AN INTRODUCTION TO MATH
Unit 1 of 4

This course is an adaptation of *The Numbers Game* from College Guild, a non-profit organization offering free, general interest correspondence courses to prisoners (their contact information and course offerings are attached to the end of each unit). They have no religious or political affiliation. We will be doing these units from their math course as class assignments.

Directions:

1) Write your full name at the top of this page.
2) Break up into groups of three-to-five students.
3) Answer all the questions in bold print.
4) Take time to read the question thoroughly and find the most creative way to word your answers.
5) Discuss your answers within the group; decide who has the best answer for each question to share with the class. (We will have a group discussion at the end of the assignment.)

****

As everyone knows, the only way to really learn anything – be it football, piano playing, or Latin – is to do it. This is true above all else in learning math. But, like football, the doing can be fun, and it can be play. With this object in mind, consider learning to play with mathematics.

Our spotter counts reps, and we push to get in one more number; the microwave counts backward to snack time …

1. Make a list of counts you have made, or could make.

Undoubtedly, these will include months/years in prison.

2. How many *days* have you been in prison so far? Were there any leap years?

3. How many steps can you take in the length of your cell? How many in the width?

Adults can have fun learning math, but many of us were turned off by unhappy early training.

4. Were you turned off by math earlier in life? If you know why, explain what happened.

Some of the best ways to get into the numbers game are by thinking about math and math concepts and asking questions. Try asking yourself and then asking others in your group, "What is the biggest number you can think of?" This isn't very interesting unless you connect it to something. Depending on personal experience, a person might think of "a hundred," "a thousand," or "a million," but if you also ask, "Can you think of something that *is* a million?" the question becomes more difficult (and interesting!).

5. What can you think of that *is* 10?

6. What is 100?

7. What is 1,000?

8. Now try … What is the biggest number you know of that actually goes with something?

This opens up a whole world of associations: the height of a building, the distance around the world, the number of steps in a flight of stairs, the interstate highway number around a big city, the number of feet in a mile, the population of your town, the distance to the moon, the national debt. The possibilities are endless. One sixth grader came up with a great answer: "6 billion, the number of hamburgers McDonald's has sold." Scientists can go on forever – well, no, not forever but for a long time on this one. Try to think of what you are sure you can imagine. Units, tens, and hundreds are not too hard. But it is hard to imagine what a thousand of something looks like, and bigger numbers are nearly impossible to imagine. To make this manageable, we have to have a way of keeping numbers in order.

Numbers have place value. That means that a plain 7 is different from 70, because the 7 is in a different place. In the number 54,321, the last number ("1") means the number of plain units. If we changed it to 54,312, the one would mean 10, because it is in the "tens place." We have only ten different symbols for numbers – 0 (zero) through 9. When we need to write a bigger number, we use two "old" symbols and put them together to make "new" numbers that are bigger than nine, such as 10. Next, we can use up all the digits in the "units" place until we get to 19. After that, we get into 20 … 29, 30 … 39, until we reach 99. After this, we need to add a new place, so we go on to 100. The hundreds last until we get to 999. Since we have no more symbols to use, we add another place, and go on into the thousands (1,000).

9. What is the biggest number in the thousands?

Now consider the number 654,321. It means 1 unit, 2 tens (or 20), 3 hundreds, 4 thousands, 5 ten-thousands, and 6 hundred-thousands. The conventions of math notation condense all this into one number: 654,321. After we get through with the hundred-thousands, we go on to millions for three places, and then billions for three places. After that, most people don't bother with words. They use exponents, which will come later.

10. How many zeroes will there always be for millions (for example, four million)?

How about billions?

And for trillions?

Here are some examples of amounts written using numbers and, alternatively, words:

802	Eight hundred two
7,650	Seven thousand six hundred fifty
340,867	Three hundred forty thousand eight hundred sixty seven
621,000,000	Six hundred twenty one million

Going from words to numbers, “five thousand sixty seven” is 5,067. “Eight billion, nine hundred six million” is 8,906,000,000.

Before you write down your big number, check on how you read and write big numbers.

11. How would you write the number 7,392 in words?

How about the number 18,509?

And 3,852,000?

Now, let’s go the other way, from words to numbers.

12. How would you write “nine thousand twenty seven” in numbers?

How about “four hundred eighty thousand six hundred two”?

And “nine hundred eighty seven million six hundred fifty four thousand three hundred twenty one”?

And finally, “eight million four hundred two thousand”?

Usually, numbers are set off in groups of three separated by commas, so “six hundred fifty four thousand three hundred twenty one” would be 654,321. Notice that there is no “and” in this or any other large number. The only time you should use “and” is to indicate a decimal point. For example, $83.06 is read, “Eighty three dollars <u>and</u> six cents.” Decimals and decimal points will come in a later unit.

Imagining big numbers often leads us to thinking about what we would do with a big number of dollars.

13. What would you do with $100?

With $1,000?

With $10,000?

With $100,000?

Think up or look up some big numbers. Magazines, textbooks, and encyclopedias are great sources. Now, fill in the chart below with numbers you think of, look up, or find, and for each one give an example of something that *is* that number. We refer to those things as an “Association” in the chart below and provide some examples.

14. UNITS (1, 2, 3 … 9)

My number is: 4 Association: Legs on a chair

a) Your number: _____ Association: ______________________________

b) Your number: _____ Association: ______________________________

c) Your number: _____ Association: ______________________________

15. TENS (10, 11 … 99)

My number is: 88 Association: Keys on a piano

a) Your number: ______ Association: ______________________________

b) Your number: ______ Association: ______________________________

c) Your number: ______ Association: ______________________________

16. HUNDREDS

My number is: 476 Association: Date of the fall of the Roman Empire

a) Your number: _______Association: ____________________________

b) Your number: _______Association: ____________________________

c) Your number: _______Association: ____________________________

17. THOUSANDS

My number is: 1,250 Association: Height of the Empire State Building in feet

a) Your number: ________ Association: ___________________________

b) Your number: ________ Association: ___________________________

c) Your number: ________ Association: ___________________________

18. TEN THOUSANDS

My number is: 54,500 Association: Area of NY State

a) Your number: __________ Association: _________________________

b) Your number: __________ Association: _________________________

c) Your number: __________ Association: _________________________

19. HUNDRED THOUSANDS My number is: 186,250 Association: Speed of light in miles per second

a) Your number: ___________ Association: ________________________

____

b) Your number: ___________ Association: ________________________

____

20. MILLIONS My number is: 9,500,000 Association: Approximate number of U.S drivers 19 and under

a) Your number: ___________ Association: ________________________

____

b) Your number: ___________ Association: ________________________

____

21. TEN MILLIONS My number is: 93,000,000 Association: Miles from the earth to the sun

a) Your number: ___________ Association: ________________________

____

22. HUNDRED MILLIONS My number is: 300,000,000 Association: U.S. population in 2006

a) Your number: ___________ Association: ________________________

____

23. BILLIONS My number is: $1,318,474,576 Association: U.S. Humanitarian Aid to Syria in 2015

a) Your number: ___________ Association: ________________________

____

24. Now, write the biggest number you can think of with which you have some association. What is the number, and what is the association?

What all this really leads to is the nature of math, the burst of different ideas for which we use math, and all of our needs to know math and to know about the world around us. It is thinking about quantity and sizes and about ordering these.

25. Where do you use numbers now? List three different examples.

26. Now list three places where you would like to use numbers.

27. For further thought, what is the …

a. height of the Empire State Building? (Hint: You can find it in this unit.)

b. number of eggs in a dozen?

c. number of feet in a mile?

d. human population of the Garden of Eden?

e. number of minutes in a year?

f. approximate distance across the USA?

g. number of squares on a checker board?

h. year in which “Columbus sailed the ocean blue”?

28. Now, put your answers in order, from smallest to largest.

29. Write a phrase about or draw a picture of something BIG!

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Answer Key to Unit 1 of 4

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Our spotter counts reps, and we push to get in one more number; the microwave counts backward to snack time …

1. Make a list of counts you have made, or could make.

Undoubtedly, these will include months/years in prison.
Answers will vary. For example: Years in prison, years since son's birth, bag of peanuts (or other food stuff) on my shelf, reps of exercises.

2. How many *days* have you been in prison so far? Were there any leap years?
Answers will vary. For example: September 3, 2000 – April 18, 2016 =
120 + (3 x 366) + (12 x 365) + 109 = 5,707
3. How many steps can you take in the length of your cell? How many in the width?
Answers may vary. For example: Length 12.5 ft. (size 11 shoe = c. 12”)
Width 6.5 ft.
Adults can have fun learning math, but many of us were turned off by unhappy early training.

4. Were you turned off by math earlier in life? If you know why, explain what happened.

Answers will vary. For example: Dyslexic – no ability for rote memorization

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5. What can you think of that *is* 10? **Answers will vary.** For example: 10 Commandments

6. What is 100? **Answers will vary.** For example: 100 Fish Oil gelcaps in a bottle.

7. What is 1,000? **Answers will vary.** For example: 1,000 pages in a book.

8. Now try … What is the biggest number you know of that actually goes with something?
Answers will vary. For example: 100 billion – estimated number of nerve cells in your brain and spinal cord.

This opens up a whole world of associations: the height of a building, the distance around the world, the number of steps in a flight of stairs, the interstate highway number around a big city, the number of feet in a mile, the population of your town, the distance to the moon, the national debt. The possibilities are endless. One sixth grader came up with a great answer: "6 billion, the number of hamburgers McDonald's has sold." Scientists can go on forever – well, no, not forever but for a long time on this one. Try to think of what you are sure you can imagine. Units, tens, and hundreds are not too hard. But it is hard to imagine what a thousand of something looks like, and bigger numbers are nearly impossible to imagine. To make this manageable, we have to have a way of keeping numbers in order.

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9. What is the biggest number in the thousands? 9,999

Now consider the number 654,321. It means 1 unit, 2 tens (or 20), 3 hundreds, 4 thousands, 5 ten-thousands, and 6 hundred-thousands. The conventions of math notation condense all this into one number: 654,321. After we get through with the hundred-thousands, we go on to millions for three places, and then billions for three places. After that, most people don't bother with words. They use exponents, which will come later.

10. How many zeroes will there always be for millions (for example, four million)? 6 zeroes

How about billions? 1,000,000,000 = 9 zeros

And for trillions? 1,000,000,000,000 = 12 zeros

Here are some examples of amounts written using numbers and, alternatively, words:

802	Eight hundred two
7,650	Seven thousand six hundred fifty
340,867	Three hundred forty thousand eight hundred sixty seven
621,000,000	Six hundred twenty one million

Going from words to numbers, "five thousand sixty seven" is 5,067. "Eight billion, nine hundred six million" is 8,906,000,000.

Before you write down your big number, check on how you read and write big numbers.

11. How would you write the number 7,392 in words? Seven thousand three hundred ninety two

How about the number 18,509? Eighteen thousand five hundred nine

And 3,852,000? Three million eight hundred fifty two thousand

Now, let's go the other way, from words to numbers.

12. How would you write "nine thousand twenty seven" in numbers? 9,027

How about "four hundred eighty thousand six hundred two"? 480,602

And "nine hundred eighty seven million six hundred fifty four thousand three hundred twenty one"? 987,654,321

And finally, "eight million four hundred two thousand"? 8,402,000

Usually, numbers are set off in groups of three separated by commas, so "six hundred fifty four thousand three hundred twenty one" would be 654,321. Notice that there is no "and" in this or any other large number. The only time you should use "and" is to indicate a decimal point. For example, $83.06 is read, "Eighty three dollars <u>and</u> six cents." Decimals and decimal points will come in a later unit.

Imagining big numbers often leads us to thinking about what we would do with a big number of dollars.

13. What would you do with $100? Answers will vary. $100 – I'd put it on my commissary.

With $1,000? – I'd divorce my second wife.

With $10,000? $10,000 - $100,000 – I'd open a savings account for when I get out of prison (after doing the two above).

With $100,000?

Think up or look up some big numbers. Magazines, textbooks, and encyclopedias are great sources. Now, fill in the chart below with numbers you think of, look up, or find, and for each one give an example of something that *is* that number. We refer to those things as an "Association" in the chart below and provide some examples. **Answers will vary.** For example:

14. UNITS (1, 2, 3 … 9)

My number is: 4 **Association:** Legs on a chair

a) Your number: 2 **Association:** eyes on my face (arms, legs, etc.

b) Your number: 4 **Association:** wheels on a car

c) Your number: 7 **Association:** days in a week

15. TENS (10, 11 … 99)

My number is: 88 **Association:** Keys on a piano

a) Your number: 12 **Association:** months in a year

b) Your number: 50 **Association:** U.S. Governors

c) Your number: 66 **Association:** books in the Bible

16. HUNDREDS

My number is: 476 **Association:** Date of the fall of the Roman Empire

a) Your number: 100 **Association:** Senators in Congress

b) Your number: 325 **Association:** date of First Council of Nicaea

c) Your number: 701 BC **Association:** Sennacherib invaded Palestine

17. THOUSANDS

My number is: 1,250 **Association:** Height of the Empire State Building in feet

a) Your number: 1,000 **Association:** milligrams in a gram

b) Your number: 1,728 **Association:** cubic inches in 1 cubic foot

c) Your number: 2,000 **Association:** pounds in a ton

18. TEN THOUSANDS

My number is: 54,500 **Association:** Area of NY State

a) Your number: 33,000 **Association:** height in feet an airplane cabin must be pressurized

b) Your number: 56,600 **Association:** population of Agrigento (a city in Sicily, Italy) in 1990

c) Your number: 75,000 **Association:** displacement in metric tons of an aircraft carrier

19. HUNDRED THOUSANDS

My number is: 186,250 **Association:** Speed of light in miles per second

a) Your number: 117,000 **Association:** population of Hebron (a city in Israel) est. in 1995

b) Your number: 251,773 **Association:** square-mile area of Afghanistan

20. MILLIONS

My number is: 9,500,000 **Association:** Approximate number of U.S drivers 19 and under

a) Your number: 5,967,305 **Association:** pop. of the Hessen region of Germany in 1993

b) Your number: 6,560,000 **Association:** population of Beijing, ca. 1995

21. TEN MILLIONS

My number is: 93,000,000 **Association:** Miles from the earth to the sun

a) Your number: 10,000,000 **Association:** estimated number killed in WWI (50,000,000 in WWII)

22. HUNDRED MILLIONS

My number is: 300,000,000 **Association:** U.S. population in 2006

a) Your number: 395,000,000 **Association:** population of North America, ca. 1995

23. BILLIONS

My number is: $1,318,474,576 **Association:** U.S. Humanitarian Aid to Syria in 2015

a) Your number: 1,187,997,000 **Association:** population of China, ca. 1995

24. Now, write the biggest number you can think of with which you have some association. What is the number, and what is the association?

