UNIT - MODERN PHYSICS CHAPTER - 1

Dual Nature of Matter & Radiation

CATHODE RAY TUBE EXPERIMENT

Introduction

Gases are generally poor conductors of electricity because they do not have enough free charged particles. But gases can conduct electricity if large amount of free charged particles are produced continuously. This can be done by various methods. Few of them are

a. Applying a large potential difference across a gas column at very low pressure

b. By heating a metal kept in an evacuated chamber to high temperatures at which electrons are ejected from the metal

c. By passing X-rays through the gas

* **Discharge tube**

Components of discharge tube :

a. Anode and Cathode

b. Vacuum pump-to get desired low pressure

c. High voltage source-from secondary of an induction coil

• **Sparking potential**

The minimum potential difference which can cause sparks in a gas is called sparking potential. Value of sparking potential is given by Paschen's Law

$$V = f(pd) = \text{function of } (pd)$$

Where

V = sparking potential

p = pressure of the gas

d = separation between electrodes

Discharge at low pressure (at 1200V)

If we go on decreasing pressure of the discharge tube the following phenomena will take place

10 mm of Hg - Sparking : discharge takes palce in the form of luminous streaks called blue streamrs with a cracking 5 mm of Hg - Positive column

Streaks spread into aluminous column

It starts from the anode and ends upto cathode. Colour of the positive column depends on the nature of the enclosed gas.

Reddish ® air, bright red ® neon, bluish ® CO_2

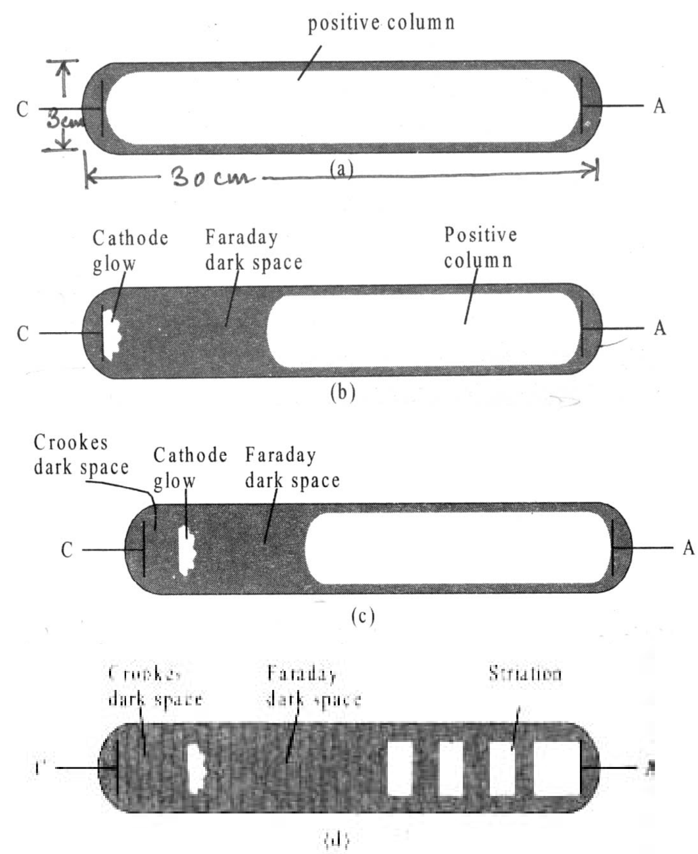
0.5 mm of Hg - the length of the positive column decreases. It starts from anode but ends well before the cathode. A bluish glow is seen around the cathode and there is dark space between this glow and the positive column. The glow around cathode is called cathode glow or negative glow. The dark space between the cathode glow and the positive column is called Faraday dark space.

0.1 mm of Hg -: the length of the positive column further decreases and length of the Faraday dark space increases. The cathode glow is detached from the cathode and a new dark space, called Crooke's dark space appears between cathode and the cathode glow. If we further decrease pressure, Crooke's dark space and cathode glow will expand.

0.05 mm of Hg -: The positive column is split into alternate bright and dark bands called striations. With further decrease in pressure striations move towards anode and vanish.

0.01 mm of Hg -: The Crooke's dark space fills the entire tube. At this stage walls, of the tube begin to glow. This is called "florescence. At this stage some invisible particles move from cathode which on striking the wat (back side of anod) cause - tube to glow

If we further decrease pressure to 10^{-4} mm the tube will .stop conducting.



