $12(3x - 4)^3$

b $40(8x + 11)^4$

c $4x(x^2 - 3)$

d $27x^2(3x^3 + 1)^2$

e $-12(1 - 3x)^3$

f $-10(3 - x)^9$

3 a $-3(1 + 3x)^{-2}$

b $-4(2x + 1)^{-3}$

c $-15(5x + 2)^{-4}$

d $-24(4x - 1)^{-3}$

e $-10x(x^2 + 1)^{-2}$

f $40(1 - x)^{-5}$

4 b $\frac{4}{7}\sqrt{7}$

5 a $\frac{5x}{\sqrt{5x^2 + 3}}$

b $(3x + 1)^{-\frac{2}{3}}$

c $-x(x^2 - 3)^{-\frac{3}{2}}$

d $-\frac{3x^2}{(x^3 + 1)^2}$

e $-4(8x + 7)^{-\frac{3}{2}}$

f $-\frac{1}{2\sqrt{x}(\sqrt{x} + 2)^2}$

6 y = 64x - 48

7 x + 18y - 1157 = 0

8 (0, 8)

9 48

10 $2y = x + 3$

11 a $3(2\sqrt{x} - 3x)^2\left(\frac{1}{\sqrt{x}} - 3\right)$ b $\frac{1}{2x^2}\left(1 - \frac{1}{x}\right)^{-\frac{1}{2}}$

c $-3\sqrt{x}(x^{\frac{3}{2}} + 2)^{-2}$

d $-\frac{1}{\sqrt{x}}(1 + \sqrt{x})^{-3}$

e $\frac{1}{2}\left(x^2 - \frac{1}{x}\right)^{-\frac{1}{2}}\left(2x + \frac{1}{x^2}\right)$

f $20x + (2x - 3)^{-\frac{1}{2}}$

12 a (1, 7) minimum and (-3, -9) maximum

13 a (1, 2)

EXERCISE 7B PAGE 143

1 $(2x + 1)(6x + 1)$

2 $2x(3x - 1)(6x - 1)$

3 $x^2(x - 1)(5x - 3)$

4 $2(x + 2)(x + 3)(2x + 5)$

- 5 $2(1-x)^2(1-4x)$ 6 $2(x-1)(10x^3-6x^2+1)$
 7 $5(1+2x)^3(10x+1)$ 8 $5(3x+1)^3(3x-7)$
 9 $\frac{\sqrt{x}}{2}(5x+6)$ 10 $\frac{(4x+1)}{2\sqrt{x}}(20x+1)$
 11 $\frac{x}{\sqrt{2x+1}}(5x+2)$ 12 $\frac{x^2}{\sqrt{4x-1}}(14x-3)$
 13 $y = 19x - 1$ 14 16 15 $2, \frac{2}{5}$
 16 a $-3, -\frac{1}{3}$
 b maximum at $x = -3$, minimum at $x = -\frac{1}{3}$
 17 a i $(-2, 0), (4, 0), (0, 32)$
 ii $(0, 32)$ maximum; $(4, 0)$ minimum
 18 a $(2, -4)$ b $(-1, -4)$
 19 i $k = 3$ ii $\frac{16}{3}$

EXERCISE 7C PAGE 147

- 1 $\frac{2}{(x+1)^2}$ 2 $-\frac{15}{(2x-1)^2}$
 3 $\frac{3x^2+2x}{(3x+1)^2}$ 4 $\frac{4-10x-4x^2}{(x^2+1)^2}$
 5 $\frac{-2(x+1)}{x^3}$ 6 $\frac{-4x-1}{(x+1)^4}$
 7 $\frac{-6(3x+2)(2x+3)}{(4x+1)^4}$ 8 $\frac{-x^3-9x^2}{(x-3)^5}$
 9 $\frac{(3x-4)^2(6x+25)}{(2x+1)^3}$ 10 $\frac{-x-2}{2x^2\sqrt{x+1}}$
 11 $\frac{2x+3}{(4x+3)^{\frac{3}{2}}}$ 12 $\frac{3x-4}{(2x-1)^{\frac{3}{2}}}$
 13 $16y = x + 1$ 14 $y = -3x - 1$
 15 $y = 4 - x$ 16 $\frac{3}{2}$
 17 $(3, -\frac{1}{6})$ minimum, $(-1, \frac{1}{2})$ maximum

EXERCISE 7D PAGE 151

- 1 a e^x b $3e^{3x}$ c $2e^x$
 d $2xe^{x^2}$ e $2e^{2x+1}$ f $-e^{-x}$
 g $20e^{4x}$ h $e^x + 2e^{2x}$ i $2x + e^{-x}$
 j $-2e^{-2x}$ k $-4e^{-x}$ l $3x^2e^{x^3} + 3x^2$
 m $(4\ln 5)5^{4x}$ n $(300\ln 2)2^{10x}$ o $(-14\ln 3)3^{-7x}$
 2 a $xe^x + e^x$ b $x^2e^x + 2xe^x$
 c $2e^{2x}(2x+1)$ d $e^x(x^2+2x+1)$
 e $4x^2e^{2x}(2x+3)$ f $3x^2e^{3x} + 2xe^{3x} - 3x^2$
 g $\frac{e^{3x}}{x^2}(3x-1)$ h $\frac{xe^x - 2e^x - 2}{x^3}$
 i $\frac{2x-x^2}{e^x}$ j $e^x(1+x)^2(4+x)$
 k $e^{-x}(3x^2-x^3-1)$ l $\frac{2e^{2x}-e^{3x}}{(1-e^x)^2}$
 3 a $3e^2$ b 3
 c 1 d $-\frac{4e^2}{(e^2-1)^2}$

- 4 $y = e^2(x-1)$
 5 $y = x$
 6 b $y + e^2x = \frac{1}{e^2}$ c $(\frac{1}{e^4}, 0)$
 7 $(-1, -\frac{1}{e})$ minimum
 8 $(1, e)$ minimum
 9 a $(0, 2)$
 10 a $(0, 0)$ minimum, $(-2, 4e^{-2})$ maximum
 16 $(\frac{1}{3}\ln 2, 8)$ minimum

EXERCISE 7E PAGE 155

- 1 a $\frac{1}{x}$ b $\frac{3}{x}$ c $\frac{6}{x}$
 d $\frac{3}{3x-1}$ e $\frac{-2}{1-2x}$ f $\frac{3x^2+1}{x^3+x}$
 g $\frac{1}{x+1}$ h $\frac{2}{x}$ i $\frac{3}{x+2}$
 j $-\frac{1}{x}$ k $\frac{1}{2x}$ l $\frac{2x+1}{x^2+x-2}$
 2 a $\ln(x+4) - \ln(x-2)$ b $\frac{1}{x+4} - \frac{1}{x-2}$
 3 a $\frac{1}{x} - \frac{1}{x+1}$ b $\frac{2}{2x+3} - \frac{4}{4x-1}$
 4 a $1 + \ln x$ b $x + 2x\ln x$
 c $\frac{x}{1+x} + \ln(1+x)$ d $\frac{1-\ln x}{x^2}$
 e $\frac{(x+1)-2x\ln x}{x(x+1)^3}$ f $\frac{\ln x - 1}{(\ln x)^2}$
 g $2x + \frac{1}{x}$ h $\frac{3}{x} - \frac{1}{x^2}$
 i $\frac{1}{2x} + \frac{1}{2\sqrt{x}}$ j $-\frac{\ln x}{x^2}$
 k $\frac{2x^3}{1+x^2} + 2x\ln(1+x^2)$ l $\frac{1}{x+1} - \frac{1}{x+2}$
 5 $x + 2x\ln x, 3 + 2\ln x$
 7 $\frac{e^x}{1+e^x}$
 8 b $3y = x + 3\ln 3 - 2$
 9 $y = -x + \ln 2 + 1$
 10 a $y = x - 2$
 b 2 square units
 11 A $(1, 0)$ B $(e, \frac{1}{e})$
 12 $(\frac{1}{e}, -\frac{1}{e})$
 13 $\frac{1}{2e}$
 14 a $(3.67, -4.57)$
 15 a $\frac{f'(x)}{f(x)}$ b $\ln(x^3+1)$
 16 $y = e(x-1)$
 17 $[1, \ln(e+1)]$

EXERCISE 7F PAGE 159

- 1 a $\cos x$ b $-3 \sin 3x$
 c $4 \sec^2 x$ d $6 \cos 6x$
 e $-\frac{3}{2} \sin \frac{3}{2}x$ f $10 \cos 2x$
 g $\frac{1}{2} \sec^2 \frac{1}{2}x$ h $-10 \sin x + \cos x$
 i $\sec^2 x + \sin x$ j $\cos(x+1)$
 k $2 \sec^2(2x-1)$ l $\frac{1}{2} \sin \frac{1}{2}x$
- 2 a $2 \sin x \cos x$ b $-6 \cos x \sin x$
 c $3 \sin^2 x \cos x$ d $-6 \cos^2 x \sin x$
 e $-\frac{1}{2}(\cos x)^{-\frac{1}{2}} \sin x$ f $-8 \cos 4x \sin 4x$
 g $18 \tan 9x \sec^2 9x$ h $1 + \frac{\cos x}{2\sqrt{\sin x}}$
 i $3(\sin 2x)^{\frac{1}{2}} \cos 2x$
- 3 a $\sin x + x \cos x$ b $\cos 2x - 2x \sin 2x$
 c $x^2 \cos x + 2x \sin x$ d $\cos^2 x - \sin^2 x$
 e $-\left(\frac{x \sin x + \cos x}{x^2}\right)$ f $\frac{2x \cos 2x - 2 \sin 2x}{x^3}$
 g $\frac{\sin x - x \cos x}{\sin^2 x}$ h $\frac{3x^2 \cos x + x^3 \sin x}{\cos^2 x}$
 i $-\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$
- 6 a -6 b 4 c -2 d $\frac{\pi}{2} + 1$

9 $y = 2x + 1 - \frac{\pi}{2}$

- 10 a $\pi \cos \pi x$ b $2\pi \sec^2 2\pi x$
 c $-\sin(x-\pi)$ d $-\frac{\pi}{2} \sin \frac{\pi}{2}x$
 e $2x \cos x^2$ f $3x^2 \sec^2 x^3$
 g $2 \cos(2x-\pi)$ h $\frac{1}{\pi} \cos \frac{x}{\pi}$
 i $4x \sin(x^2) \cos(x^2)$
- 11 $0, \sqrt{\frac{\pi}{2}}$ 13 $2y + x = \frac{3\pi}{2}$
- 15 $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ 16 $-3\sqrt{3}$
- 18 b -4 19 $\frac{\pi\sqrt{3} + 3}{6}$
- 20 -4
- 21 $\left(\frac{\pi}{4}, \sqrt{2}\right)$ maximum $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$ minimum
- 22 a $(\cos x)^3 - 2(\sin x)^2 \cos x$
 b $0.62 \quad 1.57 \quad 2.53$
- 23 a $2y + \sqrt{3} = -x + \frac{4\pi}{3}$
 b $\left(0, \frac{4\pi - 3\sqrt{3}}{6}\right)$
- 24 a $-\pi$ b $y\pi = x - 1$

EXERCISE 7G PAGE 162

- 1 a $-\operatorname{cosec} x \cot x$ b $-\operatorname{cosec}^2 x$
 c $2 \sec 2x \tan 2x$ d $-3 \operatorname{cosec}^2 3x$
 e $-4 \operatorname{cosec} 4x \cot 4x$ f $-2 \operatorname{cosec}^2 x$
 g $2 \sec(2x+1) \tan(2x+1)$
 h $2x + \cos x + \sec x \tan x$
- 2 a $-2 \cot x \operatorname{cosec}^2 x$ b $2 \sec^2 x \tan x$
 c $\sec x \tan^2 x + \sec^3 x$ d $6 \sec^2 3x \tan 3x$

