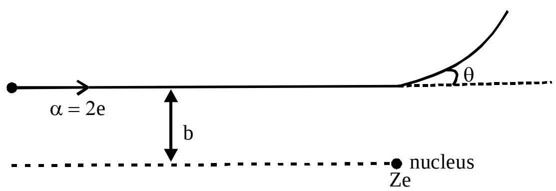
UNIT - 8 CHAPTER - 2

Atoms, Molecules & Nuclei

RUTHERFORD MODEL

Rutherford Model



- No. of particle scattered per u area—

$$N(\theta) \propto (z^2/r^2)\sin^4\left(\frac{\theta}{2}\right)$$

The angle of scattering θ and the impact parameter b are related as →

$$b = \frac{ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi\epsilon_0 E}$$

where K = KE of the α-particles smaller the impact parameter (b) larger is θ.

Impact parameter–It is the closest distance between nucleus and α-particle in absence of scattering.

BOHR'S ATOMIC MODEL

Bohr made following conclusions to support his atomic model.

CIRCULAR ORBITS

The atom consists of central nucleus, containing the entire positive charge and almost all mass of the atom. The electrons revolve around the nucleus in certain discrete circular orbits. The necessary centripetal force for circular orbit is provided by Coulomb's attraction between the electron and nucleus. So,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$$

where, m = mass of electron,

r = radius of circular orbit,

v = speed of electron in circular orbit,

Ze = charge on nucleus,

Z = atomic number,

e = charge on electron = -1.6×10^{-19}C

STATIONARY ORBITS

The allowed orbits for electron are those in which the electron does not radiate energy. These orbits are also called **stationary orbits**.

Quantum Condition (Bohr's Quantisation Rule)

The stationary orbits are those in which angular momentum of electron is an integral multiple of $\frac{h}{2\pi}(=\hbar)$ i.e., $mvr = n\left(\frac{h}{2\pi}\right)$, n being integer or the principle quantum number.

Stationary Nucleus

The nucleus is so heavy, that its motion may be neglected.

Constancy of Mass

The mass of the electron in motion is assumed to be constant.

Bohr's Transition Rule

When an electron jumps from one stationary orbit to another, a photon is emitted or absorbed having energy equal to the difference of energies between initial and final states and being given by

$$E_i - E_f = hv \quad \Rightarrow \quad v = \frac{E_i - E_f}{h}$$

RADIUS OF ORBIT

Since, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \quad \text{..... (1)}$$

and $mvr = \frac{nh}{2\pi}$ ..... (2)

From (2), $v = \frac{nh}{2\pi mr}$

Putting in (1), we get

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2Z} \quad \Rightarrow \quad r_n = (0.53)\frac{n^2}{Z}\text{Å}$$

So, for H-like atoms

$$r_n \propto \frac{n^2}{Z}$$

Velocity of Electron in *n*th Orbit

Since $v = \frac{nh}{2\pi mr}$

and $r = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$ $\Rightarrow$ $v = \left(\frac{e^2}{2h\varepsilon_0}\right)\frac{Z}{n}$

$\Rightarrow$ $v = \left(\frac{e^2}{2h\varepsilon_0 c}\right)\left(\frac{cZ}{n}\right)$ $\Rightarrow$ $v = \alpha\left(\frac{cZ}{n}\right)$

where $\alpha = \frac{e^2}{2h\varepsilon_0 c}$ is the fine structure constant (a pure number) whose value is $\frac{1}{137}$

$\Rightarrow$ $v = \left(\frac{1}{137}\right)\frac{cZ}{n}$

i.e. velocity of electron in Bohr's First Orbit is $\frac{c}{137}$, in Second Orbit is $\frac{c}{274}$ and so on.

Kinetic Energy of Electron (E_K)

Since, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$$

$\Rightarrow$ $\frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$ $\Rightarrow$ $E_K = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$

Potential Energy (U) of Electron in *n*th Orbit

$$u = -\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r} \quad \Rightarrow \quad u = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

Total Energy (E) of electron in *n*th Orbit

Total Energy = K.E. + P.E.

$\Rightarrow$ $E = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$ $\Rightarrow$ $E = \frac{-Ze^2}{8\pi\varepsilon_0 r}$

So, we conclude that

Total Energy = $-K.E. = \frac{1}{2}(P.E.)$

Further, since $r = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$

$\Rightarrow$ $E = -\left(\frac{me^4}{8h^2\varepsilon_0^2}\right)\frac{Z^2}{n^2}$ $\Rightarrow$ $E = (-13.6)\frac{Z^2}{n^2}eV$

Also, $E = -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right)ch\frac{Z^2}{n^2}$

$\Rightarrow$ $E = -(Rch)\frac{Z^2}{n^2}$

where R = Rydberg's constant

$$= \frac{me^4}{8\varepsilon_0^2 ch^3} = 1.097 \times 10^7\, \text{m}^{-1} \text{ and}$$

Rch = Rydberg's Energy $\simeq 2.17\times10^{-18}\, J \simeq 13.6eV$ is the electron energy in first orbit of H atom.

Frequency of emitted Radiation

If electron jumps from initial state n_i to a final state n_f, then frequency of emitted or absorbed radiation v is given by $E_i - E_f = hv$

or $v = \frac{E_i - E_f}{h} = Z^2 Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

If c is the speed of light and λ the wavelength of emitted or absorbed radiation, then

$$v = \frac{c}{\lambda} = Z^2 Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

So, wave number $(\overline{v})$ is given by

$$\overline{v} = \frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

HYDROGEN SPECTRUM

	Initial state	Final State	Wavelength formula	First Member-Second Member	Series Limit $n_i \to \infty$ To n_f	Maximum wavelength (n_1+1) To n_f	Lines found in
Lyman	n_i = 2, 3, 4, 5, 6,....	n_f=1	$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n_i^2}\right)$	n_i = 2 to n_f = 1 n_i = 3 to n_f = 1	From ∞ to 1 $\lambda=\frac{1}{R}$ λ = 911 Å	From 2 to 1 $\lambda=\frac{4}{3R}$ λ = 1216 Å	UV Region
Balmer	n_i = 3, 4, 5, 6, 7,....	n_f=2	$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n_i^2}\right)$	n_i = 3 to n_f = 2 n_i = 4 to n_f = 2	From ∞ to 2 $\lambda=\frac{4}{R}$ λ = 3646 Å	From 3 to 2 $\lambda=\frac{36}{5R}$ λ = 6563 Å	Visible Region
Paschen	n_i = 4, 5, 6, 7, 8,....	n_f=3	$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n_i^2}\right)$	n_i = 4 to n_f = 3 n_i = 5 to n_f = 3	From ∞ to 3 $\lambda=\frac{9}{R}$ λ = 8204 Å	From 4 to 3 $\lambda=\frac{144}{7R}$ λ = 18753 Å	IR Region
Brackett	n_i = 5, 6, 7, 8, 9,....	n_f=4	$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n_i^2}\right)$	n_i = 5 to n_f = 4 n_i = 6 to n_f = 4	From ∞ to 4 $\lambda=\frac{16}{R}$ λ = 14585 Å	From 5 to 4 $\lambda=\frac{400}{9R}$ λ = 40515Å	IR Region
Pfund	n_i = 6, 7, 8, 9, 10,....	n_f=5	$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n_i^2}\right)$	n_i = 6 to n_f = 5 n_i = 7 to n_f = 5	From ∞ to 5 $\lambda=\frac{25}{R}$ λ = 22790 Å	From 6 to 5 $\lambda=\frac{900}{11R}$ λ = 74583Å	Far IR Region

SUMMARY

BOHR'S MODEL

Spectral series :

Lyman : $\bar{v}_{lyman} = R\left[\frac{1}{1^2}-\frac{1}{n_2^2}\right]; n_2 > 1$

Balmer : $\bar{v}_{balmer} = R\left[\frac{1}{2^2}-\frac{1}{n_2^2}\right]; n_2 > 2$

Paschen : $\bar{v}_{paschen} = R\left[\frac{1}{3^2}-\frac{1}{n_2^2}\right]; n_2 > 3$

Brackett : $\bar{v}_{brackett} = R\left[\frac{1}{4^2}-\frac{1}{n_2^2}\right]; n_2 > 4$

P fund : $\bar{v}_{P\ fund} = R\left[\frac{1}{5^2}-\frac{1}{n_2^2}\right]; n_2 > 5$

