The Numbers Game
HiSET Math Skills

Mathematics Skills You Need to Pass the HiSET Exam
***Units 1 and 2** (TDOC Curriculum Level 2.1-2.2)*

By Garry W. Johnson *et al.*

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The Numbers Game
Unit 1 of 4

Student Name ___________________________________ Date __________________________

Skill Set: *Unit 1 – Number Associations, Place Values, Writing Numbers as Words*
Unit 2 – Addition, Multiplication, Palindromes, Inverse Operations

Directions:
1) Answer all the questions in bold print.
2) Take time to read the question thoroughly and find the most creative way to word your answers.
3) Check you answers for accuracy.

****

As everyone knows, the only way to really learn anything – be it poker, chess, singing, guitar playing, or Latin – is to do it. This is true above all else in learning math. But, like chess, the doing can be fun, and it can be described as play. With this object in mind, consider learning to play with mathematics.

Our spotter counts reps, and we push to get in one more number; the microwave counts backward to snack time …

1. Make a list of counts you have made, or could make.

Undoubtedly, these will include months/years in prison.

2. How many *days* have you been in prison so far? Were there any leap years?

3. How many steps can you take in the length of your cell? How many in the width?

Adults can have fun learning math, but many of us were turned off by unhappy early training.

4. Were you turned off by math earlier in life? If you know why, explain what happened.

Some of the best ways to get into the numbers game are by thinking about math and math concepts and asking questions. Try asking yourself and then asking others: "What is the biggest number you can think of?" This isn't very interesting unless you connect it to something. Depending on personal experience, a person might think of "a hundred," "a thousand," or "a million," but if you also ask, "Can you think of something that *is* a million?" the question becomes more difficult (and interesting!).

5. What can you think of that *is* 10?

6. What is 100?

7. What is 1,000?

8. Now try … What is the biggest number you know of that actually goes with something?

This opens up a whole world of associations: the height of a building, the distance around the world, the number of steps in a flight of stairs, the interstate highway number around a big city, the number of feet in a mile, the population of your town, the distance to the moon, the national debt. The possibilities are endless. One sixth grader came up with a great answer: "6 billion, the number of hamburgers McDonald's has sold." Scientists can go on forever – well, no, not forever but for a long time on this one. Try to think of what you are sure you can imagine. Units, tens, and hundreds are not too hard. But it is hard to imagine what a thousand of something looks like, and bigger numbers are nearly impossible to imagine. To make this manageable, we have to have a way of keeping numbers in order.

Numbers have place value. That means that a plain 7 is different from 70, because the 7 is in a different place. In the number 54,321, the last number ("1") means the number of plain units. If we changed it to 54,312, the one would mean 10, because it is in the "tens place." We have only ten different symbols for numbers – 0 (zero) through 9. When we need to write a bigger number, we use two "old" symbols and put them together to make "new" numbers that are bigger than nine, such as 10. Next, we can use up all the digits in the "units" place until we get to 19. After that, we get into 20 … 29, 30 … 39, until we reach 99. After this, we need to add a new place, so we go on to 100. The hundreds last until we get to 999. Since we have no more symbols to use, we add another place, and go on into the thousands (1,000).

9. What is the biggest number in the thousands?

Now consider the number 654,321. It means 1 unit, 2 tens (or 20), 3 hundreds, 4 thousands, 5 ten-thousands, and 6 hundred-thousands. The conventions of math notation condense all this into one number: 654,321. After we get through with the hundred-thousands, we go on to millions for three places, and then billions for three places. After that, most people don't bother with words. They use exponents, which will come later.

10. How many zeroes will there always be for millions (for example, four million)?

How about billions?

And for trillions?

Here are some examples of amounts written using numbers and, alternatively, words:

802	Eight hundred two
7,650	Seven thousand six hundred fifty
340,867	Three hundred forty thousand eight hundred sixty seven
621,000,000	Six hundred twenty one million

Going from words to numbers, “five thousand sixty seven” is 5,067. “Eight billion, nine hundred six million” is 8,906,000,000.

Before you write down your big number, check on how you read and write big numbers.