100 billion – estimated number of nerve cells in your brain and spinal cord.

What all this really leads to is the nature of math, the burst of different ideas for which we use math, and all of our needs to know math and to know about the world around us. It is thinking about quantity and sizes and about ordering these.

25. Where do you use numbers now? List three different examples.

Figuring commissary balances; telling time; explaining math to students in ABE

26. Now list three places where you would like to use numbers.

Figuring my release date; figuring my income/taxes; filling my gas tank

27. For further thought, what is the …

a. height of the Empire State Building? (Hint: You can find it in this unit.) 1,250 ft.

b. number of eggs in a dozen? 12

c. number of feet in a mile? 5,280 ft.

d. human population of the Garden of Eden? 2

e. number of minutes in a year? 525,600 min.

f. approximate distance across the USA? 3,000 miles

g. number of squares on a checker board? 64

h. year in which "Columbus sailed the ocean blue"? 1492

28. Now, put your answers in order, from smallest to largest.

2
12
64
1,250
1,492
3,000
5,280
525,600

29. Write a phrase about or draw a picture of something BIG!

Answers will vary. For example: God "stretchest out the heavens like a curtain ..." and the universe is still expanding in every direction. That's big!

AN INTRODUCTION TO MATH
Unit 2 of 4

To be proficient in math, one must know and be able to use the basic number facts; that is, the simple addition and multiplication facts using the numbers 0 through 9. They do not have to be dreary. In fact, some really neat relationships can be found within them. Noticing these relationships makes them a lot more interesting. As elementary school children, we were not shown many of these patterns. In this unit and the next ones, some number oddities are shown that should help you to learn these basic facts – and have some fun at the same time.

PALINDROMIC NUMBERS

Some numbers read the same going left to right as they do going right to left. 56765 is one example. The same is true for words like “radar”; or names like “Anna,” “Otto,” and “Hannah”; or phrases like “Poor Dan is in a droop,” or “Able was I ere I saw Elba.” These are called *palindromes*. You have probably heard some. Language lovers like to collect them and love to invent them.

1. Make a list of words and/or names that are palindromes. Next, write down some phrases that you have heard, read, or made up yourself that are palindromes.

There has been a good deal of study on palindromic numbers. It is part of a branch of mathematics called ***Number Theory***. Palindromic numbers are kind of fun to notice on a car’s odometer, but there’s more to them than just the pattern. For example, take the number 742.

Reverse the digits to get a second number and then add them together:

```
  742
+ 247
-----
  989  It came out
       palindromic!
```

2. Try this with the numbers:

a. 423

b. 621

c. 238

As you can see, it doesn't work all the time …

But, try taking it another step with 238. Reverse the digits in your answer, and add that number to your answer. Did you get a palindrome?

In case that was confusing or you didn't get a palindrome, let's do an example together with the number 561:

Step One:	561 +165 726	Reverse the 561 to 165 and add the numbers together.
Step Two:	+ 627 1353	Reverse the answer 726 to 627 and add the numbers together. (*Uh oh … still no palindrome … but, if we go one more step …*)
Step Three:	+3531 4884	Reverse the answer 1353 to 3531 and add the numbers together Bingo! We got a palindrome.

3. Now you try it with the numbers:

a. 43

b. 56

c. 78

d. 35

e. 67

It is easy to check that it will always work for two digit numbers. If the sum of the two digits is less than 10, it is clear that it will work in one step, as you saw when working with 43 and 35. If the sum of the digits is 10, 11, 12, 13, 14, 15, 16, or 18 (as it was with 56, 78, and 67) it works in six steps or less. If the digits add up to 17, however, it can take a while.

4. What two digit numbers have digits that add up to 17?

5. Why don't we have to worry about instances where the two digits add up to 19?

With three digit numbers, it can get quite big sometimes before it works. It will still work, though!

6. Try it with the numbers:
a. 597 **b. 876**

Mathematicians (those who like to sit around and add forever or who program computers to do it for them) have found that there are 249 numbers less than 10,000 which do NOT generate palindromes after 100 steps. Aside from these 249 exceptions and 89, all integers (whole numbers and zero) produce a palindrome in less than 24 steps. The smallest of these exceptions is 196. If you reverse the digits and add (and have oodles of time), at the end of 230,310 additions you will come out with a palindromic number! The largest palindrome found from integers less than 10,000 is generated by 6,999 in 20 steps.

7. Now, you try it with 89. The process is started for you on the next page. Be sure to keep your numbers in line – ones in the ones place, tens in the tens place, etc. If you get a palindromic number before you have done 24 steps, it means you made an error. Here are some "check points" to help you stay on track:

After the 10th step, you should have 8,872,688.

After the 15th step, you should have 1,317,544,822.

After the 20th step, you should have 93,445,163,438.

The last/24th step starts 8,813 … and it has 13 digits.

			8	9
		+	9	8
	=	1	8	7
	+	7	8	1
	=	9	6	8
	+			
=				
+				
=				

MATH JEOPARDY

Math Jeopardy is a game in which someone says a number, and the responder must ask a question for which this is the answer. If we were playing with names instead of numbers, I might say, “The answer is George Washington,” and you would say, “Who was the first president of the United States?” In Math Jeopardy, I might say, “The answer is 54.” You would have to come up with a question whose answer is 54, such as: “What is 47 + 7?”

Of course, there could be more than one right question for any given answer. If the answer is 54, another perfectly good question is, “What is 60 - 6?” Below, you will be asked to come up with still another question whose answer is 54.

The answers can be refined, restricted, or set up with different rules, such as “the question must be about a multiplication fact,” but it’s less fun this way. It also means you might not get the chance to come up with an extremely imaginative question like: “What is the square root of 2,916?” (Which also happens to be 54.) The game has good possibilities for letting a show off be just that without being pretentious. It also has the virtue of teaching that there can be more than one correct response to a math question.

Below are some answers. You write the questions. Make up several for each answer. Yes, it’s hard work to make up more than 2 or 3 questions for each answer – but you will also find that it becomes more fun and more interesting the more questions you try to write. Good luck!

8. The answer is 17.

9. The answer is 54.

10. The answer is 81.

11. The answer is 256.

12. The answer is _______? (You make one up). Now, create some questions to go with it.

13. The answer is _______ ? Create some questions to go with it.

NOTES ON THE FOUR BASIC ARITHMETIC PROCESSES

We are ordinarily taught about addition, subtraction, multiplication, and division. It is easier to think of subtraction and division as "un-adding" and "un-multiplying." In subtracting, what we are doing is asking the question, "What must I add to a number to get the given total?" For example, 13 – 9 really means: "What must I add to 9 in order to get 13?" This is called an ***inverse process***. ("Inverse" means something like "opposite of.") Subtraction is the inverse of addition.

Likewise, division is the inverse of multiplication. 18 divided by 3 asks the question: "What must I multiply by 3 to get 18?" So, subtraction and division are inverse processes. We also use the word "inverse" for numbers. For example, negative 8 (-8) is the inverse of positive 8. So, all subtraction problems are merely ones of adding the inverse number, as in this example:

12 – 8 = 4 is exactly the same as 12 + (-8) = 4

For a cool subtraction trick, try the following. Think of a number between 100 and 1,000. It should not end in 00, and the difference between the first and last digits should be greater than 1. A number like 842 would be fine. Reverse the digits and subtract the smaller from the larger, like this:

$$\begin{array}{r} 842 \\ \underline{-\ 248} \\ 594 \end{array}$$

Now reverse the digits of the answer and add, like this:

$$\begin{array}{r} 594 \\ \underline{+\ 495} \\ 1089 \end{array}$$

14. Now, you try it with another number. (Remember, it must be 3 digits, shouldn't end in 00, and the difference between the first and last digits should be greater than 1.) **What is your final answer?**

15. Try three more numbers, and write your final answers.

a.

b.

c.

16. Any ideas on why this happens? Take your best shot at finding an explanation.

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Answer Key to Unit 2 of 4

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```

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+ 126
  747
```

c.
```
  238
+ 832
 1070
+0701
 1771
```

As you can see, it doesn't work all the time …

But, try taking it another step with 238. Reverse the digits in your answer, and add that number to your answer. Did you get a palindrome?

In case that was confusing or you didn't get a palindrome, let's do an example together with the number 561:

Step One:	561 +165 726	Reverse the 561 to 165 and add the numbers together.
Step Two:	+ 627 1353	Reverse the answer 726 to 627 and add the numbers together. (*Uh oh … still no palindrome … but, if we go one more step …*)
Step Three:	+3531 4884	Reverse the answer 1353 to 3531 and add the numbers together Bingo! We got a palindrome.

3. Now you try it with the numbers:

a. 43

a. 43
+ 34
77

b. 56

b. 56
+ 65
121

c. 78

c. 78
+ 87
165
561
726
627
1353
3531
4884

d. 35

d. 35
+ 53
88

e. 67

e. 67
+ 76
143
341
484

It is easy to check that it will always work for two digit numbers. If the sum of the two digits is less than 10, it is clear that it will work in one step, as you saw when working with 43 and 35. If the sum of the digits is 10, 11, 12, 13, 14, 15, 16, or 18 (as it was with 56, 78, and 67) it works in six steps or less. If the digits add up to 17, however, it can take a while.

4. What two digit numbers have digits that add up to 17?

98, 89
(9 + 8 = 8 + 9 = 17)

5. Why don't we have to worry about instances where the two digits add up to 19?

9 + 9 = 18 (The maximum possible sum is 18.)

With three digit numbers, it can get quite big sometimes before it works. It will still work, though!

6. Try it with the numbers:

a. 597

