The Josephus problem

Suppose we have the numbers 1 - 100 arranged in a circle and we start deleting every second number. Which number survives?

In[9]:=
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<< Combinatorica`
```

The table below shows the order in which each number is deleted ie 1 is deleted on the 88th go.The survivor is 73

In[11]:=
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TableForm[Table[{i, Josephus[100, 2][[i]]}, {i, 1, 100}],
 TableHeadings → {None, {"n", "Deletion order"}}]
```

Out[11]//TableForm=

n	Deletion order
1	88
2	1
3	51
4	2
5	76
6	3
7	52
8	4
9	99
10	5
11	53
12	6
13	77
14	7
15	54
16	8
17	89
18	9
19	55
20	10
21	78
22	11
23	56
24	12
25	95
26	13
27	57
28	14
29	79
30	15
31	58
32	16
33	90
34	17
35	59
36	18
37	80
38	19
39	60
40	20
41	98
42	21
43	61
44	22

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45    81
46    23
47    62
48    24
49    91
50    25
51    63
52    26
53    82
54    27
55    64
56    28
57    96
58    29
59    65
60    30
61    83
62    31
63    66
64    32
65    92
66    33
67    67
68    34
69    84
70    35
71    68
72    36
73    100
74    37
75    69
76    38
77    85
78    39
79    70
80    40
81    93
82    41
83    71
84    42
85    86
86    43
87    72
88    44
89    97
90    45
91    73
92    46
93    87
94    47
95    74
96    48
97    94
98    49
99    75
100   50
```

The following list shows the order in which each number is deleted ie 2 is deleted first, then 4 etc. Again 73 is the survivor.

In[12]:=
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InversePermutation[Josephus[100, 2]]
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Out[12]=
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{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42,
 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80,
 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, 3, 7, 11, 15, 19, 23, 27, 31, 35, 39,
 43, 47, 51, 55, 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, 99, 5, 13, 21, 29, 37,
 45, 53, 61, 69, 77, 85, 93, 1, 17, 33, 49, 65, 81, 97, 25, 57, 89, 41, 9, 73}
```

A formula for calculating the survivor when each second number is deleted is

$$L(n, 2) = 1 + 2n - 2^{1+\lfloor \mathrm{Log}(n,2) \rfloor}$$

where $\lfloor \mathrm{Log}(n,2) \rfloor$ is the floor function of Log to base 2 of the number n.

Log(100, 2) is the power to which 2 must be raised to give 100; this is 6.64386 and the floor function of this number (the integer below) is 6.

Hence L(100, 2) = 1 + 200 - 2^7 = 73