- e $\sec x(x \tan x + 1)$
 f $2x \cot 2x - 2x^2 \operatorname{cosec}^2 2x$
 g $\frac{\sec x(x \tan x - 1)}{x^2}$
 h $\frac{2x \cot x + x^2 \operatorname{cosec}^2 x}{\cot^2 x}$
 i $2 \sec x \tan x(1 + \sec x)$
 6 $-2 \cot x - \operatorname{cosec} x + c$

EXERCISE 7H PAGE 163

- 1 a $\frac{2}{\sqrt{1-(2x)^2}}$ b $\frac{-1}{\sqrt{1-(x+1)^2}}$
 c $\frac{1}{3\left(1+\left(\frac{x}{3}\right)^2\right)}$ d $\frac{1}{2\sqrt{1-\left(\frac{x+1}{2}\right)^2}}$
 e $\frac{2x}{1+x^4}$ f $\frac{-1}{x\sqrt{x^2-1}}$
 g $\frac{-1}{1+x^2}$ h $\frac{1}{\sqrt{1-(3-x)^2}}$
 i $3x^2 \arcsin(x) + \frac{x^3}{\sqrt{1-x^2}}$ j $\frac{-e^{\arccos(x)}}{\sqrt{1-x^2}}$

EXERCISE 7I PAGE 165

- 1 a $\frac{1}{x}$ b $\frac{1}{x}$ c $\frac{2}{x}$
 d $1 + \frac{1}{x}$ e $5e^x + \frac{1}{x}$ f $2e^x + \frac{5}{x}$
- 2 a $\left(\frac{1}{3}, \ln \frac{1}{3}\right)$
- 3 a $(1, \ln 4 - 3)$
- 4 b $4x + 12y = 5$
- 5 $y = 6x - 2$
- 6 a $5e^x$ b $7e^x$ c $20e^{4x}$
 d $\frac{1}{x+1}$ e $\frac{-4}{e^x}$ f $6e^{3x}(e^{3x} + 1)$
 g $2e^x + \frac{1}{x}$ h $5e^x$ i $\frac{-2e^x}{(e^x - 1)^2}$
- 7 e^x
- 8 a $\frac{2}{x}$ b $\frac{3}{x}$ c $\frac{1}{2x}$
 d $-\frac{1}{x}$ e $\frac{1}{2x}$ f $-\frac{1}{3x}$
- 9 a $2(x+1)$ b $14(2x+1)^6$
 c $24(3x-5)^7$ d $45(3x-7)^2$
 e $-12(4x+3)^{-4}$ f $-20(5x-2)^{-5}$
 g $2(4x+11)^{-\frac{1}{2}}$ h $5(15x-17)^{-\frac{2}{3}}$
 i $na(ax+b)^{n-1}$
- 11 $(1, 1)$
- 12 $(0, -1)$
- 13 a $1 + \ln x$ b $x^2 e^x + 2xe^x$
 c $\cos^2 x - \sin^2 x$ d $e^x \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$
 e $e^x(\sec^2 x + \tan x)$ f $\cot x - x \operatorname{cosec}^2 x$
 g $e^x \operatorname{cosec} x(1 - \cot x)$ h $x^2 \sec x(x \tan x + 3)$
 i $x^2(3 \cos x - x \sin x)$

15 a $\frac{1 - \ln x}{x^2}$ b $\frac{e^x}{x^3}(x - 2)$
 c $\frac{\sec x}{e^x}(\tan x - 1)$ d $\sec^2 x$
 e $-\operatorname{cosec}^2 x$ f $\frac{x}{e^x}(2 - x)$
 g $\frac{x - 1}{2x\sqrt{x}}$ h $\frac{x^2(3 \cos x + x \sin x)}{\cos^2 x}$
 i $-\frac{(x \operatorname{cosec}^2 x + 2 \cot x)}{x^3}$

17 $a = 1, b = -1, c = -1$

18 a $2 \cos 2x$ b $5e^{5x}$
 c $\cos x e^{\sin x}$ d $-2x \sin(x^2)$
 e $\sec x \tan x e^{\sec x}$
 f $-3 \operatorname{cosec}\left(3x + \frac{\pi}{3}\right) \cot\left(3x + \frac{\pi}{3}\right)$
 g $-3 \operatorname{cosec}^2 3x$ h $\frac{3x^2}{x^3 + 3}$
 i $2 \cot 2x$ j $-\tan x$
 k $3x^2 e^{x^3}$ l $\tan x$
 m $6x(x^2 + 1)^2$ n $3 \sin^2 x \cos x$
 o $6 \tan^2(2x) \sec^2 2x$

19 $-\frac{1}{\sqrt{1 - x^2}}$

20 $\frac{1}{2\sqrt{1 - x^2}}$

21 a $\frac{1}{1 + x^2}$

b $\frac{1}{3(1 + x^2)}$

c $\frac{1}{2x}$

d $-\frac{1}{x}$

e $\frac{e^x}{5}$

f $\frac{e^{\frac{x}{2}}}{2}$

g $\frac{1}{2\sqrt{1 - x^2}}$

22 a $\frac{1}{2\sqrt{x}}$

b $\frac{1}{2\sqrt{x}}$

23 a $nf(x)^{n-1}f'(x)$

b $f'(x)e^{f(x)}$

c $f'(x) \cos[f(x)]$

24 a $x = a^p, a = e^q, x = e^r$

c $\log_a x = \frac{\log_e x}{\log_e a}$

REVIEW EXERCISE 7] PAGE 168

1 a $12(3x - 1)^3$ b $45(3x - 4)^2$

c $-8(4x + 3)^{-3}$ d $-15(5x - 1)^{-4}$

e $12(2 - 3x)^{-5}$ f $\frac{3}{2}(3x + 1)^{-\frac{1}{2}}$

2 a $8x(x^2 + 3)^3$ b $-15x^2(2 - x^3)^4$

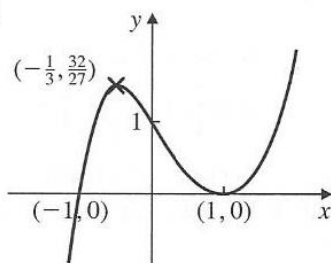
c $30x(3x^2 + 1)^4$ d $4(2x + 1)(x^2 + x)^3$

e $-\frac{3}{x^2}\left(1 + \frac{1}{x}\right)^2$ f $-3\left(x^2 + \frac{1}{x}\right)^{-4}\left(2x - \frac{1}{x^2}\right)$

3 a $(x + 1)^2(4x + 1)$ b $5(2x - 3)^3(2x + 1)$

c $(x + 1)^2(x - 1)(5x - 1)$ d $4x(3x - 2)(3x - 1)$

4



5 a $(-3, 0)$ maximum, $(\frac{1}{3}, -\frac{500}{27})$ minimum

6 a $\frac{x^2 + 2x}{(x + 1)^2}$ b $\frac{-(x + 2)}{2x^2\sqrt{x + 1}}$

c $-\frac{1}{(x + 1)^2}$ d $\frac{e^{3x}}{x^3}(3x - 2)$

e $\frac{2x - 6}{x^3}$ f $\frac{x \cos x - \sin x}{x^2}$

8 a e^x b $-3e^x$

c $\frac{1}{2}e^x$ d $6x^2 - 4e^x$

e $3e^x - \frac{3}{2}x^{-\frac{1}{2}}$ f $\frac{1}{6}x^{-\frac{2}{3}} - \frac{1}{4}e^x$

9 a $\frac{1}{x}$ b $\frac{4}{x}$ c $\frac{1}{x}$ d $\frac{2}{x}$

e $\frac{1}{2x}$ f $\frac{1}{x}$ g $-\frac{1}{x}$

10 a $e^2 - 1$ b 1 c 0 d 9

11 a $y = x + 1$ b $y = x - 1$

c $y = e^2 + 1 - e^{2x}$

12 a $ey = 1 + e^2 - x$ b $y = \frac{9}{2} + \ln 9 - \frac{3}{2}x$

13 a $(0, 1)$ minimum

b $(0, 2)$ minimum

c $(1, -1)$ maximum

d $(\frac{1}{2}, \frac{1}{2} + \ln 2)$ minimum

e $(\frac{1}{8}, \ln \frac{1}{8})$ maximum

14 A $(1, 0)$ B $(e, \frac{1}{e})$

16 a $\cos x$ b $-2 \sin x$

c $\cos x - \sin x$ d $2 \cos x + 3 \sin x$

e $\sec^2 x + 2x$ f $3 \cos x - \sec^2 x$

17 a $\frac{1}{2}$ b -1 c 0 d $\frac{1}{2}$ e 1

18 $y = 2x - \frac{\pi}{2}$ 19 $y = x + 2 - \frac{\pi}{2}$

20 a $3 \cos 3x$ b $-4 \sin 4x$

c $5 \sec^2 5x$ d $2 \sin x \cos x$

e $4 \tan^3 x \sec^2 x$ f $2 \cos\left(2x + \frac{\pi}{4}\right)$

g $\frac{1}{2} \frac{\cos x}{\sqrt{\sin x}}$ h $6 \sin 3x \cos 3x$

i $-6 \cos^2 2x \sin 2x$

21 a $2e^{2x}$ b $-3e^{-3x}$

c $2x e^{x^2}$ d $-e^{-x}$

e $\cos x e^{\sin x}$ f $\frac{2x}{x^2 + 1}$

g $\frac{3x^2}{x^3 - 2}$ h $\frac{3x^2 + 2}{x^3 + 2x - 1}$

i $\frac{2x}{x^2 + 1} - \frac{1}{x - 1}$ j $-\left(\frac{2x}{x^2 + 3}\right)$

k 1 l $\frac{1}{2\sqrt{x}} e^{\sqrt{x}}$

23 $\frac{e^x \cos x}{(1 + \sin x)} + e^x \ln(1 + \sin x)$

24 a $e^x(x + 1)$ b $1 + \ln x$

c $e^x\left(\frac{1}{x} + \ln x\right)$ d $x \cos x + \sin x$

e $-x^2 \sin x + 2x \cos x$ f $e^x \sec^2 x + e^x \tan x$

$$25 \frac{2\pi\ell^3\sqrt{3}}{27} \text{ at } \theta = \tan^{-1}\sqrt{2}$$

$$26 \left(\frac{\pi}{6}, \frac{3\sqrt{3}}{16}\right) \text{ maximum } \left(\frac{\pi}{2}, 0\right) \text{ shoulder}$$

$$\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{16}\right) \text{ minimum}$$

$$27 \text{ a } \sin x (2 \cos^2 x - \sin^2 x)$$

$$\text{b } \frac{2}{2x+5}$$

$$\text{c } 2 \cos 2x \cos 3x - 3 \sin 2x \sin 3x$$

$$\text{d } 2xe^{x^2+2}$$

$$\text{e } \frac{2}{x} \ln x$$

$$\text{f } 2x \cos x^2$$

$$28 \text{ a } \sec x \tan x$$

$$\text{b } -\operatorname{cosec} x \cot x$$

$$\text{c } -\operatorname{cosec}^2 x$$

$$29 6\sqrt{3}$$

$$30 (2, 4 - \ln 16) \text{ minimum}$$

$$31 \text{ a } \frac{1}{\sqrt{1-x^2}}$$

$$\text{b } \frac{1}{3x}$$

$$\text{c } e^x$$

$$\text{d } \frac{1}{2}e^x$$

$$32 \text{ a } y^2 + 3y^2 \ln 2y$$

$$\text{b } \frac{1}{(4e^2 + 3e^2 \ln 2)}$$

$$33 -1 \text{ and } 3$$

EXAMINATION EXERCISE 7 PAGE 171

$$1 \text{ a i } 5x(x^2+1)^{\frac{3}{2}} \quad \text{ii } 2$$

$$\text{b } 2, -\frac{1}{2}$$

$$2 \text{ i } \text{ a } 3x^2 \ln 2x + x^2$$

$$\text{b } 3(x + \sin 2x)^2(1 + 2\cos 2x)$$

$$3 \text{ a } 2(4x+1)^3 \cos 2x + 12(4x+1)^2 \sin 2x$$

$$\text{b } p = -2$$

$$\text{c } \frac{-2x}{(2x^2+3)(3x^2+4)}$$

$$4 \text{ a } 4x^3 \tan 2x + 2x^4 \sec^2 2x$$

$$\text{b } \frac{3}{4}$$

$$5 \text{ i } 0.12 \quad \text{ii } 1.6$$

$$6 \text{ i } \ln(2y+3) + \frac{2(y+4)}{2y+3}$$

$$\text{ii } \text{A: } 0.27, \text{B: } 0.17$$

$$7 28$$

$$8 \text{ b } (5, 7)$$

$$9 \text{ a } 3e^{3x} + \frac{1}{x}$$

$$10 \text{ a } x^2 + 3x^2 \ln x$$

$$\text{b i } y - e^3 = 4e^2(x - e)$$

$$\text{ii } x = \frac{3}{4}e$$

$$11 y = 12x + 15$$

$$12 \text{ a i } 2xe^{2x} + e^{2x}$$

$$\text{ii } y - e^2 = 3e^2(x - 1)$$

$$\text{b } k = 6$$

$$13 27x + 3y - 32 = 0$$

$$14 \text{ a } \frac{-2}{2e-x}$$

$$\text{b } y = \frac{e}{2}x - \frac{e^2}{2} + 2$$

$$15 y - \frac{\pi}{4} = -8(x - 2\sqrt{3})$$

$$16 \text{ a } \frac{2\pi}{9} \quad \text{b } y = -\frac{1}{3}x$$

$$17 \text{ d i } x = 1, 2$$

$$\text{ii } \frac{-(2x-2)}{(x^2-2x+2)^2} + \frac{1}{x^2}$$

$$\text{iii at } x = 1, \frac{d^2y}{dx^2} > 0 \therefore \text{min, and } y = 0$$