11. How would you write the number 7,392 in words?

How about the number 18,509?

And 3,852,000?

Now, let’s go the other way, from words to numbers.

12. How would you write “nine thousand twenty seven” in numbers?

How about “four hundred eighty thousand six hundred two”?

And “nine hundred eighty seven million six hundred fifty four thousand three hundred twenty one”?

And finally, “eight million four hundred two thousand”?

Usually, numbers are set off in groups of three separated by commas, so “six hundred fifty four thousand three hundred twenty one” would be 654,321. Notice that there is no “and” in this or any other large number. The only time you should use “and” is to indicate a decimal point. For example, $83.06 is read, “Eighty three dollars <u>and</u> six cents.” Decimals and decimal points will come in a later unit.

Imagining big numbers often leads us to thinking about what we would do with a big number of dollars.

13. What would you do with $100?

With $1,000?

With $10,000?

With $100,000?

Think up or look up some big numbers. Magazines, textbooks, and encyclopedias are great sources. Now, fill in the chart below with numbers you think of, look up, or find, and for each one give an example of something that *is* that number. We refer to those things as an "Association" in the chart below and provide some examples.

14. UNITS (1, 2, 3 … 9)

My number is: 4 Association: Legs on a chair

a) Your number: ______ Association: ______

b) Your number: ______ Association: ______

c) Your number: ______ Association: ______

15. TENS (10, 11 … 99)

My number is: 88 Association: Keys on a piano

a) Your number: ______ Association: ______

b) Your number: ______ Association: ______

c) Your number: ______ Association: ______

16. HUNDREDS

My number is: 476 Association: Date of the fall of the Roman Empire

a) Your number: ______ Association: ______

b) Your number: ______ Association: ______

c) Your number: ______ Association: ______

17. THOUSANDS

My number is: 1,250 Association: Height of the Empire State Building in feet

a) Your number: ______ Association: ______

b) Your number: ______ Association: ______

c) Your number: ______ Association: ______

18. TEN THOUSANDS

My number is: 54,500 Association: Area of NY State

a) Your number: ______ Association: ______

b) Your number: ______ Association: ______

c) Your number: ______ Association: ______

19. HUNDRED THOUSANDS My number is: 186,250 Association: Speed of light in miles per second

a) Your number: ___________ Association: ________________________

____

b) Your number: ___________ Association: ________________________

____

20. MILLIONS My number is: 9,500,000 Association: Approximate number of U.S drivers 19 and under

a) Your number: ___________ Association: ________________________

____

b) Your number: ___________ Association: ________________________

____

21. TEN MILLIONS My number is: 93,000,000 Association: Miles from the earth to the sun

a) Your number: ___________ Association: ________________________

____

22. HUNDRED MILLIONS My number is: 300,000,000 Association: U.S. population in 2006

a) Your number: ___________ Association: ________________________

____

23. BILLIONS My number is: $1,318,474,576 Association: U.S. Humanitarian Aid to Syria in 2015

a) Your number: ___________ Association: ________________________

____

24. Now, write the biggest number you can think of with which you have some association. What is the number, and what is the association?

What all this really leads to is the nature of math, the burst of different ideas for which we use math, and all of our needs to know math and to know about the world around us. It is thinking about quantity and sizes and about ordering these.

25. Where do you use numbers now? List three different examples.

26. Now list three places where you would like to use numbers.

27. For further thought, what is the …

a. height of the Empire State Building? (Hint: You can find it in this unit.)

b. number of eggs in a dozen?

c. number of feet in a mile?

d. human population of the Garden of Eden?

e. number of minutes in a year?

f. approximate distance across the USA?

g. number of squares on a checker board?

h. year in which "Columbus sailed the ocean blue"?

28. Now, put your answers in order, from smallest to largest.

29. Write a phrase about or draw a picture of something BIG!

Answer Key for Unit 1

Skill Set: *Unit 1 – Number Associations, Place Values, Writing Numbers as Words*
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Our spotter counts reps, and we push to get in one more number; the microwave counts backward to snack time …

1. Make a list of counts you have made, or could make.

Undoubtedly, these will include months/years in prison.
Answers will vary. For example: Years in prison, years since son's birth, bag of peanuts (or other food stuff) on my shelf, reps of exercises.

