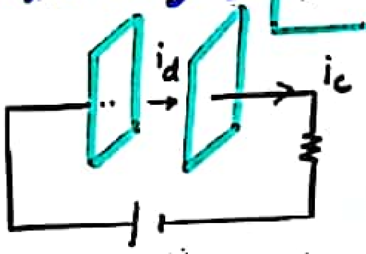
EM WAVES

Displacement current:-

Current b/w plates of capacitor while charging or discharging $\boxed{i_d = \epsilon_0 \frac{d\phi_E}{dt}}$



as capacitor gets charged E b/w plates & hence ϕ through plate = constant

$\therefore i_d = 0$

Imp points :-

1) Conduction current i_c & displacement current i_d are equal in mag $i_c = i_d$

Proof :- $i_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d(EA)}{dt}$

$= \epsilon_0 \frac{d}{dt}\left(\frac{\sigma}{\epsilon_0} A\right) = \frac{d(\sigma A)}{dt} = \frac{dq}{dt} = i_c$

ii) $i_c = 0$ where $i_d \neq 0$ and $i_d = 0$ where $i_c \neq 0$

iii) i_c and i_d together provide continuity to circuit

B at a very small distance r from axis of plates $= \frac{\mu_0 I_d}{2\pi r}$

Maxwell's equations:-

These four Eqns prove that Electric and magnetic field co-exist in space in form of EM Waves

1) Gauss Law in Electrostatics

$\oint \vec{E} \cdot d\vec{s} = q_{enclosed}/\epsilon_0$

ii) Gauss Law in magnetism

$\oint \vec{B} \cdot d\vec{s} = 0$

iii) Ampere circuital Law

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left[i_c + \epsilon_0 \frac{d\phi_E}{dt}\right]$

iv) Faradays Law of EMI

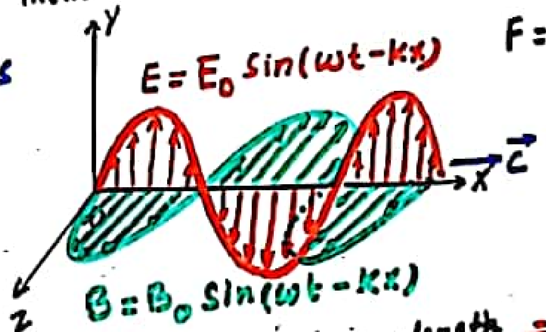
$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left[\int \vec{B} \cdot d\vec{s}\right]$

EM WAVES & their Properties:-

1) In lab EM Waves can be produced by accelerated or retarded or oscillating charge (Hertz Experiment)

2) The frequency of EM Waves produced in Hertz experiment is $f = \frac{1}{2\pi\sqrt{LC}}$

3) EMW are transverse waves composed of sinusoidally varying Electric and magnetic fields vibrating in $\perp$ directions and the direction of propagation of EM waves is $\perp$ to both of them.



4) The frequency/wave length of variation of E and B are equal to that of EM waves

5) They exert pressure on the surface on which they are incident and this pressure is called radiation pressure

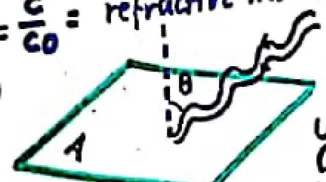
6) They propagate with speed of light $c_0 = \frac{E}{B} = \frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8$ m/s (vacuum)

In other media,

$c = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \mu_r \epsilon_r}} = \frac{c_0}{\sqrt{\mu_r \epsilon_r}}$

$n = \frac{c}{c_0}$ = refractive index of medium $= \sqrt{\mu_r \epsilon_r}$

7)



Intensity $I = \frac{dU}{Adt}$

U = Energy (per photon) of EMW $= h\nu = mc^2$

i) If EMW are reflected completely from surface then momentum transferred

Incident normal to surf.

$\star \Delta p = \frac{2U}{c}$ $\therefore$ force exerted

$F = \frac{dp\cos\theta}{dt} = \frac{dp}{dt}\cos\theta = \frac{2}{c}\frac{dU}{dt}\cos\theta$

$F = \frac{2}{c} IA\cos^2\theta$ $\therefore$ radn. pr. $\frac{dF}{dA} = \frac{2I\cos^2\theta}{c}$

ii) If EMW are completely absorbed from surface

$\Delta p = U/c$, $F = \frac{IA\cos^2\theta}{c}$, $\frac{dF}{dt} = \frac{I\cos^2\theta}{c}$

Energy Density of EMW:-

av. energy density of EMW

$u_{av} = u_E + u_B$

$u_{av} = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$

$u_{av} = \frac{1}{2}\epsilon_0 E^2 = B^2/2\mu_0$

but $u_E = u_B = \frac{1}{2}\epsilon_0 E^2 = B^2/2\mu_0$

$E, B \rightarrow$ rms values

$u_{av} = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2} B^2/\mu_0$

$u_{av} = \frac{1}{4}\epsilon_0 E_0^2 + \frac{1}{4} B_0^2/\mu_0$

$u_{av} = \epsilon_0 E^2 = B^2/\mu_0$

$u_{av} = \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2} B_0^2/\mu_0$

poynting vector:- ($\vec{S}$)