EXERCISE 8A PAGE 177

$$1 \text{ b } 3.4$$

$$2 \text{ b } 4.0$$

$$3 \text{ a } -1, -2$$

$$\text{b } -1.2$$

$$\text{c } 1.4$$

$$4 \text{ c } 1.8$$

$$\text{d } (-3, -2)$$

$$5 \text{ b } a = 9$$

$$\text{c } 0.9$$

$$6 \text{ a } 5.85, -173, -8030$$

$$\text{b } (1, 2)$$

$$\text{c } 1.52$$

$$7 \text{ b } 0.6, 8.4$$

EXERCISE 8B PAGE 180

$$1 \text{ a } 2.571$$

$$\text{b } -2.714$$

$$\text{c } 0.143$$

$$\text{d } 8.602$$

$$\text{e } 2.714$$

$$\text{f } 1.935$$

$$2 \text{ a } x^3 - 7x + 1 = 0$$

$$\text{b } x^3 - 7x + 1 = 0$$

$$\text{c } x^3 - 7x + 1 = 0$$

$$\text{d } x^2 - 74 = 0$$

$$\text{e } x^3 - 20 = 0$$

$$\text{f } x - \sin x - 1 = 0$$

$$3 \text{ a i } 0.562$$

$$\text{ii } -7.140$$

$$\text{iii } 0.372$$

$$\text{b i } x^2 + 3x - 2 = 0$$

$$\text{ii } x^2 + 7x - 1 = 0$$

$$\text{iii } x^2 + 5x - 2 = 0$$

$$\text{c i } \frac{\sqrt{17}-3}{2}$$

$$\text{ii } \frac{-7-\sqrt{53}}{2}$$

$$\text{iii } \frac{-5+\sqrt{33}}{2}$$

$$4 \text{ a } f(-2) = 2, f(-3) = -14$$

$$\text{c } -2.196$$

$$5 \text{ c } 1.7$$

$$6 \text{ c } 5.82$$

$$7 p = 7.2\%$$

$$8 \text{ c } 6.61$$

$$9 \text{ b } 0.572 \text{ hrs, } 201 \text{ km/h}$$

EXERCISE 8C PAGE 184

$$1 \text{ a } 3x^2 + 5$$

$$\text{b } 1.42$$

$$2 6.662$$

$$3 1.378$$

$$4 2.88$$

$$5 2.285$$

$$6 1.856$$

$$7 \text{ a } (2, 3)$$

$$\text{b } 2.807$$

$$8 2.646$$

$$9 4.123$$

$$10 1.360$$

$$11 1.252$$

EXERCISE 8D PAGE 187

$$1 \text{ a } 21.5$$

$$\text{b } \text{above}$$

$$\text{c } 21$$

$$2 5.83$$

$$3 58.0$$

$$4 \text{ a } 0.977$$

$$\text{b } 0.994$$

$$5 \text{ a } 0.076$$

$$\text{b } 0.720$$

$$\text{c } 0.785$$

$$6 \text{ a } 3.92$$

$$\text{b } 2.78$$

$$\text{c } 3.39$$

$$7 0.937$$

REVIEW EXERCISE 8E PAGE 188

- 2 b 0.629
 3 1.78
 4 c 2.21
 5 -1.67 6 0.697 7 1.70
 8 1.31 9 0.821 10 -1.84
 11 1.87 12 -2.279 13 -3.080
 14 a (2, 3) b 2.15 15 1.824
 16 a 4.5×10^6 b 5.3 c 0.95

EXAMINATION EXERCISE 8 PAGE 189

- 1 iii 5.828, 5.557
 2 b 0.485, 0.492, 0.489
 c 0.49
 3 a 3 and 4 b 3.24
 4 c 2.236, 2.054
 5 b 0.80219, 0.80133, 0.80167
 6 i $y = 9 - x^2$
 ii b $\alpha = 2.156$
 7 b 1.41, 1.20, 1.31
 8 iii 1.917 iv (3.92, 2.60)
 9 b 3.880, 3.918
 10 a -1.439, 0.268
 b 1.384
 11 ii 39.59
 12 b 2.219
 13 a at a stationary point $f'(x) = 0$
 b 0.622
 14 i a 4.146
 b staircase diagram will always move to upper root
 ii b 1.159
 15 i $x_2 = -1.5$
 16 ii values alternate
 17 a $y = 0.6595$ b 1.083
 18 6.39
 19 i 21.4
 ii more/narrower strips
 20 ii curve is above tops of trapezia
 iii $69\frac{1}{3}$
 21 i 6.97
 ii tops of trapezia are below the curve

EXERCISE 9A PAGE 198

- 1 a $y = (x - 1)^2$, $x, y \in \mathbb{R}$, $y \geq 0$
 b $y^2 = x^3$, $x, y \in \mathbb{R}$, $x \geq 0$
 c $y = \frac{9}{x}$, $x, y \in \mathbb{R}$, $x \neq 0$, $y \neq 0$
 d $y = x^3 - 4x$, $x, y \in \mathbb{R}$
 e $y = x^2 + 2x + 2$, $x, y \in \mathbb{R}$, $y \geq 1$
 f $y^2 = \frac{1}{x}$, $x, y \in \mathbb{R}$, $x > 0$, $y \neq 0$
 g $y = \frac{18}{x^2} + 1$, $x, y \in \mathbb{R}$, $x > 0$, $y > 1$
 h $y = \frac{3x^4}{16} + 4$, $x, y \in \mathbb{R}$, $x > 0$, $y > 4$
 i $y = \frac{x}{2x - 1}$, $x, y \in \mathbb{R}$, $x \neq 0$, $\frac{1}{2}$, $y \neq 0$, $\frac{1}{2}$

j $y = 3x^2 - 10x + 9$, $x, y \in \mathbb{R}$, $y \geq \frac{2}{3}$

2 a $y = t^2 + 1$

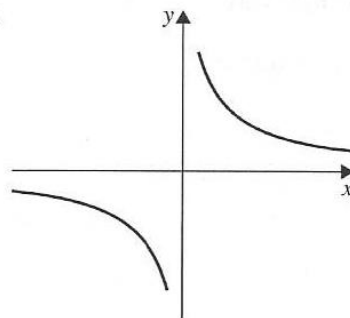
b $y = \sin t$

c $y = \frac{2}{t}$

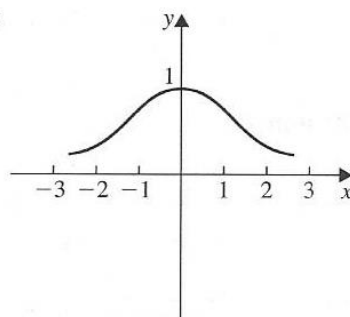
d $y = 9t^3 - t$

3 $y = 2x - 3$

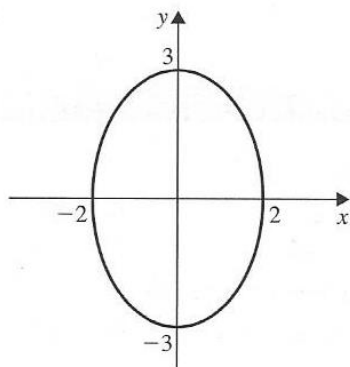
4 a



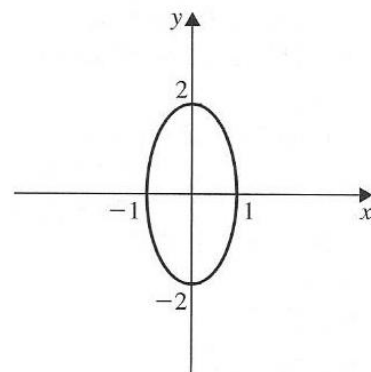
b



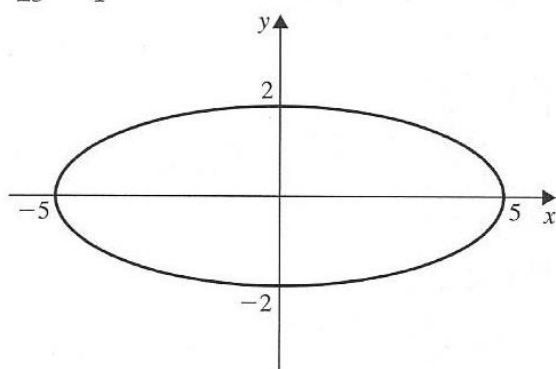
c



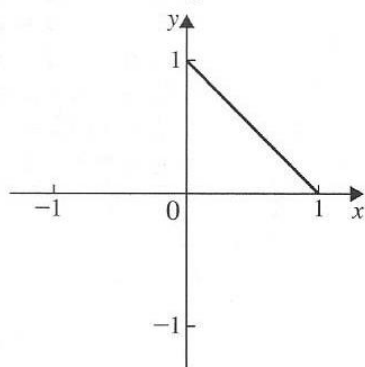
5 a $x^2 + \frac{y^2}{4} = 1$



6 a $\frac{x^2}{25} + \frac{y^2}{4} = 1$



7 a $x + y = 1$
b $0 \leq x \leq 1, 0 \leq y \leq 1$



8 $x = 2y^2 - 1$
9 a $y = 1 - 2x^2$
10 $(x + y)(x - y)^2 = 8$

EXERCISE 9B PAGE 201

1 a $\frac{t}{2}$ b $-\frac{3}{2}\tan t$
2 a $\frac{1}{t}$ b $-\frac{3}{5t^2}$ c $-\frac{4}{7}\cot t$
d $\frac{1}{2\sin t}$ e $\frac{e^t + e^{-t}}{e^t}$
3 b $3y = x + 1$
4 a $t = -1$ b $\frac{2}{3}$ c $2y + 3x = 10$
5 b $y = x$ c $16y = -9x + \frac{9}{4}$
6 a $(-\frac{20}{9}, -\frac{40}{27}), (-3, -9)$ b $(-4, -8)$
7 a $\frac{t^2 - 1}{t}$ b $(8, -4)$ and $(8, 4)$
8 $(1, 0)$ and $(-1, -4)$
9 $y + x = \sqrt{2}$
11 $4y = 4x + a$
12 $\frac{2y}{\sin \theta} - \frac{x}{\cos \theta} = 3$
13 a $-\frac{\cos 2t}{\sin t}$ b $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
14 $(0, 0), (\pi, 2), (2\pi, 0)$
15 a $(x - 2)^2 + (y - 2)^2 = 9$; circle, centre $(2, 2)$, radius 3
b $x + y = 4 + 3\sqrt{2}$

