

KENDERDINE MATHS TUTORING

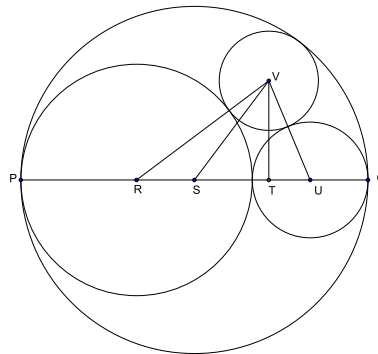
ABN 49 278 719 954

6 Braeside Dr
BOWRAL NSW 2576
AUSTRALIA

4862 2827 mob 0429 317 491
rdkdine@acenet.com.au
www.kenderdinemathstutoring.com.au

TANGENTIAL CIRCLES

Prepared by Dr Richard Kenderdine



Let S be the centre of circle C_0 , R the centre of C_1 , U the centre of C_2 and V the centre of C_3 .

Draw VT perpendicular to PQ . Let $x = TU$, $h = TV$ and r the radius of C_3 .

Now consider the 3 right-angled triangles:

1. RTV with $RV = \frac{1}{3} + r$, since the radius of C_1 is $\frac{1}{3}$, $RT = \frac{1}{2} - x$ and $VT = h$

2. STV with $ST = \frac{1}{3} - x$ since radii of C_0 and C_2 are $\frac{1}{2}$ and $\frac{1}{6}$ respectively, $SV = \frac{1}{2} - r$

3. TUV with $UV = \frac{1}{6} + r$

Now use Pythagoras' Theorem with these triangles to form three equations with three unknowns, h, r, x . Use two of these equations to eliminate h ; then use one of these equations together with the remaining equation to eliminate h again. Now there are two equations with two unknowns, r and x . Eliminate x to find $r = \frac{1}{7}$. For the record, $h = \frac{2}{7}$ and $x = \frac{5}{42}$.

This problem finds the centre and radius of the first circle in a *Pappus Chain*. The area enclosed between circles C_0, C_1 and C_2 in the top semi-circle, without C_3 , is called an *arbelos*.

Extension: (1) Research Pappus chain and arbelos. (2) Find an expression for the radius of C_3 when the radii of C_1 and C_2 are in the ratio $p : 1$.