Mean free path :- Electrons, emitted from the cathode, travel in straight lines before they collide with other particles. Average distance travelled by electrons in between successive collisions is known as mean free path. Electrons ionise gas molecules on collision. This ionised gas molecules emit light which appears as cathode glow. Thus cathode glow appears some distance away from the cathode and Crooke's dark space appear between cathode and cathode glow. If we further decrease pressure mean free path of an electron increases as $I \propto \frac{1}{\text{electron density}}$ $n \propto \frac{p}{KT}$ thus length of Crooke' s dark space increases. At very low pressure when ^ mean free path becomes larger than the length of the tube, the Crooke's dark space fills the entire tube and no cathode glow or positive column is observed.

* **Cathode Rays** :- It is stream of electrons that are emitted from the cathode. This phenomenon occurs when Crooke's dark space fills the tube. It was discovered by Crooke and Thomson

* **Important properties of cathode rays**

a. The direction of cathode rays is independent of the position of the anode

b. Cathode rays travel in straight Ines (why ? #1) and produce shadow of an object as they do not collide with themselves as l increases

c. Cathode rays produce heat when they strike a material surface. (Why ? #2)

d. Cathode rays produce florescence when they strike a number of crystals, minerals and salts. (Why ? #3) due to loss of excited state.

e. When cathode rays strike a solid object specially a metal, X-rays are emitted from the object. (Why ? #4)

f. Cathode rays can penetrate thin foils of metal. (Why ? #5)

* **Canal Rays or Positive Rays ?**

If the cathode of a discharge tube has holes in it and the pressure of the gas is around 1 mm of Hg, faint luminous glow comes out from each hole on the back side of the cathode. This is called canal rays or positive rays. Canal rays are positive charged ions.

* **Use of Cathode rays**

Cathode rays are used mainly in cathode-ray oscilloscope and in the production of X-rays.

RELATIVITY

- If l' and l are length of object in motion and at rest then $l' = l\sqrt{1-\frac{v^2}{c^2}}$
- If Δt and $\Delta t'$ are time intervals at rest and in motion with velocity v then $\Delta t' = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}$
- Variation of mass $m' = \frac{\Delta m}{\sqrt{1-\frac{v^2}{c^2}}}$
- Relativistic formula of $KE = mc^2 - m_0c^2$
- Equations for photon or particles with zero rest mass

$$m = \frac{E}{c^2} = \frac{hv}{c^2};\ p = \frac{E}{c} = \frac{hv}{c}$$

- Total Energy $E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{c^2p^2 + m_0{}^2c^4}$
- For Ultra relativistic particle $v \sim c$

$\therefore\ c^2p^2 >> m_0^2c^4$

$\therefore\ \boxed{E = pc}$ valid for particles with zero rest mass like neutrino, photon etc.

THOMSONS ELEMENTARY EXPERIMENT

Elementary idea

J.J. Thomson's experiment proved negative charge of cathode rays. When the cathode rays were subjected to electric field, it get deflected, similarly application of magnetic field also deflected cathode rays.

Simultaneous application of both the fields of appropriate strengths helped keep cathode rays undeviated at this point

$F_e = F_B \Rightarrow CE = BeV$

$V = E / B$ .... (i)

$\therefore$ Velocity of cathode rays = E / B

Magnetic field alone bends the particles along a circle of radius R_B.

$$\therefore\ \frac{mv^2}{R_B} = Bev$$

$$Be = \frac{mv}{R_B} \quad .... \text{(ii)}$$

Substituting expression of velocity from (i) in (ii)

$$Be = \frac{mE}{BR_B}$$

$$\Rightarrow \boxed{\frac{e}{m} = \frac{E}{B^2 R_B}}$$

MATTER WAVES

- Light possesses dual nature. In some phenomena e.g. interference, diffraction and polarisation, it behaves as waves, while in some other phenomena e.g., photoelectric effect, Compton effect, it behaves as particles (photons).

de-Broglie's Postulate

- According to de-Broglie a material particle in motion must have a wave like character and the wavelength associated with it is given by

$$\lambda = \frac{h}{p} \qquad \text{..... (1)}$$

where, h = Planck's constant and

p = momentum of the particle

de-Broglie assumed this expression in analogy with photon because momentum of photon is

$$p = \frac{h}{\lambda} \qquad \Rightarrow \qquad \lambda = \frac{h}{p}$$

If m is the mass of particle and v the velocity, then momentum of particle is $p = mv$.