- In all series :

  $n_2 = n_1 + 1 = \alpha$ line

  $= n_1 + 2 = \beta$ line

  $= n_1 + 3 = \gamma$ line

  α line has maximum λ

- Each series has a frequency limit $\bar{v}_{limit} = \frac{R}{n_1^2}$

  where $n_2 = \infty$
- An electron can be stable only in certain discrete circular orbits in which is doesn't radiate energy. In these orbitals $\boxed{mvr = nh/2\pi}$
- $\lambda = \frac{h}{mv}$ or $\frac{h}{\lambda} = mv = \frac{mvr}{r} = \frac{nh}{2\pi r}$ or $\boxed{2\pi r = n\lambda}$
- An electron absorbs or emits energys whenit quantum jumps from one orbit to another. The emited energy is $\boxed{E = nhv}$
- Based on these assumptions he derived for hydrogen atom (z = 1)

  (i) L= angular momentum $\boxed{nh/2\pi}$

(ii) $\boxed{r_{nth} = 0.53n^2}\ \mathring{A}$ {0.53 Å is called Bohr's radius)

(iii) $\boxed{E_n = -Rhc/n^2}$ where Rhc = 13.6 eV

(iv) Velocity $\boxed{V_n = \frac{2.2 \times 10^6 \times z}{n} cm}$

(v) Frequency $\propto \frac{1}{r^{3/2}}$ or $T \propto r^{3/2}$

The unit for energy = $2.17 \times 10^{-18} J$ = the Binding Energy. of an electron in the innermost orbit in a hydrogen atom i.e. (n =1) = 1 Rydberg

1 Ryd = $2.17 \times 10^{-18} J = 13.6\ eV$

Net Energy is –ve means it is bound

$\boxed{\text{Binding Energy} = -\text{Net Energy}}$

(vi) $E_{n_2 \to n_1}$ (emitted energy when it jumps from n_2 to n_1 th orbit)

$$= E_{n_2} - E_{n_1} = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = eV = hv$$

(vii) $\bar{v} = \frac{1}{\lambda}$ = wave no. and $\bar{v} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

where R = $1.097 \times 10^7\ m^{-1}$ rydberg's constt of H_2

(viii) However, if Bohr's model is applied to a hydrogen like atom e.g. He^+, Li^{++}, Be^{+++} etc. Now, for an electron in nth orbit of atom having atomic no. z.

- $r_{n,z} = 0.53n^2/z$ A°
- $E_n = \frac{13.6}{n^2} z^2 (eV)$
- $R_z = z^2 \times 1.097 \times 10^7 m^{-1}$
- $v_{n,z} = 2.2 \times 10^6 \times \frac{z}{n} m/s$
- frequency $\propto \frac{z^2}{n^3}$
- **Extra points :** Excitation potential for quantum jump $n_1 \to n_2 = \frac{(E_{n_2} - E_{n_1})}{(electrons\ volts)}$
- Ionisation potential from ground state

$$= \frac{\text{ground state BE}}{e} = \frac{13.6}{e} z^2\ (eV)$$

- Electrons: $KE = e^2/8\pi\varepsilon_0 r$, $PE = -e^2/4\pi\varepsilon_0 r$, $TE = -e^2/8\pi\varepsilon_0 r$; $TE = -KE = \frac{PE}{2}$
- Velocity of electron $V = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}}$

$\Rightarrow V \propto \frac{1}{\sqrt{r}}$

- Linear momentum

$p = mv \Rightarrow p \propto \frac{1}{\sqrt{r}}$

- Angular momentum $L = mvr \propto \sqrt{r}$

Example 1. Calculate the energy of a He^+ ion in its first excited state.

Solution. E_2 = –13.6 eV

Example 2. Calculate the wavelength of radiation emitted when He^+ makes a transition from the state n = 3 to the state n = 2.

Solution. $\lambda = \frac{9}{5R}$ = 164.0 nm

Example 3. If the difference of energies of an electron in the second and the fourth orbits of an atom is E. find the ionisation energy of that atom.

Solution. $13.6\ eV \times Z^2 = \frac{16E}{3}$

Example 4. Find the ratio of magnetic movement of an electron to its angular momentum in an atomic orbit.

Solution. $\frac{M}{L} = \frac{eVr}{2 \times mVr}$

$\frac{M}{L} = \frac{e}{2m}$

Example 5. The excitation energy of a hydrogen - like ion in its first excited state is 4.08 eV. Find the energy needed to remove the electron from the ion.

Solution. E = –54.4 eV

Thus 54.4 eV is required to remove the electron from the ion.

Example 6.(a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(b) How many spectral lines are observed in the emission spectrum of the above excited system?

Solution. (a) $\lambda = 114\mathring{A}$

(b) The spectral lines emitted are due to the transition $n = 3 \rightarrow n = 2$, $n = 3 \rightarrow n = 1$ and $n = 2 \rightarrow n = 1$, thus there will be three spectral lines.

Example 7. How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n?

Solution. The total number of possible transitions is

$(n–1) + (n–2) + (n–3) + ... 2+1$

No. of transitions $= \frac{n(n-1)}{2}$

Example 8. Find the radius of Li^{++} ions in its ground state assuming Bohr's model to be valid.

Solution. $r_1 = \frac{1^2 \times 53 \times 10^{-12}}{3}$ m

$r_1 = 18 \times 10^{-12}$ m

Example 9.(a) Find the maximum wavelength λ_0 of light which can ionize a hydrogen atom in its ground state.

(b) Light of wavelength λ_0 is incident on a hydrogen atom which is in its first excited state. Find the kinetic energy of the electron coming out.

Solution. (a) $\frac{hc}{\lambda_0} = 13.6\text{eV}$

$\lambda_0 = 910$ Å

(b) K.E. = 10.2 eV

Example 10. Which state of the triply ionized Be^{+++} has the same orbital radius as that of the ground state of hydrogen? Find the ratio of energies of two states.

Soluion. $\frac{E(Be)}{E(H)} = \frac{\left(\frac{Z^2}{n^2}\right)_{Be}}{\left(\frac{Z^2}{n^2}\right)_H} = \frac{16/4}{1/1}$

$\frac{E(Be)}{E(H)} = 4$

Example 11. The total energy of electron in the first excited state of hydrogen is about –3.4eV. Find kinetic energy and potential energy of electron in this state.

Solution. K.E. = 3.4 eV

P.E. = –2 × (3.4eV) = –6.8eV

X-RAYS

Production

When fast moving electrons strike a target of high melting point and high atomic weight (like tungsten, platinum molybdenum), X-rays are produced. A modern X-ray tube consists of

- An electron source, preferably a filament heated by the passage of an electric current which may be varied.
- A heavy target of high melting point inclined at 45° to the path of electron beam, kept cooled by circulating cold water internally.
- A source of high potential difference applied across the filament and the target, keeping target positive with respect to filament. When the filament is heated, a fine beam of electrons strikes the target to produce X-rays.

Control of Intensity

The intensity of incident electrons determines the intensity of X-rays, *i.e.*, greater is the number of electrons striking the target, more intense are the X-rays produced. (Voltage and hence current in filaments circuit is increase, l_{min} remains unchanged)

Control of Penetrating Power

The potential difference across the filament and target determines the energy and hence the penetrating power of X-rays.

Need of Cooling Device

Only about 1% of incident electron's energy is converted into X-rays and the remaining 99% is converted into heat, therefore cooling device is essential with an X-ray tube.

Hard and Soft X-rays

X-rays upto 4 Å have high penetrating power and are called **hard X-rays** while those of $\lambda > 4$ Å are called soft **X-rays**.

Hard X-rays $\rightarrow$ low $\lambda \rightarrow$ high $v \rightarrow$ high penetration

soft X rays $\rightarrow$ High λ $\rightarrow$ low

v $\rightarrow$ low penetration.

Absorption of X-rays

When a beam of X-rays passes through a material of thickness x, the intensity of transmitted X-rays is given by $I = I_0 e^{-\mu x}$

μ = X ray absorption coefficient of material. Absorption is greater for materials having high at.no.

