The Numbers Game
HiSET Math Skills

Mathematics Skills You Need to Pass the HiSET Exam
***Units 3 and 4** (TDOC Curriculum Level 3.1-3.2)*

By Garry W. Johnson *et al.*

This *The Numbers Game, HiSET Math Skills* packet was adapted by tutor Garry W. Johnson in 2017 for Mrs. Teri Angel and the NECX Education Department. The base text was taken from a course offered free to prisoners by College Guild, a non-profit organization. The materials are copyrighted and were reproduced with permission of the organization. If you are interested in their wide selection of free educational materials, write to:

College Guild
Post Office Box 6448
Brunswick, ME 04011

The Numbers Game

Unit 3 of 4

Student Name ___________________________________ **Date** __________________________

Skill Set: *Unit 3 – Multiplication, Area, Rate, Inverse Operations, Division*
Unit 4 – Exponents, Order of Operations, Expressions, Division, Perimeter, Area

HELPS FOR MULTIPLYING

There is no great problem for multiplying as there is (in the palindromic number exercise in Unit 2) for addition, but there are a good many ways to learn multiplication facts. These facts are essential to working in math with ease. You have to practice them many, many times to really know them. With this in mind, your first task is to fill in the three tables below. The first one is set up for you to use and, if need be, refer to. The next three are mixed up, and you will need to think carefully to complete them.

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

x	**9**	**7**	**4**	**8**
5		35		
3				24
6				
7	63			

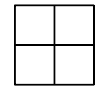
1. Table #1

x	**8**	**7**	**3**	**5**
8				
7				
3				
5				

2. Table #2

x	**2**	**7**	**0**	**6**
5				
9				
8				
3				

3. Table #3

Consider a sheet of graph or quad ruled paper, one that is covered with a grid like this: 
It may have four squares per inch or five, but that doesn't matter right now.

4. How many squares are on the paper? Just guess …

Is it more than 10? Obviously. More than 100? More than 1,000? More than 10,000? It is not so obvious when you get big numbers or many squares in the piece of graph paper. It's hard to get an answer without counting each square – and that would take a long time (and be very, very boring.) The best way to calculate the number of squares would be to multiply the number of squares going across the edge of the page horizontally by the number of squares going along the other edge vertically.

If we wanted to figure out how many squares are on a paper 8.5 inches wide by 11 inches tall that has 4 squares per inch, we would have to use multiplication. First, we would have to figure out how many squares are going across the page horizontally by multiplying the number of inches across and the number of squares per inch:

4 squares per inch x 8.5 inches horizontally = 34 squares across the paper horizontally

Next, we would use the same process to figure out how many squares are going along the paper vertically:

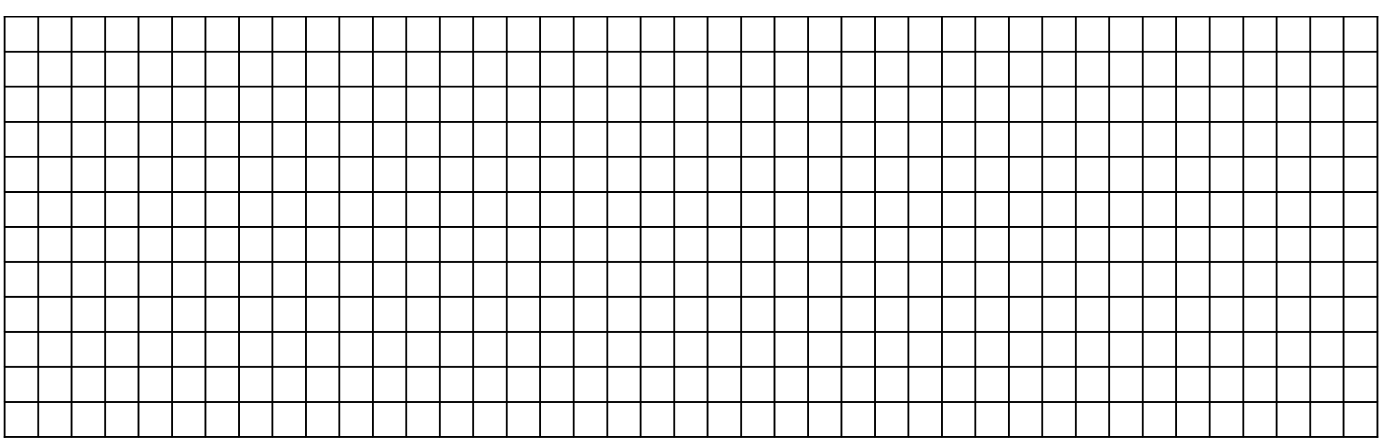
4 squares per inch x 11 inches vertically = 44 squares along the paper vertically

Now, all we have to do is multiply the horizontal number and the vertical number to get the total number of squares:

34 squares horizontally x 44 squares vertically = 1,496 total squares

On the graph below, draw with a straight edge (like the side of a book, or a heavy piece of paper) five rectangular boxes, each a different size.

5. For each box:
– Count the number of squares down one side and across the bottom edge.
– Multiply these two numbers together to get the total number of squares inside the box.
– Write the total number inside the box.



6. If you had an 8.5" x 11" piece of paper with 5 squares per inch, how many squares would be on the paper?

7. Suppose you had an 11" x 14" piece of paper.

a. How many squares would it have if there are four squares to an inch?

b. How many if there are five squares to an inch?

8. If a section of a football stadium has 28 rows, and if there are 18 seats in each row, how many people can be seated in that section? Show your multiplication in the space below.

9. If an airplane flies at 390 miles per hour and travels for 6 hours, how far will it go? Show how you figured it.

10. If the cost of oil is $43 per barrel, and the Zlickow Petroleum Company imports 5,000 barrels, what is the total cost of the oil? Show how you calculated the answer.

Questions 5 through 10 are examples of RATE. Rate means the *amount per unit*. A very commonly used rate is *miles per hour*, or distance per unit of time. To find the whole distance traveled, you multiply the rate in miles per hour by the number of hours. For example, *65 miles per hour x 2 hours = a total distance of 130 miles*. Another way to say the same thing is Distance = Rate x Time. The common formula is D = RT, sometimes abbreviated to just “Dirt.”

Another example of rate is cost per unit. *Total cost = cost per unit x number of units*. In Question 10, the cost was in dollars and the units were barrels of oil. The one about seating in a football stadium can also be thought of as a rate problem. The number of seats per row times the number of rows gives the total seating capacity of the section.

11. What other examples of rate can you think of? List three.

Rate is one of the most useful concepts we have, and it always involves either multiplication or division. Suppose a stadium section had a total of 520 seats, and there are 20 seats per row. How many rows are there in the section? You would divide 520 seats by 20 seats per row (520 ÷ 20) to get an answer of 26 rows. Here, you might call the rate *seats per row.*

Total seating, as we already know, would be the number of rows times the seats per row (26 rows x 20 seats per row = 520 total seats). We could make a formula, saying T = R x S.

12. If a hall has a total seating capacity of 360, and there are 15 rows, how many seats per row are there?

MATH MICE

The drawings which follow may not actually look much like mice, but they can be a neat way to practice multiplication. The square box in the middle is the face. Put in four simple numbers, one in each cell as shown (3, 5, 7, 8). The feet are the boxes below the face, the body is to the right of the face, and the ears are the circles above the face.

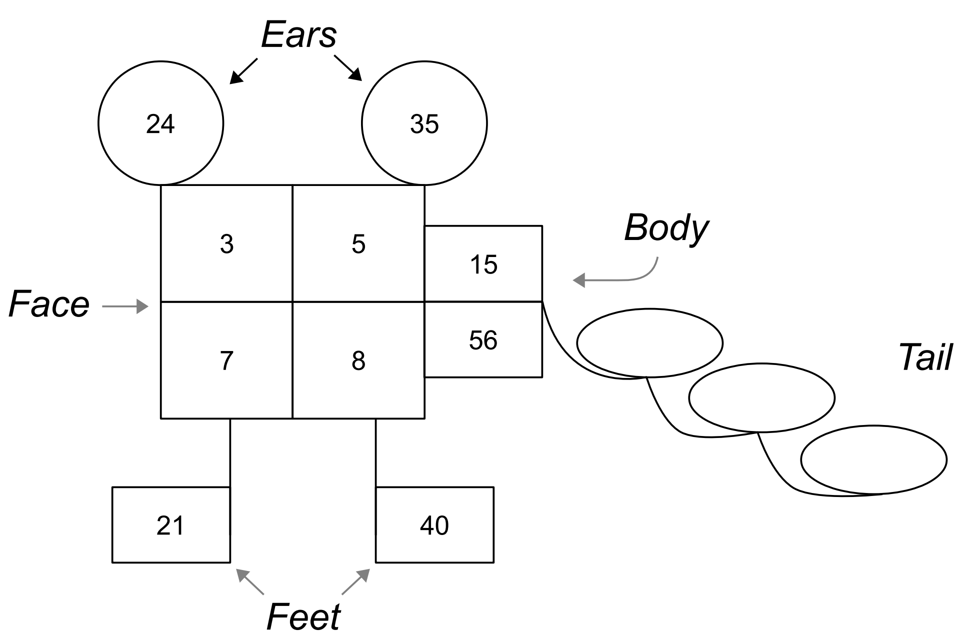
To get the left foot, multiply the two left hand numbers of the face (3 x 7 = 21). Multiply the two right hand numbers to get the right foot (5 x 8 = 40).

For the top of the body, multiply the top two numbers of the face (3 x 5 = 15). Multiply the bottom numbers of the face to get the bottom of the body (7 x 8 = 56).

To get the ears, multiply on the diagonal: 8 x 3 = 24 and 7 x 5 = 35.