```
  597
+ 795
 1392
 2931
 4323
 3234
 7557
```

b. 876

```
  876
+ 678
 1554
 4551
 6105
 5016
11121
12111
23232
```

Mathematicians (those who like to sit around and add forever or who program computers to do it for them) have found that there are 249 numbers less than 10,000 which do NOT generate palindromes after 100 steps. Aside from these 249 exceptions and 89, all integers (whole numbers and zero) produce a palindrome in less than 24 steps. The smallest of these exceptions is 196. If you reverse the digits and add (and have oodles of time), at the end of 230,310 additions you will come out with a palindromic number! The largest palindrome found from integers less than 10,000 is generated by 6,999 in 20 steps.

7. Now, you try it with 89. The process is started for you on the next page. Be sure to keep your numbers in line – ones in the ones place, tens in the tens place, etc. If you get a palindromic number before you have done 24 steps, it means you made an error. Here are some "check points" to help you stay on track:

After the 10th step, you should have 8,872,688.

After the 15th step, you should have 1,317,544,822.

After the 20th step, you should have 93,445,163,438.

The last/24th step starts 8,813 … and it has 13 digits.

																												8	9
																											+	9	8
																										=	1	8	7
																										+	7	8	1
																										=	9	6	8
																										+	8	6	9
																									=	1	8	3	7
																									+	7	3	8	1
																									=	9	2	1	8
																										8	1	2	9
																									1	7	3	4	7
																									7	4	3	7	1
																									9	1	1	1	8
																									8	1	7	1	9
																								1	7	3	4	3	7
																								7	3	4	3	7	1
																								9	0	7	8	0	8
																								8	0	8	7	0	9
																							1	7	1	6	5	1	7
																							7	1	5	6	1	7	1
																							8	**8**	**7**	**2**	**6**	**8**	**8**
																							8	8	6	2	7	8	8
																						1	7	7	3	5	4	7	6
																						6	7	4	5	3	7	7	1
																						8	5	1	8	9	2	4	7
																						7	4	2	9	8	1	5	8
																					1	5	9	4	8	7	4	0	5
																					5	0	4	7	8	4	9	5	1
																					6	6	4	2	7	2	3	5	6
																					6	5	3	2	7	2	4	6	6
																				1	**3**	**1**	**7**	**5**	**4**	**4**	**8**	**2**	**2**
																				2	2	8	4	4	5	7	1	3	1
																				3	6	0	2	0	0	1	9	5	3
																				3	5	9	1	0	0	2	0	6	3
																				7	1	9	3	0	0	4	0	1	6
																				6	1	0	4	0	0	3	9	1	7
																			1	3	2	9	7	0	0	7	9	3	3
																			3	3	9	7	0	0	7	9	2	3	1
																			4	7	2	6	7	0	8	7	1	6	4
																			4	6	1	7	8	0	7	6	2	7	4
																			9	**3**	**4**	**4**	**5**	**1**	**6**	**3**	**4**	**3**	**8**
																			8	3	4	3	6	1	5	4	4	3	9
																		1	7	6	8	8	1	3	1	7	8	7	7
																		7	7	8	7	1	3	1	8	8	6	7	1
																		9	5	5	5	9	4	5	0	6	5	4	8
																		8	4	5	6	0	5	4	9	5	5	5	9
																	1	8	0	1	2	0	0	0	0	2	1	0	7
																	7	0	1	2	0	0	0	0	2	1	0	8	1
																	8	**8**	**1**	**3**	**2**	**0**	**0**	**0**	**2**	**3**	**1**	**8**	**8**

MATH JEOPARDY

Math Jeopardy is a game in which someone says a number, and the responder must ask a question for which this is the answer. If we were playing with names instead of numbers, I might say, "The answer is George Washington," and you would say, "Who was the first president of the United States?" In Math Jeopardy, I might say, "The answer is 54." You would have to come up with a question whose answer is 54, such as: "What is 47 + 7?"

Of course, there could be more than one right question for any given answer. If the answer is 54, another perfectly good question is, "What is 60 - 6?" Below, you will be asked to come up with still another question whose answer is 54.

The answers can be refined, restricted, or set up with different rules, such as "the question must be about a multiplication fact," but it's less fun this way. It also means you might not get the chance to come up with an extremely imaginative question like: "What is the square root of 2,916?" (Which also happens to be 54.) The game has good possibilities for letting a show off be just that without being pretentious. It also has the virtue of teaching that there can be more than one correct response to a math question.

Below are some answers. You write the questions. Make up several for each answer. Yes, it's hard work to make up more than 2 or 3 questions for each answer – but you will also find that it becomes more fun and more interesting the more questions you try to write. Good luck!

8. The answer is 17.

What is the square root of 289?
What is the cubed root of 4,913?
What is one times 17?; What is ten plus 7?

9. The answer is 54.

What is 18 times 3?
What is 9 times 6?
What is the square root of 2,916?

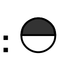
10. The answer is 81.

What is 9^2?
What is 486 divided by 6?
What is 567 minus 486?
What is 40 plus 41?

11. The answer is 256.

What is 16^2
What is128 times 2?
What is $\sqrt{65536}$?
What is $255\ ^3/_3$?

12. The answer is __½__? (You make one up). Now, create some questions to go with it.

What is 50%?; What is 0.5?; What is 4/8?; What fraction is shaded:  **?**

13. The answer is ___-5___ ? Create some questions to go with it.

Other than 5, what equals │5│? (absolute value); What is 0 minus 5?; What is 10 minus 15?; What is -5 times 1?; What is 5 divided by -1?

NOTES ON THE FOUR BASIC ARITHMETIC PROCESSES

We are ordinarily taught about addition, subtraction, multiplication, and division. It is easier to think of subtraction and division as "un-adding" and "un-multiplying." In subtracting, what we are doing is asking the question, "What must I add to a number to get the given total?" For example, 13 – 9 really means: "What must I add to 9 in order to get 13?" This is called an ***inverse process***. ("Inverse" means something like "opposite of.") Subtraction is the inverse of addition.

Likewise, division is the inverse of multiplication. 18 divided by 3 asks the question: "What must I multiply by 3 to get 18?" So, subtraction and division are inverse processes. We also use the word "inverse" for numbers. For example, negative 8 (-8) is the inverse of positive 8. So, all subtraction problems are merely ones of adding the inverse number, as in this example:

12 – 8 = 4 is exactly the same as 12 + (-8) = 4

For a cool subtraction trick, try the following. Think of a number between 100 and 1,000. It should not end in 00, and the difference between the first and last digits should be greater than 1. A number like 842 would be fine. Reverse the digits and subtract the smaller from the larger, like this:

$$\begin{array}{r} 842 \\ -\underline{248} \\ 594 \end{array}$$

Now reverse the digits of the answer and add, like this:

$$\begin{array}{r} 594 \\ +\underline{495} \\ 1089 \end{array}$$

14. Now, you try it with another number. (Remember, it must be 3 digits, shouldn't end in 00, and the difference between the first and last digits should be greater than 1.) **What is your final answer?**

$$\begin{array}{r} 753 \\ -\underline{357} \\ 396 \end{array} \qquad \begin{array}{r} 396 \\ +\underline{693} \\ 1089 \end{array}$$

15. Try three more numbers, and write your final answers.

a.

$$\begin{array}{r} 975 \\ -\underline{579} \\ 396 \end{array} \qquad \begin{array}{r} 396 \\ +\underline{693} \\ 1089 \end{array}$$

b.

$$\begin{array}{r} 962 \\ -\underline{269} \\ 693 \end{array} \qquad \begin{array}{r} 693 \\ +\underline{396} \\ 1089 \end{array}$$

c.

$$\begin{array}{r} 753 \\ -\underline{357} \\ 396 \end{array} \qquad \begin{array}{r} 396 \\ +\underline{693} \\ 1089 \end{array}$$

16. Any ideas on why this happens? Take your best shot at finding an explanation.

Suppose the number is xyz, i.e. 100x+10y+z. Reverse the digits to get x+10y+100z. Subtract to get 99x-99y or 99(x-y). Now 99 times a number k will have 3 digits, the first being k-1, the middle being 9, the final digit being 10-k. Reverse and add to get 900+180+9=1089

AN INTRODUCTION TO MATH
Unit 3 of 4

HELPS FOR MULTIPLYING

There is no great problem for multiplying as there is (in the palindromic number exercise) for addition, but there are a good many ways to learn multiplication facts. These facts are essential to working in math with ease. You have to practice them many, many times to really know them. With this in mind, your first task is to fill in the three tables below. The first one is set up for you to use and, if need be, refer to. The next three are mixed up, and you will need to think carefully to complete them.

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

x	**9**	**7**	**4**	**8**
5		35		
3				24
6				
7	63			

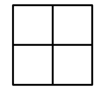
1. Table #1

x	**8**	**7**	**3**	**5**
8				
7				
3				
5				

2. Table #2

x	**2**	**7**	**0**	**6**
5				
9				
8				
3				

3. Table #3

Consider a sheet of graph or quad ruled paper, one that is covered with a grid like this: 
It may have four squares per inch or five, but that doesn't matter right now.

4. How many squares are on the paper? Just guess …

Is it more than 10? Obviously. More than 100? More than 1,000? More than 10,000? It is not so obvious when you get big numbers or many squares in the piece of graph paper. It's hard to get an answer without counting each square – and that would take a long time (and be very, very boring.) The best way to calculate the number of squares would be to multiply the number of squares going across the edge of the page horizontally by the number of squares going along the other edge vertically.

If we wanted to figure out how many squares are on a paper 8.5 inches wide by 11 inches tall that has 4 squares per inch, we would have to use multiplication. First, we would have to figure out how many squares are going across the page horizontally by multiplying the number of inches across and the number of squares per inch:

4 squares per inch x 8.5 inches horizontally = 34 squares across the paper horizontally

Next, we would use the same process to figure out how many squares are going along the paper vertically:

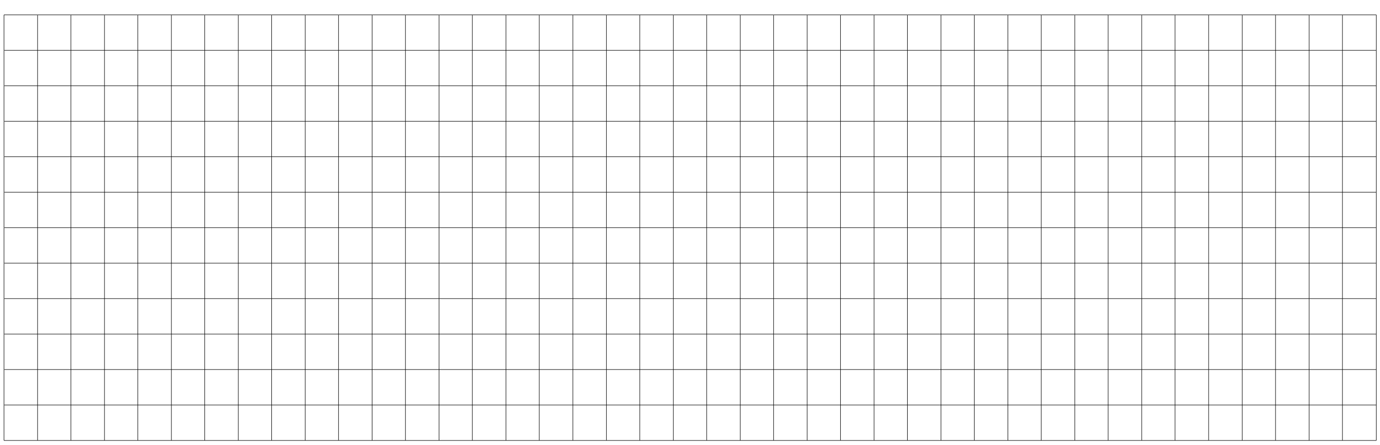
4 squares per inch x 11 inches vertically = 44 squares along the paper vertically