So, de-Broglie wavelength $\lambda = \dfrac{h}{mv}$ ..... (2)

For electrons accelerated through a potential difference of V volts

de-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2meV}}$

Substituting m = 9.1 × 10^{-31} kg, h = 6.62 × 10^{-34} Js, e = 1.6 × 10^{-19} C, we get

$$\lambda = \sqrt{\frac{150}{V}} \times 10^{-10}$$

$$\Rightarrow \lambda = \sqrt{\frac{150}{V}}\,\text{Å} = \frac{12.27}{\sqrt{V}}\,\text{Å}$$

$$\Rightarrow \boxed{V = \frac{150.6}{\lambda^2}} \text{ volts}$$

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m(qV)}}} \quad \boxed{\lambda_\alpha = \frac{0.101}{\sqrt{V}}\,Å}$$

and $\boxed{\lambda_p = \dfrac{0.286}{\sqrt{V}}\,Å}$ put V in volts every where

V = potential through which they are accelerated.

- de Broglie waves of thermal neutrons at the ordinary temperature T kelvin is

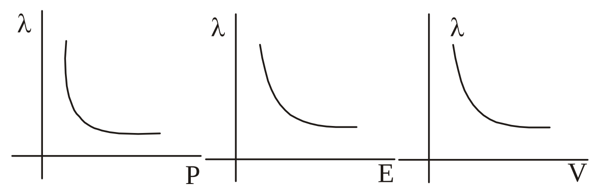
$$\boxed{\lambda = \frac{h}{\sqrt{2mkT}} = \frac{30.8}{\sqrt{T}}}$$

K = Boltzmann constt.

- de-Broglie waves of a gas molecule

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}} = \frac{h}{mC_{rms}}}$$

- In all these case $\lambda \propto \dfrac{1}{\sqrt{energy}}$ or $E \propto \dfrac{1}{\lambda^2}$



Example 1. The stopping potential for the photoelectorns emitted from a metal surface of work function 1.7eV is 10.4 V. Find the wave length of radiation used.

Solution. $\dfrac{hc}{\lambda} eV_0 W_0 \quad \lambda = 1022.7\text{Å}$

Example 2. What is the de broglie wavelength associated with (a) an electron moving with a speed of 5.4 × 10^6 m/s and (b) a ball of mass 150g, travelling at 30.0 m/s.

Solution. (a) $\lambda_d = \dfrac{h}{p} = \dfrac{6.63\times10^{-34}\,\text{Js}}{4.92\times10^{-24}\,\text{Kg m/s}}$

$\lambda_d = 0.135\,\text{nm}$

(b) $\lambda'_d = \dfrac{h}{p'} = \dfrac{6.63\times10^{-34}\,\text{Js}}{4.5}$

$\lambda'_d = 1.47\times10^{-34}\,\text{m}$

Example 3. What is the debroglie wave length associated through a potential of 100 volts?

Solution. $\lambda_d = \dfrac{h}{p} = \dfrac{12.27}{\sqrt{\text{V}}}\text{Å} = \dfrac{12.27}{\sqrt{100}}$

$\lambda_d = 1.227\text{Å}$

Example 4. For what kinetic energy of a neutron will the associated de broglie wavelength be 1.40 × 10^{-10} m?

Ans. K = 6.69 × 10^{-21} Joule

Quantum Nature of Light

Some phenomena like photoelectric effect, Compton effect, Raman effect could not be explained by Wave theory of light. Therefore quantum theory of light was proposed by Einstein who extended the Planck's hypothesis to explain Black Body radiation. According to quantum theory of light "light is propagated in bundles of small energy, each bundle being called a photon and possessing energy", given by

$$E = h\nu = \frac{hc}{\lambda} \qquad \text{.... (1)}$$

where ν is frequency, λ is wavelength of light and h is Planck's constant whose value is 6.62 × 10^{-34} Js and $c = 3 \times 10^{8}$ ms^{-1}

Momentum of photon

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \qquad \text{..... (2)}$$

Rest mass of photon = 0 ..... (3)

Dynamic or kinetic mass of a photon,

$$m = \frac{h\nu}{c^2} = \frac{h}{c\lambda} \qquad \text{..... (4)}$$

The number of photons emitted from a source of monochromatic radiation of wavelength λ and energy W and power P

$$N = \frac{W}{E} = \frac{W}{h\nu} = \frac{Pt}{h\nu} \qquad \text{..... (5)}$$

PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface by the use of light (or radiant) energy of certain minimum frequency (or maximum wavelength) is called **photoelectric effect**.