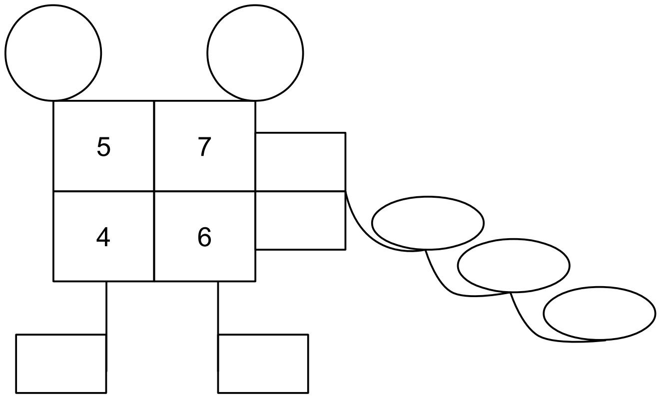
Now, multiply the two ears together (24 x 35), and put the answer in the first loop of the tail. Do the same for the feet (21 x 40), and put it in the second loop. Finally, do the same thing with the body (56 x 15) for the third loop. If you have done the calculations correctly, something quite neat happens …

13. What do you notice about the tail?



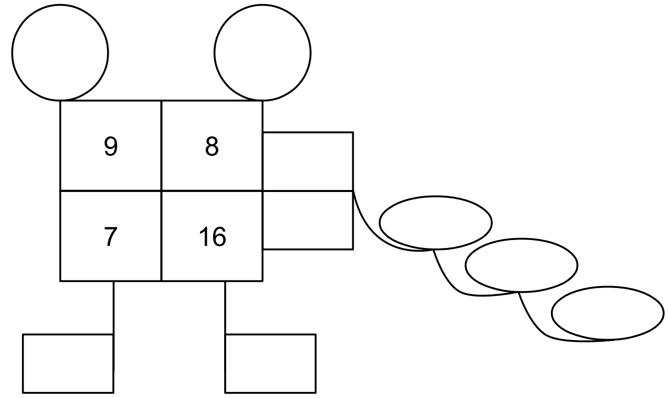
Ears: 24 x 35 120 + 720 840	Feet: 40 x 21	Body: 56 x15

14. Mouse #2



Ears: Feet: Body:

15. Mouse #3



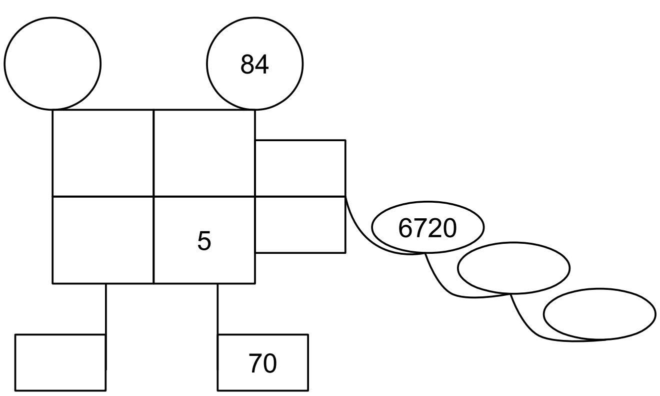
Ears: Feet: Body:

16. Make up two different "multiplication mice" of your own and solve them. Show your work on the calculations you made for each part of the tail (not just the answers).

Incidentally, mice work for addition, too. Just try it. They can also work for division and subtraction – just think about in the latter examples.

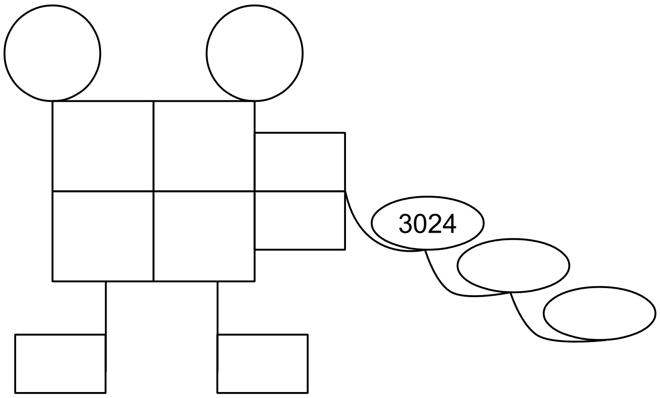
The following mice will require division. If a foot is filled, and you know one of the numbers above it, you must divide (or un-multiply) the foot number by the face number above it to get the missing part. The same applies if an ear or a body part is filled in.

17. Keeping this in mind, fill in all the missing parts on this multiplication mouse. Make sure to show your work!



It is possible to construct a mouse given only the tail number.

18. The tail is 3024, and there are several correct solutions. Find one!



Now we can look at some notable products. (Product is just a fancy math term for the answer you get when multiplying two or more numbers together.) The products below require two-digit multiplication. You should get some special answers. **Show your work.**

19. 12345679	**20. 12345679**	**21. 12345679**	**22. 12345679**
x 9	**x 27**	**x 63**	**x 54**

23. What is the missing number in 12345679 x __??__ = 888,888,888?

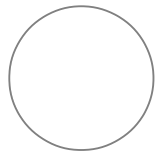
24. And here are two final tables for you to fill in. Please show your work on a separate sheet.

24. 9 x 7 =	________	**8 x 1 + 1 =**	________
99 x 77 =	________	**8 x 12 + 2 =**	________
999 x 777 =	________	**8 x 123 + 3 =**	________
9999 x 7777 =	________	**8 x 1234 + 4 =**	________
99999 x 77777 =	________	**8 x 12345 + 5 =**	________
		8 x 123456 + 6 =	________
		8 x 1234567 + 7 =	________
		8 x 12345678 + 8 =	________
		8 x 123456789 + 9 =	________

These last pages are for super thinkers …

This is commonly known as "The Pancake Problem." Consider a large circle, or a pancake. If you make one cut across it, you will get 2 sections. If you make two cuts, you get 4 sections.

25. Show how many sections you get with three cuts of this pancake.



26. Are there any special ways to make the cuts to change your answer to #25?

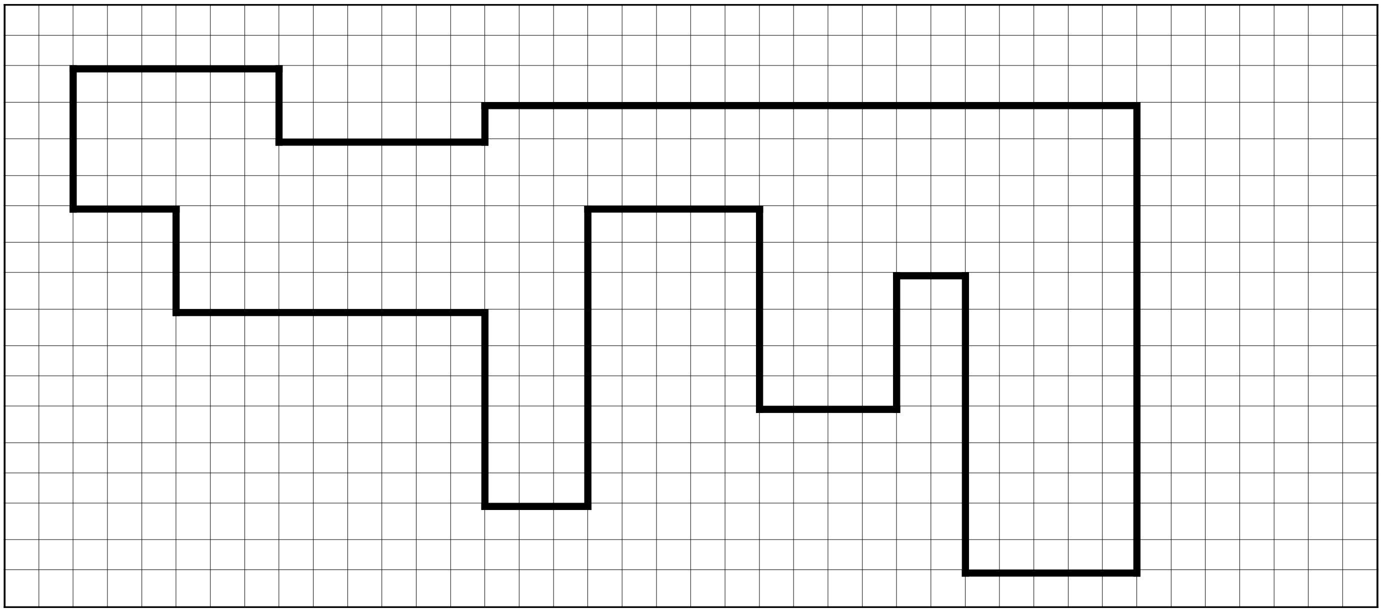
27. The cuts do NOT have to go through the center of the circle, nor do they have to be the same size. Use the space below to show more circles with different cuts.

28. How many sections can you get with 4 cuts? How many with 5 cuts?

29. Fill in the table below:

Number of Cuts	Number of Sections

30. Last question … The number of squares inside each of the boxes you drew is called the *area* of the box, or in math terms, the *area of the rectangle*. Can you find the area of the shape outlined below without counting all of the individual squares within it?



Answer Key for Unit 3

HELPS FOR MULTIPLYING

There is no great problem for multiplying as there is (in the palindromic number exercise) for addition, but there are a good many ways to learn multiplication facts. These facts are essential to working in math with ease. You have to practice them many, many times to really know them. With this in mind, your first task is to fill in the three tables below. The first one is set up for you to use and, if need be, refer to. The next three are mixed up, and you will need to think carefully to complete them.

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

x	9	7	4	8
5	**45**	35	**20**	**40**
3	**27**	**21**	**12**	24
6	**54**	**42**	**24**	**48**
7	63	**49**	**28**	**56**

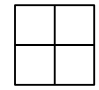
1. Table #1

x	8	7	3	5
8	**64**	**56**	**24**	**40**
7	**56**	**49**	**21**	**35**
3	**24**	**21**	**9**	**15**
5	**40**	**35**	**15**	**25**

2. Table #2

x	2	7	0	6
5	**10**	**35**	**0**	**30**
9	**18**	**63**	**0**	**54**
8	**16**	**56**	**0**	**48**
3	**6**	**21**	**0**	**18**

3. Table #3

Consider a sheet of graph or quad ruled paper, one that is covered with a grid like this: 
It may have four squares per inch or five, but that doesn't matter right now.

4. How many squares are on the paper? Just guess …

Is it more than 10? Obviously. More than 100? More than 1,000? More than 10,000? It is not so obvious when you get big numbers or many squares in the piece of graph paper. It's hard to get an answer without counting each square – and that would take a long time (and be very, very boring.) The best way to calculate the number of squares would be to multiply the number of squares going across the edge of the page horizontally by the number of squares going along the other edge vertically.