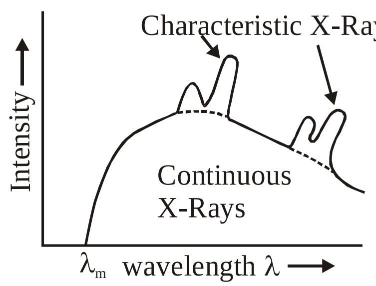
I_0 = intensity of falling X-ray

X-RAY SPECTRA

A detailed study of X-ray spectra reveals two distinct types of spectra:

Continuous Spectrum

It consists of radiations of all possible wavelengths within a definite wavelength range having a definite short wavelength limit. These are produced due to deceleration of electrons near heavy nucleus. These decelerated electrons emit radiations which lie in X-ray region. As the electrons suffer collisions at all angles, right from the glancing collision to the direct hit, they suffer varying decelerations and hence radiations of all possible wavelengths within a certain range are emitted, forming the continuous spectrum.



The maximum limiting frequency v_{max} or minimum limiting wavelength λ_{min} is obtained when entire kinetic energy of bombarding electron is converted to X-ray energy. If v_0 is the accelerating potential, then

$$eV_0 = hv_{max} \text{ or } v_{max} = \frac{eV_0}{h}$$

In terms of λ_{min}

$$eV_0 = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV_0}$$

$$\lambda_{min} = \frac{hc}{eV_0} = \frac{12400}{V_0(\text{involts})} \text{Å} \rightarrow$$ "Duane Hunt relation"

This relation gives the short wavelength limit of continuous X-ray spectrum.

Note: If the bombarding electrons knocks off an electron from the K shell and if the vacancy is filled in from :

(i) L shell : $hv_{k\alpha} = E_L - E_K \rightarrow$ this spectral line is called K_α line

(ii) M shell : $hv_{k\beta} = E_M - E_K \rightarrow$ this spectral line is called K_β line

Characteristic Spectrum

It consists of radiations of definite wavelengths superimposed on continuous spectrum. These lines are characteristic of the material used as the target.

When the fast moving electrons knock off bound electrons from the inner orbits of an atom, the vacancies so caused are filled up either by free electrons from space or bound electrons from outer orbits, thus releasing energy in the form of X-rays, having wavelengths characteristics of target atom. According to Moseley, for a given transition, the frequencies of characteristic lines emitted by different elements obey the relation

$$\sqrt{v} = a(Z - \sigma)$$

$$\boxed{\frac{v_1}{v_2} = \left(\frac{z_1 - 1}{z_2 - 1}\right)^2 = \frac{\lambda_2}{\lambda_1}}$$

also called **Moseley's Law**.

where a and σ are constants for a given transition.

σ = 1 for K_α - radiations of all substances.

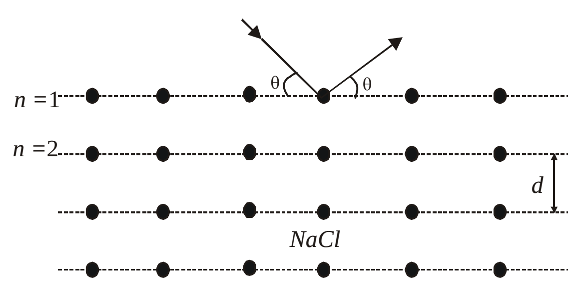
σ = 7.4 for L_α - radiations of all substances.

BRAGG'S LAW

Solid crystals eg. NaCl act as diffraction grating as inter atomic spacing is comparable to wavelength of X rays. By using monochromatic X rays beam and noting the angles of strong reflection, interplanar spacing d, several information can be obtained about structure of the solid.

Condition for strong reflection of X-rays :

$$\boxed{2d \sin\theta = n\lambda}$$



n = order of diffraction

Example 12. Find the maximum frequency of the X-rays emitted by an X-ray tube operating at 30 kV.

Solution. $\frac{30}{4.14} \times 10^{18}\,\text{Hz}$ = 7.2 × 10^{18} Hz.

Example 13. An X-ray tube is operated at 20 Kv and the current through the tube is 0.5 mA. Find (a) the number of electrons hitting the target per second (b) the cut off wavelength of the X-rays emitted.

Solution. (a) 3.1 × 10^{15} per sec

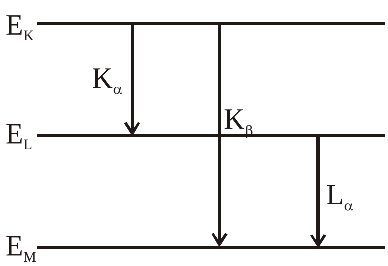
(b) $\lambda_{\min} = 0.6188$ Å

Example 14. Show that the frequency of K_β X-ray of a material equals the sum of the frequencies of K_α and L_α X-rays of the same material.

Solution. Energy of K_α X-ray is

$$E_{K_a} = E_L - E_K$$

Energy of K_β X-ray



$$E_{Kb} = E_M - E_K$$

and Energy of L_α X-ray

$$E_{La} = E_M - E_L$$

Thus, $E_{K\beta} = E_{k\alpha} + E_{L\alpha}$ or $h\nu_{K\beta} = h\nu_{K\alpha} + h\nu_{L\alpha}$

$$\nu_{K\beta} = \nu_{K\alpha} + \nu_{L\alpha}$$

$$l_{kb} = \frac{l_{ka}\, l_{La}}{l_{ka} + l_{La}}$$

Example 15 :

Calculate the energy of a He^+ ion in its first excited state.

Solution :

The energy is $E_n = \frac{-RhcZ^2}{n^2} = \frac{-(13.6eV)Z^2}{n^2}$

For He^+ ion, $Z = 2$ and for the first excited state, n = 2 so that the energy of He^+ ion.

$$E_2 = \frac{-13.6\text{eV} \times 2^2}{2^2}$$

$$E_2 = -13.6 \text{ eV}$$

Example 16 :

Calculate the wavelength of radiation emitted when He^+ makes a transition from the state n = 3 to the state n = 2.

Solution :

The wavelength λ is given by $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$n_i = 3$, $n_f = 2$, Z = 2, R = 1.097×10^7/m

$\Rightarrow \frac{1}{\lambda} = 4R\left(\frac{1}{4} - \frac{1}{9}\right)$ $\Rightarrow$ $\frac{1}{\lambda} = \frac{5}{9}R$

$\Rightarrow \lambda = \frac{9}{5R}$ = 164.0 nm

Example 17 :

Find the ratio of magnetic movement of an electron to its angular momentum in an atomic orbit.

Solution :

Magnetic moment M = IA

$$M = \frac{e}{T} \cdot \pi r^2 \quad \text{..... (1)}$$

We know that velocity of electron $V = \frac{2\pi r}{T}$ or

$$T = \frac{2\pi r}{\text{V}}$$

Putting the value of T in equation (1)

$$\Rightarrow \text{M} = e\frac{\pi r^2}{2\pi r / \text{V}}$$

$$\text{M} = \frac{e\text{V}r}{2}$$

Angular momentum of an electron in n^{th} orbit

L = mVr

$\therefore \frac{\text{M}}{\text{L}} = \frac{e\text{V}r}{2 \times m\text{V}r}$ $\qquad \frac{\text{M}}{\text{L}} = \frac{e}{2m}$

Example 18:

(a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(b) How many spectral lines are observed in the emission spectrum of the above excited system?

Solution :

(a) The energy in the first orbit

$$E_1 = Z^2 E_0$$

where E_0 = –13.6eV is the energy of a hydrogen atom in ground state. Thus for Li^{++}, Z = 3

$E_1 = 9E_0 = 9 \times (-13.6\text{eV})$

The energy in the third orbit is

$$E_3 = \frac{E_1}{n^2} \quad = \frac{E_1}{9} = -13.6\text{eV}$$

Thus, $E_3 - E_1 = 8 \times 13.6\text{eV} = 108.8\text{eV}$

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1$$

$$\lambda = \frac{hc}{E_3 - E_1}$$

$$\lambda = \frac{12375}{108.8\text{eV}}(\text{Å})$$

$$\lambda = 114\text{Å}$$

(b) The spectral lines emitted are due to the transition $n = 3 \rightarrow n = 2$, $n = 3 \rightarrow n = 1$ and $n = 2 \rightarrow n = 1$, thus there will be three spectral lines.

Example 19 :

How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number *n*?

