RATIONAL EXPRESSIONS SOLUTIONS

1. ANSWER:

A. FALSE – The largest exponent on a quadratic function is a 2.
B. FALSE – A polynomial has one unique y-value for every 'x' in its domain. It can only cross the y-axis once.
C. **TRUE** – A function may cross the x-axis 0 times, or up to a maximum of number of times equal to the degree of the function.
D. FALSE – The maximum number of turning points a function may have is equal to it's degree *minus* one.

2. ANSWER: Domain: $\{x \in R\}$ Range: $\{y \geq -6\}$

Domain -> The arrows on the ends of the function indicate that it continues forever in the left and right directions. Meaning that the function is defined for all real numbers.
Range -> The function has a minimum value at y = -6. There are no lower values, but it continues in the upward direction, so may be any value greater than -6.

3. ANSWER: 0

The polynomial has no x-term, only a constant. That means it could have been:

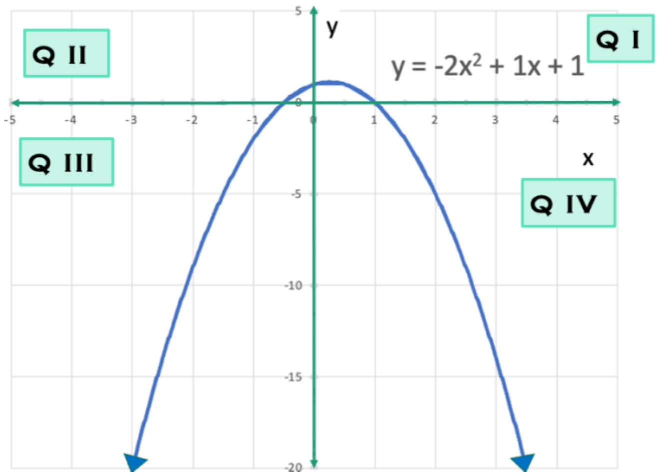
$$y = -2x^0$$

And since $x^0 = 1$, it simplifies to: $y = -2$.

4. ANSWER: More than one solution is possible. An example would be:

$$\boldsymbol{f(x) = -2x^2 + x + 1}$$

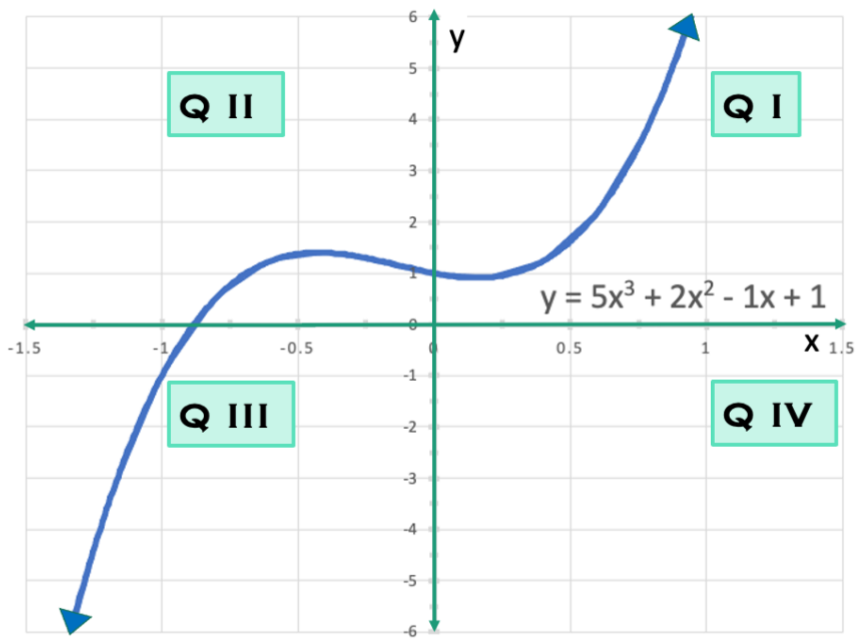
- For there to be 1 turning point, the highest exponent must be a 2.
- For the end behaviour to be from Quadrant III to Quadrant IV, the leading coefficient must be a negative number, so that the quadratic opens down. Our example solution uses a negative 2.
- The following graph is for reference only, not required for solution.