If we wanted to figure out how many squares are on a paper 8.5 inches wide by 11 inches tall that has 4 squares per inch, we would have to use multiplication. First, we would have to figure out how many squares are going across the page horizontally by multiplying the number of inches across and the number of squares per inch:

4 squares per inch x 8.5 inches horizontally = 34 squares across the paper horizontally

Next, we would use the same process to figure out how many squares are going along the paper vertically:

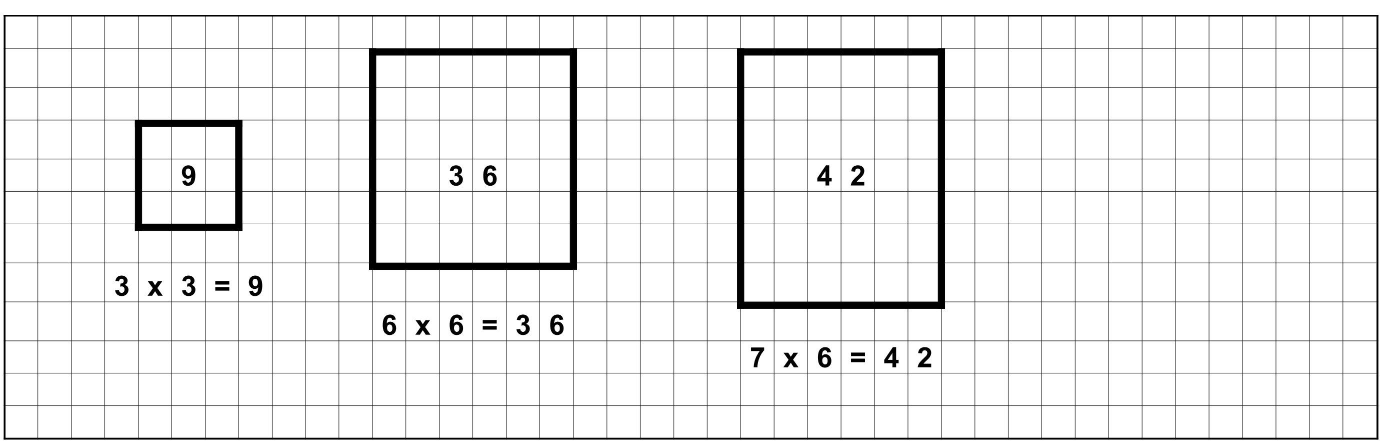
4 squares per inch x 11 inches vertically = 44 squares along the paper vertically

Now, all we have to do is multiply the horizontal number and the vertical number to get the total number of squares:

34 squares horizontally x 44 squares vertically = 1,496 total squares

On the graph below, draw with a straight edge (like the side of a book, or a heavy piece of paper) five rectangular boxes, each a different size.

5. For each box:
– Count the number of squares down one side and across the bottom edge.
– Multiply these two numbers together to get the total number of squares inside the box.
– Write the total number inside the box. **Answers will very.** For example:



6. If you had an 8.5" x 11" piece of paper with 5 squares per inch, how many squares would be on the paper?

(8.5 x 11) x 5 = 93.5 x 5 = 467.5

7. Suppose you had an 11" x 14" piece of paper.

a. How many squares would it have if there are four squares to an inch?

(11 x 14) x 4 = 154 x 4 = 616

b. How many if there are five squares to an inch?

(11 x 14) x 5 = 154 x 5 = 770

8. If a section of a football stadium has 28 rows, and if there are 18 seats in each row, how many people can be seated in that section? Show your multiplication in the space below.

28 x 18 = 504 seats/people

9. If an airplane flies at 390 miles per hour and travels for 6 hours, how far will it go? Show how you figured it.

390 x 6 = 2,340 miles

10. If the cost of oil is $43 per barrel, and the Zlickow Petroleum Company imports 5,000 barrels, what is the total cost of the oil? Show how you calculated the answer.

```
    5000
   x  43
   15000
   20000
$215,000
```

Questions 5 through 10 are examples of RATE. Rate means the *amount per unit*. A very commonly used rate is *miles per hour*, or distance per unit of time. To find the whole distance traveled, you multiply the rate in miles per hour by the number of hours. For example, *65 miles per hour x 2 hours = a total distance of 130 miles*. Another way to say the same thing is Distance = Rate x Time. The common formula is D = RT, sometimes abbreviated to just "Dirt."

Another example of rate is cost per unit. *Total cost = cost per unit x number of units*. In Question 10, the cost was in dollars and the units were barrels of oil. The one about seating in a football stadium can also be thought of as a rate problem. The number of seats per row times the number of rows gives the total seating capacity of the section.

11. What other examples of rate can you think of? List three.

Answers will vary. For example:

1) *Atmospheric pressure: 14.7* **lbs. per. sq. in.**
2) **knots per. hour**
3) **price per. gallon (gas/milk)**

Rate is one of the most useful concepts we have, and it always involves either multiplication or division. Suppose a stadium section had a total of 520 seats, and there are 20 seats per row. How many rows are there in the section? You would divide 520 seats by 20 seats per row (520 ÷ 20) to get an answer of 26 rows. Here, you might call the rate *seats per row.*

Total seating, as we already know, would be the number of rows times the seats per row (26 rows x 20 seats per row = 520 total seats). We could make a formula, saying T = R x S.

12. If a hall has a total seating capacity of 360, and there are 15 rows, how many seats per row are there?

360 ÷ 15 = 24 seats per row

MATH MICE

The drawings which follow may not actually look much like mice, but they can be a neat way to practice multiplication. The square box in the middle is the face. Put in four simple numbers, one in each cell as shown (3, 5, 7, 8). The feet are the boxes below the face, the body is to the right of the face, and the ears are the circles above the face.

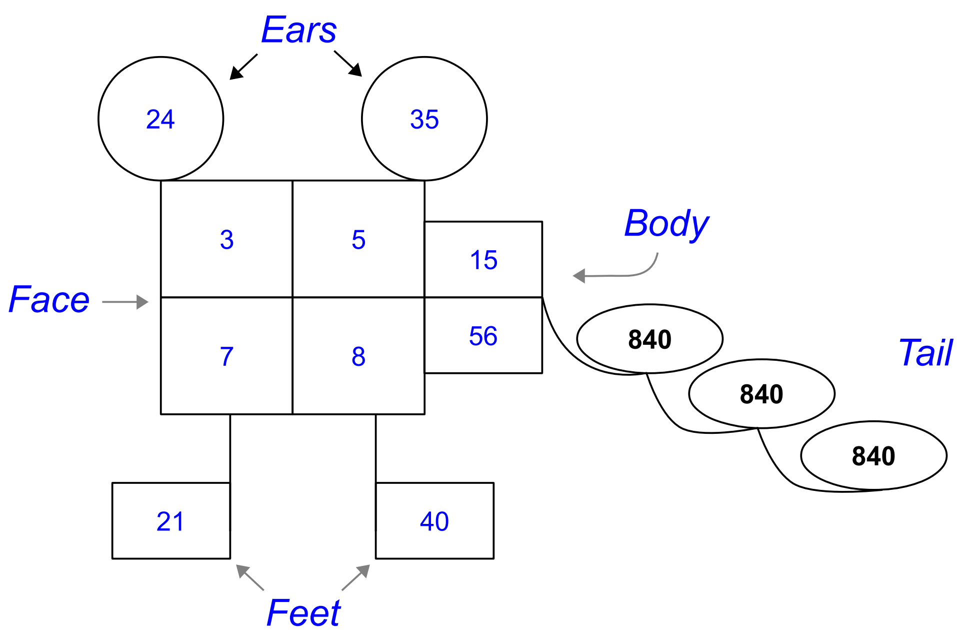
To get the left foot, multiply the two left hand numbers of the face (3 x 7 = 21). Multiply the two right hand numbers to get the right foot (5 x 8 = 40).

For the top of the body, multiply the top two numbers of the face (3 x 5 = 15). Multiply the bottom numbers of the face to get the bottom of the body (7 x 8 = 56).

To get the ears, multiply on the diagonal: 8 x 3 = 24 and 7 x 5 = 35.

Now, multiply the two ears together (24 x 35), and put the answer in the first loop of the tail. Do the same for the feet (21 x 40), and put it in the second loop. Finally, do the same thing with the body (56 x 15) for the third loop. If you have done the calculations correctly, something quite neat happens …

13. What do you notice about the tail?

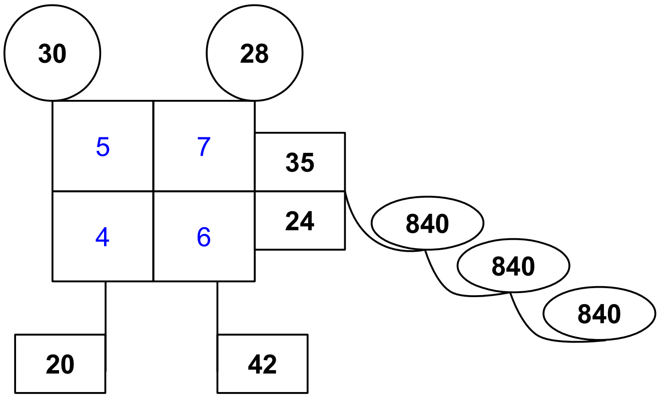


Ears: 24
x 35
120
+ 720
840

Feet: 40
x 21

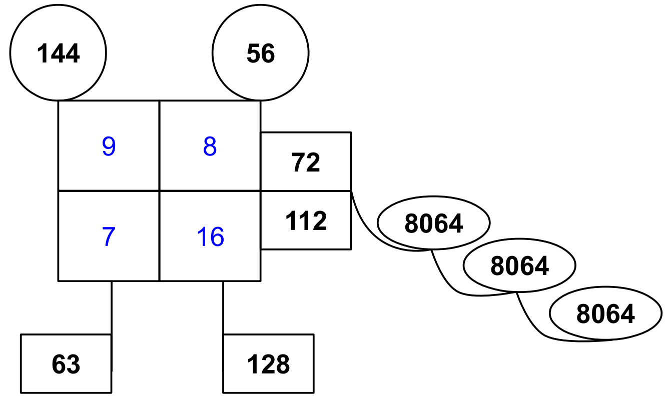
Body: 56
x15

14. Mouse #2



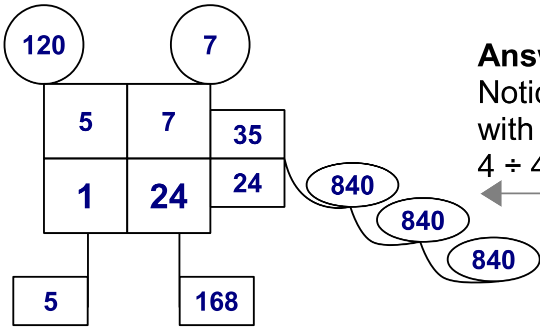
Ears: Feet: Body:

15. Mouse #3



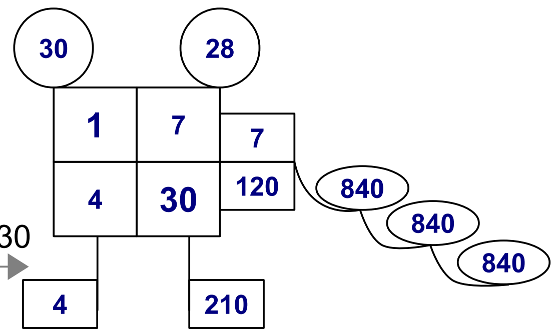
Ears: Feet: Body:

16. Make up two different "multiplication mice" of your own and solve them. Show your work on the calculations you made for each part of the tail (not just the answers).