Solution :

From the *n*th state, the atom may go to (*n*–1)th state, ..., 2nd state or Ist state. So there are (*n*–1) possible transitions starting from the *n*th state. The atoms reaching (*n*–1)th state may make (*n*–2) different transitions. Similarly for other lower states. The total number of possible transitions is

$(n-1) + (n-2) + (n-3) + \dots\dots\dots 2+1$

$$\text{No. of transitions } = \frac{n(n-1)}{2}$$

Example 20 :

Find the radius of Li^{++} ions in its ground state assuming Bohr's model to be valid.

Solution :

For hydrogen-like ions, the radius of the n^{th} orbit is

$$r_n = \frac{n^2 r_0}{Z}$$

For Li^{++}, Z = 3 and in ground state *n* = 1

The radius is

$$r_1 = \frac{1^2 \times 53 \times 10^{-12}}{3}\text{m}$$

$$r_1 = 18 \times 10^{-12}\text{ m}$$

Example 21 :

Which state of the triply ionized Be^{+++} has the same orbital radius as that of the ground state of hydrogen? Find the ratio of energies of two states.

Solution :

Radius of n^{th} orbit is given by $r = \dfrac{n^2h^2}{4\pi^2 m K Z e^2}$

$$r \propto \frac{n^2}{Z}$$

For hydrogen Z = 1, *n* = 1 in ground state.

$$\frac{n^2}{Z} = \frac{1}{1} = 1$$

For Beryllium, Z = 4 as orbital radius is same

$$\frac{n^2}{Z} = 1$$

$n^2 = Z$

$n^2 = 4$

$n = 2$

Hence *n* = 2 level of Be^{+++} has same radius as *n* = 1 level of hydrogen.

Now energy of electron in *n*th orbit

$$E_n = \frac{-13.6(\text{eV})Z^2}{n^2}$$

Ratio of energy of Be to that of hydrogen

$$\frac{E(Be)}{E(H)} = \frac{\left(\frac{Z^2}{n^2}\right)_{Be}}{\left(\frac{Z^2}{n^2}\right)_H} = \frac{16/4}{1/1}$$

$$\frac{E(Be)}{E(H)} = 4$$

Example 22 :

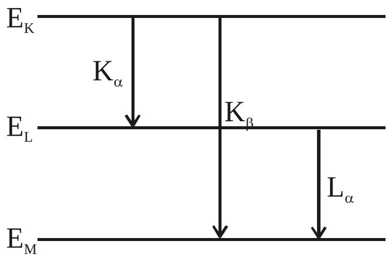
Show that the frequency of K_β X-ray of a material equals the sum of the frequencies of K_α and L_α X-rays of the same material.

Solution :

Energy of K_α X-ray is

$$E_{K_\alpha} = E_K - E_L$$

Energy of K_β X-ray



$$E_{K\beta} = E_K - E_M$$

and Energy of L_α X-ray

$$E_{L\alpha} = E_L - E_M$$

Thus, $E_{K\beta} = E_{k\alpha} + E_{L\alpha}$

or $h\nu_{K\beta} = h\nu_{K\alpha} + h\nu_{L\alpha}$

$$\nu_{K\beta} = \nu_{K\alpha} + \nu_{L\alpha}$$

TEST YOUR SELF - 1

1. The ratio of the radii of sulphur and helium atoms in the ground state will be
   (1) 1 : 8 (2) 1 : 4
   (3) $1:\sqrt{8}$ (4) 1 : 3
2. The wavelength of spectral line due to electron transition is inversely proportional to
   (1) The difference of energies associated with the transition levels
   (2) To the nuclear charge of atom
   (3) The number of electrons taking part in transition
   (4) The velocity of electrons taking part in transitions
3. There are seven orbitals in a subshell then the value of l for it will be
   (1) $l = 4$ (2) $l = 3$
   (3) $l = 2$ (4) $l = 1$
4. In Rutherford's scattering experiment of α -particles by metallic foils, with the increase of atomic number of nucleus, the scattering angle
   (1) Remains unchanged (2) Decreases
   (3) Increases (4) None of these
5. Which of the following statements is correct for hard X-rays ?
   (1) Penetrating power is more and wavelength is less than that of soft X-rays
   (2) Penetrating power is more and wavelength is more than that of soft X-rays
   (3) Penetrating power is equal to that of soft X-rays and wavelength is less than that of soft X-rays
   (4) Penetrating power is equal to that of soft X-rays wavelength is more than that of soft X-rays

ANSWER

1. (1) 2. (1) 3. (2) 4. (3)
5. (1)

NUCLEAR STRUCTURE

The nucleus of an atom consists of two types of particles, protons and neutrons. A proton has a positive charge equal to 1.6×10^{-19} C and a mass equal to 1.6726×10^{-27} kg. A neutron has no charge and its mass is 1.6749×10^{-27} kg. Thus a neutron is slightly heavier than a proton. Protons or neutrons, being the particles present inside nucleus, together are called **Nucleons**.

The total number of protons in the nucleus is called its **atomic number (Z)**. The total number of nucleons in the nucleus is called its **mass number (A)**. If N is the number of neutrons, then,

$$A = Z + N$$

No electrons are present inside the nucleus.

If X is the chemical symbol for an element then its nucleus is represented as ${}^{A}_{Z}X$ or as ${}_{Z}X^{A}$

Isotopes

Nuclides having the same charge number (Z) but different mass number (A) are called **isotopes**. All the isotopes are chemically similar and hence they occupy the same position in the periodic table.

Isobars

Nuclides having the same mass number (A) but different atomic number (Z) are called **isobars**.

Isotones

Nuclides having the same neutron number (A–Z) but different mass number (A) are called **isotones**.

Atomic Mass Unit (u or amu)

Atomic and nuclear masses are generally expressed in terms of atomic mass unit (a.m.u.)

$$1u = \frac{1}{12}\text{ (mass of one atom of }{}_6C^{12}) = 1.66056 \times 10^{-27}\text{ kg}$$

Nuclear Radius

Assuming that the nuclei are spherical, their radii are well represented by the empirical formula

$$R = R_0 A^{1/3}$$

where $R_0 = 1.1\times10^{-15}$ m = 1.1 fermi (fm)

Nuclear Density

The density of a nucleus of mass M and mass number A can be written as

$$\rho=\frac{M}{\frac{4}{3}\pi R^3}=\frac{A\,amu}{\frac{4}{3}\pi\left(R_0A^{1/3}\right)^3}=\frac{\left(A\times1.67\times10^{-27}\right)kg}{\frac{4}{3}\pi\left(1.1\times10^{-15}m\right)^3A}$$

$$\Rightarrow\quad \rho\cong2.9\times10^{17}\,kgm^{-3}$$

This comes out to be ~10^{17} kgm^{-3}, which is extremely large as compared to the density of ordinary matter which is ~10^3 kgm^{-3}.

Note : Since density is independent of mass number A, so all nuclei same density. So, whether two nuclei are isobars, or isotopes or isotones. They must possess same density.

THE NUCLEAR FORCE

The force which binds the protons and neutrons inside the nucleus is neither electrical nor gravitational. It is an entirely different kind of force called the **strong nuclear force**. This force is extremely complex in nature. Some of its main characteristics are mentioned below.

- Nuclear forces are attractive in nature. Their magnitude, which depends upon inter nucleon distance is of very high order.
- Nuclear forces are charge independent. Nature of force remains the same whether we consider force between two protons, between two neutrons or between a proton and a neutron.
- These are short range forces. Nuclear forces operate between two nucleons situated in close neighbourhood only.
- Nuclear forces decrease very quickly with distance between two nucleons. Their rate of decrease is much more rapid than that of inverse square law forces (Coulombic forces). The forces become negligible when the nucleons are more than 10^{-14} cm apart.
- Nuclear forces are spin dependent. Nucleons having parallel spin are more strongly bound to each other than those having anti-parallel spin.

YUKAWA THEORY OR MESON THEORY OF NUCLEAR FORCE

According to this theory, a nucleon consists of a core surrounded by a cloud of mesons, which may be charged or neutral. The mesons constantly get exchanged, back and forth, between two neighbouring nucleons. In this process the two nucleons remain bound to each other.

Proton-proton interaction

It is the force between two neighbouring protons. It is due to the exchange of π^0 meson between them. It is represented in the form of a reaction as follows.