Einstein's Explanation of Photo-electric Effect

- The wave theory of light could not explain the observed characteristics of photoelectric effect. Einstein extended Planck's quantum idea for light to explain photo-electric effect.
- According to his idea, the energy of electromagnetic radiation is not continuously distributed over the wave front like the energy of water waves but remains concentrated in packets of energy content hv, where v is frequency of radiations and h is universal Planck's constant (= 6.625 × 10^{-34} Js).
- Each packet of energy moves with the speed of light. The assumptions of Einstein's theory are
  - The photoelectric effect is the result of collision of two particles, one of which is a photon of incident light and the other is an electron of photo-metal.
  - The electron of photo-metal is bound with the nucleus by Coulomb attractive forces. The minimum energy required to free an electron from its bondage is called work function $(\phi_0 = hv_0)$.
  - The incident photon interacts with a single electron and loses its energy in two parts
    - (a) Firstly, in getting the electron released from the bondage of the nucleus.
    - (b) Secondly, to impart kinetic energy to emitted electron.

Accordingly, if hv is the energy of incident photon, then

$$hv = \phi_0 + E_K$$

$$\Rightarrow \quad E_K = hv - \phi_0 \qquad \text{..... (1)}$$

This is Einstein's photoelectric equation, where ϕ_0 is work function and

$$E_K = \frac{1}{2} m v_{\max}^2 = eV_s$$

$$= \mathrm{h}[v - v_0] = \mathrm{hc}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

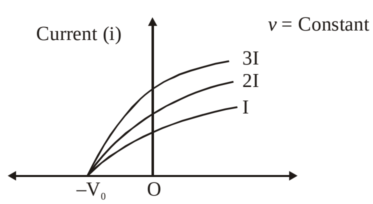
is the maximum kinetic energy of photo-electrons emitted.

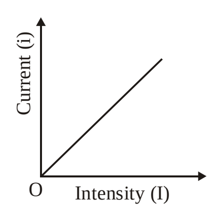
Characteristics of Photo Electric Effect

Effect of Intensity

Intensity of light means the energy incident per second per unit area. For a given frequency, if intensity of incident light is increased, the photoelectric current increases and with decrease of intensity, the photoelectric current decreases, but the stopping potential remains the same.

In photoelectric effect **current (i) is directly proportional to intensity (I) of incident light.**

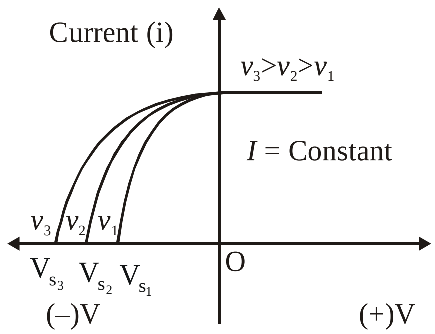




This means that the intensity of incident light affects the photoelectric current but leaves the maximum kinetic energy of photoelectrons unchanged.

Effect of Frequency

When the intensity of incident light is kept fixed and frequency is increased, the photoelectric current remains the same but the stopping potential increases.

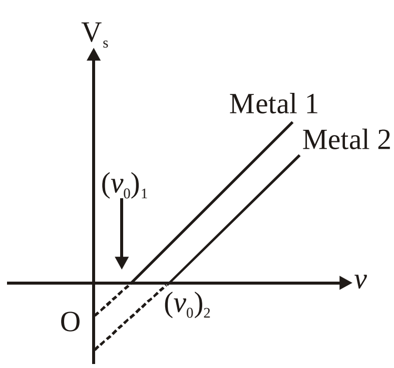


If the frequency is decreased, the stopping potential decreases and at a particular frequency of incident light, the stopping potential becomes zero. This value of frequency of incident light for which the stopping potential is zero is called threshold frequency v_0. If the frequency of incident light (v) is less than the threshold frequency (v_0), no photo-electric emission takes place.

Thus, the increase of frequency increases maximum kinetic energy of photoelectrons but leaves the photoelectric current unchanged.

Effect of Photo-Metal

When frequency and intensity of incident light are kept fixed and photo-metal is changed, we observe that stopping potential (V_s) versus frequency (v) graphs are parallel straight lines, cutting, frequency axis at different points. This shows that threshold frequency are different for different metals, the slope $\left(\frac{V_s}{\nu}\right)$ for all the metals is same and hence universal constant.



In figure threshold frequency and work function are greater for Metal 2 as compared to Metal 1.

Effect of Time

There is no time lag between incident of light and the emission of photo-electrons.

Note: No. of photo-electrons emited per sec = No. of Photons incident per sec = intensity of incident radiation × area of surface/energy of one photon

$$=\frac{IA}{hn}=\frac{IAl}{hc}$$

Example 5. Monochromatic light of frequency 6.0×10^{14}Hz is produced by a laser. The power emitted is 2.0×10^{-3} w.

(1) What is the energy of a photon in the light beam?

(2) How many photons per second, on the average, are emitted by the source?