Now, all we have to do is multiply the horizontal number and the vertical number to get the total number of squares:

34 squares horizontally x 44 squares vertically = 1,496 total squares

On the graph below, draw with a straight edge (like the side of a book, or a heavy piece of paper) five rectangular boxes, each a different size.

5. For each box:
– Count the number of squares down one side and across the bottom edge.
– Multiply these two numbers together to get the total number of squares inside the box.
– Write the total number inside the box.



6. If you had an 8.5" x 11" piece of paper with 5 squares per inch, how many squares would be on the paper?

7. Suppose you had an 11" x 14" piece of paper.

a. How many squares would it have if there are four squares to an inch?

b. How many if there are five squares to an inch?

8. If a section of a football stadium has 28 rows, and if there are 18 seats in each row, how many people can be seated in that section? Show your multiplication in the space below.

9. If an airplane flies at 390 miles per hour and travels for 6 hours, how far will it go? Show how you figured it.

10. If the cost of oil is $43 per barrel, and the Zlickow Petroleum Company imports 5,000 barrels, what is the total cost of the oil? Show how you calculated the answer.

Questions 5 through 10 are examples of RATE. Rate means the *amount per unit*. A very commonly used rate is *miles per hour*, or distance per unit of time. To find the whole distance traveled, you multiply the rate in miles per hour by the number of hours. For example, *65 miles per hour x 2 hours = a total distance of 130 miles*. Another way to say the same thing is Distance = Rate x Time. The common formula is D = RT, sometimes abbreviated to just “Dirt.”

Another example of rate is cost per unit. *Total cost = cost per unit x number of units*. In Question 10, the cost was in dollars and the units were barrels of oil. The one about seating in a football stadium can also be thought of as a rate problem. The number of seats per row times the number of rows gives the total seating capacity of the section.

11. What other examples of rate can you think of? List three.

Rate is one of the most useful concepts we have, and it always involves either multiplication or division. Suppose a stadium section had a total of 520 seats, and there are 20 seats per row. How many rows are there in the section? You would divide 520 seats by 20 seats per row (520 ÷ 20) to get an answer of 26 rows. Here, you might call the rate *seats per row.*

Total seating, as we already know, would be the number of rows times the seats per row (26 rows x 20 seats per row = 520 total seats). We could make a formula, saying T = R x S.

12. If a hall has a total seating capacity of 360, and there are 15 rows, how many seats per row are there?

MATH MICE

The drawings which follow may not actually look much like mice, but they can be a neat way to practice multiplication. The square box in the middle is the face. Put in four simple numbers, one in each cell as shown (3, 5, 7, 8). The feet are the boxes below the face, the body is to the right of the face, and the ears are the circles above the face.

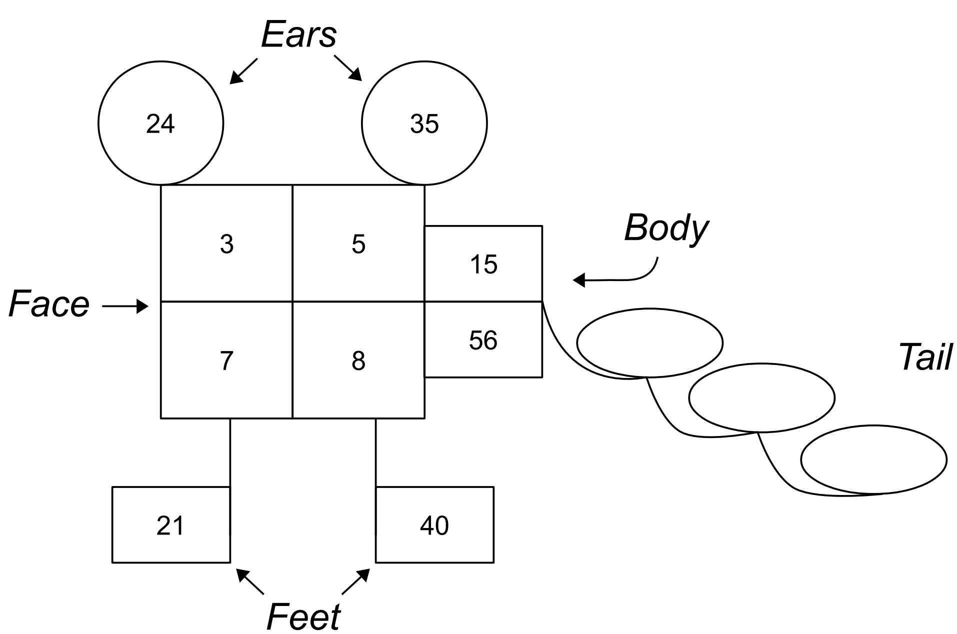
To get the left foot, multiply the two left hand numbers of the face (3 x 7 = 21). Multiply the two right hand numbers to get the right foot (5 x 8 = 40).

For the top of the body, multiply the top two numbers of the face (3 x 5 = 15). Multiply the bottom numbers of the face to get the bottom of the body (7 x 8 = 56).

To get the ears, multiply on the diagonal: 8 x 3 = 24 and 7 x 5 = 35.

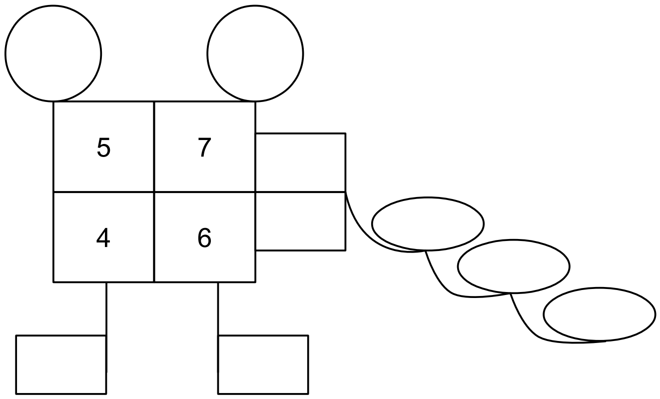
Now, multiply the two ears together (24 x 35), and put the answer in the first loop of the tail. Do the same for the feet (21 x 40), and put it in the second loop. Finally, do the same thing with the body (56 x 15) for the third loop. If you have done the calculations correctly, something quite neat happens …

13. What do you notice about the tail?



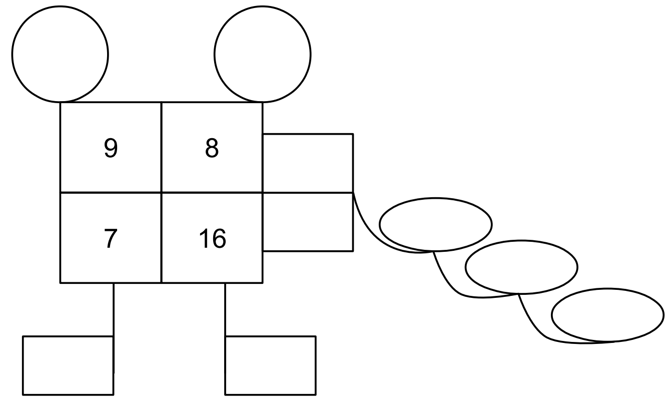
Ears:	Feet:	Body:
24	40	56
x 35	x 21	x15
120		
+ 720		
840		

14. Mouse #2



Ears: Feet: Body:

15. Mouse #3



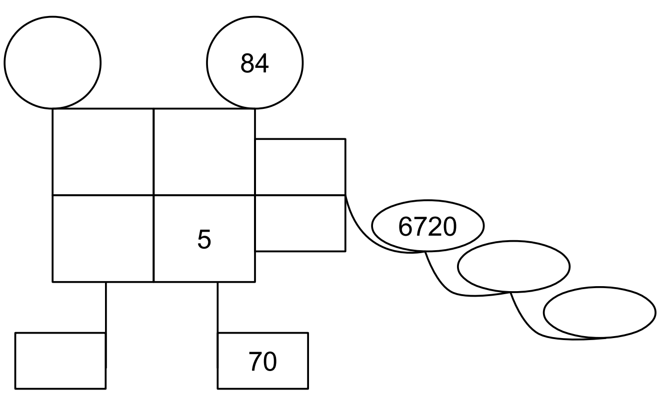
Ears: Feet: Body:

16. Make up two different "multiplication mice" of your own and solve them. Show your work on the calculations you made for each part of the tail (not just the answers).

Incidentally, mice work for addition, too. Just try it. They can also work for division and subtraction – just think about in the latter examples.

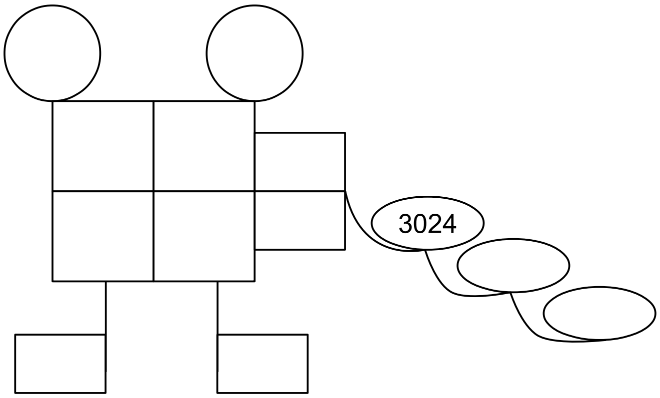
The following mice will require division. If a foot is filled, and you know one of the numbers above it, you must divide (or un-multiply) the foot number by the face number above it to get the missing part. The same applies if an ear or a body part is filled in.

17. Keeping this in mind, fill in all the missing parts on this multiplication mouse. Make sure to show your work!



It is possible to construct a mouse given only the tail number.

18. The tail is 3024, and there are several correct solutions. Find one!



Now we can look at some notable products. (Product is just a fancy math term for the answer you get when multiplying two or more numbers together.) The products below require two-digit multiplication. You should get some special answers. **Show your work.**

19. 12345679	**20. 12345679**	**21. 12345679**	**22. 12345679**
x 9	**x 27**	**x 63**	**x 54**

23. What is the missing number in 12345679 x __??__ = 888,888,888?

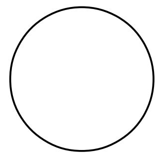
24. And here are two final tables for you to fill in. Please show your work on a separate sheet.

24.	**9 x 7 =**	__________	**8 x 1 + 1 =**	__________
	99 x 77 =	__________	**8 x 12 + 2 =**	__________
	999 x 777 =	__________	**8 x 123 + 3 =**	__________
	9999 x 7777 =	__________	**8 x 1234 + 4 =**	__________
	99999 x 77777 =	__________	**8 x 12345 + 5 =**	__________
			8 x 123456 + 6 =	__________
			8 x 1234567 + 7 =	__________
			8 x 12345678 + 8 =	__________
			8 x 123456789 + 9 =	__________

These last pages are for super thinkers …

This is commonly known as “The Pancake Problem.” Consider a large circle, or a pancake. If you make one cut across it, you will get 2 sections. If you make two cuts, you get 4 sections.

25. Show how many sections you get with three cuts of this pancake.



26. Are there any special ways to make the cuts to change your answer to #25?

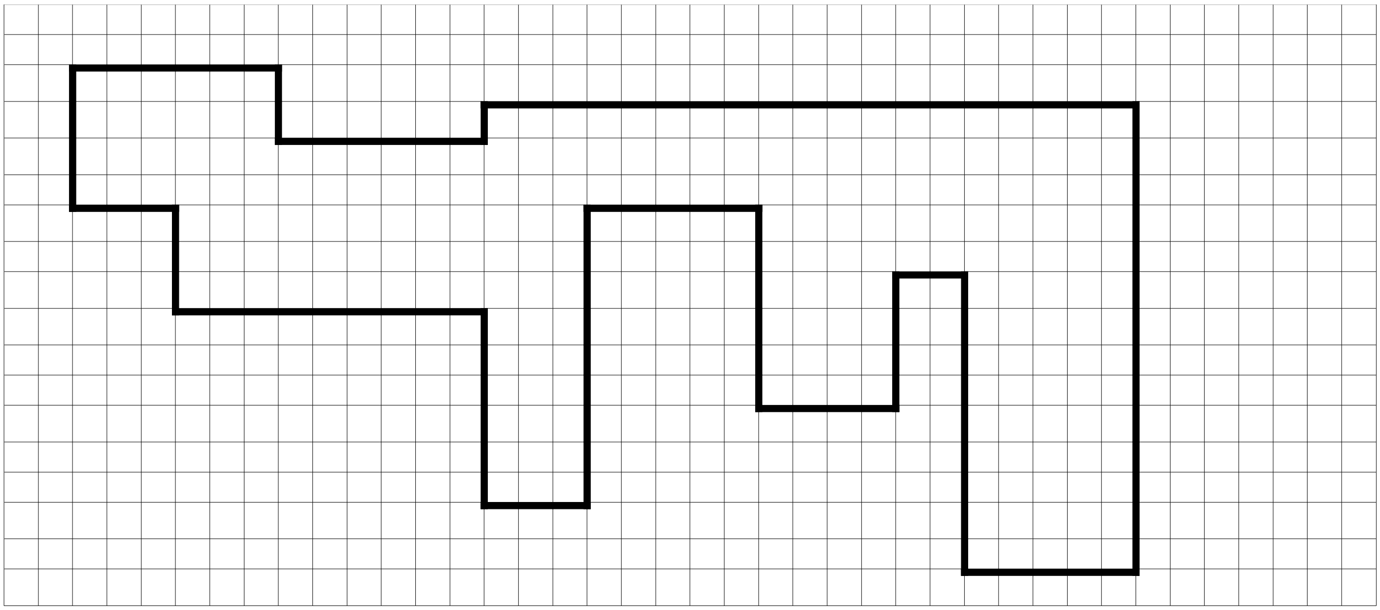
27. The cuts do NOT have to go through the center of the circle, nor do they have to be the same size. Use the space below to show more circles with different cuts.

28. How many sections can you get with 4 cuts? How many with 5 cuts?

29. Fill in the table below:

Number of Cuts	Number of Sections

30. Last question … The number of squares inside each of the boxes you drew is called the *area* of the box, or in math terms, the *area of the rectangle*. Can you find the area of the shape outlined below without counting all of the individual squares within it?



AN INTRODUCTION TO MATH

Answer Key to Unit 3 of 4

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0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

x	9	7	4	8
5	**45**	35	**20**	**40**
3	**27**	**21**	**12**	24
6	**54**	**42**	**24**	**48**
7	63	**49**	**28**	**56**

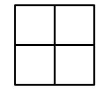
1. Table #1

x	8	7	3	5
8	**64**	**56**	**24**	**40**
7	**56**	**49**	**21**	**35**
3	**24**	**21**	**9**	**15**
5	**40**	**35**	**15**	**25**

2. Table #2

x	2	7	0	6
5	**10**	**35**	**0**	**30**
9	**18**	**63**	**0**	**54**
8	**16**	**56**	**0**	**48**
3	**6**	**21**	**0**	**18**

3. Table #3

Consider a sheet of graph or quad ruled paper, one that is covered with a grid like this: 
It may have four squares per inch or five, but that doesn't matter right now.

4. How many squares are on the paper? Just guess …

Is it more than 10? Obviously. More than 100? More than 1,000? More than 10,000? It is not so obvious when you get big numbers or many squares in the piece of graph paper. It's hard to get an answer without counting each square – and that would take a long time (and be very, very boring.) The best way to calculate the number of squares would be to multiply the number of squares going across the edge of the page horizontally by the number of squares going along the other edge vertically.

If we wanted to figure out how many squares are on a paper 8.5 inches wide by 11 inches tall that has 4 squares per inch, we would have to use multiplication. First, we would have to figure out how many squares are going across the page horizontally by multiplying the number of inches across and the number of squares per inch:

4 squares per inch x 8.5 inches horizontally = 34 squares across the paper horizontally

Next, we would use the same process to figure out how many squares are going along the paper vertically:

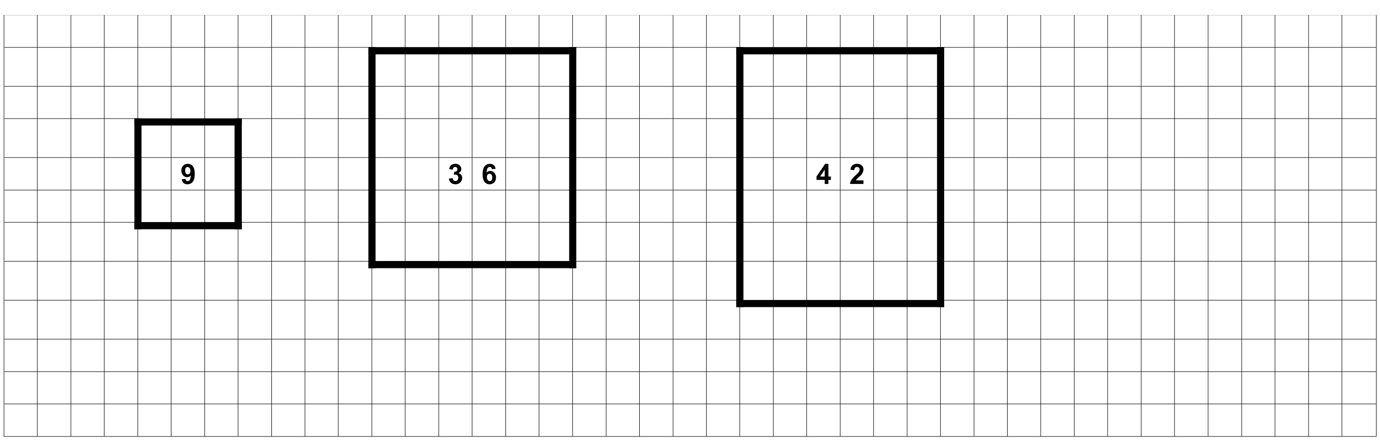
4 squares per inch x 11 inches vertically = 44 squares along the paper vertically

Now, all we have to do is multiply the horizontal number and the vertical number to get the total number of squares:

34 squares horizontally x 44 squares vertically = 1,496 total squares

On the graph below, draw with a straight edge (like the side of a book, or a heavy piece of paper) five rectangular boxes, each a different size.

5. For each box:
– Count the number of squares down one side and across the bottom edge.
– Multiply these two numbers together to get the total number of squares inside the box.
– Write the total number inside the box. **Answers will very.** For example:



6. If you had an 8.5" x 11" piece of paper with 5 squares per inch, how many squares would be on the paper?

(8.5 x 11) x 5 = 93.5 x 5 = 467.5

7. Suppose you had an 11" x 14" piece of paper.

a. How many squares would it have if there are four squares to an inch?

(11 x 14) x 4 = 154 x 4 = 616

b. How many if there are five squares to an inch?

(11 x 14) x 5 = 154 x 5 = 770

8. If a section of a football stadium has 28 rows, and if there are 18 seats in each row, how many people can be seated in that section? Show your multiplication in the space below.

28 x 18 = 504 seats/people

9. If an airplane flies at 390 miles per hour and travels for 6 hours, how far will it go? Show how you figured it.

390 x 6 = 2,340 miles

10. If the cost of oil is $43 per barrel, and the Zlickow Petroleum Company imports 5,000 barrels, what is the total cost of the oil? Show how you calculated the answer.