Proton P_1 emits π^0 and gets converted into a proton P_1', having different co-ordinates. So,

$$P_1 \longrightarrow P_1' + \pi^0$$

This π^0 is absorbed by P_2 which also gets converted into a new proton P_2'. Hence,

$$P_2 + \pi^0 \longrightarrow P_2'$$

Neutron-Neutron interaction

It is the force between two neutrons. It is also due to exchange of π^0 between them

$$N_1 \longrightarrow N_1' + \pi^0$$

$$N_2 + \pi^0 \longrightarrow N_2'$$

Proton-Neutron Interaction

It is the force between a proton and a neutron situated close to each other. It can take place in following two ways.

(a) Due to exchange of π^+ meson.

Proton emits π^+ meson and gets converted to a neutron. Another neutron absorbs this π^+ meson to get converted to a proton. So,

$$P \longrightarrow N' + \pi^+$$

$$N' + \pi^+ \longrightarrow P'$$

(b) Due to exchange of π^- meson.

π^- meson is emitted by the neutron which is absorbed by the proton.

$$N \longrightarrow P' + \pi^-$$

$$P + \pi^- \longrightarrow N'$$

RADIOACTIVITY

The phenomenon of spontaneous emission of radiations (α,β,γ etc.) by certain nuclei is called **radioactivity**.

The phenomenon of radioactivity was discovered by **Bacquerel**.

Law of Radioactive Disintegration

Rutherford-Soddy Laws

(a) Radioactivity is nuclear disintegration phenomenon. It is independent of all physical and chemical conditions.

(b) The disintegration is random and spontaneous statistical process. It is a matter of chance for any atom to disintegrate first.

(c) The radioactive substances emit α or β particles along with γ -rays. These rays originate from the nuclei of disintegrating atoms and form fresh radioactive products with different physical and chemical properties.

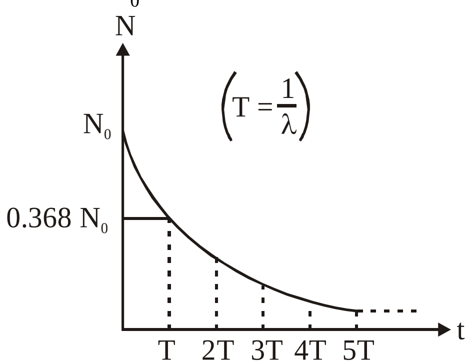
The rate of decay of nuclei $\left(-\frac{dN}{dt}\right)$ is directly proportional to the number of undecayed nuclei (N) in the sample at time *t*.

$$\Rightarrow -\frac{dN}{dt} \propto N \qquad \Rightarrow \qquad \frac{dN}{dt} = -\lambda N$$

where λ is constant of proportionality called **Decay Constant** or **Disintegration Constant**.

$$\Rightarrow \int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \qquad \Rightarrow \qquad \log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow N = N_0 e^{-\lambda t}$$



where

N = Number of undecayed nuclei in the sample at time t.

N_0 = Number of undecayed nuclei in the sample at time t = 0 (initially)

So, we conclude that the number of undecayed nuclei in the sample decays exponentially with time.

Displacement Laws

(a) When a nuclide emits α -particle, its mass number is reduced by four and atomic number by two, i.e.

$$_zX^A \longrightarrow {}_{z-2}Y^{A-4} + {}_2He^4 + \text{Energy}$$

(b) When a nuclide emits a β -particle, its mass number remains unchanged but atomic number increases by one, i.e.,

$$_zX^A \longrightarrow {}_{z+1}Y^A + {}_{-1}\beta^0 + \bar{v} + \text{Energy}$$

where $\bar{v}$ is the antineutrino

The β -particle is not present initially in the nucleus but is produced due to disintegration of neutron into a proton i.e.,

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{v} \text{ (antineutrino)}$$

When a proton is converted into a neutron, positive β -particle or positron is emitted.

$$_1H^1 \longrightarrow {}_0n^1 + {}_1\beta^0 + v \text{(neutrino)}$$

(c) When a nuclide emits a gamma photon, neither the atomic number nor the mass number changes.

Half Life ($T_{1/2}$)

The half life period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 is initial number of radioactive atoms present, then in a half life time $T_{1/2}$, the number of undecayed radioactive atoms will be $\frac{N_0}{2}$ and in next half life $\frac{N_0}{4}$ and so on.

Note : So at $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$N_0 \xrightarrow{1T_{1/2}} \frac{N_0}{2} \xrightarrow{2T_{1/2}} \frac{N_0}{2^2} \xrightarrow{3T_{1/2}} \frac{N_0}{2^3} \longrightarrow .... \xrightarrow{nT_{1/2}} \frac{N_0}{2^n}$$

where $n = \frac{t}{T_{1/2}} = \frac{\text{Time Lapsed}}{\text{Half Life}}$

Since $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \qquad \Rightarrow \qquad e^{-T_{1/2}} = \frac{1}{2}$$

$$\Rightarrow \lambda T_{1/2} = \log_e 2 = 0.693 \qquad \Rightarrow \qquad T_{1/2} = \frac{0.693}{\lambda}$$

Mean Life (τ)

The mean life or average life of a radioactive substance is equal to the average time for which the nuclei of atoms of the radioactive substance exist.

The average life of a sample can be calculated by finding the total life of all the nuclei of the substance and then dividing it by the total number of nuclei present in the sample initially. Mathematically

$$\tau = \frac{\int_0^{N_0} t dN}{\int_0^{N_0} dN} = \frac{1}{N_0}\int_0^{N_0} t dN = \frac{N_0/\lambda}{N_0}$$

$$\Rightarrow \quad \tau = \frac{1}{\lambda}$$

Note : So, we conclude that

$$T_{1/2} = 0.693\tau \qquad \Rightarrow \qquad T_{1/2} < \tau$$

Activity of Radioactive Substance (A)

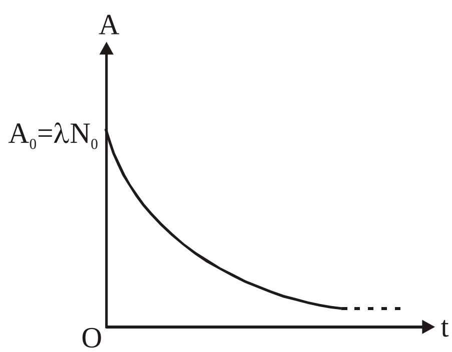
The activity of a radioactive substance means the rate of decay (or the number of disintegration/sec). This is denoted by

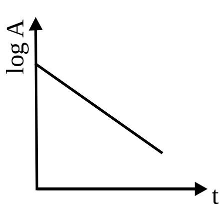
$$A = -\frac{dN}{dt} = -\frac{d}{dt}\left(N_0 e^{-\lambda t}\right) = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$$

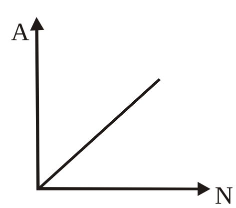
where

$A_0 = (\lambda N_0)$ is the activity at time $t = 0$.

So, activity of a radioactive sample decreases exponentially with time







MASS ENERGY EQUIVALENCE

According to Einstein the mass and energy are equivalent i.e., mass can be converted into energy and vice-versa. The mass energy equivalence relation is

$$\Delta E = c^2 \Delta m$$

Accordingly, annihilation of 1 kg mass is equivalent to energy given by

$$\Delta E = 1 \times (3\times 10^8)^2 \quad \Rightarrow \quad \Delta E = 9\times 10^{16}\,J$$

Energy corresponding to annihilation of 1 amu of mass is

$$\Delta E = \left(1.67\times 10^{-27}\right)\left(9\times 10^{-16}\right)\text{J}$$

$$\Rightarrow \quad \Delta E = \frac{\left(1.67\times 10^{-27}\right)\left(9\times 10^{-16}\right)}{1.6\times 10^{-19}}\text{eV}$$

$$\Rightarrow \quad \Delta E = 931.5 \quad \text{MeV}$$

Mass Defect and Binding Energy

It has been observed that the mass of a nucleus is always less than the mass of its constituent nucleons (i.e. protons + neutrons). This difference of mass is called the **mass defect** (Δm). Let m ($_Z X^A$) be the mass of nucleus, m_p = the mass of proton and m_n = mass of neutron, then the mass defect is given by