EXERCISE 9C PAGE 203

1 a $\frac{26}{3}$ b 818.4 c $25\ln 3$
2 a $\frac{28}{3}$ b 9 c 8
3 a $9\ln 4$ b 174 c $7\frac{1}{2}$ d 261.2
4 A = 3π

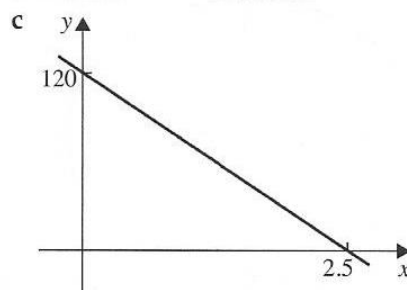
EXERCISE 9D PAGE 205

1 a $(18, 0)$ b $(-0.73, 0), (2.73, 0)$
c $(\pm 3, 0)$
2 a $(0, 2)$ b $(0, 3)$ c $(0, 0.37)$
3 a $(2.2, 0), (0, -4)$ b $(-0.33, 0)$
c $(1, 0), (0, -1)$ d $(4, 0), (0, -4)$

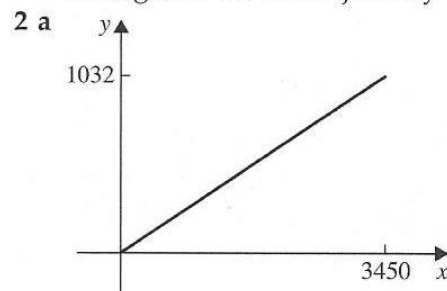
e $(\pm 2, 0), (0, \pm 4)$ f $(\pm 3, 0), (0, \pm 2)$
4 $(4, 16)$
5 $(1, \frac{\pm 5\sqrt{3}}{2})$
6 $(1, 4), (6, 9)$
7 $(0.317, 5.32), (-6.32, -1.32)$
9 a $2y + x - 3 = 0$ b $(-6, 4.5)$
10 $(\frac{729}{32}, -\frac{97}{8})$

EXERCISE 9E PAGE 208

1 a 120 km b 2.5 hrs



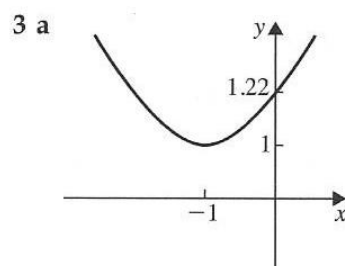
d 48 km/h
e No – constant speed is unlikely throughout the whole journey.



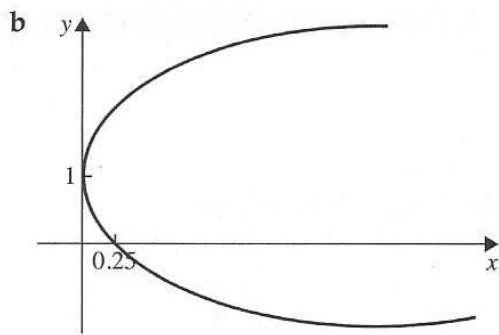
b 59.8 km
c The plane cannot keep climbing indefinitely.
3 a 20 m b 2.0 sec c 7.07 m
4 a 2.96 sec b 12.7 m c 10.7 m
5 a $r = 40$ m b $(40, 80)$
c $t = 6\pi = 18.8$ mins d 0.2 m/s
6 a $(3, -2.5)$ b $(0, 2.5)$
c $4\pi = 12.6$ mins

REVIEW EXERCISE 9F PAGE 210

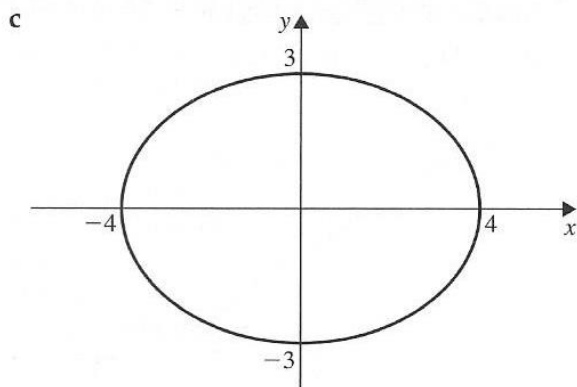
1 a $y = (x - 1)^2$ b $y = \frac{3}{x + 1}$ c $xy = 8$
d $y = 1 - x$ e $\frac{x^2}{25} + y^2 = 1$ f $y = 4x^2 - 2$



$x, y \in \mathbb{R}, y \geq 1$



$$x, y \in \mathbb{R}, x \geq 0$$



$$x, y \in \mathbb{R}, -4 \leq x \leq 4 \\ -3 \leq y \leq 3$$

4 a $\frac{1}{2t}$ **b** $-\tan t$ **c** $\frac{2}{3t}$

5 $(0, -1)$ **6** $(1, -2), (-1, 2)$

7 a $y = -2x - 1$ **b** $y + x = \sqrt{2}$

c $x + y = 2$ **d** $9y = 24\sqrt{3} - 4\sqrt{3}x$

e $2y = 9 - 3x$ **f** $y = x(1 + \sqrt{2}) + 3$

8 $(\sqrt{3}, 1 + \sqrt{3}), (-\sqrt{3}, 1 - \sqrt{3})$

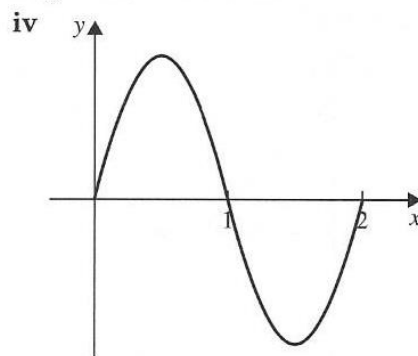
9 $(-3, 8)$

10 a 1500 m **b** 187.5 sec **c** 12.2 km

11 b $\frac{\pi}{4} = 0.79$ sec **c** 2.6 m/s

12 a $(8, -5\sqrt{3})$ **b** $\pi = 3.14$ mins

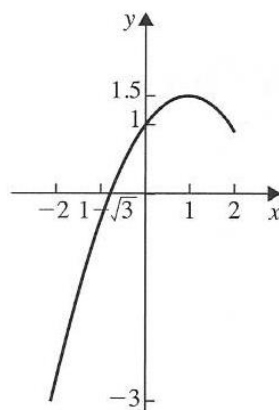
iii $y = 2x^3 - 6x^2 + 4x$



8 i $(1, 1\frac{1}{2})$

ii $y = 1 + x - \frac{1}{2}x^2$

iii $-2 \leq x \leq 2$



9 a $\frac{1}{2}e^4$

b $y = \frac{e}{2\sqrt{1-x}(1 - \ln 2\sqrt{1-x})}$

EXAMINATION EXERCISE 9 PAGE 212

1 b $a = 3, b = 12$

2 ii $(\frac{4}{9}, \frac{64}{27})$ **iv** $y^2 = x(4 - x)^2$

3 i $\frac{2}{t} - 2t^2$

ii $(0, 3)$ minimum

iii $y = \frac{2}{x+1} + (x+1)^2$

4 b $x = 1$ **c** $y = 3 + \frac{1}{8\ln 2}x$

5 a $\frac{\pi}{3}$ **b** $k = \frac{17}{16}$

6 a $k = -\frac{2}{3}$ **b** $y = 2x - 2$

c $x^2 = 3y(1 - \frac{y}{4})$ OR $\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$

7 i $(0, 0), (1, 0), (2, 0)$

ii $(1 + \frac{1}{\sqrt{3}}, \frac{-4}{3\sqrt{3}}), (1 - \frac{1}{\sqrt{3}}, \frac{4}{3\sqrt{3}})$

EXERCISE 10A PAGE 217

- 1 a A, B positive, C, D, E negative
 b B, E positive, A, C negative, D zero
 c A, C, D, E positive, B negative

- 2 a all $x \in \mathbb{R}$ b $x \geq 1$
 c $x \leq -\frac{1}{2}, x \geq 1$ d all $x \in \mathbb{R}$

- 3 a $x \leq -\frac{5}{3}$
 b $\frac{-1-\sqrt{3}}{2} \leq x \leq \frac{-1+\sqrt{3}}{2}$
 c $0 \leq x \leq \frac{\pi}{2}, \frac{3\pi}{2} \leq x \leq 2\pi$
 d $x \leq -\frac{3}{2}$

- 4 a $\frac{2}{3}$ b $e^{-\frac{5}{6}}$ c $\frac{-5 \pm \sqrt{5}}{2}$

- 6 a (1.25, 3.16) max
 (4.39, -3.16) min
 b (2.82, 0), (5.96, 0)

EXERCISE 10B PAGE 219

- 1 a $-\frac{x}{y}$ b $-\frac{y}{x}$ c $-\left(\frac{2x+y}{x+2y}\right)$
 d $\frac{3-x}{y+4}$ e $-\frac{x}{3y-2}$ f $\frac{2x}{3y^2}$
 g $-\left(\frac{9x^2+4y^2}{8xy+3y^2}\right)$

EXERCISE 10A

(alternative answers for OCR & MEI)

- 2 a all $x \in \mathbb{R}$ b $x > 1$
 c $x < -\frac{1}{2}, x > 1$ d all $x \in \mathbb{R}$

- 3 a $x < -\frac{5}{3}$
 b $\frac{-1-\sqrt{3}}{2} < x < \frac{-1+\sqrt{3}}{2}$
 c $0 < x < \frac{\pi}{2}, \frac{3\pi}{2} < x < 2\pi$
 d $x < -\frac{3}{2}$

$$\begin{array}{lll}
 2a - 3 & b - 1 & c - \frac{3}{2} \\
 3x + y = 4 & 4y = x & 5x + y = 4 \\
 6y = x & 74y + 3x = 20 & 8y = x \\
 10a - 1 & b 6 & c - 4 \\
 11b 2, -2 & c (2, 4), (-2, -4) & d - 8 \\
 12(-4, 2), (4, -2) & & \\
 13a \frac{2-x+y}{3y-x} & c (5, 3) & \\
 14(-7, 7) & & \\
 15a -\frac{17}{8} & b \frac{1}{7} & c -\frac{4}{3} \\
 16a i \frac{-1}{\sin y} & ii -\frac{1}{\sqrt{1-x^2}} & b -\frac{1}{\sqrt{1-x^2}} \\
 17a i \cos^2 y & ii \frac{1}{1+x^2} & b \frac{1}{1+x^2} \\
 18x^{x^2}(x + 2x \ln x) & & \\
 19a \frac{1}{x} \text{ or } \frac{1}{e^y} & b -\frac{y}{x} & c -\frac{y^2}{x^2} \\
 & d \tan x \tan y & e \frac{1}{\cos y - \sin y}
 \end{array}$$

EXERCISE 10C PAGE 222

$$\begin{array}{lll}
 1a 3^x \ln 3 & b 5^{x-1} \ln 5 & c 3(4^x) \ln 4 \\
 d 2x(\ln 2)2^{x^2} & e -\frac{2x}{3}e^{-\frac{x}{3}} & \\
 2a 7^t(\ln 7) & b 4^t(\ln 4) & c -3^{-t} \ln 3 \\
 3a -15e^{-\frac{t}{10}} & b 2e^{-2t} & c 6^t(10 \ln 6) \\
 4 372^\circ\text{C/min} & & \\
 5a 220 & b 1.36 \text{ g/year} & \\
 6a 2000 & b 12.8 \text{ min} & \\
 c \frac{180}{e} (= 66.2^\circ\text{C/min}) & & \\
 7a 190^\circ\text{C} & b 22.0 \text{ min} & c 11.2^\circ\text{C/min}
 \end{array}$$