2. How many *days* have you been in prison so far? Were there any leap years?
Answers will vary. For example: September 3, 2000 – April 18, 2016 =
120 + (3 x 366) + (12 x 365) + 109 = 5,707
3. How many steps can you take in the length of your cell? How many in the width?
Answers may vary. For example: Length 12.5 ft. (size 11 shoe = c. 12”)
Width 6.5 ft.
Adults can have fun learning math, but many of us were turned off by unhappy early training.

4. Were you turned off by math earlier in life? If you know why, explain what happened.

Answers will vary. For example: Dyslexic – no ability for rote memorization

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5. What can you think of that *is* 10? **Answers will vary.** For example: 10 Commandments

6. What is 100? **Answers will vary.** For example: 100 Fish Oil gelcaps in a bottle.

7. What is 1,000? **Answers will vary.** For example: 1,000 pages in a book.

8. Now try … What is the biggest number you know of that actually goes with something?
Answers will vary. For example: 100 billion – estimated number of nerve cells in your brain and spinal cord.

This opens up a whole world of associations: the height of a building, the distance around the world, the number of steps in a flight of stairs, the interstate highway number around a big city, the number of feet in a mile, the population of your town, the distance to the moon, the national debt. The possibilities are endless. One sixth grader came up with a great answer: "6 billion, the number of hamburgers McDonald's has sold." Scientists can go on forever – well, no, not forever but for a long time on this one. Try to think of what you are sure you can imagine. Units, tens, and hundreds are not too hard. But it is hard to imagine what a thousand of something looks like, and bigger numbers are nearly impossible to imagine. To make this manageable, we have to have a way of keeping numbers in order.

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9. What is the biggest number in the thousands? 9,999

Now consider the number 654,321. It means 1 unit, 2 tens (or 20), 3 hundreds, 4 thousands, 5 ten-thousands, and 6 hundred-thousands. The conventions of math notation condense all this into one number: 654,321. After we get through with the hundred-thousands, we go on to millions for three places, and then billions for three places. After that, most people don't bother with words. They use exponents, which will come later.

10. How many zeroes will there always be for millions (for example, four million)? 6 zeroes

How about billions? 1,000,000,000 = 9 zeros

And for trillions? 1,000,000,000,000 = 12 zeros

Here are some examples of amounts written using numbers and, alternatively, words:

802	Eight hundred two
7,650	Seven thousand six hundred fifty
340,867	Three hundred forty thousand eight hundred sixty seven
621,000,000	Six hundred twenty one million

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Before you write down your big number, check on how you read and write big numbers.

11. How would you write the number 7,392 in words? Seven thousand three hundred ninety two

How about the number 18,509? Eighteen thousand five hundred nine

And 3,852,000? Three million eight hundred fifty two thousand

Now, let's go the other way, from words to numbers.

12. How would you write "nine thousand twenty seven" in numbers? 9,027

How about "four hundred eighty thousand six hundred two"? 480,602

And "nine hundred eighty seven million six hundred fifty four thousand three hundred twenty one"? 987,654,321

And finally, "eight million four hundred two thousand"? 8,402,000

Usually, numbers are set off in groups of three separated by commas, so "six hundred fifty four thousand three hundred twenty one" would be 654,321. Notice that there is no "and" in this or any other large number. The only time you should use "and" is to indicate a decimal point. For example, $83.06 is read, "Eighty three dollars and six cents." Decimals and decimal points will come in a later unit.

Imagining big numbers often leads us to thinking about what we would do with a big number of dollars.

13. What would you do with $100? Answers will vary. $100 – I'd put it on my commissary.

With $1,000? – I'd divorce my second wife.

With $10,000? $10,000 - $100,000 – I'd open a savings account for when I get out of prison (after doing the two above).

With $100,000?