Solution. (1) $E = n\nu_0 \Rightarrow E = 3.98 \times 10^{-19}$ J

(2) $N = \frac{P}{h\nu}$ per sec

$N = 5.0 \times 10^{15}$ photons per second.

Example 6. The work function of caesium metal is 2.14 eV, when light of frequency 6×10^{14} Hz is incident on the metal surface, photo emission of electrons occurs

what is

(1) Maximum kinetic energy

(2) Stopping potential

(3) Maximum speed of the emitted photoelectrons.

Solution. (1) $E = eV_0 = 0.35$ eV

(2) $V_0 = 0.35$ volt

(3) 3.5×10^5 m/s

Example 7. Light of two different frequencies whose photons have energies 1 eV and 2.5 eV respectively. Successively illuminate a metal whose work function is 0.5 eV. Find the ratio of maximum speed of electrons for both light frequencies.

Solution. $$\frac{\frac{1}{2}m\ v_1^2}{\frac{1}{2}mv_2^2 - w} = \frac{1}{2}\quad 1:2$$

Example 8. The threshold frequency for a certain metal is v_o. When light of frequency $v = 2v_o$ is incident on it, the maximum velocity of photo electrons is 4×10^6 m/s. If the frequency of incident radiation is increased to $5v_0$, then find the maximum velocity of the electron.

Solution. $$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_1^2} = \frac{h2\nu_0 - h\nu_0}{h5\nu_0 - h\nu_0} \Rightarrow v_2 = 8\times10^6 \text{m/s}$$

Example 9. When a certain metallic surface is illuminated with monochromatic light of wave length λ, the stopping potential for photo electric current is $3V_0$ and when the same surface is illuminated with light of wavelength 2λ. The stopping potential is V_0. Find the thresold wavelength of this surface for photoelectric effect.

Solution. $$\frac{hc}{\lambda_0} = \frac{hc}{4\lambda} \qquad \Rightarrow \lambda_0 = 4\lambda$$

Example 10. Two separate monochromatic light beam A and B of the same intensity are falling normally on a unit area of a metallic surface. Their wavelengths are λ_A and λ_B respectively. Assuming that all the incident light is used in ejecting the photo electrons. Find the ratio of the number of photoelectrons from beam A to that of beam B.

Solution.

$$\frac{\text{No. of photo electron of beam A}}{\text{No. of photoelectron of beam B}} = \frac{\frac{I}{hc}\cdot\lambda_A}{\frac{I}{hc}\cdot\lambda_B} = \frac{\lambda_A}{\lambda_B}$$

Example 11. 1.5 mW of 4000 Å light is directed at a photo-electric cell. If 0.1% of the incident photons produce photoelectrons, find the current in the cell.

$h = 6.6\times10^{-34}$ J-S, C = 3×10^8 m/s, $e = 1.6\times10^{-19}$ C

Solution. $n_e \times e = 3\times10^{12}\times1.6\times10^{-19}$

$i = 0.48\ \mu$A.

Compton Effect

When high energetic radiations fall on a target containing free electrons it is scattered, recoiling the electrons and the scattered radiation has wavelength longer than incident one. This effect is called **Compton effect** and the change in wavelength is called Compton Shift and is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

where λ is wavelength of incident photon

λ' is wavelength of scattered photon

θ is the angle of scattering

m_0 is rest mass of electron and

c is the speed of light.

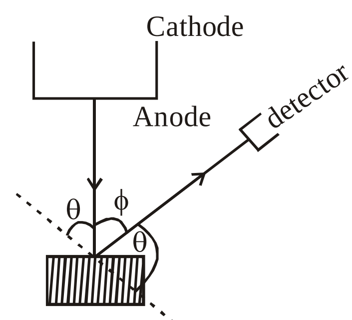
kE of recoiled electron $E = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda'}\right]$

direction of recoiled electron $\tan\phi = \frac{\lambda\sin\theta}{\lambda' - \lambda\cos\theta}$

Davision and Germer Experiment

Davision & Germer performed an experiment to show the wave nature of electrons. In the experiment, a benne of electrons in allowed to strike on a nickel crystal. The electrons are scattered in all directions by the atoms of the crystal. The intensity of the electron beam scattered in a ponticulus direction, say at an angle ϕ to the direction of incident because is measured. A graph is pleated between the values of ϕ and the intensity of Scotland election beans. The curve slows a bump at $\phi = 50°$. And the size of units bump is max when the potential of anode is 54 V.