Answers will vary.
Notice the correlations with the mouse in 14:
4 ÷ 4 = 1; 4 x 6 = 24

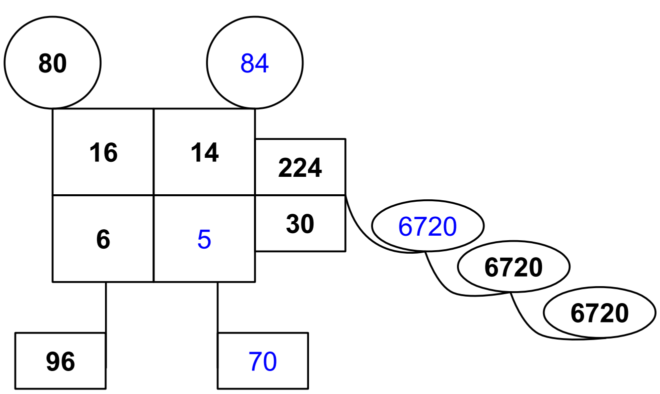
5 ÷ 5 = 1; 5 x 6 = 30



Incidentally, mice work for addition, too. Just try it. They can also work for division and subtraction – just think about in the latter examples.

The following mice will require division. If a foot is filled, and you know one of the numbers above it, you must divide (or un-multiply) the foot number by the face number above it to get the missing part. The same applies if an ear or a body part is filled in.

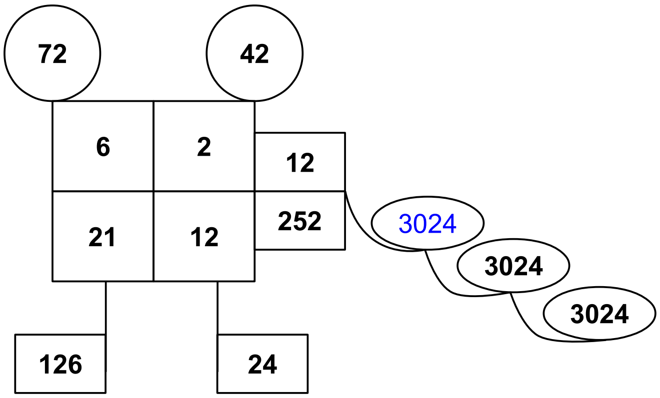
17. Keeping this in mind, fill in all the missing parts on this multiplication mouse. Make sure to show your work!



70 ÷ 5 = 14
84 ÷ 14 = 6
6720 ÷ 30 = 224
224 ÷ 14 = 16

It is possible to construct a mouse given only the tail number.

18. The tail is 3024, and there are several correct solutions. Find one!



3024 ÷ 12 = 252
252 ÷ 12 = 21

Now we can look at some notable products. (Product is just a fancy math term for the answer you get when multiplying two or more numbers together.) The products below require two-digit multiplication. You should get some special answers. **Show your work.**

19. 12345679 x 9

111111111

(9 x 1 = 9)

20. 12345679 x 27

86419753
24691358
333333333

(9 x 3 = 27)

21. 12345679 x 63

37037037
74074074
777777777

(9 x 7 = 63)

22. 12345679 x 54

666666666

(54 ÷ 9 = 6)

23. What is the missing number in 12345679 x __72__ = 888,888,888? (9 x 8 = 72)

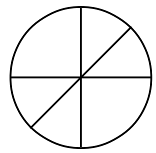
24. And here are two final tables for you to fill in. Please show your work on a separate sheet.

24. 9 x 7 =	63	**8 x 1 + 1 =**	9
99 x 77 =	7623	**8 x 12 + 2 =**	98
999 x 777 =	776223	**8 x 123 + 3 =**	987
9999 x 7777 =	77762223	**8 x 1234 + 4 =**	9876
99999 x 77777 =	7777622223	**8 x 12345 + 5 =**	98765
		8 x 123456 + 6 =	987654
		8 x 1234567 + 7 =	9876543
		8 x 12345678 + 8 =	98765432
		8 x 123456789 + 9 =	987654321

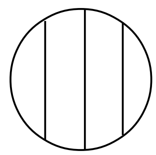
These last pages are for super thinkers …

This is commonly known as "The Pancake Problem." Consider a large circle, or a pancake. If you make one cut across it, you will get 2 sections. If you make two cuts, you get 4 sections.

25. Show how many sections you get with three cuts of this pancake.

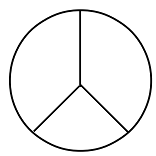


26. Are there any special ways to make the cuts to change your answer to #25?



27. The cuts do NOT have to go through the center of the circle, nor do they have to be the same size. Use the space below to show more circles with different cuts.

Answers will vary. For example –



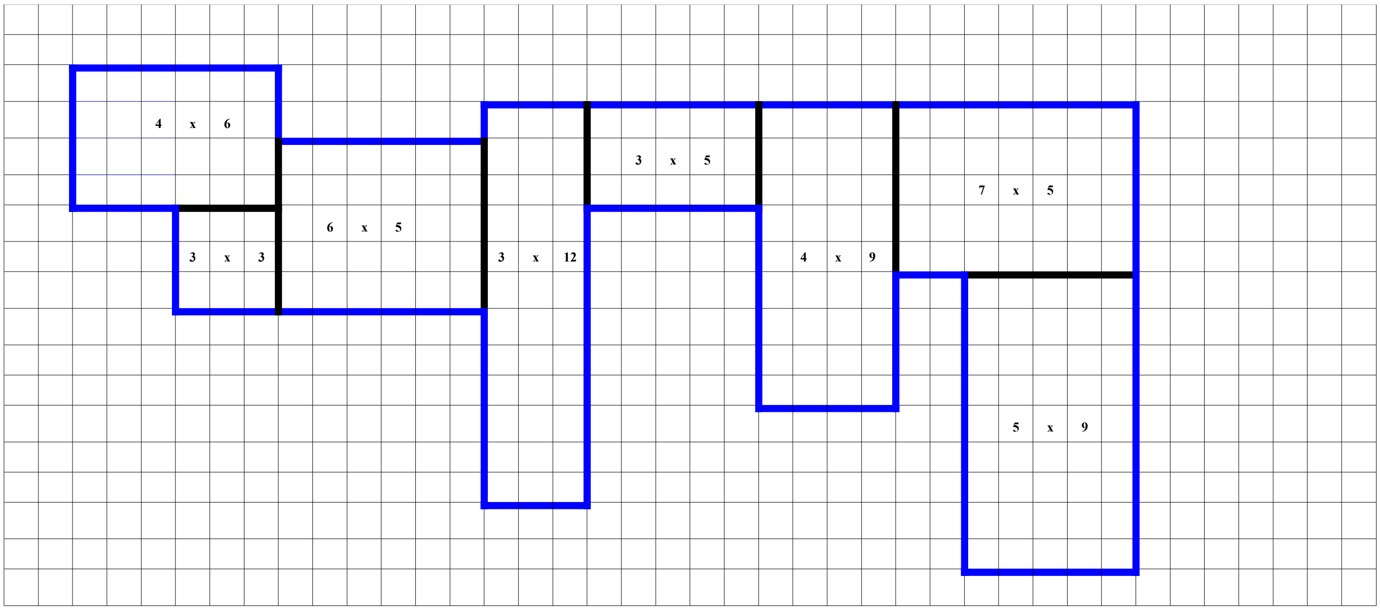
28. How many sections can you get with 4 cuts? How many with 5 cuts?

Answers will vary. Check for number of cuts, one set of 4, one set of five.

29. Fill in the table below:

Number of Cuts	Number of Sections
6	6-12
7	7-14
8	8-16
9	9-18
10	10-20
11	11-22
12	12-24

30. Last question … The number of squares inside each of the boxes you drew is called the *area* of the box, or in math terms, the *area of the rectangle*. Can you find the area of the shape outlined below without counting all of the individual squares within it?



Yes, by dividing the shape into individual boxes. Area = L x W (or side x side)
If they do the math, calculations will vary depending on how the shape is divided:

4 x 6 =	24
3 x 3 =	9
5 x 6 =	30
3 x 12 =	36
3 x 5 =	15
4 x 9 =	36
5 x 7 =	35
5 x 9 =	+ 45

A = 320

The Numbers Game
Unit 4 of 4

Beyond the four basic processes, there are other expressions one needs to understand to gain facility in math. These include ***exponents***. Exponents are numbers written above and to the right of any ordinary number. They express the number of times a number is multiplied by itself.

For example, 5^2 is read "five to the second power" or "five squared." It means 5 x 5. In the expression "5 x 5" we are multiplying two numbers; each number is called a ***factor***. In "5 x 5" then, there are two factors, each of which is a 5. In 5^2, the exponent "2" tells the number of factors of the base "5" that are to be multiplied together.

So, 5^3 means 5 x 5 x 5. This is also called "five cubed." The exponent (in this case 3) tells the number of fives to multiply together. Here are some other examples:

4^2 = 4 x 4 = 16 6^3 = 6 x 6 x 6 = 216 2^4 = 2 x 2 x 2 x 2 = 16

3^5 = 3 x 3 x 3 x 3 x 3 = 243 8^5 = 8 x 8 x 8 x 8 x 8 = 4,096 7^3 = 7 x 7 x 7 = 343

Notice how fast these numbers go up. This is what "increasing exponentially" means.

1. Find the following:

3^4 = **5^3 =** **9^3 =** **10^4 =**

2. Notice the number of zeroes in 10^4. With that in mind, what is 10^6?

The number of zeroes in any power of ten is the same as the exponent to which 10 is raised. This is a basic property of number systems. It may intrigue you to know how other number systems work, but it is not essential. Knowing basic squares and cubes is helpful, so your next task is to complete the tables below.

3. Table of Squares

1^2 =		**5^2 =**		**9^2 =**		**13^2 =**		**17^2 =**	
2^2 =		**6^2 =**		**10^2 =**		**14^2 =**		**18^2 =**	
3^2 =		**7^2 =**		**11^2 =**		**15^2 =**		**19^2 =**	
4^2 =		**8^2 =**		**12^2 =**		**16^2 =**		**20^2 =**	

4. Table of Cubes

1^3 =		**2^3 =**		**3^3 =**		**4^3 =**		**5^3 =**	
6^3 =		**7^3 =**		**8^3 =**		**9^3 =**		**10^3 =**	

Notice that one to any power is one. And of course, zero to any power is zero. Another set of numbers worth noticing is the powers of 2.