```
    5000
  x   43
   15000
   20000
$215,000
```

Questions 5 through 10 are examples of RATE. Rate means the *amount per unit*. A very commonly used rate is *miles per hour*, or distance per unit of time. To find the whole distance traveled, you multiply the rate in miles per hour by the number of hours. For example, *65 miles per hour x 2 hours = a total distance of 130 miles*. Another way to say the same thing is Distance = Rate x Time. The common formula is D = RT, sometimes abbreviated to just "Dirt."

Another example of rate is cost per unit. *Total cost = cost per unit x number of units*. In Question 10, the cost was in dollars and the units were barrels of oil. The one about seating in a football stadium can also be thought of as a rate problem. The number of seats per row times the number of rows gives the total seating capacity of the section.

11. What other examples of rate can you think of? List three.

Answers will vary. For example:

1) Atmospheric pressure: 14.7 lbs. per. sq. in.
2) **knots per. hour**
3) **price per. gallon (gas/milk)**

Rate is one of the most useful concepts we have, and it always involves either multiplication or division. Suppose a stadium section had a total of 520 seats, and there are 20 seats per row. How many rows are there in the section? You would divide 520 seats by 20 seats per row (520 ÷ 20) to get an answer of 26 rows. Here, you might call the rate *seats per row.*

Total seating, as we already know, would be the number of rows times the seats per row (26 rows x 20 seats per row = 520 total seats). We could make a formula, saying T = R x S.

12. If a hall has a total seating capacity of 360, and there are 15 rows, how many seats per row are there?

360 ÷ 15 = 24 seats per row

MATH MICE

The drawings which follow may not actually look much like mice, but they can be a neat way to practice multiplication. The square box in the middle is the face. Put in four simple numbers, one in each cell as shown (3, 5, 7, 8). The feet are the boxes below the face, the body is to the right of the face, and the ears are the circles above the face.

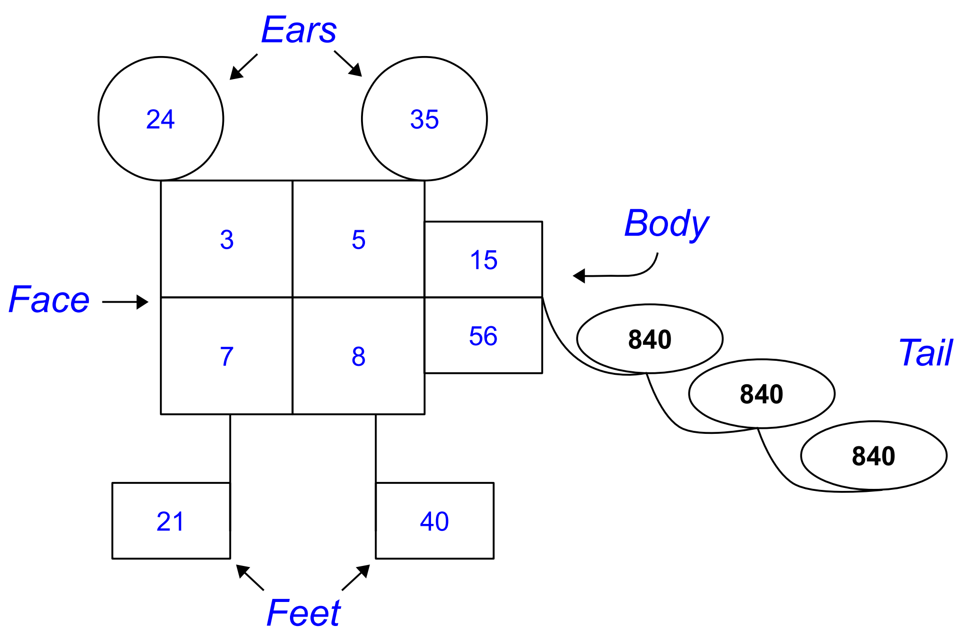
To get the left foot, multiply the two left hand numbers of the face (3 x 7 = 21). Multiply the two right hand numbers to get the right foot (5 x 8 = 40).

For the top of the body, multiply the top two numbers of the face (3 x 5 = 15). Multiply the bottom numbers of the face to get the bottom of the body (7 x 8 = 56).

To get the ears, multiply on the diagonal: 8 x 3 = 24 and 7 x 5 = 35.

Now, multiply the two ears together (24 x 35), and put the answer in the first loop of the tail. Do the same for the feet (21 x 40), and put it in the second loop. Finally, do the same thing with the body (56 x 15) for the third loop. If you have done the calculations correctly, something quite neat happens …

13. What do you notice about the tail?

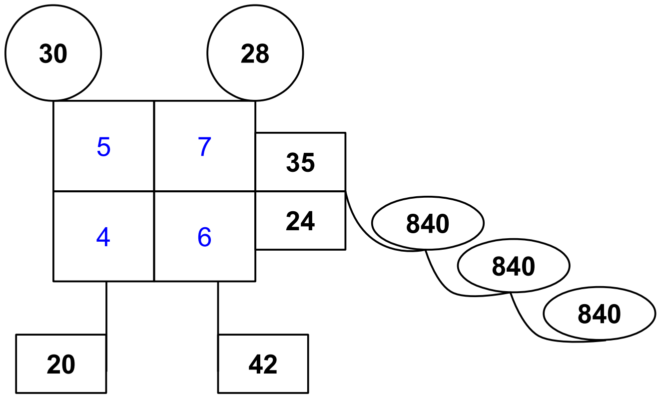


Ears: 24
x 35
120
+ 720
840

Feet: 40
x 21

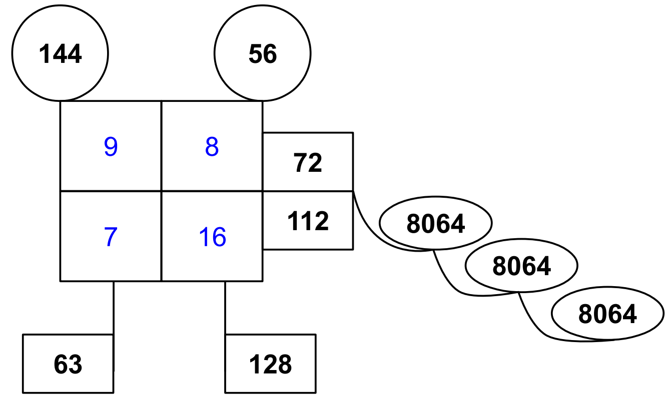
Body: 56
x15

14. Mouse #2



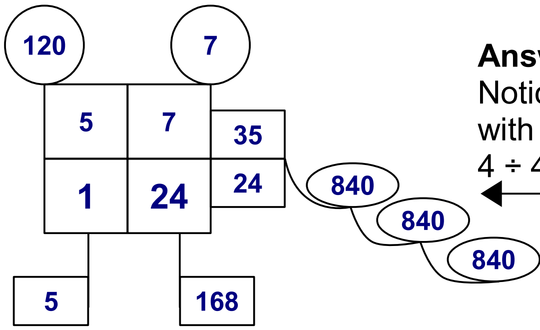
Ears: Feet: Body:

15. Mouse #3



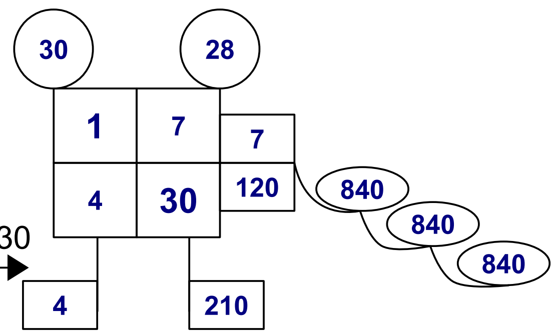
Ears: Feet: Body:

16. Make up two different "multiplication mice" of your own and solve them. Show your work on the calculations you made for each part of the tail (not just the answers).



Answers will vary.
Notice the correlations with the mouse in 14:
4 ÷ 4 = 1; 4 x 6 = 24

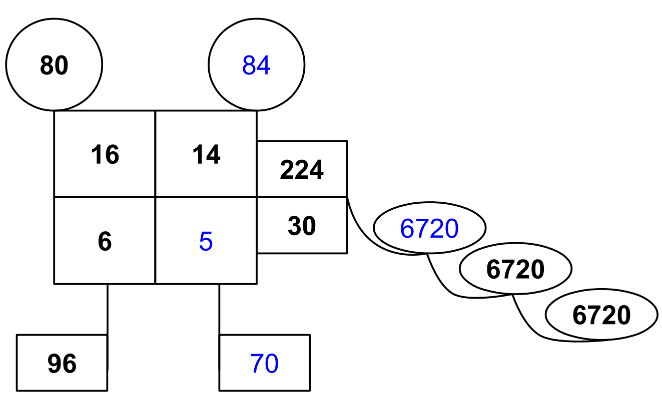
5 ÷ 5 = 1; 5 x 6 = 30



Incidentally, mice work for addition, too. Just try it. They can also work for division and subtraction – just think about in the latter examples.

The following mice will require division. If a foot is filled, and you know one of the numbers above it, you must divide (or un-multiply) the foot number by the face number above it to get the missing part. The same applies if an ear or a body part is filled in.

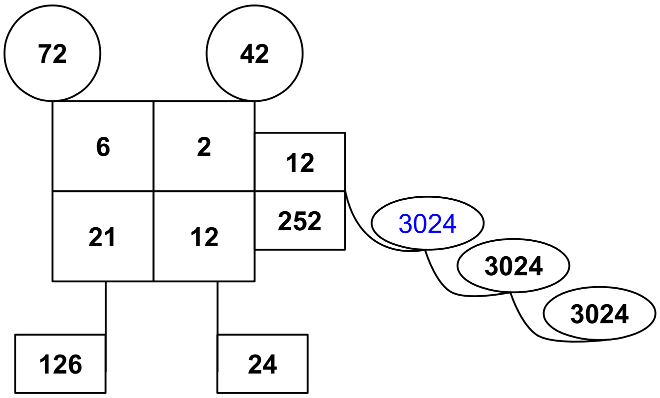
17. Keeping this in mind, fill in all the missing parts on this multiplication mouse. Make sure to show your work!



70 ÷ 5 = 14
84 ÷ 14 = 6
6720 ÷ 30 = 224
224 ÷ 14 = 16

It is possible to construct a mouse given only the tail number.

18. The tail is 3024, and there are several correct solutions. Find one!



3024 ÷ 12 = 252
252 ÷ 12 = 21

Now we can look at some notable products. (Product is just a fancy math term for the answer you get when multiplying two or more numbers together.) The products below require two-digit multiplication. You should get some special answers. **Show your work.**

19. 12345679	**20. 12345679**	**21. 12345679**	**22. 12345679**
x 9	**x 27**	**x 63**	**x 54**
111111111	**86419753**	**37037037**	
	24691358	**74074074**	
(9 x 1 = 9)	**333333333**	**777777777**	**666666666**
	(9 x 3 = 27)	**(9 x 7 = 63)**	**(54 ÷ 9 = 6)**

23. What is the missing number in 12345679 x __72__ = 888,888,888? (9 x 8 = 72)

24. And here are two final tables for you to fill in. Please show your work on a separate sheet.

24.

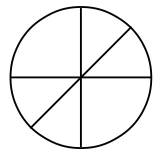
9 x 7 =	**63**
99 x 77 =	**7623**
999 x 777 =	**776223**
9999 x 7777 =	**7762223**
99999 x 77777 =	**7777622223**

8 x 1 + 1 =	**9**
8 x 12 + 2 =	**98**
8 x 123 + 3 =	**987**
8 x 1234 + 4 =	**9876**
8 x 12345 + 5 =	**98765**
8 x 123456 + 6 =	**987654**
8 x 1234567 + 7 =	**9876543**
8 x 12345678 + 8 =	**98765432**
8 x 123456789 + 9 =	**987654321**

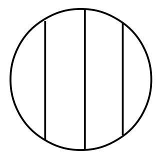
These last pages are for super thinkers …

This is commonly known as "The Pancake Problem." Consider a large circle, or a pancake. If you make one cut across it, you will get 2 sections. If you make two cuts, you get 4 sections.

25. Show how many sections you get with three cuts of this pancake.

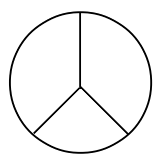


26. Are there any special ways to make the cuts to change your answer to #25?



27. The cuts do NOT have to go through the center of the circle, nor do they have to be the same size. Use the space below to show more circles with different cuts.

Answers will vary. For example –



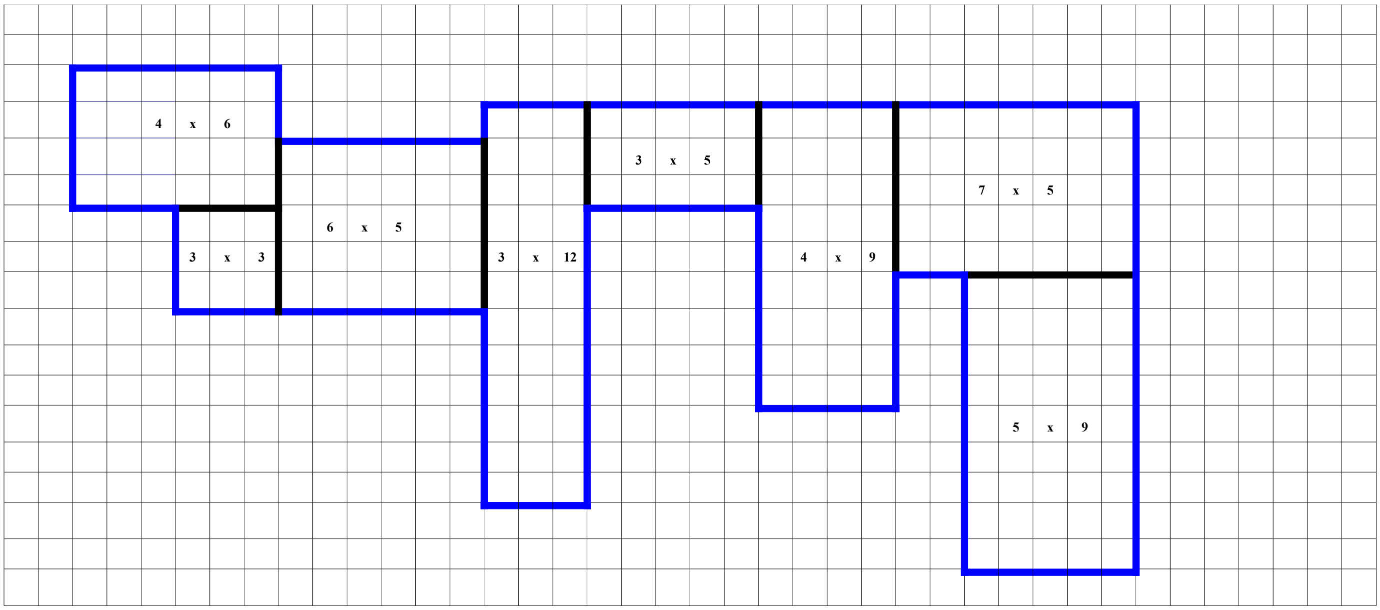
28. How many sections can you get with 4 cuts? How many with 5 cuts?

Answers will vary. Check for number of cuts, one set of 4, one set of five.

29. Fill in the table below:

Number of Cuts	Number of Sections
6	6-12
7	7-14
8	8-16
9	9-18
10	10-20
11	11-22
12	12-24

30. Last question … The number of squares inside each of the boxes you drew is called the *area* of the box, or in math terms, the *area of the rectangle*. Can you find the area of the shape outlined below without counting all of the individual squares within it?



Yes, by dividing the shape into individual boxes. Area = L x W (or side x side)
If they do the math, calculations will vary depending on how the shape is divided:

4 x 6 =	24
3 x 3 =	9
5 x 6 =	30
3 x 12 =	36
3 x 5 =	15
4 x 9 =	36
5 x 7 =	35
5 x 9 =	+ 45
A =	320

AN INTRODUCTION TO MATH

Unit 4 of 4

Beyond the four basic processes, there are other expressions one needs to understand to gain facility in math. These include ***exponents***. Exponents are numbers written above and to the right of any ordinary number. They express the number of times a number is multiplied by itself.

For example, 5^2 is read "five to the second power" or "five squared." It means 5 x 5. In the expression "5 x 5" we are multiplying two numbers; each number is called a ***factor***. In "5 x 5" then, there are two factors, each of which is a 5. In 5^2, the exponent "2" tells the number of factors of the base "5" that are to be multiplied together.

So, 5^3 means 5 x 5 x 5. This is also called "five cubed." The exponent (in this case 3) tells the number of fives to multiply together. Here are some other examples:

4^2 = 4 x 4 = 16 6^3 = 6 x 6 x 6 = 216 2^4 = 2 x 2 x 2 x 2 = 16

3^5 = 3 x 3 x 3 x 3 x 3 = 243 8^5 = 8 x 8 x 8 x 8 x 8 = 4,096 7^3 = 7 x 7 x 7 = 343

Notice how fast these numbers go up. This is what "increasing exponentially" means.

1. Find the following:

3^4 = **5^3 =** **9^3 =** **10^4 =**

2. Notice the number of zeroes in 10^4. With that in mind, what is 10^6?

The number of zeroes in any power of ten is the same as the exponent to which 10 is raised. This is a basic property of number systems. It may intrigue you to know how other number systems work, but it is not essential. Knowing basic squares and cubes is helpful, so your next task is to complete the tables below.

3. Table of Squares

1^2 =		**5^2 =**		**9^2 =**		**13^2 =**		**17^2 =**	
2^2 =		**6^2 =**		**10^2 =**		**14^2 =**		**18^2 =**	
3^2 =		**7^2 =**		**11^2 =**		**15^2 =**		**19^2 =**	