$|\vec{S}|$ = intensity $I_{av} = u_{av} \cdot c$

direcn. of $\vec{S}$ along wave propagation $\hat{S} = \frac{\vec{E}\times\vec{B}}{|\vec{E}\times\vec{B}|}$

$\vec{S} = I \frac{\vec{E}\times\vec{B}}{|\vec{E}\times\vec{B}|}$

Power of radiation

$P_{av} = \frac{dU}{dt}$ $\therefore I = \frac{P_{av}}{4\pi r^2}$

r = dist from source

E M WAVES

$I_{av} = u_{av} \cdot c = \frac{P_{av}}{4\pi r^2} \propto \frac{1}{r^2}$

for a 3D propagating wave

$I_{av} = u_{av} \cdot c = \frac{P_{av}}{2\pi r l} \propto \frac{1}{r}$

for a linearly propagating wave (produced from a linear source)

$I_{peak} = u_{peak} \cdot c$

$= 2u_{av} \cdot c$

$$\boxed{\begin{aligned} u_{E\,peak} &= \epsilon_0 E^2 = \tfrac{1}{2}\epsilon_0 E_0^2 \\ u_{B\,peak} &= B^2/\mu_0 = \tfrac{1}{2} B_0^2/\mu \\ u_{net\,peak} &= 2\epsilon_0 E^2 = 2B^2/\mu_0 \\ &= \epsilon_0 E_0^2 = B_0^2/\mu_0 \end{aligned}}$$

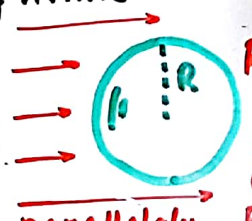
Relation b/w Range of transmission and ht. of antenna :-

$d = \sqrt{2R_e h}$ → ht. of antenna

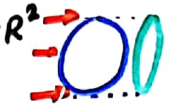
R_e → radius of earth

no. of viewers $= \pi d^2 \times$ population density

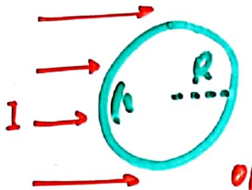
Eg Radiation of int. I is falling on a sphere parallelaly. Find force exerted on sphere assuming complete absorption



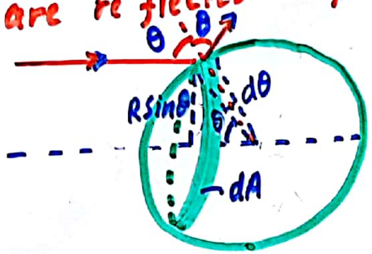
Ans $F = \frac{I}{c} \cdot A_{projected} = \frac{I\pi R^2}{c}$

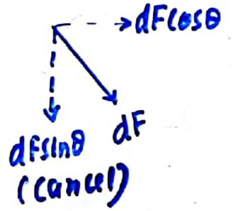


Eg Find force on sphere exerted by parallel beam of intensity I, if radiations are reflected completely



Ans:-



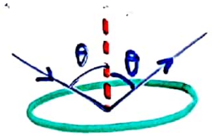


$F_{net} = \int dF \cos\theta \quad -(1)$

$dA = (2\pi R\sin\theta)\, R\,d\theta$

$dA = 2\pi R^2 \sin\theta\, d\theta$

$dF = \frac{2I\cos^2\theta}{c}\, 2\pi R^2 \sin\theta\, d\theta$



here,

no. of photons/sec

$\frac{dn}{dt} = \frac{IA_\perp \lambda}{hc} = \frac{IA_\perp}{h\nu}$

$\frac{dn}{dt} = \frac{IA\cos\theta\,\lambda}{hc}$

$F = \frac{dP}{dt}\cos\theta = \frac{2}{c}\frac{dU}{dt}\cos\theta \quad \left\{\frac{dU}{dt} = \frac{dn}{dt} h\nu\right\}$

$F = \frac{2}{c}(IA\cos\theta)\cos\theta \qquad \boxed{F = \frac{2IA\cos^2\theta}{c}}$

now $F_{net} = \int dF\cos\theta$

$= \frac{4I\pi}{c} R^2 \int_0^{\pi/2} \cos^3 d\theta \sin\theta\, d\theta$

Let $\cos\theta = z$

$-\sin\theta\, d\theta = dz$

$F = -\frac{4\pi IR^2}{c}\int_1^0 z^3 dz$

$F = \frac{4\pi IR^2}{c}\left[\frac{z^4}{4}\right]_0^1$

$$\boxed{F = I\pi R^2/c}$$