5. Find the following. (The first one, 2^0 has been done for you.)

$2^0 = 1$

$2^1 =$

$2^2 =$

$2^3 =$

$2^4 =$

$2^5 =$

$2^6 =$

$2^7 =$

$2^8 =$

With this in mind, here is a famous problem:

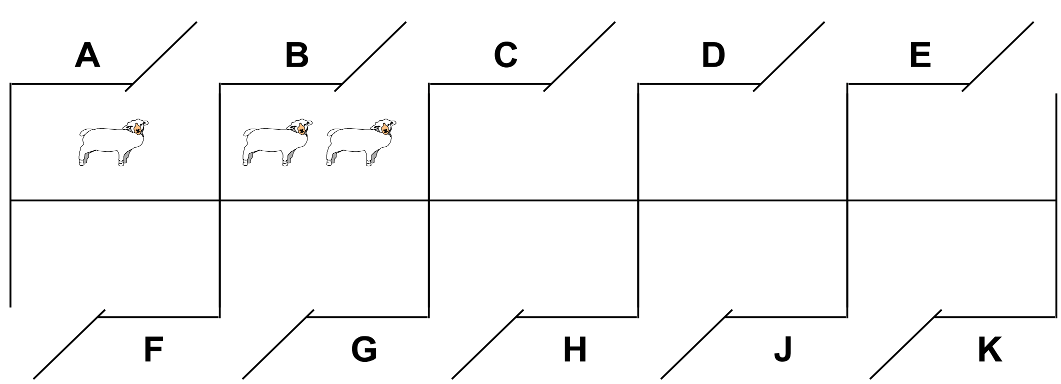
ONE THOUSAND SHEEP

A farmer has a thousand sheep. He wants to build pens and place the sheep in the pens so that anyone can come along and say, "I'd like 23 sheep," and the farmer can let out the correct number of sheep by opening up exactly the right gates. If, for example, a customer wanted just one sheep, there would have to be a pen with just one sheep in it. Of course, if a customer wanted two sheep, the farmer would need a pen with two sheep in it. Once the farmer opens a gate, all the sheep in that pen will come out; he can't empty part of a pen.

6. Would the farmer need a separate pen with 3 sheep in it? How about 4 sheep?

7. How would the farmer get 6 sheep?

8. *Write* the number of sheep (you do NOT have to draw them!) the farmer must put in each pen to enable him to open gates and get out any exact number of sheep from 1 up to 1,000. (Hint: None of these, other than B, will be ***prime numbers***. Primes are whole numbers greater than 1 whose only two factors are 1 and itself.)



9. What gates would the farmer open to get 13 sheep? How many sheep would come out of each pen that he opened?

10. To get 48 sheep? List the gates he needs to open, and how many sheep will come out of each pen. Now, add up the number of sheep that came out of each opened pen to check your work.

11. To get 73 sheep? List the gates opened, the number of sheep from each of the opened pens, and show the sum of your numbers.

12. Do the same for 86 sheep.

13. Do the same for 127 sheep.

14. Do the same for 259 sheep.

15. Do the same for 683 sheep.

16. How many sheep are in the last pen? (Be careful … the answer isn't especially obvious.)

If you have put the right number of sheep in each pen, you will have written the powers of 2 up to and including 28. But if you wrote the number for 29 in the last pen (K), you are not right.

17. Why would it be incorrect to write the number for 29 in the last pen (K)?

This problem of the thousand sheep is good for seeing some mathematical thinking. It is not of particular practical value (what farmer ever had exactly 1,000 sheep, and how can you get a sheep to do anything right in the first place?). But math is full of fun questions to play with like this one. Meanwhile, you now have the powers of 2 at hand.

18. Fill in the table below.

2^0 =		2^1 =		2^2 =		2^3 =		2^4 =	
2^5 =		2^6 =		2^7 =		2^8 =		2^9 =	
2^{10} =									

Knowing the use of powers and exponents enables us to establish the rules for the order of operations. This is very important. The expression 8 x 2 + 4 might mean 20 or 48, depending on the order in which you do the arithmetic. For example, if you start with 8 x 2, you get 16. Add that to 4 and you get 20. However, if you add 2 + 4 first, you get 6. 8 x 6 is 48. Clearly, we need to have some rules about what to do first so that we get the same answer every time.

Fortunately, just such a set of rules has been created and agreed to. They are called the **Order of Operations** and are often referred to as PEMDAS. This is just an abbreviation for the words Parentheses, Exponents, Multiplication/Division, Addition/Subtraction. (Some people find it easier to remember as "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally".)

The Order of Operations says that we have to do calculations in a certain order, every time:

1 . ***P****arentheses*: Make any calculations inside parentheses before doing anything else.
Example: 8 x (2 + 4) … Start inside the parentheses with 2 + 4 to get 6. Then multiply by 8 to get 48.

2. ***E****xponents*: Once the parentheses are done (or if there aren't any) calculate any exponents.
Example: 8 x 3^2 ...Start with 3^2, which is 3 x 3, or 9. Then, multiply by eight to get 72.

3. ***M****ultiplication/***D***ivision*: Once the exponents are done (or if there aren't any) do any multiplication or division, working from left to right.
Example: 8 ÷ 4 x 3 … Since we have both multiplication and division, we have to work from left to right. Starting on the left, then, 8 ÷ 4 = 2. Continuing to the right, multiply by 3 to get 6.

4. ***A****ddition/***S***ubtraction*: Once the multiplying and dividing are done (or if there isn't any) do any addition or subtraction, working left to right.
Example: 9 – 5 + 2 … Since we have both addition and subtraction, we have to work from left to right. Starting on the left, then, 9 – 5 = 4. Continuing to the right, add 2 to get 6.

I know that's a lot to take in, so let's do an example together before you try some on your own. **You fill in the blanks.**

$$6 \times (12 - 9)^2 - 8 \div 4 + 5 = ?$$

Step 1. ***P****arentheses first*. (12 – 9) = _____. Now, we have 6 x 3^2 – 8 ÷ 4 + 5 = ?

Step 2. ***E****xponents second*. 3^2 = ______. Now, we have 6 x 9 – 8 ÷ 4 + 5 = ?

Step 3. ***M****ultiplication and* ***D****ivision third*. Reading our equation left to right, multiplication comes first, and then division. 6 x 9 = ___ and 8 ÷ 4 = ___. Now, we are left with 54 – 2 + 5 = ?

Step 4. ***A****ddition and* ***S****ubtraction last*. Reading our equation left to right, subtraction comes first, and then addition. 54 – 2 = ____. Now, we are left with 52 + 5. So, the final answer is _____.

19.7 + 6 x 2 =

20. 47 + 5 x 0 =

21. 4 x (10 ÷ 5) =

22. 8 + (32 – 24) ÷ 4 =

23. 8 x (9 – 7) ÷ 4 + 2 =

24. $(8 - 3)^2$ x (7 – 3) =

25. 5 + 3^2 x (9 – 4) =

26. 8 + 43 – 8 x 2 – 42 – 6 =

Try a few more with exponents. Remember "P-E-MD-AS." Do the work inside the parentheses first, and then calculate the exponents.

27. $3 \times 5^2 =$

28. $(3 \times 5)^2 =$

29. $3^2 \times 5 =$

30. $3^2 + 5^2 =$

31. $(3 + 5)^2 =$

32. $(3^2 + 5) =$

33. $(3^2 + 5)^2 =$

34. $(3 \times 5)^3 =$

Notice how adding parentheses changes the answer. For example, the equations in questions 27 and 28 use the same base numbers and exponents (3 and 5^2), but the answers are very different because parentheses were added. The same principle can be seen at work in questions 30 and 33.

35. For each equation below, add in parentheses to make the statement true.

a. $24 \div 6 + 6 \times 3 - 3 = 27$

b. $24 \div 6 + 6 \times 3 - 3 = 4$

c. $24 \div 6 + 6 \times 3 - 3 = 0$

d. $24 \div 6 + 6 \times 3 - 3 = 19$

e. $24 \div 6 + 6 \times 3 - 3 = 3$

f. $72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 20$

g. $72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 16$

h. $72 \times 2 \div 6 - 3 \times 2 + 8 \div 4 = 26$

i. $6 \div 2 \times 3 + 7 \times 8 - 2 = 55$

j. $6 \div 2 \times 3 + 7 \times 8 - 2 = 180$

That was challenging! The next few will be a little easier. Figure out the ***numerators*** (the part above the dividing line), then the ***denominators*** (the part below the line), and then do the division. For example, given: $\frac{8 \times 9}{3 \times 4}$

Start with the top part (the numerator): 8 x 9 = 72 …

Then do the bottom part (the denominator): 3 x 4 = 12 …

Then divide. $\frac{72}{12}$ is the same as 72 ÷ 12, which is equal to 6

Remember to follow P-E-MD-AS very carefully! You should get "easy" answers (no decimals or fractions).

36. $\frac{(36 \div 18 + 6 \times 2)}{7 \times (6 - 4)} =$

37. $(2^2 + 3^2) \div 13 =$

38. $(7 - 5)^2 - (4 - 3)^8 =$

39. $\frac{(17 \times 6 - 4 \times 20) \times 10 \div 5}{8 \times 5 + 4} =$

40. $(5 - 3)^4 \div 4 =$

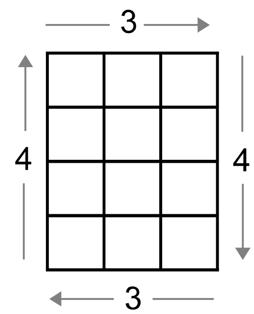
41. $\frac{9 \times 9 + 9}{6 + 36 \div 6 + 3 \times 6} =$

42. $9 \times (3 + 4) \div [(7 + 3) - 3] =$

43. $\frac{2^2 \times (5 + 6)}{25 \div 5 + 3 \times 2} =$

The previous pages gave some practice in the Order of Operations. It is an area that gives many people trouble and requires a lot of practice. Keep at it! Facility with the Order of Operations makes all work in mathematics easier.

To conclude this Unit, consider a bit of geometry. Unit 3 concluded with an irregular rectilinear shape, and you found its area. (“Rectilinear” is just a fancy math term that describes a shape made of lines that meet at right angles.) The perimeter was never mentioned. ***Perimeter*** merely means the distance around the figure. For a simple rectangle like the one below, which is 3 units wide and 4 units long, the perimeter would be 3 (across the top) + 4 (down one side) + 3 (across the bottom) + 4 (up the other side) = 14 units total.



Using “P” for perimeter, “W” for width, and “L” for length, this could be stated as P = W + L + W + L. To make it shorter and easier to write, we could also say P = 2W + 2L (two times the width, plus 2 times the length). We could make it even shorter by using parentheses, like this: P = 2(W + L).

P = 2(W + L) is the ***formula*** for perimeter. We can use it to figure out the perimeter of any rectangle, no matter how big or small! Just change the “W” to the number of units in the width and change the “L” to the number of units in the length. Using the numbers from our rectangle above, we would go from P = 2(W + L) to P = 2(3 + 4). Thus, P (perimeter) = 2 x 7 = 14 units.

At the end of Unit 3, you learned about area, or the number of units *inside* a rectangle. The formula for area is A = WL, or Area = Width x Length. Applied to our rectangle above, A = WL becomes A = 3 x 4 = 12 square units.