Think up or look up some big numbers. Magazines, textbooks, and encyclopedias are great sources. Now, fill in the chart below with numbers you think of, look up, or find, and for each one give an example of something that *is* that number. We refer to those things as an "Association" in the chart below and provide some examples. **Answers will vary.** For example:

14. UNITS (1, 2, 3 … 9)

My number is: 4 **Association:** Legs on a chair

a) Your number: 2 **Association:** eyes on my face (arms, legs, etc.

b) Your number: 4 **Association:** wheels on a car

c) Your number: 7 **Association:** days in a week

15. TENS (10, 11 … 99)

My number is: 88 **Association:** Keys on a piano

a) Your number: 12 **Association:** months in a year

b) Your number: 50 **Association:** U.S. Governors

c) Your number: 66 **Association:** books in the Bible

16. HUNDREDS

My number is: 476 **Association:** Date of the fall of the Roman Empire

a) Your number: 100 **Association:** Senators in Congress

b) Your number: 325 **Association:** date of First Council of Nicaea

c) Your number: 701 BC **Association:** Sennacherib invaded Palestine

17. THOUSANDS

My number is: 1,250 **Association:** Height of the Empire State Building in feet

a) Your number: 1,000 **Association:** milligrams in a gram

b) Your number: 1,728 **Association:** cubic inches in 1 cubic foot

c) Your number: 2,000 **Association:** pounds in a ton

18. TEN THOUSANDS

My number is: 54,500 **Association:** Area of NY State

a) Your number: 33,000 **Association:** height in feet an airplane cabin must be pressurized

b) Your number: 56,600 **Association:** population of Agrigento (a city in Sicily, Italy) in 1990

c) Your number: 75,000 **Association:** displacement in metric tons of an aircraft carrier

19. HUNDRED THOUSANDS **My number is:** 186,250 **Association:** Speed of light in miles per second

a) Your number: 117,000 **Association:** population of Hebron (a city in Israel) est. in 1995

b) Your number: 251,773 **Association:** square-mile area of Afghanistan

20. MILLIONS **My number is:** 9,500,000 **Association:** Approximate number of U.S drivers 19 and under

a) Your number: 5,967,305 **Association:** pop. of the Hessen region of Germany in 1993

b) Your number: 6,560,000 **Association:** population of Beijing, ca. 1995

21. TEN MILLIONS **My number is:** 93,000,000 **Association:** Miles from the earth to the sun

a) Your number: 10,000,000 **Association:** estimated number killed in WWI (50,000,000 in WWII)

22. HUNDRED MILLIONS **My number is:** 300,000,000 **Association:** U.S. population in 2006

a) Your number: 395,000,000 **Association:** population of North America, ca. 1995

23. BILLIONS **My number is:** $1,318,474,576 **Association:** U.S. Humanitarian Aid to Syria in 2015

a) Your number: 1,187,997,000 **Association:** population of China, ca. 1995

24. Now, write the biggest number you can think of with which you have some association. What is the number, and what is the association?

100 billion – estimated number of nerve cells in your brain and spinal cord.

What all this really leads to is the nature of math, the burst of different ideas for which we use math, and all of our needs to know math and to know about the world around us. It is thinking about quantity and sizes and about ordering these.

25. Where do you use numbers now? List three different examples.

Figuring commissary balances; telling time; explaining math to students in ABE

26. Now list three places where you would like to use numbers.

Figuring my release date; figuring my income/taxes; filling my gas tank

27. For further thought, what is the …

a. height of the Empire State Building? (Hint: You can find it in this unit.) 1,250 ft.

b. number of eggs in a dozen? 12

c. number of feet in a mile? 5,280 ft.

d. human population of the Garden of Eden? 2

e. number of minutes in a year? 525,600 min.

f. approximate distance across the USA? 3,000 miles

g. number of squares on a checker board? 64

h. year in which "Columbus sailed the ocean blue"? 1492

28. Now, put your answers in order, from smallest to largest.

2
12
64
1,250
1,492
3,000
5,280
525,600

29. Write a phrase about or draw a picture of something BIG!

Answers will vary. For example: God "stretchest out the heavens like a curtain ..." and the universe is still expanding in every direction. That's big!