4^2 =		**8^2 =**		**12^2 =**		**16^2 =**		**20^2 =**	

4. Table of Cubes

1^3 =		**2^3 =**		**3^3 =**		**4^3 =**		**5^3 =**	
6^3 =		**7^3 =**		**8^3 =**		**9^3 =**		**10^3 =**	

Notice that one to any power is one. And of course, zero to any power is zero. Another set of numbers worth noticing is the powers of 2.

5. Find the following. (The first one, 2^0 has been done for you.)

$2^0 = 1$

$2^1 =$

$2^2 =$

$2^3 =$

$2^4 =$

$2^5 =$

$2^6 =$

$2^7 =$

$2^8 =$

With this in mind, here is a famous problem:

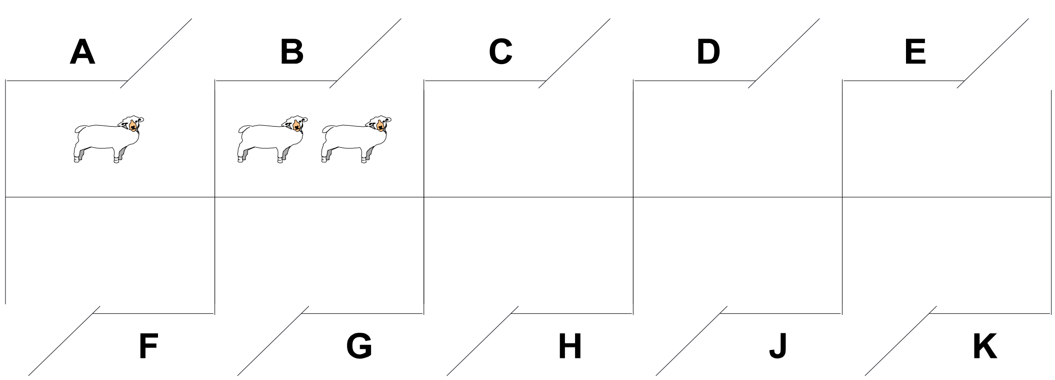
ONE THOUSAND SHEEP

A farmer has a thousand sheep. He wants to build pens and place the sheep in the pens so that anyone can come along and say, “I’d like 23 sheep,” and the farmer can let out the correct number of sheep by opening up exactly the right gates. If, for example, a customer wanted just one sheep, there would have to be a pen with just one sheep in it. Of course, if a customer wanted two sheep, the farmer would need a pen with two sheep in it. Once the farmer opens a gate, all the sheep in that pen will come out; he can’t empty part of a pen.

6. Would the farmer need a separate pen with 3 sheep in it? How about 4 sheep?

7. How would the farmer get 6 sheep?

8. *Write* the number of sheep (you do NOT have to draw them!) the farmer must put in each pen to enable him to open gates and get out any exact number of sheep from 1 up to 1,000. (Hint: None of these, other than B, will be ***prime numbers***. Primes are whole numbers greater than 1 whose only two factors are 1 and itself.)



9. What gates would the farmer open to get 13 sheep? How many sheep would come out of each pen that he opened?

10. To get 48 sheep? List the gates he needs to open, and how many sheep will come out of each pen. Now, add up the number of sheep that came out of each opened pen to check your work.

11. To get 73 sheep? List the gates opened, the number of sheep from each of the opened pens, and show the sum of your numbers.

12. Do the same for 86 sheep.

13. Do the same for 127 sheep.

14. Do the same for 259 sheep.

15. Do the same for 683 sheep.

16. How many sheep are in the last pen? (Be careful … the answer isn't especially obvious.)

If you have put the right number of sheep in each pen, you will have written the powers of 2 up to and including 28. But if you wrote the number for 29 in the last pen (K), you are not right.

17. Why would it be incorrect to write the number for 29 in the last pen (K)?

This problem of the thousand sheep is good for seeing some mathematical thinking. It is not of particular practical value (what farmer ever had exactly 1,000 sheep, and how can you get a sheep to do anything right in the first place?). But math is full of fun questions to play with like this one. Meanwhile, you now have the powers of 2 at hand.

18. Fill in the table below.

$2^0 =$		$2^1 =$		$2^2 =$		$2^3 =$		$2^4 =$	
$2^5 =$		$2^6 =$		$2^7 =$		$2^8 =$		$2^9 =$	
$2^{10} =$									

Knowing the use of powers and exponents enables us to establish the rules for the order of operations. This is very important. The expression 8 x 2 + 4 might mean 20 or 48, depending on the order in which you do the arithmetic. For example, if you start with 8 x 2, you get 16. Add that to 4 and you get 20. However, if you add 2 + 4 first, you get 6. 8 x 6 is 48. Clearly, we need to have some rules about what to do first so that we get the same answer every time.

Fortunately, just such a set of rules has been created and agreed to. They are called the **Order of Operations** and are often referred to as PEMDAS. This is just an abbreviation for the words Parentheses, Exponents, Multiplication/Division, Addition/Subtraction. (Some people find it easier to remember as "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally".)

The Order of Operations says that we have to do calculations in a certain order, every time:

1 . ***P****arentheses*: Make any calculations inside parentheses before doing anything else.
Example: 8 x (2 + 4) … Start inside the parentheses with 2 + 4 to get 6. Then multiply by 8 to get 48.

2. ***E****xponents*: Once the parentheses are done (or if there aren't any) calculate any exponents.
Example: 8 x 3^2 ...Start with 3^2, which is 3 x 3, or 9. Then, multiply by eight to get 72.

3. ***M****ultiplication/****D****ivision*: Once the exponents are done (or if there aren't any) do any multiplication or division, working from left to right.
Example: 8 ÷ 4 x 3 … Since we have both multiplication and division, we have to work from left to right. Starting on the left, then, 8 ÷ 4 = 2. Continuing to the right, multiply by 3 to get 6.

4. ***A****ddition/****S****ubtraction*: Once the multiplying and dividing are done (or if there isn't any) do any addition or subtraction, working left to right.
Example: 9 – 5 + 2 … Since we have both addition and subtraction, we have to work from left to right. Starting on the left, then, 9 – 5 = 4. Continuing to the right, add 2 to get 6.

I know that's a lot to take in, so let's do an example together before you try some on your own. **You fill in the blanks.**

$$6 \times (12 - 9)^2 - 8 \div 4 + 5 = ?$$

Step 1. ***P****arentheses first*. (12 – 9) = _____. Now, we have 6 x 3^2 – 8 ÷ 4 + 5 = ?

Step 2. ***E****xponents second*. 3^2 = ______. Now, we have 6 x 9 – 8 ÷ 4 + 5 = ?

Step 3. ***M****ultiplication and* ***D****ivision third*. Reading our equation left to right, multiplication comes first, and then division. 6 x 9 = ___ and 8 ÷ 4 = ___. Now, we are left with 54 – 2 + 5 = ?

Step 4. ***A****ddition and* ***S****ubtraction last*. Reading our equation left to right, subtraction comes first, and then addition. 54 – 2 = ____. Now, we are left with 52 + 5. So, the final answer is _____.

19.7 + 6 x 2 =

20. 47 + 5 x 0 =

21. 4 x (10 ÷ 5) =

22. 8 + (32 – 24) ÷ 4 =

23. 8 x (9 – 7) ÷ 4 + 2 =

24. (8 – 3)^2 x (7 – 3) =

25. 5 + 3^2 x (9 – 4) =

26. 8 + 43 – 8 x 2 – 42 – 6 =

Try a few more with exponents. Remember "P-E-MD-AS". Do the work inside the parentheses first, and then calculate the exponents.

27. $3 \times 5^2 =$

28. $(3 \times 5)^2 =$

29. $3^2 \times 5 =$

30. $3^2 + 5^2 =$

31. $(3 + 5)^2 =$

32. $(3^2 + 5) =$

33. $(3^2 + 5)^2 =$

34. $(3 \times 5)^3 =$

Notice how adding parentheses changes the answer. For example, the equations in questions 27 and 28 use the same base numbers and exponents (3 and 5^2), but the answers are very different because parentheses were added. The same principle can be seen at work in questions 30 and 33.

35. For each equation below, add in parentheses to make the statement true.

a. $24 \div 6 + 6 \times 3 - 3 = 27$

b. $24 \div 6 + 6 \times 3 - 3 = 4$

c. $24 \div 6 + 6 \times 3 - 3 = 0$

d. $24 \div 6 + 6 \times 3 - 3 = 19$

e. $24 \div 6 + 6 \times 3 - 3 = 3$

f. $72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 20$

g. $72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 16$

h. $72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 26$

i. $6 \div 2 \times 3 + 7 \times 8 - 2 = 55$

j. $6 \div 2 \times 3 + 7 \times 8 - 2 = 180$

That was challenging! The next few will be a little easier. Figure out the ***numerators*** (the part above the dividing line), then the ***denominators*** (the part below the line), and then do the division. For example, given: $\frac{8 \times 9}{3 \times 4}$

Start with the top part (the numerator): 8 x 9 = 72 …

Then do the bottom part (the denominator): 3 x 4 = 12 …

Then divide. $\frac{72}{12}$ is the same as 72 ÷ 12, which is equal to 6

Remember to follow P-E-MD-AS very carefully! You should get "easy" answers (no decimals or fractions).

36. $\frac{(36 \div 18 + 6 \times 2)}{7 \times (6 - 4)} =$

37. $(2^2 + 3^2) \div 13 =$

38. $(7 - 5)^2 - (4 - 3)^8 =$

39. $\frac{(17 \times 6 - 4 \times 20) \times 10 \div 5}{8 \times 5 + 4} =$

40. $(5 - 3)^4 \div 4 =$

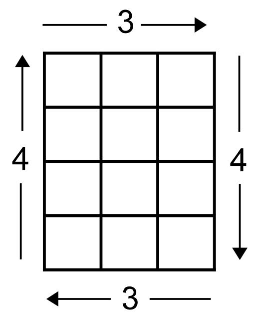
41. $\frac{9 \times 9 + 9}{6 + 36 \div 6 + 3 \times 6} =$

42. $9 \times (3 + 4) \div [(7 + 3) - 3] =$

43. $\frac{2^2 \times (5 + 6)}{25 \div 5 + 3 \times 2} =$

These pages give some practice in the Order of Operations. It is an area that gives many people trouble and requires a lot of practice. Keep at it! Facility with the order of operations makes all work in mathematics easier.

To conclude this Unit, consider a bit of geometry. Unit 3 concluded with an irregular rectilinear shape, and you found its area. ("Rectilinear" is just a fancy math term that describes a shape made of lines that meet at right angles.) The perimeter was never mentioned. ***Perimeter*** merely means the distance around the figure. For a simple rectangle like the one below, which is 3 units wide and 4 units long, the perimeter would be 3 (across the top) + 4 (down one side) + 3 (across the bottom) + 4 (up the other side) = 14 units total.