44. Using these formulas, find the perimeter and area of the following rectangles:

a. width = 5, length = 7

b. width = 8, length = 15

c. width = 9, length = 4

d. width = 3, length = 48

45. Consider the last rectangle, “d” above. Is there a rectangle with a different width and length, but the same area? Find one.

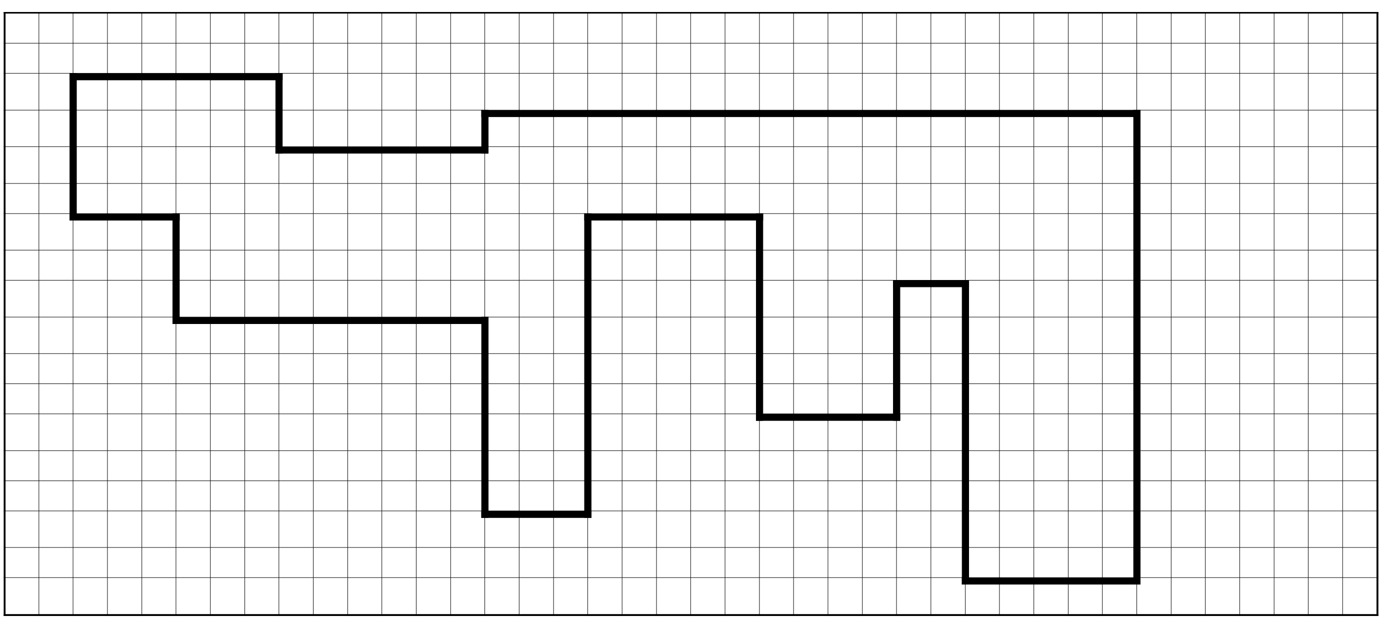
46. In fact, there are eight rectangles with different widths and lengths that have the same area as rectangle “d.” Find all eight and give the perimeter for each rectangle. Show your results in the table on the next page.

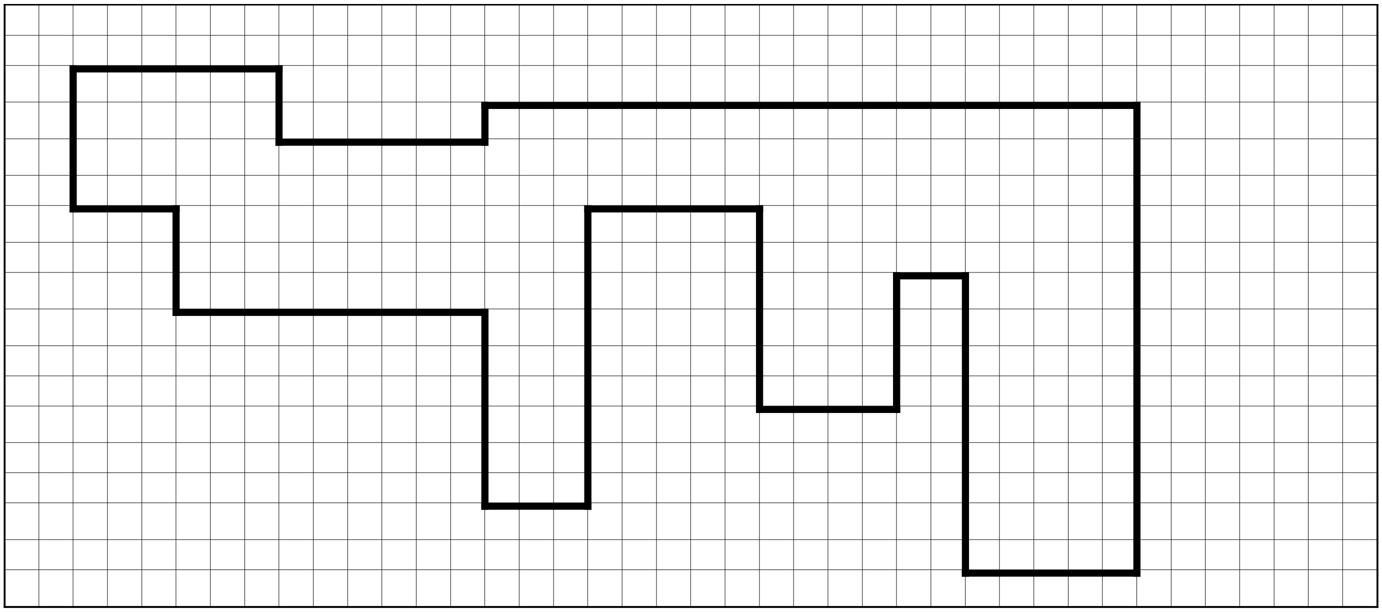
Width	Length	Perimeter

Remember … all the rectangles must have an ***area*** of 144 square units!

47. If you were going to build a rectilinear pen for a herd of sheep, what shape would require the least amount of fence for a given area? Would it be a square, a rectangle, or something like the one below?

The figure from the end of Unit 3 appears below. In Unit 3, you found the area, or the number of square units inside the figure.





48. Find the perimeter of this figure, and show the calculations you made to figure it out.

49. Draw some straight lines inside the figure above so that it becomes a group of 7-10 boxes. Find the area of each box using the A = WL formula. Add up the areas of all the boxes. Does the number match what you got in Unit 3?

50. How would you feel about building a house of this shape? Explain why you feel that way.

Answer Key for Unit 4

Beyond the four basic processes, there are other expressions one needs to understand to gain facility in math. These include ***exponents***. Exponents are numbers written above and to the right of any ordinary number. They express the number of times a number is multiplied by itself.

For example, 5^2 is read "five to the second power" or "five squared." It means 5 x 5. In the expression "5 x 5" we are multiplying two numbers; each number is called a ***factor***. In "5 x 5" then, there are two factors, each of which is a 5. In 5^2, the exponent "2" tells the number of factors of the base "5" that are to be multiplied together.

So, 5^3 means 5 x 5 x 5. This is also called "five cubed." The exponent (in this case 3) tells the number of fives to multiply together. Here are some other examples:

4^2 = 4 x 4 = 16

6^3 = 6 x 6 x 6 = 216

2^4 = 2 x 2 x 2 x 2 = 16

3^5 = 3 x 3 x 3 x 3 x 3 = 243

8^5 = 8 x 8 x 8 x 8 x 8 = 4,096

7^3 = 7 x 7 x 7 = 343

Notice how fast these numbers go up. This is what "increasing exponentially" means.

1. Find the following:

3^4 = 81

5^3 = 125

9^3 = 729

10^4 = 10,000

2. Notice the number of zeroes in 10^4. With that in mind, what is 10^6?

10^6 = 1,000,000 (6 zeros)

The number of zeroes in any power of ten is the same as the exponent to which 10 is raised. This is a basic property of number systems. It may intrigue you to know how other number systems work, but it is not essential. Knowing basic squares and cubes is helpful, so your next task is to complete the tables below.

3. Table of Squares

1^2 =	**1**	**5^2 =**	**25**	**9^2 =**	**81**	**13^2 =**	**169**	**17^2 =**	**289**
2^2 =	**4**	**6^2 =**	**36**	**10^2 =**	**100**	**14^2 =**	**196**	**18^2 =**	**324**
3^2 =	**9**	**7^2 =**	**49**	**11^2 =**	**121**	**15^2 =**	**225**	**19^2 =**	**361**
4^2 =	**16**	**8^2 =**	**64**	**12^2 =**	**144**	**16^2 =**	**256**	**20^2 =**	**400**

4. Table of Cubes

1^3 =	**1**	**2^3 =**	**8**	**3^3 =**	**27**	**4^3 =**	**64**	**5^3 =**	**125**
6^3 =	**216**	**7^3 =**	**343**	**8^3 =**	**512**	**9^3 =**	**729**	**10^3 =**	**1000**

Notice that one to any power is one. And of course, zero to any power is zero. Another set of numbers worth noticing is the powers of 2.

5. Find the following. (The first one, 2^0 has been done for you.)

$2^0 = 1$

$2^1 =$ **2**

$2^2 =$ **4**

$2^3 =$ **8**

$2^4 =$ **16**

$2^5 =$ **32**

$2^6 =$ **64**

$2^7 =$ **128**

$2^8 =$ **256**

With this in mind, here is a famous problem:

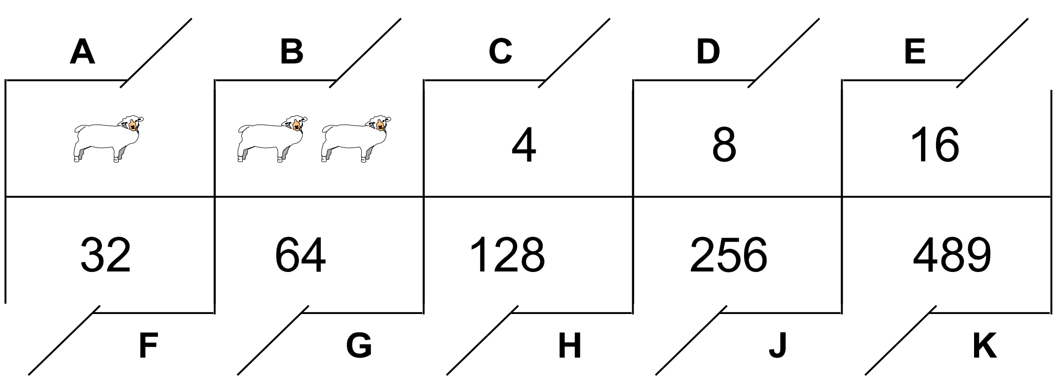
ONE THOUSAND SHEEP

A farmer has a thousand sheep. He wants to build pens and place the sheep in the pens so that anyone can come along and say, “I’d like 23 sheep,” and the farmer can let out the correct number of sheep by opening up exactly the right gates. If, for example, a customer wanted just one sheep, there would have to be a pen with just one sheep in it. Of course, if a customer wanted two sheep, the farmer would need a pen with two sheep in it. Once the farmer opens a gate, all the sheep in that pen will come out; he can’t empty part of a pen.