In the given figure



In figure $\theta = \frac{1}{2}(180° - \phi)$

$\therefore$ at $\phi = 50°, \theta = 65°$

2d sin θ = nλ

for nickel atom d = 0.91 – 19

n = 1

$\therefore$ $2d\sin\theta = \lambda$

or λ = 1.66 Å

Also wavelength of wave associated with moving election at potential 54 V

$$\lambda = \frac{12.27}{\sqrt{v}} = 1.67\text{Å}$$

The results are in perfect agrement thus experiment pries wave nature of electron.

Example 12 :

Monochromatic light of frequency 6.0×10^{14}Hz is produced by a laser. The power emitted is 2.0×10^{-3} w.

(1) What is the energy of a photon in the light beam?

(2) How many photons per second, on the average, are emitted by the source?

Solution :

(1) Each photon has an energy

$E = hv$

$= (6.63 \times 10^{-34}\ \text{Js})\ (6.0 \times 10^{14}\ \text{Hz})$

$E = 3.98 \times 10^{-19}$ J

(2) If N is the number of photons emitted by the source per second, the power P transmitted in the beam equals N times the energy per photon E, so that

$P = NE$

$$\Rightarrow\ N = \frac{P}{E}$$

$$\Rightarrow\ N = \frac{2.0\times10^{-3}\,\text{W}}{3.98\times10^{-19}\,\text{J}}$$

$\Rightarrow$ $N = 5.0 \times 10^{15}$ photons per second.

Example 13:

The work function of caesium metal is 2.14 eV, when light of frequency 6×10^{14} Hz is incident on the metal surface, photo emission of electrons occurs

what is

(1) Maximum kinetic energy

(2) Stopping potential

(3) Maximum speed of the emitted photoelectrons.

Solution :

Given $\phi_0 = 2.14\text{eV}$

(1) According to photo electric equation, the maximum kinetic energy of electron.

$K_{max.} = hv - \phi_0$

Energy of incident photon

$E = hv$

$$= \frac{6.63\times10^{-34}\,(\text{J-s})\times6\times10^{14}\,\text{Hz}}{1.6\times10^{-19}}\text{eV}$$

= 2.49 eV

Thus,

K_{max} . = (2.49 – 2.14)eV

$K_{max.}$ = 0.35 eV

(2) Stopping potential $V_0 = \dfrac{K_{max.}}{e} = \dfrac{0.35\,\text{eV}}{e}$

V_0 = 0.35 volt

(3) Maximum speed of electron ($V_{max.}$)

$$\frac{1}{2}m_eV_m^2 = K_{max.}$$

$$\frac{1}{2}\times9.1\times10^{-31}\times V_m^2 = 0.56 \times 10^{-19}$$

$\Rightarrow$ $V_m = 3.5 \times 10^5$ m/s

Example 14 :

A beam of light has three wavelength 4144 Å, 4972Å and 6216Å with a total intensity of 3.6×10^{-3} w/m^2 equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photo-electrons liberated in two seconds.

Solution :

Work function of metallic surface

$\phi_0 = 2.3\text{eV}$

Corresponding threshold wave length,

$$\lambda_0 = \frac{12375}{\varphi_0(\text{eV})}\text{Å} = \frac{12375}{2.3}$$

$\lambda_0 = 5380\text{Å}$

Hence only 4144 Å and 4972 Å wave length of light will emit the electron from the surface of metal.

Total energy incident on the surface of metal having area 1.0 cm^2 in 2 sec.

$E = I \times A \times t$

{where I - intensity of light incident}

$$= \left(3.6\times10^{-3}\frac{\text{w}}{\text{m}^2}\right)\times\left(1\times10^{-4}\,\text{m}^2\right)\times2\text{s}$$

$$= \left(3.6\times10^{-3}\frac{\text{Joule}}{\text{m}^2\text{-s}}\right)\times\left(1\times10^{-4}\,\text{m}^2\right)\times2\,\text{s}$$

$E = 7.2 \times 10^{-7}$ Joule

Energy incident per wave length $= \dfrac{E}{3}$

$$E' = \frac{7.2}{3} \times 10^{-7} \text{ Joule}$$

$E' = 2.4 \times 10^{-7}$ Joule

No. of photo electron ejected for wave length λ = No. of photon incident

$$= \frac{\text{Total energy incident per wavelength}}{\text{Energy of single photon having wavelength } \lambda}$$

Energy of photon for $\lambda = 4144 \text{Å}$

$$E_1 = \frac{12375}{4144} \text{ eV} = 2.98 \text{ eV}$$

$= 2.96 \times 1.6 \times 10^{-19}$ Joule

$E_1 = 4.78 \times 10^{-19}$ Joule

No. of photo electron $n_1 = \dfrac{2.4 \times 10^{-7}}{4.78 \times 10^{-19}}$

$n_1 = 0.502 \times 10^{12}$

Energy of photon having wavelength $\lambda = 4972$ Å

$$E_2 = \frac{12375}{4972} = 2.48 \text{eV} = 3.98 \times 10^{-19} \text{ Joule}$$