EXERCISE 10D PAGE 224

$$\begin{array}{ll}
 1 0.6 \text{ cm}^2 \text{ s}^{-1} & 2 10\pi \text{ cm}^2 \text{ s}^{-1} \\
 3 0.2 \text{ cm s}^{-1} & \\
 4a 1.92 \text{ cm}^3 \text{ s}^{-1} & b 0.96 \text{ cm}^2 \text{ s}^{-1} \\
 5 300\pi \text{ cm}^3 \text{ s}^{-1} & 6 0.0637 \text{ cm s}^{-1} \\
 7 800\pi \text{ cm}^3 \text{ s}^{-1} & 8 225 \text{ cm}^3 \text{ s}^{-1} \\
 9 2 \text{ cm s}^{-1} & 10 5 \text{ cm s}^{-1} \\
 11 \frac{4}{\pi} \text{ cm s}^{-1} & 12 b 1500 \text{ cm}^3 \text{ s}^{-1} \\
 13 \frac{3}{2} \text{ units s}^{-1} & 14 \frac{1}{3\pi} \text{ cm s}^{-1} (\approx 0.106) \\
 15 \frac{2}{\pi} \text{ cm s}^{-1} &
 \end{array}$$

REVIEW EXERCISE 10E PAGE 226

$$\begin{array}{ll}
 1a \left(\frac{2}{9}, -\frac{205}{243}\right) & b (0.908, 0), (2.48, 0) \\
 2a i x \leq -\frac{\sqrt{30}}{6}, x \geq \frac{\sqrt{30}}{6} & \\
 ii -\frac{\sqrt{30}}{6} \leq x \leq \frac{\sqrt{30}}{6} & \\
 b i x \geq -1.6 & ii x \leq -1.6
 \end{array}$$

$$\begin{array}{lll}
 3a \frac{1}{2y+1} & b -\left(\frac{y}{1+x+2y}\right) & c -\frac{x}{y} \\
 4a -\frac{x}{3y} & b \frac{2+8x}{3y^2-3} & \\
 c \frac{3x+2y}{5y-2x} & d \frac{1-y^3-3x^2y}{1+3xy^2+x^3} & \\
 e \frac{\cos x}{2 \sin y} & f \frac{2xy^2}{1-2x^2y} & \\
 5 \frac{3}{4} & & \\
 6a -\left(\frac{2x+7y}{7x+6y}\right) & b -\frac{16}{19} & \\
 7 7x + 11y - 32 = 0 & & \\
 8 16x - 10y - 33 = 0 & & \\
 9a \frac{2-x}{4y-4} & & \\
 b \text{ At } (2, 2) \text{ gradient} = 0. \text{ At } (0, 1) \text{ gradient of curve is undefined.} & & \\
 12b 3 & & \\
 13a -\frac{3}{2} & b -\frac{4}{3} & \\
 14a 8^x \ln 8 & b (2 \ln 3)3^x & c (2x \ln 4)4^{x^2} \\
 d -12e^{-3x} & e x^x(1 + \ln x) & \\
 16 0.121^\circ\text{C/min} & & \\
 17 100\pi \text{ cm}^3 \text{ s}^{-1} & & \\
 18 216 \text{ cm}^3 \text{ s}^{-1} & & \\
 19 64\pi \text{ cm}^2 \text{ s}^{-1} & & \\
 20 3 \text{ cm s}^{-1} & & \\
 21 0.25 \text{ cm s}^{-1} & & \\
 22b \frac{45}{2\pi} \text{ cm s}^{-1} & & \\
 23a 3.08 \times 10^{-7} \text{ cm s}^{-1} & b 7.4 \times 10^{-6} \text{ cm}^2 \text{ s}^{-1} & \\
 24 2 \text{ cm s}^{-1} & & \\
 25 12 \text{ cm s}^{-1} & &
 \end{array}$$

EXAMINATION EXERCISE 10 PAGE 228

$$\begin{array}{ll}
 1 \frac{1-2x^2y \cos 2x}{(\sin 2x + 2y)x^2} & \\
 2 -\frac{3}{8} & \\
 3a \frac{1}{4} - \ln 2 & b \frac{11}{8} \\
 4 \frac{1-x^2}{2y}, (1, \sqrt{2}), (1, -\sqrt{2}) & \\
 5a -\frac{4}{9} & b 9x - 4y + 13 = 0 \\
 6a i \frac{-3ye^{3x}}{e^{3x} - 2 \sin 2y} & ii -\pi \\
 b y = \frac{\pi}{4} - \frac{\ln 2}{\pi} & \\
 7i \frac{2x-y-4}{x-2y+3} & \\
 ii Denominator is zero $\Rightarrow$ tangents are parallel to y -axis. & \\
 iii $8x - 9y = 48$ & \\
 & $-4xy - 2$ \\
 8a $\frac{2x^2 + 4 + \pi \sin(\pi y)}{\pi + 22}$ & \\
 b $\frac{3\pi + 62}{\pi + 22}$ &
 \end{array}$$

Alternative answer for Q4:

$$\frac{dy}{dx} = \frac{2x - y^2}{2xy}$$

- 9 ii 0.28 cm/min
 10 a 0.0149 cm s^{-1} b $1.5 \text{ cm}^2 \text{ s}^{-1}$
 11 0.06 m/h
 12 1.25 cm s^{-1}
 13 i 12.17 ii $0.18 \text{ m}^3/\text{h}$
 14 $(10, 2), (-2, 2)$

EXERCISE 11A PAGE 233

- 1 a $\frac{(1+x)^4}{4} + c$ b $\frac{1}{7}(x+4)^7 + c$
 c $\frac{1}{10}(2x+1)^5 + c$ d $-\frac{1}{5}(1-x)^5 + c$
 e $-\frac{1}{(x+3)} + c$ f $\frac{2}{9}(3x-1)^3 + c$
 g $-2(x+1)^{-2} + c$ h $-\frac{1}{5}(5x+3)^{-1} + c$
 i $7e^x + c$ j $\frac{x^2}{2} + 5e^x + c$
 k $\frac{1}{5}e^{5x} + c$ l $\frac{e^{6x}}{6} - x + c$
 m $\frac{3}{2}e^{2x} + c$ n $-9e^{-x} + c$
 o $-\frac{4}{e^x} + c$ p $-\frac{1}{e^x} + x^2 + c$
 2 a $-\cos x + c$ b $\frac{1}{4}\sin 4x + c$
 c $-\frac{1}{10}\cos 10x + c$ e $\frac{2}{5}\sin 5x + c$
 e $-\frac{1}{12}\cos 6x + c$ f $\frac{1}{2}\sin 2x + x^2 + c$
 g $-\frac{1}{4}\cos(4x-1) + c$ h $\frac{x^2}{2} - \sin(x+1) + c$
 i $x^2 + \ln x + c$ j $\sin x + 2\ln x + c$
 k $\frac{1}{4}\ln x + c$ l $\ln(x+3) + c$
 m $6\ln(1+x) + c$ n $\frac{1}{3}\ln(3x+2) + c$
 o $\frac{1}{2}e^{2x} + 2\ln x + c$
 3 a $6(4x+1)^{\frac{1}{2}}$ b $\frac{1}{6}(4x+1)^{\frac{1}{2}} + c$
 4 a $\frac{5}{2}(5x-2)^{-\frac{1}{2}}$ b $\frac{2}{5}(5x-2)^{\frac{1}{2}} + c$
 5 a $\frac{1}{8}(6x+1)^{\frac{4}{3}} + c$ b $2(x+2)^{\frac{1}{2}} + c$
 c $\frac{1}{5}(1+2x)^{\frac{5}{2}} + c$
 6 $e^2 - 1$
 7 a $\frac{1}{3}e^{3x+1} + c$ b $2e^{2x-3} + c$
 c $-\frac{5}{e^x} + c$ d $\frac{1}{8}(2x+1)^4 + c$
 e $-\frac{1}{4}(4x-3)^{-1} + c$ f $e^x - e^{-x} + c$
 g $\frac{1}{3}(2x+1)^{\frac{3}{2}} + c$ h $\frac{1}{2}(4x+5)^{\frac{1}{2}} + c$
 i $\frac{1}{2}e^{2x} + 2e^x + x + c$
 8 a $2\ln(2x+7) + c$ b $2x + e^x - e^{-x} + c$
 c $-\frac{2}{3}(3x-2)^{-1} + c$ d $\frac{1}{6}(4x+3)^{\frac{3}{2}} + c$
 e $\frac{1}{4}(8x-1)^{\frac{1}{2}} + c$ f $\frac{1}{8}(6x+5)^{\frac{4}{3}} + c$
 9 $\frac{x^2}{2} + 5\ln x - \frac{1}{x} + c$
 10 a $x - \frac{4}{x} - 4\ln x + c$ b $x + 5\ln x - \frac{3}{x} + c$

- c $\frac{1}{2}(e^x + x) + c$ d $x - 5\ln x - \frac{6}{x} + c$
 e $\frac{x^2}{2} - 25\ln x + c$ f $e^{x+2} - \frac{1}{2}e^{-2x} + c$
 11 a $\frac{\pi}{4}$ b $\frac{1}{2}$
 12 $\frac{1}{2}$ 13 $\frac{1}{3}(e^3 - 1)$ 14 $\ln \frac{5}{2}$
 15 $-\frac{1}{12}$ 16 $\frac{1}{4}$ 17 $\frac{1}{2}e^3 - \frac{1}{2}e^{-1}$
 18 $10\ln 2$ 19 $\ln \frac{3}{2}$ 20 $12\frac{2}{3}$
 21 $0.0339 = \frac{1}{2}(\sin 2^\circ - \sin 1^\circ)$ 22 $8 + \frac{1}{2}\ln 3$
 23 1 24 $3 - \sqrt{3}$ 25 $4\frac{5}{6}$
 26 $\frac{1}{2}\ln 5$ 27 $\frac{1}{2} + \frac{3\pi}{4}$ 28 $\frac{7}{12}$
 30 a $2, 3$ b $6\ln \frac{3}{2} - \frac{1}{2}$
 31 $4 - 3\ln 3$
 32 b $0, 5$ c $\frac{1}{6}$
 34 a $\ln 2, \ln 6$ b $16 - 12\ln 3$

EXERCISE 11B PAGE 236

- 1 $\ln(x+3) + c$ 2 $\ln(2x+1) + c$ 3 $\ln(x^2+5) + c$
 4 $\ln(x^3+2) + c$ 5 $\frac{1}{4}\ln(4x+1) + c$ 6 $\frac{1}{7}\ln(7x-1) + c$
 7 $\ln(e^x+3) + c$ 8 $\frac{1}{2}\ln(x^2+3) + c$ 9 $\ln(\sin x) + c$
 10 $\ln(x+1)(x-2) + c$
 11 $\ln(2x+1)(5x+2) + c$
 12 $3\ln(2x+1) - \ln(x+1) + c$
 13 $\frac{1}{3}\ln \frac{5}{2}$
 14 a $2\ln 4$ b $\ln 2$ c $6 + \ln 2$
 d $12 - \ln 2$ e $\ln 4$ f $2\ln 3$
 g $\ln \frac{27}{10}$ h $-\ln 2$ i $10 + \ln 3$
 j $\ln 30$ k $10 + 3\ln 2$ l $4 - 4\ln 3$
 15 $\ln \left(\frac{3}{2}\right)$
 16 a -1.10 b -0.24 c -0.32
 17 a $-\ln \cos x + c$ b $\frac{1}{2}\ln \sin 2x + c$
 c $\ln(\tan x) + c$ d $-\ln(\cos x + 3) + c$
 e $-\frac{1}{2}\ln \cos 2x + c$ f $\ln(\ln x) + c$

EXERCISE 11C PAGE 240

- 1 a $\frac{(x+2)^4}{4} - \frac{2(x+2)^3}{3} + c$
 b $\frac{(x-3)^4}{4} + (x-3)^3 + c$
 c $\frac{2(x+4)^5}{5} - 2(x+4)^4 + c$
 d $\frac{(x-1)^4}{4} + \frac{(x-1)^3}{3} + c$
 e $\frac{2(x+3)^{\frac{5}{2}}}{5} - 2(x+3)^{\frac{3}{2}} + c$
 f $\frac{2(x-2)^{\frac{5}{2}}}{5} + \frac{8(x-2)^{\frac{3}{2}}}{3} + 8(x-2)^{\frac{1}{2}} + c$