The Numbers Game
Unit 2 of 4

To be proficient in math, one must know and be able to use the basic number facts; that is, the simple addition and multiplication facts using the numbers 0 through 9. They do not have to be dreary. In fact, some really neat relationships can be found within them. Noticing these relationships makes them a lot more interesting. As elementary school children, we were not shown many of these patterns. In this unit and the next ones, some number oddities are shown that should help you to learn these basic facts – and have some fun at the same time.

PALINDROMIC NUMBERS

Some numbers read the same going left to right as they do going right to left. 56765 is one example. The same is true for words like "radar"; or names like "Anna," "Otto," and "Hannah"; or phrases like "Poor Dan is in a droop," or "Able was I ere I saw Elba." These are called *palindromes*. You have probably heard some. Language lovers like to collect them and love to invent them.

1. Make a list of words and/or names that are palindromes. Next, write down some phrases that you have heard, read, or made up yourself that are palindromes.

There has been a good deal of study on palindromic numbers. It is part of a branch of mathematics called ***Number Theory***. Palindromic numbers are kind of fun to notice on a car's odometer, but there's more to them than just the pattern. For example, take the number 742.

Reverse the digits to get a second number and then add them together:

$$\begin{array}{r} 742 \\ +\ 247 \\ \hline 989 \end{array}$$

It came out palindromic!

2. Try this with the numbers:

a. 423

b. 621

c. 238

As you can see, it doesn't work all the time …

But, try taking it another step with 238. Reverse the digits in your answer, and add that number to your answer. Did you get a palindrome?

In case that was confusing or you didn't get a palindrome, let's do an example together with the number 561:

Step One:	561 +165 726	Reverse the 561 to 165 and add the numbers together.
Step Two:	+ 627 1353	Reverse the answer 726 to 627 and add the numbers together. (*Uh oh … still no palindrome … but, if we go one more step …*)
Step Three:	+3531 4884	Reverse the answer 1353 to 3531 and add the numbers together Bingo! We got a palindrome.

3. Now you try it with the numbers:

a. 43

b. 56

c. 78

d. 35

e. 67

It is easy to check that it will always work for two digit numbers. If the sum of the two digits is less than 10, it is clear that it will work in one step, as you saw when working with 43 and 35. If the sum of the digits is 10, 11, 12, 13, 14, 15, 16, or 18 (as it was with 56, 78, and 67) it works in six steps or less. If the digits add up to 17, however, it can take a while.

4. What two digit numbers have digits that add up to 17?

5. Why don't we have to worry about instances where the two digits add up to 19?

With three digit numbers, it can get quite big sometimes before it works. It will still work, though!

6. Try it with the numbers:
a. 597 **b. 876**

Mathematicians (those who like to sit around and add forever or who program computers to do it for them) have found that there are 249 numbers less than 10,000 which do NOT generate palindromes after 100 steps. Aside from these 249 exceptions and 89, all integers (whole numbers and zero) produce a palindrome in less than 24 steps. The smallest of these exceptions is 196. If you reverse the digits and add (and have oodles of time), at the end of 230,310 additions you will come out with a palindromic number! The largest palindrome found from integers less than 10,000 is generated by 6,999 in 20 steps.

7. Now, you try it with 89. The process is started for you on the next page. Be sure to keep your numbers in line – ones in the ones place, tens in the tens place, etc. If you get a palindromic number before you have done 24 steps, it means you made an error. Here are some "check points" to help you stay on track:

After the 10th step, you should have 8,872,688.

After the 15th step, you should have 1,317,544,822.

After the 20th step, you should have 93,445,163,438.

The last/24th step starts 8,813 … and it has 13 digits.

			8	9
		+	9	8
	=	1	8	7
	+	7	8	1
	=	9	6	8
	+			
=				
+				
=				

MATH JEOPARDY

Math Jeopardy is a game in which someone says a number, and the responder must ask a question for which this is the answer. If we were playing with names instead of numbers, I might say, “The answer is George Washington,” and you would say, “Who was the first president of the United States?” In Math Jeopardy, I might say, “The answer is 54.” You would have to come up with a question whose answer is 54, such as: “What is 47 + 7?”

Of course, there could be more than one right question for any given answer. If the answer is 54, another perfectly good question is, “What is 60 - 6?” Below, you will be asked to come up with still another question whose answer is 54.