No. of photo electron $n_2 = \dfrac{E'}{E^2}$

$$= \frac{2.4 \times 10^{-7}}{3.98 \times 10^{-19}} = 0.60 \times 10^{12}$$

Total photo electron ejected = $n_1 + n_2$

$= 5.02 \times 10^{11} + 6.0 \times 10^{11}$

$n = 11.02 \times 10^{11} = 1.102 \times 10^{12}$

Example 15 :

Light of wavelength 0.6 μm from a sodium lamp falls on a photo cell and causes the emission of photo electron for which the stopping potential is 0.5 volt. With light of wavelength 0.4 μm from mercury vapour lamp the stopping potential is 1.5 volts. Find the value of *h/e*.

Solution :

According to Einstien photo electric equation

$$K_{min} = \frac{hc}{\lambda} - \phi_0$$

for $\lambda_1 = 0.6$ μm $\quad K_{max.} = 0.5$eV

$$0.5\text{eV} = \frac{hc}{0.6 \times 10^{-6}} - \phi_0 \qquad \text{..... (1)}$$

for $\lambda_2 = 0.40\ \mu m \quad K_{max.} = 1.5$ eV

$$1.5\text{eV} = \frac{hc}{0.4 \times 10^{-6}} - \phi_0 \qquad \text{..... (2)}$$

From equation (1) and (2)

$$(1.5 - 0.5) \text{ eV} = hc\left(\frac{1}{0.4 \times 10^{-6}} - \frac{1}{0.6 \times 10^{-6}}\right)$$

$$\Rightarrow 1\text{V} = \frac{hc}{e}\left(\frac{10^6}{0.4} - \frac{10^6}{0.6}\right)$$

$$\Rightarrow 1\text{V} = \frac{h}{e} \times 3 \times 10^8 \left(\frac{10^6}{0.4} - \frac{10^6}{0.6}\right)$$

$$\Rightarrow \frac{h}{e} = 4 \times 10^{-15} \text{V}$$

Example 16:

Light of two different frequencies whose photons have energies 1 eV and 2.5 eV respectively. Successively illuminate a metal whose work function is 0.5 eV. Find the ratio of maximum speed of electrons for both light frequencies.

Solution :

According to photo electric equation

$$K_{max.} = hv - \phi_0$$

$$\frac{1}{2} m V_m^2 = hv - \phi_0$$

For hv = 1 eV.

Let the maximum speed of electron be V_1

$$\frac{1}{2} m V_1^2 = 1.0\text{eV} - 0.5\text{eV}$$

$$\frac{1}{2} m V_1^2 = 0.5\text{eV} \qquad \text{..... (1)}$$

For hv = 2.5 eV maximum speed of electron = V_2

$$\frac{1}{2} m V_2^2 = hv - \phi_0$$

= (2.5 – 0.5) eV

$$\frac{1}{2}mV_2^2 = 2eV \quad ..... (2)$$

From equation (1) and (2)

$$\frac{V_1^2}{V_2^2} = \frac{0.5}{2} \qquad \frac{V_1}{V_2} = \sqrt{\frac{1}{4}}$$

$V_1 : V_2 = 1 : 2$

Example 17:

When a certain metallic surface is illuminated with monochromatic light of wave length λ, the stopping potential for photo electric current is $3V_0$ and when the same surface is illuminated with light of wavelength 2λ. The stopping potential is V_0. Find the thresold wavelength of this surface for photoelectric effect.

Solution :

Let the work function of metal be ϕ_0

According to Einstein equation $hv = \phi_0 + K_{max.}$

$$\frac{hc}{\lambda} = \phi_0 + K_{max.}$$

For wavelength of λ the stopping potential is $3V_o$.

$\Rightarrow K_{max.} = 3eV_0$

$$\Rightarrow \frac{hc}{\lambda} = \phi_0 + 3eV_0 \quad ..... (1)$$

For $\lambda = 2\lambda$, $\qquad K_{max.} = 1eV_0$

$$\Rightarrow \frac{hc}{2\lambda} = \phi_0 + 1eV_0 \quad ..... (2)$$

putting the value of $1eV_0$ in equation (1)

we have

$$\frac{hc}{\lambda} = \phi_0 + 3\left(\frac{hc}{2\lambda} - \phi_0\right)$$

$$\frac{hc}{\lambda} = \phi_0 + \frac{3hc}{2\lambda} - 3\phi_0$$

$$2\phi_0 = \frac{3hc}{2\lambda} - \frac{hc}{\lambda} = \frac{hc}{\lambda}\left(\frac{3}{2} - 1\right)$$

$$2\phi_0 = \frac{hc}{2\lambda} \qquad \phi_0 = \frac{hc}{4\lambda}$$

$$\frac{hc}{\lambda_0} = \frac{hc}{4\lambda} \quad \Rightarrow \quad \lambda_0 = 4\lambda$$

Example 18 :

The stopping potential for the photoelectorns emitted from a metal surface of work function 1.7eV is 10.4 V. Find the wave length of radiation used.