Note: Instructions do not say answers must be factorised.

$$2 \text{ a } \frac{(x+1)^5}{5} - \frac{(x+1)^4}{4} + c$$

$$\text{b } \frac{(x-1)^7}{7} + \frac{(x-1)^6}{6} + c$$

$$\text{c } \frac{(2x+1)^5}{20} - \frac{(2x+1)^4}{16} + c$$

$$\text{d } \frac{(4x-1)^{\frac{5}{2}}}{40} + \frac{(4x-1)^{\frac{3}{2}}}{24} + c$$

$$\text{e } \frac{2(5x+1)^{\frac{3}{2}}}{75} - \frac{2(5x+1)^{\frac{1}{2}}}{25} + c$$

$$\text{f } \frac{(3x-2)^6}{54} + \frac{(3x-2)^5}{9} + c$$

$$3 \text{ a } \frac{3}{2}(x^2 + 1)^4 + c$$

$$\text{b } 2(x^3 - 3)^3 + c$$

$$\text{c } \frac{1}{4}(e^x - 1)^4 + c$$

$$\text{d } 2(e^x + 2)^{\frac{1}{2}} + c$$

$$\text{e } \frac{1}{3}\sin^3 x + c$$

$$\text{f } -\frac{1}{4}\cos^4 x + c$$

$$\text{g } \frac{1}{4}\tan^4 x + c$$

$$\text{h } \sin^{-1} x + c$$

$$4 \text{ a } \frac{49}{20}, u = x + 1$$

$$\text{b } \frac{23}{30}, u = x - 3$$

$$\text{c } 21\frac{11}{15}, u = x - 2$$

$$\text{d } 3\frac{1}{3}, u = 2x + 1$$

$$5 \text{ a } e - \frac{1}{e}$$

$$\text{b } 2\sqrt{3} - \frac{2}{3}\sqrt{5}$$

$$\text{c } \frac{1}{2}\ln 2$$

$$\text{d } \frac{2}{3}$$

$$7 \text{ 0.18}$$

EXERCISE 11D PAGE 243

$$1 \text{ a } \frac{x}{3}(1+x)^3 - \frac{(1+x)^4}{12} + c$$

$$\text{b } x \sin x + \cos x + c$$

$$2 \text{ a } x e^x - e^x + c$$

$$\text{b } \frac{x}{4}(1+x)^4 - \frac{(1+x)^5}{20} + c$$

$$\text{c } -x e^{-x} - e^{-x} + c$$

$$\text{d } \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + c$$

$$\text{e } -x \cos x + \sin x + c \quad \text{f } \frac{3x}{2}\sin 2x + \frac{3}{4}\cos 2x + c$$

$$\text{g } \frac{x}{2}(x-1)^4 - \frac{1}{10}(x-1)^5 + c$$

$$\text{h } \frac{2x}{3}(x+1)^{\frac{3}{2}} - \frac{4}{15}(x+1)^{\frac{5}{2}} + c$$

$$\text{i } \frac{x^2}{2}\ln 2x - \frac{x^2}{4} + c \quad \text{j } \frac{x^3}{3}\ln x - \frac{x^3}{9} + c$$

$$\text{k } -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + c \quad \text{l } e^x(x^2 - 2x + 2) + c$$

$$3 \text{ 1} - \frac{2}{e} (= 0.264)$$

$$4 \text{ a } \frac{1}{9} - \frac{4}{9e^3}$$

$$\text{b } \frac{1}{4}$$

$$\text{c } \frac{17}{6}$$

$$\text{d } 71\frac{11}{15}$$

$$\text{e } \frac{2e^3}{9}$$

$$\text{f } \pi - 2$$

$$5 \text{ a } 7\frac{1}{10}$$

$$7 \text{ a } (1, 0), (0, 1) \quad \text{b } e - 2$$

$$8 \text{ a } 2x e^{x^2} \quad \text{b } \frac{e^{x^2}}{2}(x^2 - 1) + c$$

$$9 \text{ b } -e^x \cos x + \int e^x \cos x \, dx$$

$$\text{c } \frac{e^x}{2}(\sin x + \cos x) + c$$

$$10 \text{ a } \frac{1}{34}e^{5x}[-3 \cos 3x + 5 \sin 3x] + c$$

$$\text{b } \frac{e^{ax}}{a^2 + b^2}[a \sin bx - b \cos bx] + c$$

EXERCISE 11E PAGE 245

$$1 \text{ a } \frac{1}{2}x - \frac{1}{4}\sin 2x + c \quad \text{b } \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

$$\text{c } \tan x - x + c \quad \text{d } \frac{1}{2}x - \frac{1}{12}\sin 6x + c$$

$$\text{e } \frac{1}{2}x + \frac{1}{8}\sin 4x + c \quad \text{f } 3x + 4 \cos x - \sin 2x + c$$

$$\text{g } \frac{5}{2}x + \frac{1}{4}\sin 2x + \tan x + c$$

$$\text{h } \frac{1}{2}\tan 2x - x + c \quad \text{i } \tan x - 2 \ln \cos x + c$$

$$2 \text{ a } \frac{\pi}{8} - \frac{1}{4} \quad \text{b } \frac{1}{2} + \frac{\pi}{4} \quad \text{c } \sqrt{3} - \frac{\pi}{3}$$

$$\text{d } \frac{3\pi}{4} + 2 \quad \text{e } \frac{\pi}{12}$$

EXERCISE 11F PAGE 246

$$1 \text{ a } \frac{1}{1+x} - \frac{1}{1+2x}$$

$$\text{b } \ln(1+x) - \frac{1}{2}\ln(1+2x) + c$$

$$2 \text{ a } \ln(x-1) - \ln(x+1) + c$$

$$\text{b } 2 \ln(x+4) - \ln(x+5) + c$$

$$\text{c } 5 \ln(x-7) - 4 \ln(x-2) + c$$

$$4 \text{ 0.235}$$

$$5 \text{ a } \frac{4}{2x+1} + \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

$$6 \text{ a } x - \frac{1}{3} - \frac{2}{2x-1} + \frac{26}{3(3x-1)}$$

$$9 \text{ a } (x+1)(x+2)(x+3)$$

$$\text{b } -\frac{2}{x+2} + \frac{1}{x+3} + \frac{1}{x+1}$$

$$\text{c } \ln \frac{32}{27}$$

Misprint in Q6b:

Should say $-\left(\frac{2x+1}{4e^{2x}}\right) + c$

[Not $\left(\frac{2x+1}{4e^{2x}}\right) + c$ **]**

EXERCISE 11G PAGE 249

Section A

- 1 $\frac{1}{4}(x+2)^4 + c$
- 2 $\frac{1}{3}\sin 3x + c$
- 3 $4e^x + c$
- 4 $\ln x + c$
- 5 $-\frac{1}{x} + c$
- 6 $\ln\left(\frac{x-1}{x+1}\right) + c$
- 7 $x \sin x + \cos x + c$
- 8 $\ln(5x-1) + c$
- 9 $\frac{1}{2}\tan 2x + c$
- 10 $\ln(x^2 + a) + c$
- 11 $\frac{(x+4)^3}{12}(3x-4) + c$
or $\frac{(x+4)^4}{4} - \frac{4(x+4)^3}{3} + c$
or $\frac{x(x+4)^3}{3} - \frac{(x+4)^4}{12} + c$
- 12 $\frac{x}{5}e^{5x} - \frac{1}{25}e^{5x} + c$
- 13 $\frac{1}{3}\ln \sin 3x + c$
- 14 $\frac{1}{3}e^{3x+2} + c$
- 15 $\frac{1}{6}(x^4 - 1)^{\frac{3}{2}} + c$
- 16 $5 \ln(x-7) + c$
- 17 $2 \ln(x+1) + 3 \ln(x+2) + c$
- 18 $\frac{1}{2}x + \frac{1}{4}\sin 2x + c$
- 19 $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$
- 20 $x + \ln x + c$
- 21 $\ln(1+x) + c$
- 22 $e^x(x^2 - 2x + 2) + c$
- 23 $\frac{1}{4}\cos(3-4x) + c$
- 24 $4 \tan x + c$
- 25 $\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + c$
- 26 $\frac{x^2}{2} + 3x + \ln x + c$
- 27 $-\ln \cos x + c$
- 28 $\tan x - x + c$
- 29 $\frac{1}{8}(x^2 + 1)^4 + c$
- 30 $-\frac{1}{2}x \cos 2x + \frac{1}{4}\sin 2x + c$
- 31 a $e^{x^3} + c$ b $e^{\sin x} + c$ d $\frac{34}{3}$
- 32 a $e - 1$ b $\frac{1}{2}(e^2 - 1)$
c $\ln 3$ d $\frac{1}{2}\ln 4$
- 33 a $2 + \ln 3$ b $3 + 4 \ln 2$
c $6 \ln 2 + 27\frac{1}{6}$ d $2e^2 + 4e - 2$

$$34 y = \frac{1}{2} \ln x + 3$$

$$35 y = \frac{1}{3}(5 \ln x + 1)$$

$$36 y = \frac{1}{6}(x^2 + 5 + 2 \ln x)$$

$$37 y = e^x + x - 1$$

$$38 \text{ a } \cos 2x = 1 - 2 \sin^2 x \quad \text{b } \frac{\pi}{8} + \frac{1}{4}$$

$$39 \text{ b } \sqrt{3} - 1 - \frac{\pi}{12}$$

$$40 2x \sin x + (2 - x^2) \cos x + c$$

Section B

$$1 \text{ a } 2 \quad \text{b } 0 \quad \text{c } \frac{1}{3} - \frac{\sqrt{2}}{6}$$

$$\text{d } \sqrt{3} \quad \text{e } \frac{e}{2}(e^2 - 1) \quad \text{f } \frac{1}{3}$$

$$\text{g } 44\frac{1}{3} \quad \text{h } \frac{98}{3} \quad \text{i } 4$$

$$2 \frac{3}{2} \ln(2x + 5) + c$$

$$3 e^{x^2} + c$$

$$4 \ln(4 + \sin x) + c$$

$$5 2 \ln x + \ln(x - 3) + c$$

$$6 \frac{3}{2} x^2 \ln x - \frac{3}{4} x^2 + c$$

$$7 \frac{(1+x)^{11}}{132} (11x - 1) + c$$

$$8 x \ln x - x + c$$

$$9 - e^{-3x} + c$$

$$10 -2 \cos\left(\frac{x}{2}\right) + c$$

$$11 x - 3 \ln(x + 3) + c$$

$$12 \sin x + \ln \cos x + c$$

$$13 \frac{1}{2} x - \frac{1}{8} \sin 4x + c$$

$$14 \frac{2}{15}(5x + 1)^{\frac{3}{2}} + c$$

$$15 2(x - 4)^{\frac{1}{2}} + c$$

$$16 2x^3 - \frac{3x^2}{2} + c$$

$$17 e - \frac{1}{e}$$

$$18 \frac{1}{2} e^4 + \frac{3}{2} e^2 - 2e + 1$$

$$19 2 + \frac{1}{e} - \frac{1}{e^3}$$

$$20 38$$

$$21 64\frac{3}{4}$$

$$22 11005$$

$$23 \frac{8}{9}$$

$$24 \frac{2}{9}$$

$$25 \frac{4}{5} \ln 4$$

$$26 \frac{2e^3 + 1}{9}$$

$$27 -2$$

$$28 2 \ln 2 - \frac{3}{4}$$

$$29 29.6 = \frac{148}{5}$$

$$30 2 \ln 4$$

$$31 \ln 2$$

$$32 \text{ b } \frac{8}{3}$$

$$33 \frac{1}{2}$$

$$34 \text{ b } 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$$

$$35 \text{ a } -\frac{1}{3}(1 - x^2)^{\frac{3}{2}}$$

$$\text{b } -\frac{x^2}{3}(1 - x^2)^{\frac{3}{2}} - \frac{2}{15}(1 - x^2)^{\frac{5}{2}}$$