$\Delta m =$ mass of constituent nucleons – mass of nucleus

$$\Delta m = \left[Zm_p + (A-Z)m_n\right] - m\left(_Z X^A\right)$$

Packing fraction (f) is the mass defect per nucleon. So,

$$f = \frac{\Delta m}{A}$$

This mass defect exists in the form of binding energy of nucleus, which is responsible for binding the nucleons into a small nucleus. So,

Binding energy of nucleus $= (\Delta m)c^2 = (931.5)\Delta m$

(in MeV) and binding energy per nucleon $= \dfrac{(\Delta m)c^2}{A}$

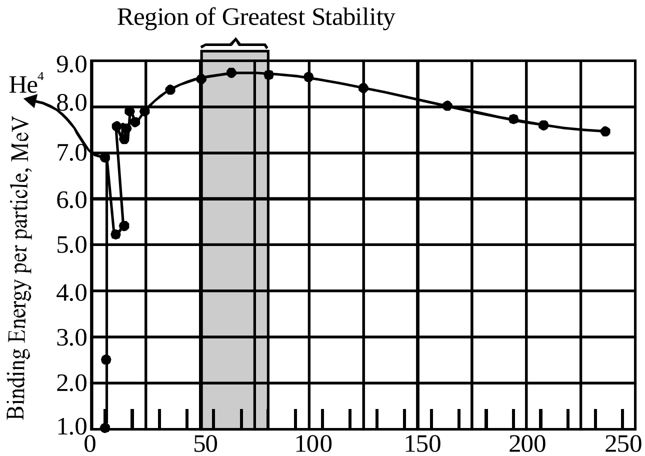
Problem Solving Trick

(a) It is not the binding energy which accounts for the stability of nucleus.

(b) The stability of nucleus is governed by binding energy per nucleon. The more the binding energy per nucleon, the more stable a nucleus is.

VARIATION OF BINDING ENERGY PER NUCLEON WITH MASS NUMBER A

The graph represents the average binding energy per nucleon in MeV against mass number A. It is observed that the binding energy for nuclei (except $_2He^4$, $_6C^{12}$ and $_8O^{16}$) rises first sharply, reaches a maximum value 8.5 MeV at A = 50 and then falls slowly, decreasing to 7.6 MeV for elements of higher mass number A = 240. Following facts can be concluded from this curve.



(a) The binding energy per nucleon for light nuclei, such as $_1H^2$, is very small ($\simeq 1$ MeV).

(b) The binding energy per nucleon increases rapidly for nuclei upto mass number 20 and the curve possesses peaks corresponding to nuclei $_2He^4$, $_6C^{12}$ and $_8O^{16}$. The peaks indicate that these nuclei are more stable than those in their neighbourhood. It confirms the reason for extraordinary stability of α -particle.

(c) After mass number 20, binding energy per nucleon increases gradually and for mass number between 40 and 120, the curve becomes more or less flat. The average value of binding energy per nucleon in this region is about 8.5 MeV. For A = 56 ($_{26}Fe^{56}$), the binding energy per nucleon is maximum and it is equal to 8.8 MeV.

(d) After mass number 120, binding energy per nucleon starts decreasing and drops to 7.6 MeV for uranium. This low value of binding energy per nucleon in case of heavy nuclei is unable to have control over the repulsion between the large number of protons. Such nuclei are unstable and are found to disintegrate by emitting α - particles. The emission of α -particle not only decreases repulsive force inside the nucleus but also increases the value of B.E./A of the nucleus due to its extraordinary stable structure (α -particle has large binding energy). It is called **α -decay**.

Sometimes, the heavy nuclei increase the value of their B.E./A by emitting an electron. It is called **β -decay.** Inside the nucleus, an electron does not exist. It is created at the time of β -decay due to conversion of a neutron into proton. The β -decay leads to increase in Coulomb's repulsive force, but it increases B.E./A and also improves the neutron-proton ratio.

All such nuclei, which undergo α and β -decay are called **radioactive nuclei**.

(e) The binding energy per nucleon has a low value for both very light and very heavy nuclei. In order to attain higher value of binding energy per nucleon, the lighter nuclei may unite together to form a heavier nucleus (process of **nuclear fusion**) or a heavier nucleus may split into lighter nuclei (process of **nuclear fission**). In both the nuclear processes, the resulting nucleus acquires greater value of binding energy per nucleon along with the liberation of enormous amount of energy.

NUCLEAR FISSION

The splitting of heavy nucleus into two or more fragments of comparable masses, with an enormous release of energy is called **nuclear fission**.

Note : When slow neutrons are bombarded on $_{92}U^{235}$, the fission takes place according to reaction

$$_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3(_0n^1) + 200\text{MeV}$$

(a) In nuclear fission the sum of masses before reaction is greater than the sum of masses after reaction, the difference in mass being released in the form of fission energy.

(b) The phenomenon of nuclear fission was discovered by Otto Hans and F. Strassmann in 1939 and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(c) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments Ba^{141} and Kr^{92} are formed but they may be any stable isotopes of middle weight atoms. The most probable division is into two fragments containing about 40% and 60% of the original nucleus with the emission of 2 or 3 neutrons per fission. So, average number of neutrons produced per fission is 2.5.

(d) Most of energy released appears in the form of kinetic energy of fission fragments.

(e) The fission of U^{238} takes place by fast neutrons.

CHAIN REACTION

If on the average more than one of the neutrons produced in each fission are capable of causing further fission, the number of fissions taking place at successive stages goes on increasing at a rapid rate, giving rise to self sustained sequence of fission known as **chain reaction**. The chain reaction takes place only if the size of the fissionable material is greater than a certain size called the **critical size**.

There are two types of chain reactions.

Uncontrolled Chain Reaction

In this process the number of fissions in a given interval on the average goes on increasing and the system will have the explosive tendency. This forms the principle of atom bomb. If a nuclear reaction is uncontrolled then in about 1 μs, energy of order of 2×10^3 J is released.

Controlled Chain Reaction (As in a Nuclear Reactor)

In this process the number of fissions in a given interval is maintained constant by absorbing a desired number of neutrons. This forms the principle of nuclear reactor, consisting of the following parts :

(a) **Fuel :** The fuel is U^{235} or U^{233} or Pu^{239}

(b) **Moderator :** A moderator is a suitable material to slow down neutrons produced in the fission. The best choice as moderators are heavy water (D_2O) and graphite (C).

(c) **Controller :** To maintain the steady rate of fission, the neutron absorbing material known as **controller** is used. The control rods are made of Cadmium or Boron-steel.

(d) **Coolant :** To remove the considerable amount of heat produced in the fission process, suitable cooling fluids known as coolants, are used. The usual coolants are water, carbon-dioxide, air etc.

(e) Reactor shield : The intense neutrons and gamma radiation produced in nuclear reactors are harmful for human body. To protect the workers from such radiations, the reactor core is surrounded by concrete wall, called the **reactor shield**.

Critical Mass

If the amount of uranium is too small, then the liberated neutrons have large scope to escape from the surface and the chain reaction may stop before enough energy is released for explosion. Therefore, in order for explosion to occur, the mass uranium has to be greater than some minimum value, called the **critical mass**.

Reproduction Factor

It is the ratio of the rate of neutron production and the rate at which the neutrons disappear. Whether a mass of active material will sustain a chain reaction or not is determined by the reproduction factor (K). If $K \geq 1$, the chain reaction will be sustained. If K = 1, the mass is said to be critical.

NUCLEAR FUSION

The phenomenon of combination of two or more light nuclei to form a heavy nucleus with release of enormous amount of energy is called the **nuclear fusion**. The sum of masses before fusion must be greater than the sum of masses after fusion, the difference in mass appearing as fusion energy. The fusion of two deuterium nuclei into helium is expressed as

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + 23.8\text{MeV}$$

It may be pointed out that this fusion reaction does not actually occur. Due to huge quantity of energy release, the helium nucleus ${}_2He^4$ has got such a large value of excitation energy that it breaks up by the emission of a proton or a neutron as soon as it is formed, giving rise to the following reactions.

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1 + Q(=3.26\text{MeV})$$

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + Q(=4.04\text{MeV})$$

The fusion process occurs at extremely high temperature and high pressure just as it takes place at sun where temperature is 10^7K. So, fusion reactions are also called **Thermo-nuclear reactions**.

Nuclear fusion has the possibility of being a much better source of energy than fission due to the following reasons.