Solution :

According to Einstein equation

$K_{max.} = hv - \phi_0$

we have $K_{max.} = eV_s$.

{Where V_s - stopping potential}

$= 10.4$ eV $\qquad \phi_0 = 1.7$ eV

$\Rightarrow hv = K_{max.} + \phi_0$

$$\Rightarrow \frac{hc}{\lambda} = 10.4eV + 1.7eV$$

$$\Rightarrow \frac{hc}{\lambda} = 12.1 \text{ eV}$$

$$\Rightarrow \lambda = \frac{12375}{12.1}eV \quad \Rightarrow \quad \lambda = 1022.7Å$$

Example 19:

For what kinetic energy of a neutron will the associated de broglie wavelength be 1.40×10^{-10} m?

Solution :

We have $\lambda_d = \dfrac{h}{p} \qquad \lambda_d = \dfrac{h}{\sqrt{2m_n \cdot K}}$

$$\Rightarrow 2m_n \cdot K = \frac{h^2}{\lambda_d^2} \qquad K = \frac{h^2}{\lambda_d^2 2m_n}$$

$$= \frac{\left(6.63 \times 10^{-34}\right)^2}{\left(1.4 \times 10^{-10}\right)^2 \times 2 \times 1.675 \times 10^{-27} \text{kg}}$$

$K = 6.69 \times 10^{-21}$ Joule

Example 20:

A particle is moving three times as fast as an electron. The ratio of the de broglie wavelength of the particle to that of an electron is 1.813×10^{-4}. Calculate the particle's mass and identify the particle.

Solution :

Let the mass of the particle be m.

Debroglie wavelength of particle $\lambda_{d_1} = \frac{h}{m V_1}$ ..... (1)

Debroglie wavelength of the electron $\lambda_{d_2} = \frac{h}{m_e \cdot V_2}$ .. (2)

given $3V_2 = V_1$

from equation (1) and (2) {Where m_e - mass of the electron}

$$\frac{\lambda_{d_1}}{\lambda_{d_2}} = \frac{\frac{h}{mV_1}}{\frac{h}{m_e V_2}} \qquad 1.813 \times 10^{-4} = \frac{m_e}{m} \times \frac{V_2}{V_1}$$

$$1.813 = \frac{9.1 \times 10^{-31}}{m} \times \frac{V_2}{3V_2} \times 10^4$$

$m = 1.675 \times 10^{-27}$ kg

particle is neutron.

TEST YOUR SELF

1. If the energy of a photon is 25eV and the work function of the material is 7eV, then the value of stopping potential will be

   (1) 32V (2) 18V
   (3) 3.3V (4) zero

2. When the source of light is kept at a distance of 1m from photoelectric cell then the value of stopping potential is 4 volt. If it is kept at a distance of 4m then stopping potential will be

   (1) 2 Volt (2) 1 Volt
   (3) 4 Volt (4) 16 Volt

3. The light photons of energy 1eV and 2.5eV respectively are made incident on a metallic plate of work function 0.5 eV one after the other. The ratio of maximum kinetic energies of photoelectrons emitted by them will be

   (1) 4 : 1 (2) 1 : 4
   (3) 1 : 5 (4) 1 : 2

4. A photoelectric cell is illuminated by a small intense source distant 1m from it. When the same source is placed at a distance 2m then the number of photoelectrons emitted from the cathode will be

   (1) half
   (2) one fourth
   (3) each carries one fourth of its previous energy
   (4) each carries one fourth of its previous momentum

5. An electron and a photon posses the same de Broglie wavelength. If E_c and E_{ph} are respectively the energies of electron and photon and v and c are their velocities respectively, then

   (1) $\frac{E_e}{E_{ph}} = \frac{v}{e}$ (2) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

   (3) $\frac{E_e}{E_{ph}} = \frac{c}{v}$ (4) $\frac{E_e}{E_{ph}} = \frac{c}{2v}$

ANSWER

1. (2) 2. (3) 3. (2) 4. (2)
5. (1)