Using "P" for perimeter, "W" for width, and "L" for length, this could be stated as P = W + L + W + L. To make it shorter and easier to write, we could also say P = 2W + 2L (two times the width, plus 2 times the length). We could make it even shorter by using parentheses, like this: P = 2(W + L).

P = 2(W + L) is the ***formula*** for perimeter. We can use it to figure out the perimeter of any rectangle, no matter how big or small! Just change the "W" to the number of units in the width and change the "L" to the number of units in the length. Using the numbers from our rectangle above, we would go from P = 2(W + L) to P = 2(3 + 4). Thus, P (perimeter) = 2 x 7 = 14 units.

At the end of Unit 3, you learned about area, or the number of units *inside* a rectangle. The formula for area is A = WL, or Area = Width x Length. Applied to our rectangle above, A = WL becomes A = 3 x 4 = 12 square units.

44. Using these formulas, find the perimeter and area of the following rectangles:

a. width = 5, length = 7

b. width = 8, length = 15

c. width = 9, length = 4

d. width = 3, length = 48

45. Consider the last rectangle, "d" above. Is there a rectangle with a different width and length, but the same area? Find one.

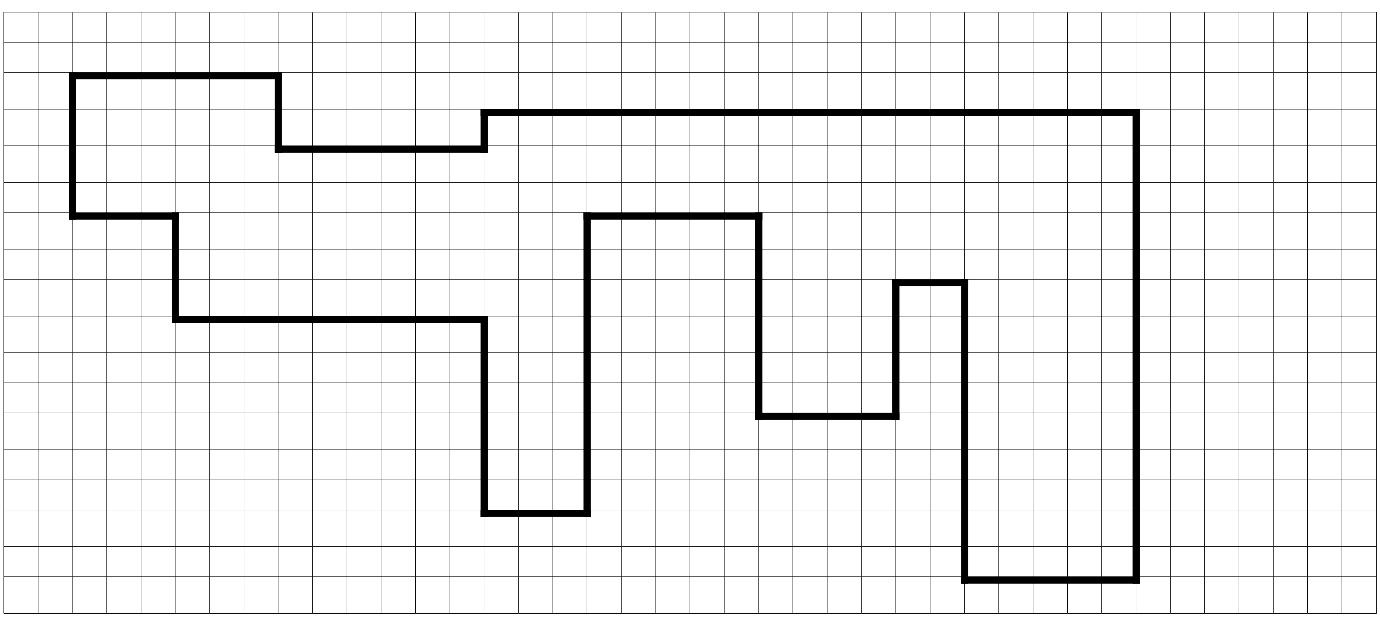
46. In fact, there are eight rectangles with different widths and lengths that have the same area as rectangle "d." Find all eight and give the perimeter for each rectangle. Show your results in the table on the next page.

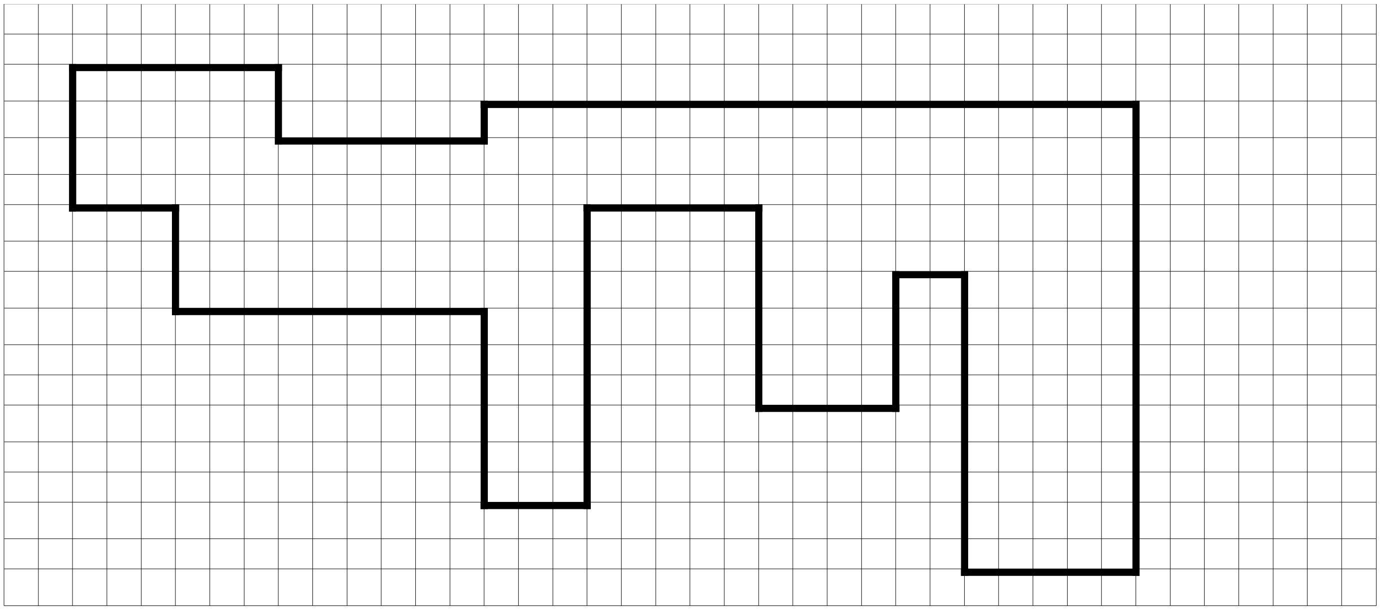
Width	Length	Perimeter

Remember … all the rectangles must have an ***area*** of 144 square units!

47. If you were going to build a rectilinear pen for a herd of sheep, what shape would require the least amount of fence for a given area? Would it be a square, a rectangle, or something like the one below?

The figure from the end of Unit 3 appears below. In Unit 3, you found the area, or the number of square units inside the figure.





48. Find the perimeter of this figure, and show the calculations you made to figure it out.

49. Draw some straight lines inside the figure above so that it becomes a group of 7-10 boxes. Find the area of each box using the A = WL formula. Add up the areas of all the boxes. Does the number match what you got in Unit 3?

50. How would you feel about building a house of this shape? Explain why you feel that way.

Congratulations! This is the last Unit in the *An Introduction to Math* course!

AN INTRODUCTION TO MATH

Answer Key to Unit 4 of 4

Beyond the four basic processes, there are other expressions one needs to understand to gain facility in math. These include ***exponents***. Exponents are numbers written above and to the right of any ordinary number. They express the number of times a number is multiplied by itself.

For example, 5^2 is read "five to the second power" or "five squared." It means 5 x 5. In the expression "5 x 5" we are multiplying two numbers; each number is called a ***factor***. In "5 x 5" then, there are two factors, each of which is a 5. In 5^2, the exponent "2" tells the number of factors of the base "5" that are to be multiplied together.

So, 5^3 means 5 x 5 x 5. This is also called "five cubed." The exponent (in this case 3) tells the number of fives to multiply together. Here are some other examples:

4^2 = 4 x 4 = 16 6^3 = 6 x 6 x 6 = 216 2^4 = 2 x 2 x 2 x 2 = 16

3^5 = 3 x 3 x 3 x 3 x 3 = 243 8^5 = 8 x 8 x 8 x 8 x 8 = 4,096 7^3 = 7 x 7 x 7 = 343

Notice how fast these numbers go up. This is what "increasing exponentially" means.

1. Find the following:

3^4 = 81 **5^3 = 125** **9^3 = 729** **10^4 = 10,000**

2. Notice the number of zeroes in 10^4. With that in mind, what is 10^6?

10^6 = 1,000,000 (6 zeros)

The number of zeroes in any power of ten is the same as the exponent to which 10 is raised. This is a basic property of number systems. It may intrigue you to know how other number systems work, but it is not essential. Knowing basic squares and cubes is helpful, so your next task is to complete the tables below.

3. Table of Squares

1^2 =	1	5^2 =	25	9^2 =	81	13^2 =	169	17^2 =	289
2^2 =	4	6^2 =	36	10^2 =	100	14^2 =	196	18^2 =	324
3^2 =	9	7^2 =	49	11^2 =	121	15^2 =	225	19^2 =	361
4^2 =	16	8^2 =	64	12^2 =	144	16^2 =	256	20^2 =	400

4. Table of Cubes

1^3 =	**1**	**2^3 =**	**8**	**3^3 =**	**27**	**4^3 =**	**64**	**5^3 =**	**125**
6^3 =	**216**	**7^3 =**	**343**	**8^3 =**	**512**	**9^3 =**	**729**	**10^3 =**	**1000**

Notice that one to any power is one. And of course, zero to any power is zero. Another set of numbers worth noticing is the powers of 2.

5. Find the following. (The first one, 2^0 has been done for you.)

$2^0 = 1$ | $2^3 =$ **8** | $2^6 =$ **64**

$2^1 =$ **2** | $2^4 =$ **16** | $2^7 =$ **128**

$2^2 =$ **4** | $2^5 =$ **32** | $2^8 =$ **256**

With this in mind, here is a famous problem:

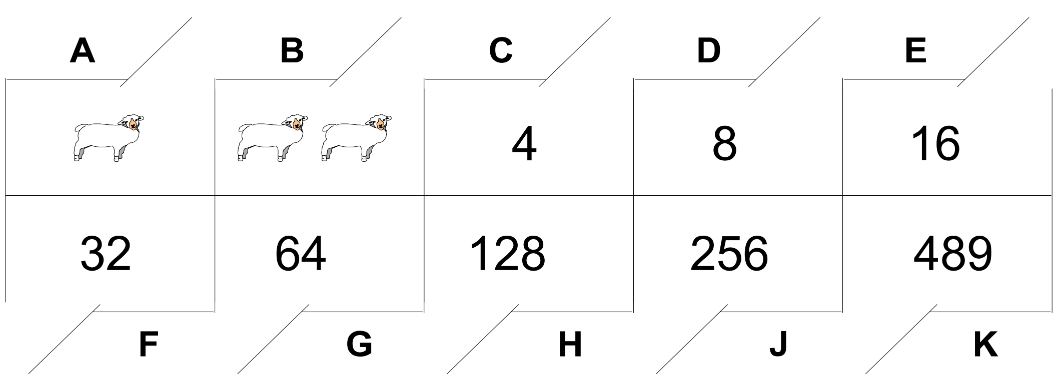
ONE THOUSAND SHEEP

A farmer has a thousand sheep. He wants to build pens and place the sheep in the pens so that anyone can come along and say, “I’d like 23 sheep,” and the farmer can let out the correct number of sheep by opening up exactly the right gates. If, for example, a customer wanted just one sheep, there would have to be a pen with just one sheep in it. Of course, if a customer wanted two sheep, the farmer would need a pen with two sheep in it. Once the farmer opens a gate, all the sheep in that pen will come out; he can’t empty part of a pen.

6. Would the farmer need a separate pen with 3 sheep in it? How about 4 sheep?
A + B = 3   C = 4

7. How would the farmer get 6 sheep?
B + C = 6

8. *Write* the number of sheep (you do NOT have to draw them!) the farmer must put in each pen to enable him to open gates and get out any exact number of sheep from 1 up to 1,000. (Hint: None of these, other than B, will be ***prime numbers***. Primes are whole numbers greater than 1 whose only two factors are 1 and itself.)



9. What gates would the farmer open to get 13 sheep? How many sheep would come out of each pen that he opened?

A gate (1) + C gate (4) + D gate (8) = 13 sheep

10. To get 48 sheep? List the gates he needs to open, and how many sheep will come out of each pen. Now, add up the number of sheep that came out of each opened pen to check your work.

E gate (16) + F gate (32) = 48 sheep

11. To get 73 sheep? List the gates opened, the number of sheep from each of the opened pens, and show the sum of your numbers.

A gate (1) + D gate (8) + G gate (64) = 73 sheep

12. Do the same for 86 sheep.

B gate (2) + C gate (4) + E gate (16) + G gate (64) = 86 sheep

13. Do the same for 127 sheep.

A gate (1) + B gate (2) + C gate (4) + D (8) + E (16) + F (32) + G (64) = 127 sheep

14. Do the same for 259 sheep.

A gate (1) + B gate (2) + J gate (256) = 259 sheep

15. Do the same for 683 sheep.

B gate (2) + G gate (64) + H gate (128) + K gate (489) = 683 sheep

16. How many sheep are in the last pen? (Be careful … the answer isn't especially obvious.) **1000**

- 511 **A + B + C + D + E + F + G + H + J + K = 511**

489 **1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 = 511**

If you have put the right number of sheep in each pen, you will have written the powers of 2 up to and including 28. But if you wrote the number for 29 in the last pen (K), you are not right.

17. Why would it be incorrect to write the number for 29 in the last pen (K)? **511**

+ 512

1023

This problem of the thousand sheep is good for seeing some mathematical thinking. It is not of particular practical value (what farmer ever had exactly 1,000 sheep, and how can you get a sheep to do anything right in the first place?). But math is full of fun questions to play with like this one. Meanwhile, you now have the powers of 2 at hand.

18. Fill in the table below.

2^0 =	**1**	2^1 =	**2**	2^2 =	**4**	2^3 =	**8**	2^4 =	**16**
2^5 =	**32**	2^6 =	**64**	2^7 =	**128**	2^8 =	**256**	2^9 =	**512**
2^{10} =	**1024**								

Knowing the use of powers and exponents enables us to establish the rules for the order of operations. This is very important. The expression 8 x 2 + 4 might mean 20 or 48, depending on the order in which you do the arithmetic. For example, if you start with 8 x 2, you get 16. Add that to 4 and you get 20. However, if you add 2 + 4 first, you get 6. 8 x 6 is 48. Clearly, we need to have some rules about what to do first so that we get the same answer every time.