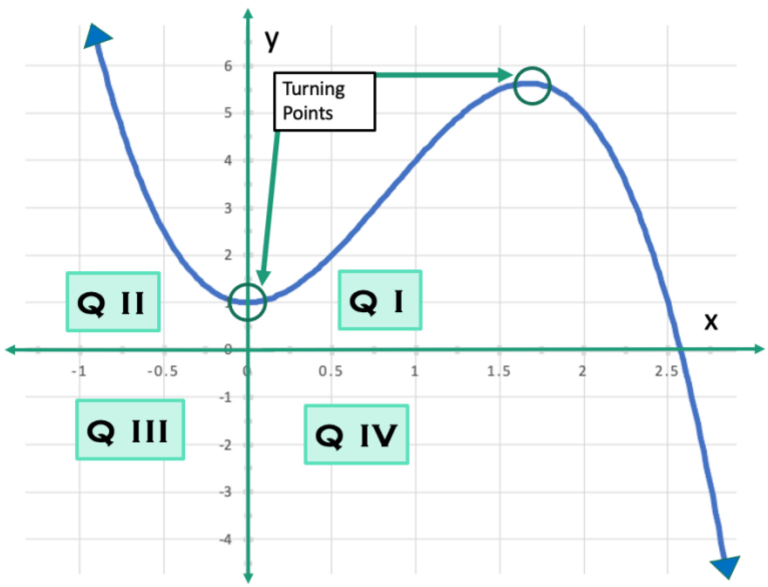
5. ANSWER: More than one solution is possible. An example would be:

$$f(x) = 5x^3 + 2x^2 - x + 1$$

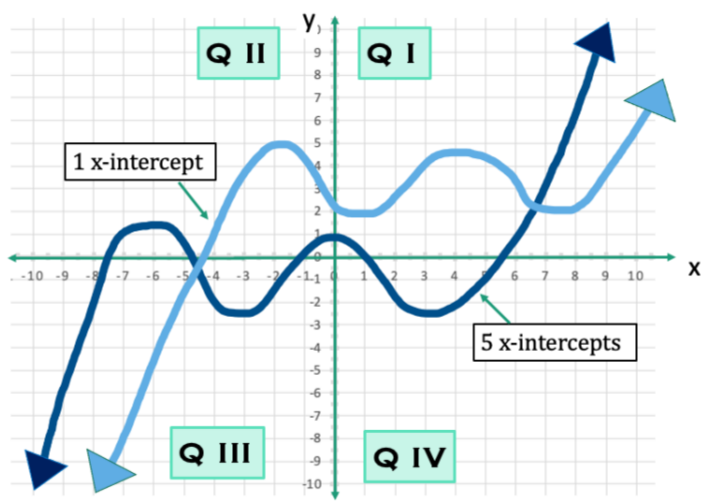
- For there to be 2 turning point, the highest exponent must be a 3.
- For the end behaviour to be from Quadrant III to Quadrant I, the leading coefficient must be a positive number. Our example solution uses a positive 5.
- The following graph is for reference only, not required for solution.



6. **ANSWER:** A rough sketch showing a shape similar to the below would be correct:



7. **ANSWER:** Maximum is 5. Minimum is 1.
The maximum number of x-intercepts a function may have is equal to its degree. Since the degree of the function is an odd number, we know the end behaviour will either go from Q III to Q I or from Q II to Q IV, which means it has to cross the x-axis at least once. The example graph below shows a rough sketch of two 5th degree polynomials. Depending on how where it is located, there may be 5 x-intercepts or 1.



8. ANSWER:

a)	$a(x) = 5x^2 - x + 2$	i, ii and v
b)	$b(x) = -x^3 - x - 5$	iii and iv
c)	$c(x) = -2x + 7$	iii and iv
d)	$d(x) = -3x^5 + 2$	iii and iv
e)	$e(x) = 3x^4 + 2x^2 - 1$	i and ii

9. ANSWER: $g(-5) = -7$

A function is 'ODD' when $-f(x) = f(-x)$, so if $g(x)$ is an odd function, then:

$$-g(5) = g(-5)$$

$$\therefore g(-5) = -7$$

10. **ANSWER:** A is the only statement with a result that is an even function.

A. $f(x)h(x)$: EVEN x EVEN = EVEN function -> **CORRECT**

B. $g(x)h(x) + f(x)$: ODD x EVEN + EVEN = resulting function is neither odd nor even

C. $f(x)g(x)h(x)$: **EVEN x ODD** x EVEN = **ODD** x EVEN = ODD function

D. $f(x) + g(x) + h(x)$: EVEN + ODD + EVEN = resulting function is neither odd nor even