The answers can be refined, restricted, or set up with different rules, such as “the question must be about a multiplication fact,” but it’s less fun this way. It also means you might not get the chance to come up with an extremely imaginative question like: “What is the square root of 2,916?” (Which also happens to be 54.) The game has good possibilities for letting a show off be just that without being pretentious. It also has the virtue of teaching that there can be more than one correct response to a math question.

Below are some answers. You write the questions. Make up several for each answer. Yes, it’s hard work to make up more than 2 or 3 questions for each answer – but you will also find that it becomes more fun and more interesting the more questions you try to write. Good luck!

8. The answer is 17.

9. The answer is 54.

10. The answer is 81.

11. The answer is 256.

12. The answer is _______? (You make one up). Now, create some questions to go with it.

13. The answer is _______ ? Create some questions to go with it.

NOTES ON THE FOUR BASIC ARITHMETIC PROCESSES

We are ordinarily taught about addition, subtraction, multiplication, and division. It is easier to think of subtraction and division as "un-adding" and "un-multiplying." In subtracting, what we are doing is asking the question, "What must I add to a number to get the given total?" For example, 13 – 9 really means: "What must I add to 9 in order to get 13?" This is called an ***inverse operation,*** *or* ***inverse process***. ("Inverse" means something like "opposite of.") Subtraction is the inverse of addition.

Likewise, division is the inverse of multiplication. 18 divided by 3 asks the question: "What must I multiply by 3 to get 18?" So, subtraction and division are inverse processes. We also use the word "inverse" for numbers. For example, negative 8 (-8) is the inverse of positive 8. So, all subtraction problems are merely ones of adding the inverse number, as in this example:

$$12 - 8 = 4 \text{ is exactly the same as } 12 + (-8) = 4$$

For a cool subtraction trick, try the following. Think of a number between 100 and 1,000. It should not end in 00, and the difference between the first and last digits should be greater than 1. A number like 842 would be fine. Reverse the digits and subtract the smaller from the larger, like this:

$$\begin{array}{r} 842 \\ \underline{-\ 248} \\ 594 \end{array}$$

Now reverse the digits of the answer and add, like this:

$$\begin{array}{r} 594 \\ \underline{+\ 495} \\ 1089 \end{array}$$

14. Now, you try it with another number. (Remember, it must be 3 digits, shouldn't end in 00, and the difference between the first and last digits should be greater than 1.) **What is your final answer?**

15. Try three more numbers, and write your final answers.

a.

b.

c.

16. Any ideas on why this happens? Take your best shot at finding an explanation.

Answer Key for Unit 2

To be proficient in math, one must know and be able to use the basic number facts; that is, the simple addition and multiplication facts using the numbers 0 through 9. They do not have to be dreary. In fact, some really neat relationships can be found within them. Noticing these relationships makes them a lot more interesting. As elementary school children, we were not shown many of these patterns. In this unit and the next ones, some number oddities are shown that should help you to learn these basic facts – and have some fun at the same time.

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```
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```

2. Try this with the numbers:

a.
```
  423
+ 324
  747
```

b.
```
  621
+ 126
  747
```

c.
```
   238
+  832
  1070
+ 0701
  1771
```

As you can see, it doesn't work all the time …

But, try taking it another step with 238. Reverse the digits in your answer, and add that number to your answer. Did you get a palindrome?

In case that was confusing or you didn't get a palindrome, let's do an example together with the number 561:

Step One:	561 +165 726	Reverse the 561 to 165 and add the numbers together.
Step Two:	+ 627 1353	Reverse the answer 726 to 627 and add the numbers together. (*Uh oh … still no palindrome … but, if we go one more step …*)
Step Three:	+3531 4884	Reverse the answer 1353 to 3531 and add the numbers together Bingo! We got a palindrome.