(a) In fusion there is no radiation hazard as no radioactive material is used.

(b) The fuel needed for fission (U-235 etc.) is not available easily whereas hydrogen needed for fusion can be obtained in huge quantity.

(c) The energy released per nucleon is much more in fusion than in fission.

However, the very high temperature and pressure required for fusion cannot be easily created and maintained and as such it has not been possible as yet to use fusion for power generation.

Example 23:

A radioactive substances has a half life period of 30 days calculate (a) time taken for $\frac{3}{4}$ of original number of atoms to disintegrate (b) time taken for $\frac{1}{8}$ of the original number of atoms to remain unchanged.

Solution :

(a) $\frac{T_{1/2}}{2} = 30 \text{ days}$

No. of atoms disintegrated $= \frac{3}{4}N_0$

No. of atoms left after time 't'

$$N = N_0 - \frac{3}{4}N_0$$

$$N = \frac{N_0}{4}$$

We have,

$$\Rightarrow N = N_0\left(\frac{1}{2}\right)^n \qquad \Rightarrow \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n \qquad \Rightarrow n = 2$$

t = 2 half life

t = 2 × 30

t = 60 days

(b) No. of atoms remain = $\frac{N_0}{8}$

$$N = N_0 \times \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^n \qquad \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$n = 3$

Time required = 3 half life = 3 × 30 = 90 days.

Example 24:

The half life of $_{92}U^{238}$ against α-decay is 4.5 ×10^9 years. What is the activity of 1g sample of $_{92}U^{238}$.

Solution :

$$T_{1/2} = 4.5\times10^9 \text{ years} = 4.5\times10^9\times3.16\times10^7\,\text{s}$$

$$T_{1/2} = 1.42\times10^{17}\,\text{s}$$

We know that

238 gm of U^{238} contains = 6.02 × 10^{23} atom

$\therefore$ 1 gm of U^{238} contains $= \frac{6.02\times10^{23}}{238}$ = 25.3 × 10^{20} atom

Activity $R = \lambda N = \frac{0.693}{T_{1/2}}\times N$

$$= \frac{0.693}{1.42\times10^{17}}\times25.3\times10^{20} = 1.23\times10^4 \text{ disintegration/s}$$

R = 1.23 × 10^4 Bq

Example 25 :

The activity of radioactive sample falls from 600/s to 500/s in 40 minutes. Calculate its half-life.

Solution :

Activity $R = R_0 e^{-\lambda t}$

$$500 = 600e^{-\lambda t} \qquad e^{-\lambda t} = \frac{5}{6}$$

$$\lambda = \frac{\log_e 6/5}{t} = \frac{\log_e 6/5}{40\,\text{min}}$$

half life $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{\log_e 2}{\log_e \frac{6}{5}}\times 40\,\text{min}$

$$T_{1/2} = 152 \text{ min}$$

Example 26 :

A radioactive isotope has a half-life of T years. After how much time is its activity reduced to 6.25% of its original activity? Given $T_{1/2} = T$

Solution :

Activity is given by $R = R_0 e^{-\lambda t}$

Let the time be t at which $R = \frac{6.25}{100}R_0$

$$\Rightarrow \frac{6.25R_0}{100} = R_0 e^{-\lambda t} \qquad \Rightarrow \frac{6.25}{100} = e^{-\lambda t}$$

$$\Rightarrow e^{\lambda t} = \frac{100}{6.25} \qquad \Rightarrow e^{\lambda t} = 16$$

$$\Rightarrow \log_e e^{\lambda t} = \log_e 16 \qquad \Rightarrow \lambda t = \log_e 16$$

$$\Rightarrow t\frac{\log_e 2}{T_{1/2}} = 4\log_e 2 \qquad \Rightarrow t = 4T$$

Example 27 :

A 1000 MW fission reactor consumes half of its fuel in 5.00 years. How much U^{235} did it contain initially? Assume that all the energy generated arises from the fission of U^{235} and that this nuclide is consumed by the fission process.

Solution :

Energy generated in fission of single atom U^{235}

= 200 MeV

= 200 × 10^6 × 1.6 × 10^{-19} Joule = 3.2 × 10^{-11} Joule

Total energy generated in 5 years in reactor

= 1000 × 10^6 × 5 years

= 1000 × 10^6 × 5 × 365 × 24 × 60 × 60 Joule

= 1.57 × 10^{17} Joule

Total No. of atom consumed

$$= \frac{\text{Total energy}}{\text{Energy generated in fission of single atom of } U^{235}}$$

$$= \frac{1.57\times10^{17}}{3.2\times10^{-11}}$$

= 4.9 × 10^{27} atom of U^{235}

Mass of 4.9×10^{27} U^{235} atom is given by

$$= \frac{235}{6\times10^{23}} \times 4.9\times10^{27}\,\text{gm} = 1920 \times 10^3\,\text{gm} = 1920\,\text{kg}.$$

Since this is half of the original U^{235} sample.

Thus,

Mass of U^{235} originally present

= 1920 × 2= 3840 kg

Example 28 :

Calculate the electric potential energy due to the electric repulsion between two nuclei of C^{12} when they 'touch' each other at the surface.

Ans. $$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} = \frac{9\times10^9\times\left(6\times1.6\times10^{-19}\right)^2}{5.04\times10^{-15}\,\text{m}}$$

$U = 1.64 \times 10^{-12}$ J

U = 10.2 MeV

RADIOACTIVTY ADDITIONAL INFORMATION AND QUESTIONS

Units of Radioactivity

Curie : amount of radioactive substance which gives 3.7×10^{10} disintegration/s which is also equal to radioactivity of 1g pure radium. In SI system the unit is becquerel (Bq) 1 Bq =1 disintegration/s

Rutherford : Amount of radioactive substance which gives rise to 10^4 disintegration/s

1. Radioactive equilibrium is the condition where the amount of the mother and daughter nucleus remains constant.

   (dN/dt) parent = (dN/dt) daughter

   $N_1\lambda_1 = N_2\lambda_2 = N_3\lambda_3 = ...$
2. Polonium is (weight to weight) 10^7 times more active than Uranium and Radium is twice as active as polonium.
3. Natural radioactivity is exhibited by heavy elements from 83 to 106 in the periodic table.
4. Radioactive elements : Uranium, polonium, radium, radon, Ionium, thorium, actinium and Mesothorium.
5. A γ ray photon disappears in the coulomb field of atomic nucleus and an electron positron pair is formed. For the production of electron position pair minimum energy of photon should be 1.02 M eV.

Artificial Transmutation

1. In 1919 Rutherford discovered artificial transmutation
2. Changing one element into another by human efforts is called *artificial transmutation*.
3. Rutherford converted nitrogen into oxygen by bombarding it with α particle coming from radium.

   $${}_7N^{14} + {}_2He^4 \longrightarrow {}_8O^{17} + {}_1H^1$$
4. In 1932 Chadwick discovered neutron. Chadwick gave the nuclear equation

   $${}_4Be^9 + {}_2He^4 \longrightarrow {}_6C^{12} + {}_0n^1$$
5. Bothe & Becker in 1932 discovered that fast moving rays come out when Berilium was bombarded with α particle and they named them as γ -rays

   $${}_4B^9 + {}_2He^4 \longrightarrow {}_6C^{13} + \gamma \quad \text{(wrong equation)}$$

   This equation ws found wrong from other experiments. Chadwick corrected the equation by discovering that fast moving rays are material particles and named them "*Neutrons*".
6. Nuclear mass is determined by "Mass *spectrographs*".
7. Nuclear charge is determined by scattering experiments and X-rays spectra.
8. **ARTIFICIAL RADIOACTIVITY :**

   In 1934 Joliot and Lnne Curie discovered artificial radioactivity or induced radioactivity. When aluminium nucleus is bombarded with α particle coming from radium. Aluminium is converted into radioactive phosphorus. This phosphorus is very unstable and its half-life period is 3.25 minutes. It changes into silicon by emitting a positron.

   $${}_3Al^{27} + {}_2He^4 \longrightarrow {}_{15}P^{30} + {}_0n^1$$
   $${}_{15}P^{30} \longrightarrow {}_{14}Si^{30} + {}_1e^0$$
9. In 1932 Anderson discovered positron in cloud chamber. A positron cannot exist in a free state for long.
10. Radio isotopes are manufactured by Nuclear reactor
11. Radio is isotopes are detected by Gieger-Muller

Uses of Radio Isotopes

I. In curing cancer, Radio cobalt and Radio gold are used instead of Radium

II. Seeds subjected to the influence of radioactivity give good results. The radio isotopes are cheap and easily available as compared to natural radio elements.