6. Would the farmer need a separate pen with 3 sheep in it? How about 4 sheep?
A + B = 3   C = 4

7. How would the farmer get 6 sheep?
B + C = 6

8. *Write* the number of sheep (you do NOT have to draw them!) the farmer must put in each pen to enable him to open gates and get out any exact number of sheep from 1 up to 1,000. (Hint: None of these, other than B, will be ***prime numbers***. Primes are whole numbers greater than 1 whose only two factors are 1 and itself.)



9. What gates would the farmer open to get 13 sheep? How many sheep would come out of each pen that he opened?

A gate (1) + C gate (4) + D gate (8) = 13 sheep

10. To get 48 sheep? List the gates he needs to open, and how many sheep will come out of each pen. Now, add up the number of sheep that came out of each opened pen to check your work.

E gate (16) + F gate (32) = 48 sheep

11. To get 73 sheep? List the gates opened, the number of sheep from each of the opened pens, and show the sum of your numbers.

A gate (1) + D gate (8) + G gate (64) = 73 sheep

12. Do the same for 86 sheep.

B gate (2) + C gate (4) + E gate (16) + G gate (64) = 86 sheep

13. Do the same for 127 sheep.

A gate (1) + B gate (2) + C gate (4) + D (8) + E (16) + F (32) + G (64) = 127 sheep

14. Do the same for 259 sheep.

A gate (1) + B gate (2) + J gate (256) = 259 sheep

15. Do the same for 683 sheep.

B gate (2) + G gate (64) + H gate (128) + K gate (489) = 683 sheep

16. How many sheep are in the last pen? (Be careful … the answer isn't especially obvious.)

1000
- 511
489

A + B + C + D + E + F + G + H + J + K = 511
1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 = 511

If you have put the right number of sheep in each pen, you will have written the powers of 2 up to and including 28. But if you wrote the number for 29 in the last pen (K), you are not right.

17. Why would it be incorrect to write the number for 29 in the last pen (K)?

511
+ 512
1023

This problem of the thousand sheep is good for seeing some mathematical thinking. It is not of particular practical value (what farmer ever had exactly 1,000 sheep, and how can you get a sheep to do anything right in the first place?). But math is full of fun questions to play with like this one. Meanwhile, you now have the powers of 2 at hand.

18. Fill in the table below.

2^0 =	**1**	2^1 =	**2**	2^2 =	**4**	2^3 =	**8**	2^4 =	**16**
2^5 =	**32**	2^6 =	**64**	2^7 =	**128**	2^8 =	**256**	2^9 =	**512**
2^{10} =	**1024**								

Knowing the use of powers and exponents enables us to establish the rules for the order of operations. This is very important. The expression 8 x 2 + 4 might mean 20 or 48, depending on the order in which you do the arithmetic. For example, if you start with 8 x 2, you get 16. Add that to 4 and you get 20. However, if you add 2 + 4 first, you get 6. 8 x 6 is 48. Clearly, we need to have some rules about what to do first so that we get the same answer every time.

Fortunately, just such a set of rules has been created and agreed to. They are called the **Order of Operations** and are often referred to as PEMDAS. This is just an abbreviation for the words Parentheses, Exponents, Multiplication/Division, Addition/Subtraction. (Some people find it easier to remember as "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally".)

The Order of Operations says that we have to do calculations in a certain order, every time:

1 . ***P****arentheses*: Make any calculations inside parentheses before doing anything else.
Example: 8 x (2 + 4) … Start inside the parentheses with 2 + 4 to get 6. Then multiply by 8 to get 48.

2. ***E****xponents*: Once the parentheses are done (or if there aren't any) calculate any exponents.
Example: 8 x 3^2 ...Start with 3^2, which is 3 x 3, or 9. Then, multiply by eight to get 72.

3. ***M****ultiplication/****D****ivision*: Once the exponents are done (or if there aren't any) do any multiplication or division, working from left to right.
Example: 8 ÷ 4 x 3 … Since we have both multiplication and division, we have to work from left to right. Starting on the left, then, 8 ÷ 4 = 2. Continuing to the right, multiply by 3 to get 6.

4. ***A****ddition/****S****ubtraction*: Once the multiplying and dividing are done (or if there isn't any) do any addition or subtraction, working left to right.
Example: 9 – 5 + 2 … Since we have both addition and subtraction, we have to work from left to right. Starting on the left, then, 9 – 5 = 4. Continuing to the right, add 2 to get 6.

I know that's a lot to take in, so let's do an example together before you try some on your own. **You fill in the blanks.**

$$6 \times (12 - 9)^2 - 8 \div 4 + 5 = ?$$

Step 1. ***P****arentheses first*. (12 – 9) = _____. Now, we have 6 x 3^2 – 8 ÷ 4 + 5 = ?

Step 2. ***E****xponents second*. 3^2 = ______. Now, we have 6 x 9 – 8 ÷ 4 + 5 = ?

Step 3. ***M****ultiplication and* ***D****ivision third*. Reading our equation left to right, multiplication comes first, and then division. 6 x 9 = ___ and 8 ÷ 4 = ___. Now, we are left with 54 – 2 + 5 = ?

Step 4. ***A****ddition and* ***S****ubtraction last*. Reading our equation left to right, subtraction comes first, and then addition. 54 – 2 = ____. Now, we are left with 52 + 5. So, the final answer is _____.

19.7 + 6 x 2 = 19

20. 47 + 5 x 0 = 47

21. 4 x (10 ÷ 5) = 8

22. 8 + (32 – 24) ÷ 4 = 10

23. 8 x (9 – 7) ÷ 4 + 2 = 6

24. (8 – 3)^2 x (7 – 3) = 100

25. 5 + 3^2 x (9 – 4) = 50

26. 8 + 43 – 8 x 2 – 42 – 6 = -13

Try a few more with exponents. Remember "P-E-MD-AS". Do the work inside the parentheses first, and then calculate the exponents.

27. 3 x 5^2 = 75

28. (3 x $5)^2$ = 225

29. 3^2 x 5 = 45

30. 3^2 + 5^2 = 34

31. (3 + $5)^2$ = 64

32. (3^2 + 5) = 14

33. (3^2 + $5)^2$ = 196

34. (3 x $5)^3$ = 3,375

Notice how adding parentheses changes the answer. For example, the equations in questions 27 and 28 use the same base numbers and exponents (3 and 5^2), but the answers are very different because parentheses were added. The same principle can be seen at work in questions 30 and 33.

35. For each equation below, add in parentheses to make the statement true.

a. (24 ÷ 6 + 6) x 3 – 3 = 27

b. (24 ÷ 6) + 6 x (3 – 3) = 4
or just: **24 ÷ 6 + 6 x (3 – 3) = 4**

c. (24 ÷ 6 + 6) x (3 – 3) = 0

d. (24 ÷ 6) + (6 x 3) – 3 = 19
or just: **24 ÷ 6 + 6 x 3 – 3 = 19**

e. 24 ÷ (6 + 6) x 3 – 3 = 3

f. (72 x 2 ÷ 6) – (3 x 2) + (8 ÷ 4) = 20
or just: **72 x 2 ÷ 6 – 3 x 2 + 8 ÷ 4 = 20**

g. (72 x 2 ÷ 6) – (3 x 2 + 8 ÷ 4) = 16

h. [(72 x 2) ÷ (6 – 3) x 2 + 8] ÷ 4 = 26

i. (6 ÷ 2 x 3) + (7 x 8) – 2 = 55
or just: **6 ÷ (2 x 3) + 7 x 8 – 2 = 55**

j. (6 ÷ 2) x (3 + 7) x (8 – 2) = 180
or just: **6 ÷ 2 x (3 + 7) x (8 – 2) = 180**

That was challenging! The next few will be a little easier. Figure out the ***numerators*** (the part above the dividing line), then the ***denominators*** (the part below the line), and then do the division. For example, given: $\frac{8 \times 9}{3 \times 4}$

Start with the top part (the numerator): 8 x 9 = 72 …

Then do the bottom part (the denominator): 3 x 4 = 12 …

Then divide. $\frac{72}{12}$ is the same as 72 ÷ 12, which is equal to 6

Remember to follow P-E-MD-AS very carefully! You should get "easy" answers (no decimals or fractions).

36. $\frac{(36 \div 18 + 6 \times 2)}{7 \times (6 - 4)} = \frac{14}{14} = 1$

37. $(2^2 + 3^2) \div 13 = 1$

38. $(7 - 5)^2 - (4 - 3)^8 = 3$

39. $\frac{(17 \times 6 - 4 \times 20) \times 10 \div 5}{8 \times 5 + 4} = \frac{44}{44} = 1$

40. $(5 - 3)^4 \div 4 = 4$

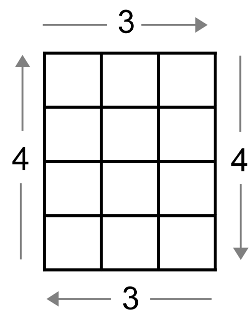
41. $\frac{9 \times 9 + 9}{6 + 36 \div 6 + 3 \times 6} = 3$

42. $9 \times (3 + 4) \div [(7 + 3) - 3] = 9$

43. $\frac{2^2 \times (5 + 6)}{25 \div 5 + 3 \times 2} = \frac{44}{11} = 4$

These pages give some practice in the Order of Operations. It is an area that gives many people trouble and requires a lot of practice. Keep at it! Facility with the order of operations makes all work in mathematics easier.

To conclude this Unit, consider a bit of geometry. Unit 3 concluded with an irregular rectilinear shape, and you found its area. ("Rectilinear" is just a fancy math term that describes a shape made of lines that meet at right angles.) The perimeter was never mentioned. ***Perimeter*** merely means the distance around the figure. For a simple rectangle like the one below, which is 3 units wide and 4 units long, the perimeter would be 3 (across the top) + 4 (down one side) + 3 (across the bottom) + 4 (up the other side) = 14 units total.



Using "P" for perimeter, "W" for width, and "L" for length, this could be stated as P = W + L + W + L. To make it shorter and easier to write, we could also say P = 2W + 2L (two times the width, plus 2 times the length). We could make it even shorter by using parentheses, like this: P = 2(W + L).