3. Now you try it with the numbers:

a. 43

a. 43
+ 34
77

b. 56

b. 56
+ 65
121

c. 78

c. 78
+ 87
165
561
726
627
1353
3531
4884

d. 35

d. 35
+ 53
88

e. 67

e. 67
+ 76
143
341
484

It is easy to check that it will always work for two digit numbers. If the sum of the two digits is less than 10, it is clear that it will work in one step, as you saw when working with 43 and 35. If the sum of the digits is 10, 11, 12, 13, 14, 15, 16, or 18 (as it was with 56, 78, and 67) it works in six steps or less. If the digits add up to 17, however, it can take a while.

4. What two digit numbers have digits that add up to 17?

98, 89
(9 + 8 = 8 + 9 = 17)

5. Why don't we have to worry about instances where the two digits add up to 19?

9 + 9 = 18 (The maximum possible sum is 18.)

With three digit numbers, it can get quite big sometimes before it works. It will still work, though!

6. Try it with the numbers:

a. 597	b. 876
597	876
+ 795	+ 678
1392	1554
2931	4551
4323	6105
3234	5016
7557	11121
	12111
	23232

Mathematicians (those who like to sit around and add forever or who program computers to do it for them) have found that there are 249 numbers less than 10,000 which do NOT generate palindromes after 100 steps. Aside from these 249 exceptions and 89, all integers (whole numbers and zero) produce a palindrome in less than 24 steps. The smallest of these exceptions is 196. If you reverse the digits and add (and have oodles of time), at the end of 230,310 additions you will come out with a palindromic number! The largest palindrome found from integers less than 10,000 is generated by 6,999 in 20 steps.

7. Now, you try it with 89. The process is started for you on the next page. Be sure to keep your numbers in line – ones in the ones place, tens in the tens place, etc. If you get a palindromic number before you have done 24 steps, it means you made an error. Here are some "check points" to help you stay on track:

After the 10th step, you should have 8,872,688.

After the 15th step, you should have 1,317,544,822.

After the 20th step, you should have 93,445,163,438.

The last/24th step starts 8,813 … and it has 13 digits.

											8	9
										+	9	8
									=	1	8	7
									+	7	8	1
									=	9	6	8
									+	8	6	9
								=	1	8	3	7
								+	7	3	8	1
								=	9	2	1	8
									8	1	2	9
								1	7	3	4	7
								7	4	3	7	1
								9	1	7	1	8
								8	1	7	1	9
							1	7	3	4	3	7
							7	3	4	3	7	1
							9	0	7	8	0	8
							8	0	8	7	0	9
						1	7	1	6	5	1	7
						7	1	5	6	1	7	1
						8	**8**	**7**	**2**	**6**	**8**	**8**
						8	8	6	2	7	8	8
					1	7	7	3	5	4	7	6
					6	7	4	5	3	7	7	1
					8	5	1	8	9	2	4	7
					7	4	2	9	8	1	5	8
				1	5	9	4	8	7	4	0	5
				5	0	4	7	8	4	9	5	1
				6	6	4	2	7	2	3	5	6
				6	5	3	2	7	2	4	6	6
			1	**3**	**1**	**7**	**5**	**4**	**4**	**8**	**2**	**2**
			2	2	8	4	4	5	7	1	3	1
			3	6	0	2	0	0	1	9	5	3
			3	5	9	1	0	0	2	0	6	3
			7	1	9	3	0	0	4	0	1	6
			6	1	0	4	0	0	3	9	1	7
		1	3	2	9	7	0	0	7	9	3	3
		3	3	9	7	0	0	7	9	2	3	1
		4	7	2	6	7	0	8	7	1	6	4
		4	6	1	7	8	0	7	6	2	7	4
		9	**3**	**4**	**4**	**5**	**1**	**6**	**3**	**4**	**3**	**8**
		8	3	4	3	6	1	5	4	4	3	9
	1	7	6	8	8	1	3	1	7	8	7	7
	7	7	8	7	1	3	1	8	8	6	7	1
	9	5	5	5	9	4	5	0	6	5	4	8
	8	4	5	6	0	5	4	9	5	5	5	9
1	8	0	1	2	0	0	0	0	2	1	0	7
7	0	1	2	0	0	0	0	2	1	0	8	1
8	**8**	**1**	**3**	**2**	**0**	**0**	**0**	**2**	**3**	**1**	**8**	**8**

MATH JEOPARDY

Math Jeopardy is a game in which someone says a number, and the responder must ask a question for which this is the answer. If we were playing with names instead of numbers, I might say, “The answer is George Washington,” and you would say, “Who was the first president of the United States?” In Math Jeopardy, I might say, “The answer is 54.” You would have to come up with a question whose answer is 54, such as: “What is 47 + 7?”

