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LOGICAL PRODUCT AND QUOTIENT RULES

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Students often find it difficult to learn the Product and Quotient Rules for differentiation and I am not surprised because they are often presented in illogical ways in textbooks.

Consider the ways the rules are defined in three textbooks:

$$y = uv \qquad y = \frac{u}{v}$$

$$\begin{array}{ll} y' = uv' + u'v & y' = \frac{vu' - uv'}{v^2} \\ y' = uv' + vu' & y' = \frac{vu' - uv'}{v^2} \\ y' = vu' + uv' & y' = \frac{vu' - uv'}{v^2} \end{array}$$

The Quotient Rule is the same in all cases but the Product Rule is different and hence there are different relationships between the two. Herein lies the seeds of confusion.

Logical representation of the Rules

- **Product Rule**
  Given $y = uv$

$$y' = u'v + uv' \tag{1}$$

- **Quotient Rule**
  Given $y = \frac{u}{v}$

$$y' = \frac{u'v - uv'}{v^2} \tag{2}$$

Why is this the best way?

1. The only difference between the Product Rule and the numerator of the Quotient Rule is that + changes to -.
2. The Quotient Rule is derived from the Product Rule and therefore should show similarity. Instead of writing $y = \frac{u}{v}$ we can write $y = uv^{-1}$ and use the Product Rule together with the Function-of-function Rule to differentiate v^{-1}. The - comes from differentiating the denominator, that's why it is $-uv'$ in the numerator.
3. The sequence u, v remains in order. So effectively you are differentiating each factor in order. For example, if $y = uvw$ then $y' = u'vw + uv'w + uvw'$
4. There is no need to write u, u', v, v' separately and then combine. Students are often taught to do this but this is inefficient in an exam situation - not the best use of time. Learn to do it the efficient way from the beginning!