$$38 3$$

$$39 \frac{1}{2} \ln(\sec 2x + \tan 2x) + c$$

EXERCISE 11H PAGE 253

$$1 \ln 8 - \ln 3 = 0.981$$

$$2 \frac{9}{4}$$

$$3 0.591$$

$$4 1.87$$

$$5 0.390$$

$$6 1.93$$

$$7 9$$

$$8 1.15$$

$$9 \text{ a } 2, 3$$

$$\text{b } \frac{5}{2} + 6 \ln \frac{2}{3}$$

$$10 4 - 3 \ln 3$$

$$11 \text{ a } 0, 5$$

$$\text{b } \frac{1}{6} u^2$$

$$13 \text{ a } \ln 2, \ln 6$$

$$\text{b } 12 \ln 3 - 16$$

EXERCISE 11I PAGE 256

$$1 \text{ a } y = x + \ln x + c$$

$$\text{b } y = x^3 - x + c$$

$$\text{c } y = e^x - x + c$$

$$\text{d } y = \sin x + c$$

$$\text{e } y = x - 3e^{-x} + c$$

$$\text{f } y = \ln(\sin x) + c$$

$$2 \text{ a } y = \frac{1}{6}(2x + 1)^3 + \frac{5}{6}$$

$$\text{b } y = \frac{2}{3}(x - 1)^{\frac{3}{2}} + \frac{4}{3}$$

$$\text{c } y = \frac{3}{2} - \frac{1}{2} \cos 2x$$

$$\text{d } y = \frac{1}{2} \ln(2x - 1) + 1$$

EXERCISE 11J PAGE 259

$$1 \text{ a } \ln y = \frac{x^3}{3} + c$$

$$\text{b } y^2 = x^2 + c$$

$$\text{c } y = kx$$

$$\text{d } y = x + \ln x + c$$

$$\text{e } y^2 = x^3 + c$$

$$\text{f } y^2 = \frac{1}{2} e^{4x} + c$$

$$\text{g } y = -\frac{1}{(e^x + c)}$$

$$\text{h } y^2 = 2 \sin x + c$$

$$\text{i } y = \ln(1 + x) + c$$

$$\text{j } y = e^{2x + c}$$

$$\text{k } y = e^{x^3 + c} - 3$$

$$\text{l } \sin y = \ln x + c$$

$$\text{m } y = \ln(2x^2 + c)$$

$$\text{n } y = e^{x^2 + c}$$

$$\text{o } y = -\frac{1}{(x + c)}$$

$$2 y = 4x^2$$

$$3 \text{ a } x = 10e^{3t}$$

$$\text{b } y = 1000e^{-10t}$$

$$\text{c } x = 15e^t + 5$$

$$\text{d } y = 18e^{-t} - 8$$

$$4 \text{ a } x = 100e^{5t}$$

$$\text{b } y = 1000e^{-2t}$$

$$\text{c } x = x_0 e^{3t}$$

$$\text{d } y = y_0 e^{-4t}$$

$$\text{e } x = x_0 e^{kt}$$

$$\text{f } y = y_0 e^{-kt}$$

$$5 \text{ a } -e^{-y} = \frac{x^2}{2} - 3 \text{ or } y = \ln\left(\frac{2}{6 - x^2}\right)$$

$$\text{b } y = e^{-\cos x}$$

$$\text{c } y = \frac{3e^{x^2} - 5}{2}$$

$$\text{d } y = \sqrt{(1 - \ln 2 - 2 \ln \cos x)}$$

$$\text{e } \tan y = 2 - \cos x$$

$$\text{f } y = 2e^{\left(\frac{x^2-1}{2}\right)} - 1$$

$$6 \text{ a } x = 20 - 15e^{-\frac{t}{10}}$$

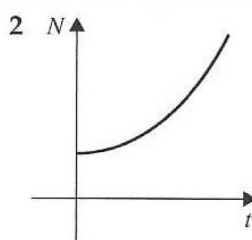
$$\text{b } 4.05$$

$$7 y = \frac{2}{1 + e^{-2x}}$$

$$8 \text{ a } \frac{1}{1+x} - \frac{1}{2+x}$$

$$\text{b } y = \frac{4}{3} \left(\frac{1+x}{2+x} \right)$$

EXERCISE 11K PAGE 262



$$4 P = 50\,000\,000 e^{0.00912t}$$

$$5 \text{ b } 46.1 \text{ min}$$

$$\text{c } 100e^3 \approx 2008$$

$$6 \text{ b } 12 \text{ min}$$

$$\text{c } 45^\circ\text{C}$$

$$7 \text{ b } £7746$$

$$\text{c } 9 \text{ years}$$

$$8 10 \text{ hours}$$

$$9 20 \text{ minutes } (c = 2, k = \frac{1}{10})$$

$$10 V = 300 e^{-\frac{t}{10}} + 200$$

$$\text{c } 200 \text{ cm}^3$$

Note: Equivalent answers acceptable.

REVIEW EXERCISE 11L PAGE 264

- 1 a $-\cos x + c$ b $\frac{1}{2}\sin 2x + c$
 c $-4\cos \frac{x}{4} + c$ d $\tan x + c$
 e $\frac{1}{7}e^{7x} + c$ f $-\frac{1}{3}e^{-3x} + c$
 g $\frac{1}{5}\sin(5x + 4) + c$ h $\frac{1}{8}(2x + 3)^4 + c$
 i $-\frac{1}{4}(1 + 4x)^{-1} + c$ j $\ln x + c$
 k $5\ln x + c$ l $\frac{3}{2}\ln(2x + 9) + c$
 m $5\ln(x - 7) + c$ n $\sin x - \cos x + c$
 o $\frac{1}{9}(6x - 1)^{\frac{3}{2}} + c$
- 3 a $\frac{1}{2} - \frac{\sqrt{3}}{4}$ b $\ln 2$ c 14
- 4 a $2\ln 2 - 2$ b maximum
- 5 a $2\ln(x + 1) + 3\ln(x + 2) + c$
 b $4\ln(x + 1) + 6\ln(x + 2) + c$
- 6 a $\ln(x^3 + 5) + c$ b $\frac{1}{4}\ln(4x^2 - 7) + c$
 c $\frac{1}{2}\ln(1 + e^{2x}) + c$ d $-\ln \cos x + c$
 e $\ln f(x) + c$ f $\frac{1}{2}\ln \sin 2x + c$
- 7 a $\frac{71}{10}$ b $\frac{41}{288}$
 c $\frac{19}{108}$ d $1 - e$
 e $18, u^2 = 2x - 1$ f $-\frac{2048}{45}, u = x - 1$
 g $e^8 - 1, u = x^3$ h $\frac{3}{16}, u = x^2 + 1$
- 8 $\frac{7}{9}$
- 9 $\frac{46}{15}$
- 11 a $\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + c$ b $\sin x - x \cos x + c$
 c $\frac{3}{2}x^2 \ln x - \frac{3}{4}x^2 + c$ d $\frac{x^3}{9}(3 \ln x - 1) + c$
- 12 $e^2 + 1$ 13 $\frac{e^4 - 1}{2}$
- 15 c $\frac{\pi}{4}$
- 16 a $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$ b $\tan x - x + c$
- 17 a $y = \left[\frac{3}{2}(x + 14)\right]^{\frac{2}{3}}$ b $y = \sqrt{40x + 4}$
 c $y = e^{x^2 - 4}$ d $y = 2e^{\frac{1}{2}(x^2 - 1)} - 1$
 e $y = x + \ln x + 2$
 f $y = \sqrt{5e^{2 \tan x} - 1}$ or $\ln\left(\frac{y^2 + 1}{5}\right) = 2 \tan x$
- 19 $y = e^{2x^2 - 2}$
- 20 c $\frac{1}{10}\ln\left(\frac{3}{2}\right)$ d 17.1 hours
- 21 $\frac{75}{2} - 50 \ln 2$
- 23 a $\sec x + c$ b $\frac{1}{3}\cot 3x + c$ c $x \ln 2x - x + c$
 d $x^2 \sin x + 2x \cos x - 2 \sin x + c$
 e $\ln(x + 2) + \frac{3}{x + 2} + c$

EXAMINATION EXERCISE 11 PAGE 267

- 2 i $-\frac{1}{24}(4 - 3x)^8 + c$
 ii $-\frac{1}{3}\ln(4 - 3x) + c$
- 3 i $4x - 4 \ln x - \frac{1}{x} + c$
 ii $\frac{3}{16}(4x + 1)^{\frac{4}{3}} + c$

- 4 b $\frac{1}{2}e^2 + 4e - \frac{1}{2}$
- 5 a $A = 2, B = 3$
 b $y = x^2 + 3 \ln(2x^2 - x + 2) + 1 - 3 \ln 5$
- 7 $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$
- 8 $\frac{3}{4}e^2 - \frac{3}{4}$
- 9 a $-\frac{1}{2x^2} \ln x - \frac{1}{4x^2} + c$
 b $\frac{3}{16} - \frac{1}{8} \ln 2$
- 10 a $a = 5, b = 3$
- 11 $\frac{13}{3}\sqrt{5} - \frac{16}{3}\sqrt{2}$
- 12 $2(x^2 - 2)^{\frac{3}{2}} + 16(x^2 - 2)^{\frac{1}{2}} + c$
- 13 $-\frac{23}{384}$
- 14 $\frac{47}{480}$
- 15 a $A = 1, B = 1, C = -4$
 b $-\frac{1}{2}e^{-2y} = -\ln(1 - 3x) + \ln(1 + x)$

$$+ \frac{4}{1 + x} - \frac{9}{2}$$
- 16 $y = \frac{1}{2} \ln(1 - 2 \ln |\sec x|)$
- 17 a $\frac{dx}{dt}$ is the rate of increase of the mass of waste products. M is the initial mass of unburned fuel.
 b $x = M(1 - e^{-kt})$
 c $x = \frac{2}{3}M$
- 18 ii $h = \sqrt[3]{\frac{729}{8}} - \frac{27t}{400\pi}$
 iii 71 minutes
- 19 b 161
- 20 a $\frac{1}{P - 2} - \frac{1}{P}$
 c 0.473