Fortunately, just such a set of rules has been created and agreed to. They are called the **Order of Operations** and are often referred to as PEMDAS. This is just an abbreviation for the words Parentheses, Exponents, Multiplication/Division, Addition/Subtraction. (Some people find it easier to remember as "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally".)

The Order of Operations says that we have to do calculations in a certain order, every time:

1 . ***P****arentheses*: Make any calculations inside parentheses before doing anything else.
Example: 8 x (2 + 4) … Start inside the parentheses with 2 + 4 to get 6. Then multiply by 8 to get 48.

2. ***E****xponents*: Once the parentheses are done (or if there aren't any) calculate any exponents.
Example: 8 x 3^2 ...Start with 3^2, which is 3 x 3, or 9. Then, multiply by eight to get 72.

3. ***M****ultiplication/****D****ivision*: Once the exponents are done (or if there aren't any) do any multiplication or division, working from left to right.
Example: 8 ÷ 4 x 3 … Since we have both multiplication and division, we have to work from left to right. Starting on the left, then, 8 ÷ 4 = 2. Continuing to the right, multiply by 3 to get 6.

4. ***A****ddition/****S****ubtraction*: Once the multiplying and dividing are done (or if there isn't any) do any addition or subtraction, working left to right.
Example: 9 – 5 + 2 … Since we have both addition and subtraction, we have to work from left to right. Starting on the left, then, 9 – 5 = 4. Continuing to the right, add 2 to get 6.

I know that's a lot to take in, so let's do an example together before you try some on your own. **You fill in the blanks.**

$$6 \times (12 - 9)^2 - 8 \div 4 + 5 = ?$$

Step 1. ***P****arentheses first*. (12 – 9) = _____. Now, we have 6 x 3^2 – 8 ÷ 4 + 5 = ?

Step 2. ***E****xponents second*. 3^2 = ______. Now, we have 6 x 9 – 8 ÷ 4 + 5 = ?

Step 3. ***M****ultiplication and* ***D****ivision third*. Reading our equation left to right, multiplication comes first, and then division. 6 x 9 = ___ and 8 ÷ 4 = ___. Now, we are left with 54 – 2 + 5 = ?

Step 4. ***A****ddition and* ***S****ubtraction last*. Reading our equation left to right, subtraction comes first, and then addition. 54 – 2 = ____. Now, we are left with 52 + 5. So, the final answer is _____.

19.7 + 6 x 2 = 19

20. 47 + 5 x 0 = 47

21. 4 x (10 ÷ 5) = 8

22. 8 + (32 – 24) ÷ 4 = 10

23. 8 x (9 – 7) ÷ 4 + 2 = 6

24. (8 – 3)^2 x (7 – 3) = 100

25. 5 + 3^2 x (9 – 4) = 50

26. 8 + 43 – 8 x 2 – 42 – 6 = -13

Try a few more with exponents. Remember "P-E-MD-AS". Do the work inside the parentheses first, and then calculate the exponents.

27. $3 \times 5^2 = 75$

28. $(3 \times 5)^2 = 225$

29. $3^2 \times 5 = 45$

30. $3^2 + 5^2 = 34$

31. $(3 + 5)^2 = 64$

32. $(3^2 + 5) = 14$

33. $(3^2 + 5)^2 = 196$

34. $(3 \times 5)^3 = 3{,}375$

Notice how adding parentheses changes the answer. For example, the equations in questions 27 and 28 use the same base numbers and exponents (3 and 5^2), but the answers are very different because parentheses were added. The same principle can be seen at work in questions 30 and 33.

35. For each equation below, add in parentheses to make the statement true.

a. $(24 \div 6 + 6) \times 3 - 3 = 27$

b. $(24 \div 6) + 6 \times (3 - 3) = 4$
or just: **$24 \div 6 + 6 \times (3 - 3) = 4$**

c. $(24 \div 6 + 6) \times (3 - 3) = 0$

d. $(24 \div 6) + (6 \times 3) - 3 = 19$
or just: **$24 \div 6 + 6 \times 3 - 3 = 19$**

e. $24 \div (6 + 6) \times 3 - 3 = 3$

f. $(72 \times 2 \div 6) - (3 \times 2) + (8 \div 4) = 20$
or just: **$72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 20$**

g. $(72 \times 2 \div 6) - (3 \times 2 + 8 \div 4) = 16$

h. $[(72 \times 2) \div (6 - 3) \times 2 + 8] \div 4 = 26$

i. $(6 \div 2 \times 3) + (7 \times 8) - 2 = 55$
or just: **$6 \div (2 \times 3) + 7 \times 8 - 2 = 55$**

j. $(6 \div 2) \times (3 + 7) \times (8 - 2) = 180$
or just: **$6 \div 2 \times (3 + 7) \times (8 - 2) = 180$**

That was challenging! The next few will be a little easier. Figure out the ***numerators*** (the part above the dividing line), then the ***denominators*** (the part below the line), and then do the division. For example, given: $\frac{8 \times 9}{3 \times 4}$

Start with the top part (the numerator): 8 x 9 = 72 …

Then do the bottom part (the denominator): 3 x 4 = 12 …

Then divide. $\frac{72}{12}$ is the same as 72 ÷ 12, which is equal to 6

Remember to follow P-E-MD-AS very carefully! You should get “easy” answers (no decimals or fractions).

36. $\frac{(36 \div 18 + 6 \times 2)}{7 \times (6 - 4)} = \frac{14}{14} = 1$

37. $(2^2 + 3^2) \div 13 = 1$

38. $(7 - 5)^2 - (4 - 3)^8 = 3$

39. $\frac{(17 \times 6 - 4 \times 20) \times 10 \div 5}{8 \times 5 + 4} = \frac{44}{44} = 1$

40. $(5 - 3)^4 \div 4 = 4$

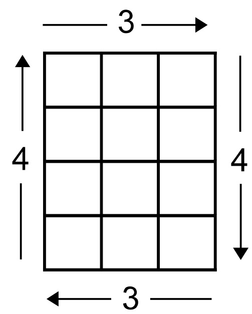
41. $\frac{9 \times 9 + 9}{6 + 36 \div 6 + 3 \times 6} = 3$

42. $9 \times (3 + 4) \div [(7 + 3) - 3] = 9$

43. $\frac{2^2 \times (5 + 6)}{25 \div 5 + 3 \times 2} = \frac{44}{11} = 4$

These pages give some practice in the Order of Operations. It is an area that gives many people trouble and requires a lot of practice. Keep at it! Facility with the order of operations makes all work in mathematics easier.

To conclude this Unit, consider a bit of geometry. Unit 3 concluded with an irregular rectilinear shape, and you found its area. ("Rectilinear" is just a fancy math term that describes a shape made of lines that meet at right angles.) The perimeter was never mentioned. ***Perimeter*** merely means the distance around the figure. For a simple rectangle like the one below, which is 3 units wide and 4 units long, the perimeter would be 3 (across the top) + 4 (down one side) + 3 (across the bottom) + 4 (up the other side) = 14 units total.



Using "P" for perimeter, "W" for width, and "L" for length, this could be stated as P = W + L + W + L. To make it shorter and easier to write, we could also say P = 2W + 2L (two times the width, plus 2 times the length). We could make it even shorter by using parentheses, like this: P = 2(W + L).

P = 2(W + L) is the ***formula*** for perimeter. We can use it to figure out the perimeter of any rectangle, no matter how big or small! Just change the "W" to the number of units in the width and change the "L" to the number of units in the length. Using the numbers from our rectangle above, we would go from P = 2(W + L) to P = 2(3 + 4). Thus, P (perimeter) = 2 x 7 = 14 units.

At the end of Unit 3, you learned about area, or the number of units *inside* a rectangle. The formula for area is A = WL, or Area = Width x Length. Applied to our rectangle above, A = WL becomes A = 3 x 4 = 12 square units.

44. Using these formulas, find the perimeter and area of the following rectangles:

a. width = 5, length = 7 **P = 2(5 + 7) = 24**
A = 5 x 7 = 35

b. width = 8, length = 15 **P = 2(8 + 15) = 46**
A = 8 x 15 = 120

c. width = 9, length = 4 **P = 2(9 + 4) = 26**
A = 9 x 4 = 36

d. width = 3, length = 48 **P = 2(3 + 48) = 102**
A = 3 x 48 = 144

45. Consider the last rectangle, "d" above. Is there a rectangle with a different width and length, but the same area? Find one.

A = 4 x 36 = 144

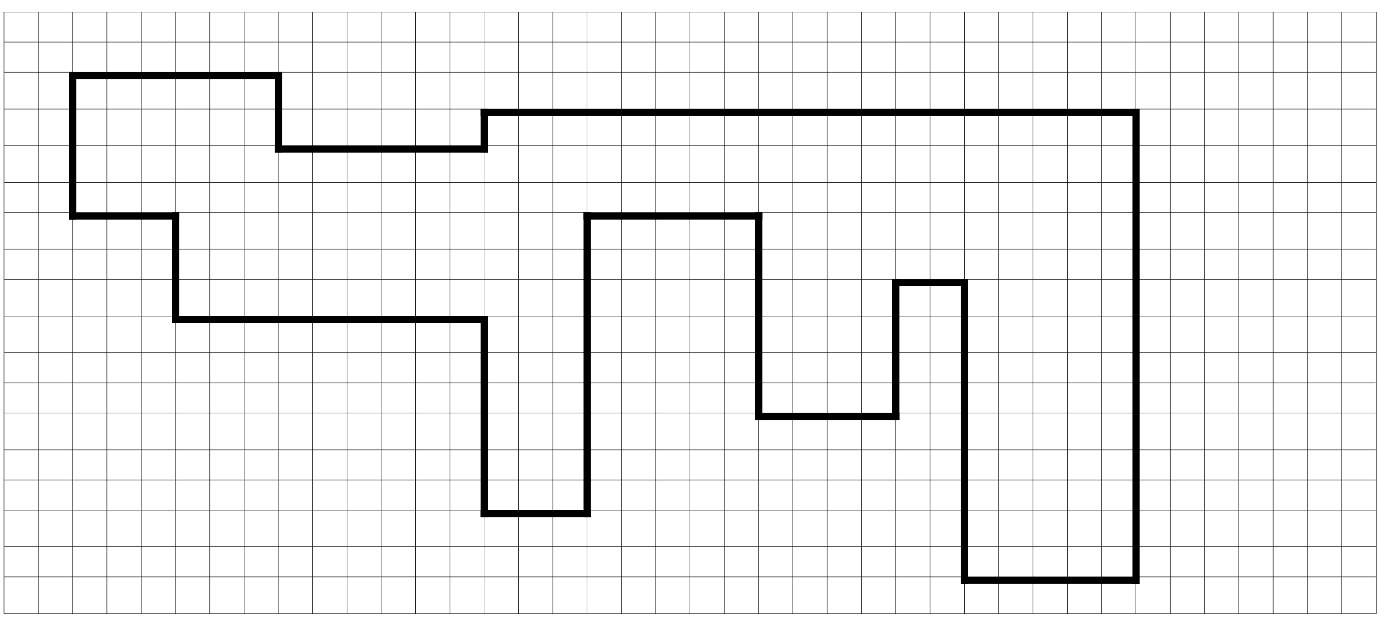
46. In fact, there are eight rectangles with different widths and lengths that have the same area as rectangle "d." Find all eight and give the perimeter for each rectangle. Show your results in the table on the next page.

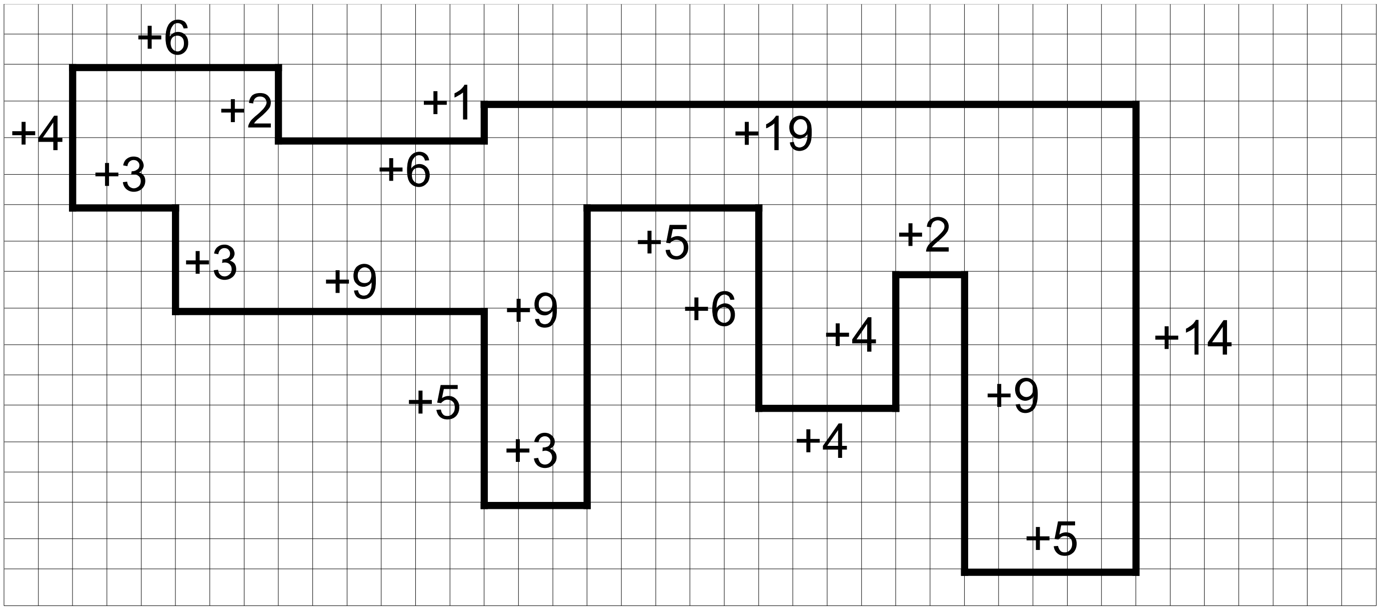
Width	Length	Perimeter
1	144	290
2	72	148
3	48	102
4	36	80
6	24	60
8	18	52
9	16	50

47. If you were going to build a rectilinear pen for a herd of sheep, what shape would require the least amount of fence for a given area? Would it be a square, a rectangle, or something like the one below?

A square

The figure from the end of Unit 3 appears below. In Unit 3, you found the area, or the number of square units inside the figure.

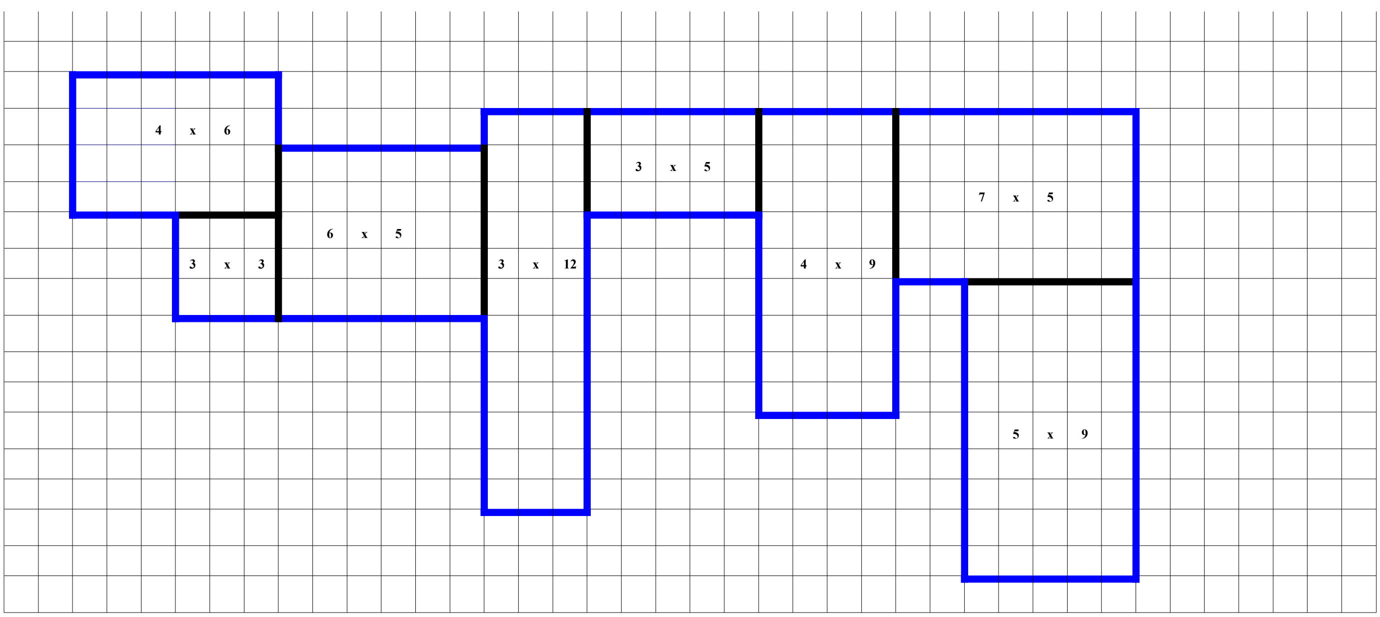




48. Find the perimeter of this figure, and show the calculations you made to figure it out.

4 + 6 + 2 + 6 + 1 + 19 + 14 + 5 + 9 + 2 + 4 + 4 + 6 + 5 + 9 + 3 + 5 + 9 + 3 + 3 = 119

49. Draw some straight lines inside the figure above so that it becomes a group of 7-10 boxes. Find the area of each box using the A = WL formula. Add up the areas of all the boxes. Does the number match what you got in Unit 3? Yes, answers should be the same as unit 3.



50. How would you feel about building a house of this shape? Explain why you feel that way. Answers will very. For example: It would be a odd house with a good bit of wasted space and a complicated roof line.

Congratulations! This is the last Unit in the *An Introduction to Math* course!