P = 2(W + L) is the ***formula*** for perimeter. We can use it to figure out the perimeter of any rectangle, no matter how big or small! Just change the "W" to the number of units in the width and change the "L" to the number of units in the length. Using the numbers from our rectangle above, we would go from P = 2(W + L) to P = 2(3 + 4). Thus, P (perimeter) = 2 x 7 = 14 units.

At the end of Unit 3, you learned about area, or the number of units *inside* a rectangle. The formula for area is A = WL, or Area = Width x Length. Applied to our rectangle above, A = WL becomes A = 3 x 4 = 12 square units.

44. Using these formulas, find the perimeter and area of the following rectangles:

a. width = 5, length = 7 **P = 2(5 + 7) = 24**
A = 5 x 7 = 35

b. width = 8, length = 15 **P = 2(8 + 15) = 46**
A = 8 x 15 = 120

c. width = 9, length = 4 **P = 2(9 + 4) = 26**
A = 9 x 4 = 36

d. width = 3, length = 48 **P = 2(3 + 48) = 102**
A = 3 x 48 = 144

45. Consider the last rectangle, "d" above. Is there a rectangle with a different width and length, but the same area? Find one.

A = 4 x 36 = 144

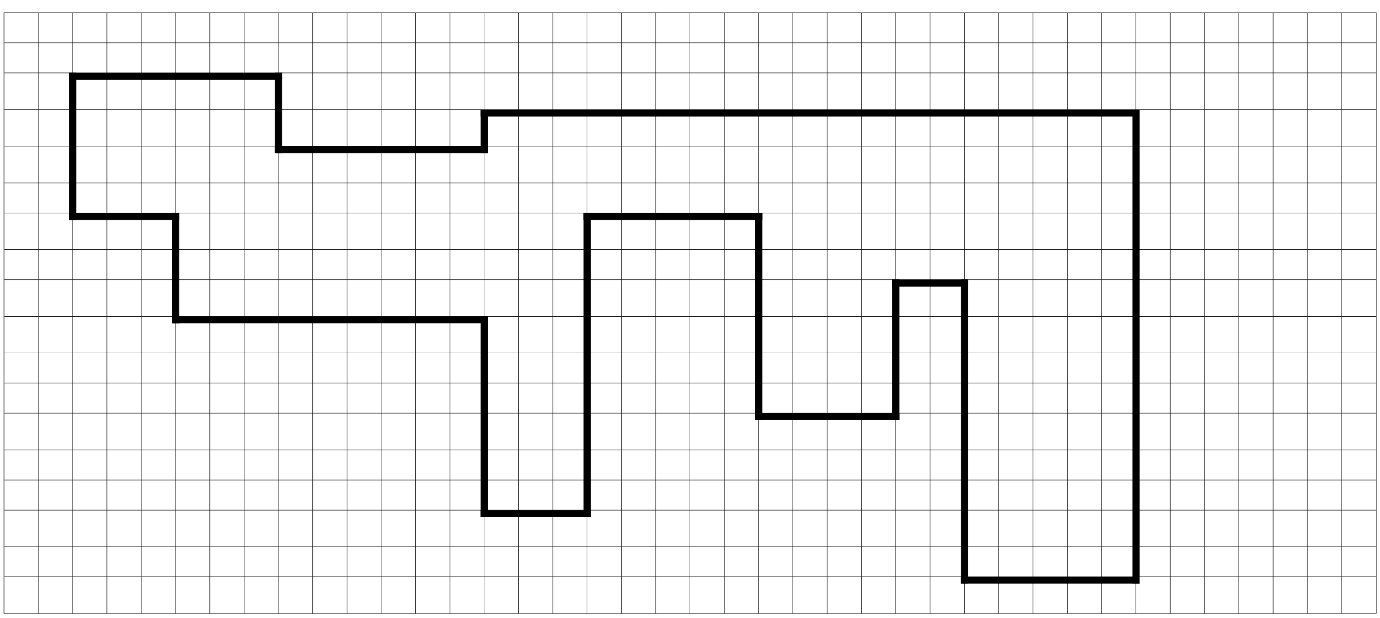
46. In fact, there are eight rectangles with different widths and lengths that have the same area as rectangle "d." Find all eight and give the perimeter for each rectangle. Show your results in the table on the next page.

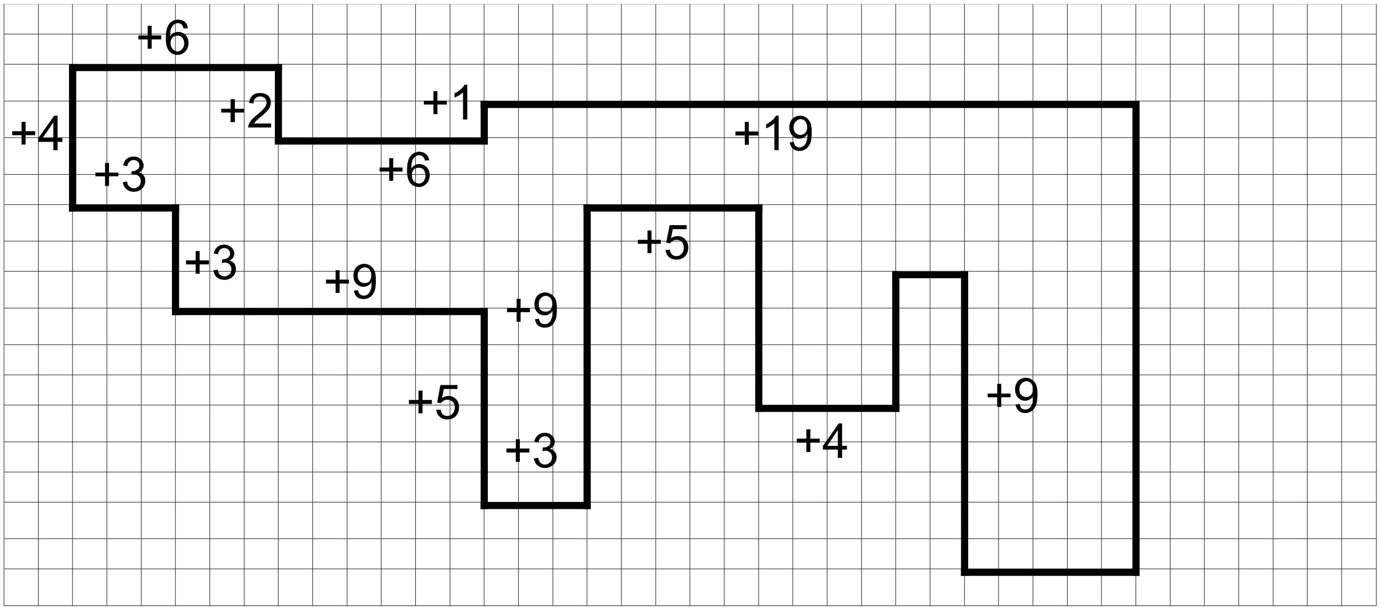
Width	Length	Perimeter
1	144	290
2	72	148
3	48	102
4	36	80
6	24	60
8	18	52
9	16	50

47. If you were going to build a rectilinear pen for a herd of sheep, what shape would require the least amount of fence for a given area? Would it be a square, a rectangle, or something like the one below?

A square

The figure from the end of Unit 3 appears below. In Unit 3, you found the area, or the number of square units inside the figure.

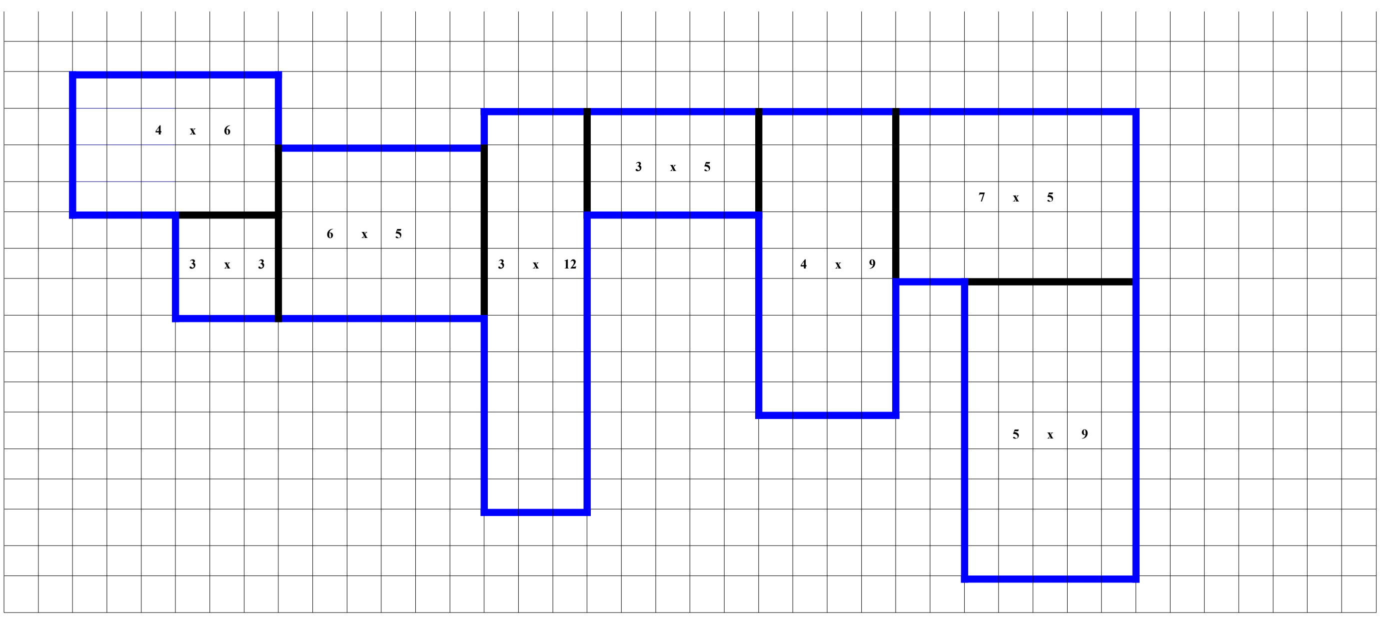




48. Find the perimeter of this figure, and show the calculations you made to figure it out.

4 + 6 + 2 + 6 + 1 + 19 + 14 + 5 + 9 + 2 + 4 + 4 + 6 + 5 + 9 + 3 + 5 + 9 + 3 + 3 = 119

49. Draw some straight lines inside the figure above so that it becomes a group of 7-10 boxes. Find the area of each box using the A = WL formula. Add up the areas of all the boxes. Does the number match what you got in Unit 3? Yes, answers should be the same as Unit 3.



50. How would you feel about building a house of this shape? Explain why you feel that way. Answers will very. For example: It would be a odd house with a good bit of wasted space and a complicated roof line.

Congratulations! This is the last Unit in *The Numbers Game* course!