Of course, there could be more than one right question for any given answer. If the answer is 54, another perfectly good question is, “What is 60 - 6?” Below, you will be asked to come up with still another question whose answer is 54.

The answers can be refined, restricted, or set up with different rules, such as “the question must be about a multiplication fact,” but it’s less fun this way. It also means you might not get the chance to come up with an extremely imaginative question like: “What is the square root of 2,916?” (Which also happens to be 54.) The game has good possibilities for letting a show off be just that without being pretentious. It also has the virtue of teaching that there can be more than one correct response to a math question.

Below are some answers. You write the questions. Make up several for each answer. Yes, it’s hard work to make up more than 2 or 3 questions for each answer – but you will also find that it becomes more fun and more interesting the more questions you try to write. Good luck!

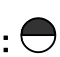
8. The answer is 17. **(Answers will vary)**
What is the square root of 289?
What is the cubed root of 4,913?
What is one times 17?; What is ten plus 7?

9. The answer is 54.
What is 18 times 3?
What is 9 times 6?
What is the square root of 2,916?

10. The answer is 81.
What is 9^2?
What is 486 divided by 6?
What is 567 minus 486?
What is 40 plus 41?

11. The answer is 256.
What is 16^2
What is128 times 2?
What is $\sqrt{65536}$?
What is 255 $^3/_3$?

12. The answer is __½__? (You make one up). Now, create some questions to go with it.

What is 50%?; What is 0.5?; What is 4/8?; What fraction is shaded:  **?**

13. The answer is ___-5___ ? Create some questions to go with it.

Other than 5, what equals │5│? (absolute value); What is 0 minus 5?; What is 10 minus 15?; What is -5 times 1?; What is 5 divided by -1?

NOTES ON THE FOUR BASIC ARITHMETIC PROCESSES

We are ordinarily taught about addition, subtraction, multiplication, and division. It is easier to think of subtraction and division as "un-adding" and "un-multiplying." In subtracting, what we are doing is asking the question, "What must I add to a number to get the given total?" For example, 13 – 9 really means: "What must I add to 9 in order to get 13?" This is called an ***inverse process***. ("Inverse" means something like "opposite of.") Subtraction is the inverse of addition.

Likewise, division is the inverse of multiplication. 18 divided by 3 asks the question: "What must I multiply by 3 to get 18?" So, subtraction and division are inverse processes. We also use the word "inverse" for numbers. For example, negative 8 (-8) is the inverse of positive 8. So, all subtraction problems are merely ones of adding the inverse number, as in this example:

12 – 8 = 4 is exactly the same as 12 + (-8) = 4

For a cool subtraction trick, try the following. Think of a number between 100 and 1,000. It should not end in 00, and the difference between the first and last digits should be greater than 1. A number like 842 would be fine. Reverse the digits and subtract the smaller from the larger, like this:

```
  842
- 248
  594
```

Now reverse the digits of the answer and add, like this:

```
  594
+ 495
 1089
```

14. Now, you try it with another number. (Remember, it must be 3 digits, shouldn't end in 00, and the difference between the first and last digits should be greater than 1.) **What is your final answer?**

```
  753          396
- 357        + 693
  396         1089
```

15. Try three more numbers, and write your final answers.

a.
```
  975          396
- 579        + 693
  396         1089
```

b.
```
  962          693
- 269        + 396
  693         1089
```

c.
```
  753          396
- 357        + 693
  396         1089
```

16. Any ideas on why this happens? Take your best shot at finding an explanation.

Suppose the number is xyz, i.e. 100x+10y+z. Reverse the digits to get x+10y+100z. Subtract to get 99x-99y or 99(x-y). Now 99 times a number k will have 3 digits, the first being k-1, the middle being 9, the final digit being 10-k. Reverse and add to get 900+180+9=1089