EXERCISE 12A PAGE 274

- 1 a $\sqrt{29}, \frac{1}{\sqrt{29}}(4i + 3j + 2k)$
 b $\sqrt{178}, \frac{1}{\sqrt{178}}(-5i + 12j + 3k)$
 c $5\sqrt{26}, \frac{1}{5\sqrt{26}}(7i - 24j - 5k)$
- 2 a $3i + 4j - 4k$ b $15i + 5j - 3k$
 c $5i - 6j$ d $-5i + 6j + 9k$
- 3 a 7 b 9 c $3\sqrt{5}$ d $3\sqrt{3}$
- 4 a $\frac{1}{3}(2i + 2j + k)$ b $\frac{1}{6}(4i - 2j + 4k)$
 c $\frac{1}{7}(3i + 6j - 2k)$ d $\frac{1}{3}(2i - j + 2k)$
- 5 a = 5, b = 2
- 6 $8i + 24j + 12k$
- 7 $12i - 3j - 24k$
- 8 a $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ b $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ c $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$

$$d \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} \quad e \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \quad f \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$9 \ n = -1$$

$$10 \ a \ 3i + 9j + 4k$$

$$b \ \sqrt{106}$$

$$c \ i + 5j - 2k$$

$$d \ i + 8j - 7k$$

$$11 \ a \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$$

$$b \ \sqrt{43}$$

$$c \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}$$

$$12 \ a \begin{pmatrix} -6 \\ -9 \\ 9 \end{pmatrix}$$

$$b \begin{pmatrix} -12 \\ -13 \\ -9 \end{pmatrix}$$

$$c \begin{pmatrix} -5 \\ -4 \\ 18 \end{pmatrix}$$

$$d \ 13$$

$$e \begin{pmatrix} -0.8 \\ 0 \\ 0.6 \end{pmatrix}$$

$$13 \ a \ -2i - 8j - 3k$$

$$b \ 2i + 8j + 3k$$

$$14 \ a \ 2i + 6j$$

$$b \ i + 3j$$

$$c \ 2i + 5j + k$$

$$15 \ a \ 3i + 4j + 4k$$

$$b \ 13$$

$$c \ (7, 6, 11)$$

$$d \ (31, 12, 19)$$

$$e \ 2:1$$

$$16 \ a \ 143^\circ$$

$$b \ 65.9^\circ$$

$$c \ 40.2^\circ$$

$$17 \ a \ (10, 1, 3)$$

$$b \ 84.5^\circ$$

EXERCISE 12B PAGE 278

$$1 \ AB = AC = \sqrt{30}$$

$$2 \ \text{Right-angle at P}$$

$$3 \ \overrightarrow{2AB} = \overrightarrow{CD}$$

$$4 \ \overrightarrow{EF} = \overrightarrow{GH} \text{ and } \overrightarrow{EG} = \overrightarrow{FH}$$

$$5 \ AB = DA = 3, \ BC = CD = \sqrt{14}$$

$$6 \ \overrightarrow{AC} = \overrightarrow{BD}, \ \overrightarrow{AB} = \overrightarrow{CD}, \ \overrightarrow{AD} = \overrightarrow{BC} = \sqrt{581}$$

$$7 \ (12, 2, 1)$$

$$8 \ D(6, 0, 5), \text{Area} = 25$$

$$9 \ V = 125, SA = 150$$

EXERCISE 12C PAGE 282

$$1 \ a \ 8i + 3j + k$$

$$b \ 3i + 6j - 7k$$

$$c \ 5i + 10j + 20k$$

$$d \ 4i + 3j + 2k$$

$$2 \ a \ 5i, 5$$

$$b \ 0.5i - 3j - k, \frac{\sqrt{161}}{4}$$

$$c \ 2i + j + k, \sqrt{6}$$

$$d \ 0.5i + 2.5j + 4k, \frac{3\sqrt{10}}{2}$$

$$3 \ a \ 3i + 2j + 2k$$

$$b \ -2i - j + 2k$$

$$c \ i + 3j + 1.75k$$

$$d \ 3i + 2j + k$$

$$4 \ a \ 8i + 3j + 5k$$

$$b \ 8i + 7j - 4k$$

$$c \ 5i + 7j + 2k$$

$$d \ 10i - 4j + 17k$$

$$5 \ a \ 4k$$

$$b \ 30i - 10j - 15k$$

$$c \ -4i + 16j + 32k$$

$$6 \ 4i - 6j - k$$

$$7 \ 8i + 2j - 11k$$

$$8 \ a \ 5 \text{ seconds}$$

$$b \ y = -11, z = 2$$

$$9 \ x = -\sqrt{651}$$

$$10 \ a \ 13i + 4k$$

$$b \ \text{No}$$

$$11 \ -4i + 2j - 8k$$

$$12 \ 5\sqrt{5}$$

$$13 \ 16.1 \text{ km}$$

$$14 \ a \ 2i + 2j + 9k$$

$$b \ i + j + 4.5k$$

$$c \ 59.0 \text{ m}$$

$$15 \ a \ a = -3, b = -3, c = 1$$

$$b \ 0.75i + 0.75j - 0.25k$$

$$c \ 7.63 \text{ ms}^{-1}$$

$$16 \ t = 4 \text{ s}, -39i + 10j - 6k$$

REVIEW EXERCISE 12D PAGE 284

$$1 \ a \ \frac{1}{\sqrt{35}}(3i - j + 5k)$$

$$b \ \frac{\sqrt{6}}{18}(-2i - 7j + k)$$

$$c \ \frac{\sqrt{26}}{26}(i + 4j - 3k)$$

$$2 \ 2.5(3i - 12j - 4k)$$

$$3 \ a \ i \ 6i - 4j - 5k$$

$$ii \ \sqrt{77}$$

$$b \ i \ 2i + 6j + 7k$$

$$ii \ \sqrt{89}$$

$$c \ i \ -13i - 12j + 4k$$

$$ii \ \sqrt{329}$$

$$4 \ a \ \sqrt{34}$$

$$b \ \sqrt{34} + \sqrt{6} = 8.28$$

$$c \ 6$$

$$d \ -5i + 20j + 5k$$

$$e \ s = 4$$

$$5 \ a \ 3i - 4j - 8k$$

$$b \ 1.5i - 2j - 4k$$

$$c \ -0.5i + j + 2k$$

$$6 \ (-0.5, 1.5, 1)$$

$$7 \ a \ 24.1^\circ$$

$$b \ 64.8^\circ$$

$$c \ 150^\circ$$

$$8 \ (10, -9, -16)$$

$$9 \ \text{scalene}$$

$$10 \ 504$$

$$11 \ a \ \text{rectangle}$$

$$b \ \text{trapezium}$$

$$c \ \text{parallelogram}$$

$$12 \ 8(2i + 3j - 6k) \text{ ms}^{-1}$$

$$13 \ (2i - 3j + k) \text{ ms}^{-1}$$

$$14 \ (13i + j + 5k) \text{ m}$$

$$15 \ (i - j + 2k) \text{ ms}^{-2}$$

$$16 \ (27i + 3j - 15k) \text{ ms}^{-1}$$

$$17 \ a \ -i - \frac{10}{3}j$$

$$b \ 111 \text{ m}$$

$$18 \ 7 \text{ seconds}, (10i - 9j - 3k) \text{ m}$$

EXERCISE 13A PAGE 288

$$1 \ \text{Assume } \exists m, n \in \mathbb{Z} \text{ s.t. } 28m + 21n = 1.$$

$$\text{Then } 4m + 3n = \frac{1}{7}.$$

$$\text{But } m, n \in \mathbb{Z} \Rightarrow 4m + 3n \in \mathbb{Z}, \text{ and } \frac{1}{7} \notin \mathbb{Z}.$$

This contradicts our assumption that $m, n \in \mathbb{Z}$, and proves that there exist no integers m and n such that $28m + 21n = 1$.

$$2 \ a \ \text{Assume } \exists m, n \in \mathbb{Z} \text{ s.t. } 27a + 36b = 1.$$

$$\text{Then } 3a + 4b = \frac{1}{9}.$$

$$\text{But } a, b \in \mathbb{Z} \Rightarrow 3a + 4b \in \mathbb{Z}, \text{ and } \frac{1}{9} \notin \mathbb{Z}.$$

This contradicts our assumption that $a, b \in \mathbb{Z}$, and proves that there exist no integers a and b such that $27a + 36b = 1$.

b e.g. $x = 7, y = -5$

3 Assume $ab \in \mathbb{Q}$, i.e. $ab = \frac{m}{n}$, $m, n \in \mathbb{Z}$.

Also, $b = \frac{c}{d}$ as $b \in \mathbb{Q}$.

Then $a \frac{c}{d} = \frac{m}{n}$

$a = \frac{dm}{cn} \Rightarrow a \in \mathbb{Q}$ as $dm, cn \in \mathbb{Z}$.

This contradicts the statement that a is irrational. Therefore the original statement that the product ab is **irrational** must be true.

4 Assume that n is odd, i.e. $n = 2m + 1$.

Then $n^2 = (2m + 1)^2 = 4m^2 + 4m + 1$
 $= 2(2m^2 + 2m) + 1$

which is odd.

This contradicts the statement that n^2 is even. Therefore the original statement that if n^2 is even n must also be even must be true.

5 Assume that $\sqrt{2}$ is rational, i.e. $\sqrt{2} = \frac{a}{b}$,

$a, b \in \mathbb{Z}$, and that this fraction is written in its simplest terms.

Then $2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2$ i.e. a^2 is even.

If a^2 is even then a must also be even, i.e. $a = 2c$, $c \in \mathbb{Z}$.

Then $a^2 = 4c^2$, $2b^2 = 4c^2$, $b^2 = 2c^2$ i.e. b^2 is even, and therefore b is also even.

But this contradicts the assumption that $\frac{a}{b}$ was a fraction written in its simplest terms.

Therefore the original statement that $\sqrt{2}$ is irrational is true.

6 Assume there does exist a greatest even number $2n$. Now consider $2n + 2$.

$2n + 2 > 2n$ and also $2n + 2 = 2(n + 1)$ so it is also even. This contradicts our assumption that $2n$ was the greatest even number, and we have therefore proved that there is no greatest even number.

7 Assume that there does exist a greatest odd number $2n + 1$. Now consider $2n + 3$.

$2n + 3 > 2n + 1$ and also $2n + 3 = 2(n + 1) + 1$ so it is also odd. This contradicts our assumption that $2n + 1$ was the greatest odd number, and proves that there is no greatest odd number.

8 Assume that there does exist a greatest rational number $q = \frac{a}{b}$, $a, b \in \mathbb{Z}$.

Now consider $q + 1 = \frac{a}{b} + 1 = \frac{a + b}{b}$.

$a, b \in \mathbb{Z} \Rightarrow a + b \in \mathbb{Z}$. $\therefore \frac{a + b}{b} \in \mathbb{Q}$ and

$\frac{a + b}{b} > q$. This contradicts the assumption

that q was the greatest rational number, and proves that there is no greatest rational number.

9 Assume that there does exist a smallest

positive rational number $q = \frac{a}{b}$, $a + b \in \mathbb{Z}^+$.

Now consider $\frac{a}{b + 1}$. $b \in \mathbb{Z}^+ \Rightarrow b + 1 \in \mathbb{Z}^+$,

so $\frac{a}{b + 1} \in \mathbb{Q}^+$, and $\frac{a}{b + 1} < \frac{a}{b}$.

This contradicts the assumption that q was the smallest positive rational number, and proves that there is no smallest positive rational number.

10 Let a^2 and b^2 be odd, and assume that $a^2 + b^2 = c^2$ for some $c \in \mathbb{Z}$.

Now a^2, b^2 odd $\Rightarrow a, b$ also odd

i.e. $a = 2m + 1$ and $b = 2n + 1$

$a^2 = 4m^2 + 4m + 1$ $b^2 = 4n^2 + 4n + 1$

Then $a^2 + b^2 = 4m^2 + 4m + 1 + 4n^2 + 4n + 1$
 $= 2(2m^2 + 2m + 2n^2 + 2n + 1)$
 $= c^2$, so c^2 is even.

Now, if c^2 is even then c must also be even, i.e. $c = 2k$ and $c^2 = 4k^2$, so c^2 is in fact a multiple of 4.

But $c^2 = 4m^2 + 4m + 4n^2 + 4n + 2$
 $= 4(m^2 + m + n^2 + n) + 2$

which is **not** a multiple of 4.

This contradiction proves that the original statement that the sum of any two odd square numbers cannot itself be a square number must be true.

11 Let $x, y \in \mathbb{Z}^+$. Then $x + y > 0$ and $2\sqrt{xy} > 0$. Now assume $x + y < 2\sqrt{xy}$. Since both sides of the inequality are positive, we can square each side to get

$$(x + y)^2 < 4xy$$

$$x^2 + 2xy + y^2 < 4xy$$

$$x^2 - 2xy + y^2 < 0$$

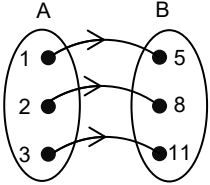
$$(x - y)^2 < 0$$

But no square can be negative.

This contradiction shows that

$x + y \geq 2\sqrt{xy} \forall x, y \in \mathbb{Z}^+$.

ERRORS IN TEXT

Page	Exercise	Question	Should be	Not
5	Functions & Mappings	Diagram	Type 1 	(Current diagram very confused!)
129	Ex 6E	15b	Find the smallest possible positive value of ...	Find the smallest possible value of ...
244	Ex 11D	6b	$-\left(\frac{2x+1}{4e^{2x}}\right)+c$	$\left(\frac{2x+1}{4e^{2x}}\right)+c$