III. Radio isotopes are artificially produced in atomic reactors. They exhibit artificial radioactivity. They are about 500 in number. Their life period is very small. Orodinary isotopes become radio isotope in nuclear reactors.

IV. Radio phosphorus is used in curing blood cancer.

V. Radio iodine is used to cure thryoid disease.

VI. Radio sodium is used to find defects in blood supply.

VII. All elements with atomic number greatr than 92 are called transuranic elements. They do not occur in nature. They are artificially produced in the laboratory.

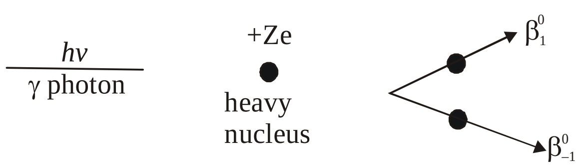
VIII. Age of rocks is estimated by Uranium dating.

IX. Radio carbon dating is used to find the age of fossils.

Important Points :

- Inside Radioactive nucleus both proton and neutron are unstable.
- Neutron is unstable outside the nucleus.

  $_0n^1 \longrightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{v}$
- **Pair production :** A γ ray photon disappears in the coulomb field of atomic nucleus and an electron position pair is formed. For this pair production energy of γ photon > 1.02 MeV.



$$\underset{\text{gamma photon}}{hv} = \underset{\text{Positron}}{{}_{+1}\beta^0} + \underset{\text{Electron}}{{}_{-1}\beta^0}$$

[rest mass energy e^- = 0.51 MeV]

If energy of γ photon < 1.02 MeV, when it falls on a material, produces photoelectric effect or Compton effect but not pair production.

Note : Energy of position > energy of electron.

Pair annihilation

$${}_{-1}\beta^0 \quad + \quad {}_{+1}\beta^0$$

before combination

← γ photon   γ photon →

after combination

- The sub orbital angular momentum $\sqrt{l(l+1)}\dfrac{h}{2\pi}$

  where l is azimuthal quantum number

  $l = 0 \Rightarrow s$ sub orbit; $l = 1 \Rightarrow p$ sub orbit

  $l = 2 \Rightarrow d$ sub orbit; $l = 3 \Rightarrow f$ sub orbit
- Orbital angular momentum $= n\dfrac{h}{2\pi}$

ATOMIC NUCLEUS, NUCLEAR FISSION &

- Proton and neutron together are called Nucleous
- Both Proton and Neutron have intrinsic angualer momentum of (h/4π) and have "small magnetic moments"
- m_H = mass of hydrogenation = 1 amu

  $= 1.673 \times 10^{-27}$ kg = 938.7 MeV
- Radii is measured in fermi : 1 Fermi 10^{-15} m

  size of nucleus : $R = R_0A^{1/3}$, $R_0 = 1.1 \times 10^{-15}$

  = 1.1 fm; A = mass number
- Order of density of matter in nuclei = 10^{14} g/cc
- Gravitational force < electrostatic force< nuclear force
- Nuclear force starts acting from a distance 4.2 fm. becomes maximum for a distance of 1 fm and turns respulsive for a distance of 0.4 fm
- Proton discovery : Rutherford

  $_7N^{14} + {}_2He^4 \longrightarrow {}_8O^{17} + {}_1H^1$
- Neutron discovery : Chdwick :

  $_4Be^+ + {}_2He^4 \longrightarrow {}_6C^{12} + {}_0n^1$

Fission & Fusion

- In 1939 Otto Hahn and Straussmen discovered Fission.
- When neutrons enter a big nucleus, the average binding energy decreases and coulomb force make the nuclei split into two parts.
- After fission of Uranium, the debris contains different elements of periodic table (A = mass number 68 to 166) the elements with A = 95 and 139 are dominant and the element with A =117 present in minority.

  $${}_{92}U^{239} + {}_0n^1 \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3({}_0n^1) + 200\text{MeV}$$
- Fission of each U^{235} releases 200 MeV of energy
- Natural Uranium has two dominant isotopes U^{238} and U^{235} in the ratio of 99.3 : 0.7.
- U^{235} is easily fissionable than U^{238}
- For U^{238} fission fast neutrons (having energy 1.0MeV or more) are necessary. No instantaneous neutrons are released.
- Plutonium $_{94}Pu^{238}$ which is radio isotope, is more fissionable than U^{235}.

Nuclear Reactor

(a) First designed by Fermi in 1942 at Chicago University

(b) Uranium fission produces high speed neutrons which does not give chain reaction

(c) **Fuel :** ${}_{92}U^{235}$ or Pu^{23}

(d) **Moderator :** for slowing down fast neutrons e.g. Heavy water, graphite or Berellium oxide.

(e) **Controller :** Cadmium or Boron rods.

(f) **Use :** (i) Production of Pu^{239} :

$${}_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}Y^{239} + \gamma \text{ (energy)}$$
$${}_{92}U^{239} \rightarrow {}_{93}Np^{239} + {}_{-1}\beta^0 + \gamma \text{ (Neptunium)}$$

(ii) Producing radio isotopes :

$${}_{92}Np^{239} \rightarrow {}_{94}Pu^{239} + {}_{-1}\beta^0 + v$$

(g) **Coolant : t**o take away heat produced in the reactor. Liquid Metals like Na are used as coolant.

Nuclear Fusion

- Two or more light nuclei form a heavy nucleus. The sum of masses before fusion is greater than the sum of masses after fusion, the difference in mass appearing as fusion energy

  e.g. fusion of two deuterium nuclei into helium is expressed as :

  $${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + 21.6\text{MeV}$$
- For the fusion to take place, the component nuclei must be brought to within a distance of 10^{14} m. For this they must be imparted high energies to overcome the repulsive force between nuclei. This is possible when temperature is enormously high.
- Energy of the sun and stars is due to 'Nuclear Fusion'.
- At very high temperature in stars all the electrons leave the atoms. Such a substance which consists of only nuclei is called "plasma".
- Bothe proposed "Carbon Cycle" to explain stellar energy.
- As a result of carbon cycloe 4 protons combine to form two positrons and one helium atom. ${}_6C^{12}$ acts as catalyst.

  $$4{}_1H^1 \longrightarrow {}_2He^4 + 2{}_{+1}\beta^0 + 2v + 26.7\text{Mev}$$
- Proton-proton cycle is sppposed to give more energy in fusion because it takes less time.
- The proton : proton cycle occurs at relatively lower temperature as compared to carbon cycle.

TEST YOUR SELF - 2

1. In a sample of radioactive material, what percentage of the initial number of active nuclei will decay during one mean life?

   (1) 37% (2) 50%

   (3) 63% (4) 69.3%

2. A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ.

   (1) $\lambda = \lambda_1 + \lambda_2$ (2) $\lambda = \frac{1}{2}(\lambda_1 + \lambda_2)$

   (3) $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (4) $\lambda = \sqrt{\lambda_1\lambda_2}$

3. If N_0 is the original mass of the substance of half-life period $t_{1/2}$ = 5 years, then the amount of substance left after 15 years is

   (1) $N_0/8$ (2) $N_0/16$

   (3) $N_0/2$ (4) $N_0/4$

4. If mass of proton = 1.008 amu and mass of neutron = 1.009 amu, then the binding energy per nucleon for ${}_4Be^9$ (mass = 9.012 a.m.u.) will be

   (1) 0.0672 MeV

   (2) 0.672 MeV

   (3) 6.724 MeV

   (4) 67.2 MeV

5. Which of the following processes represents a gamma-decay?

   (1) ${}^AX_Z + \gamma \longrightarrow {}^AX_{Z-1} + a + b$

   (2) ${}^AX_Z + {}^1n_0 \longrightarrow {}^{A-3}X_{Z-2^{+C}}$

   (3) ${}^AX_Z \longrightarrow {}^AX_Z + f$

   (4) ${}^AX_Z + e_{-1} \longrightarrow {}^AX_{Z-1} + g$

ANSWER

1. (3) 2. (1) 3. (1) 4. (3)

5. (3)