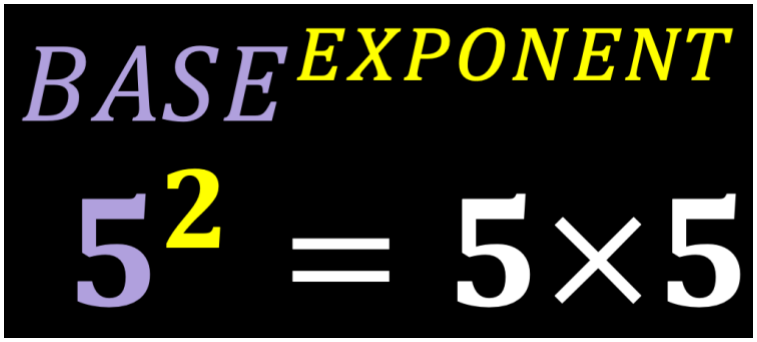
EXPONENTS REVIEW & PRACTICE PROBLEMS



- **An EXPONENT is written on the top right hand side of a BASE number**
- **The exponent tells us how many times to multiply the BASE together**
- **If the exponent is negative, we multiply the base together on the BOTTOM of a fraction,**
    - **Eg.** $\mathbf{5^{-2} = \frac{1}{5\times 5} = \frac{1}{25}}$
- **A number to the power of one is equal to that base number.**
    - **Eg.** $\mathbf{7^1 = 7, or\ y^1 = y}$
- **Any number to the power of zero equals 1.**
    - **Eg.** $\mathbf{4^0 = 1, or\ x^0 = 1}$

Exponent laws:

- Multiplying terms with exponents:

$$\boldsymbol{a^m \times a^n = a^{m+n}}$$

- Dividing terms with exponents:

$$\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$$

- An exponent on terms that are being multiplied (power of a product):

$$\boldsymbol{(ab)^m = a^m b^m}$$

- An exponent on terms that are being divided (power of a fraction):

$$\boldsymbol{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}}$$

- An exponent on a term with an exponent… (power of a power):

$$\boldsymbol{(a^m)^n = a^{m\times n}}$$

- Fractional exponents:

$$\boldsymbol{a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m}$$

<u>Practice Questions:</u>

<u>(Solutions at the end)</u>

Solve:

1. $4^3 =$
2. $(-5)^2 =$
3. $-5^2 =$
4. $7^{-2} =$
5. $2^2 \times 3^4 =$
6. $2^3 \times 4^{-2} =$

Simplify:

Multiplying numbers with exponents that have the <u>same base</u>:

$$a^m \times a^n = a^{m+n}$$

1. $4^3 \times 4^2 =$
2. $7^6 \times 7^4 \times 7 \times 6 =$
3. $9^8 \times 9^{-2} =$
4. $x^3 \times x^2 \times y^7 =$
5. $a^5 \cdot a^{-2} \cdot b \cdot c^{-4} \cdot c =$

Dividing numbers with exponents that have the <u>same base</u>:

$$\frac{a^m}{a^n} = a^{m-n}$$

EXPONENTS REVIEW & PRACTICE PROBLEMS

Simplify:

1. $\frac{13^5}{13^3} =$
2. $\frac{5^{23}}{5^2 6^3} =$
3. $\frac{x^7}{x^4} =$
4. $\frac{5x^3}{5^{-2}x^2} =$
5. $\frac{4^6}{4^3 4^2} =$

An exponent on numbers that are being multiplied (power of a product):

$$(ab)^m = a^m b^m$$

Change the expression by applying the above exponent rule:

1. $(5 \times 3)^6 =$
2. $(a \cdot b \cdot c)^{-2} =$
3. $3^4 \times 5^4 \times x^2 =$
4. $d^3 \cdot e^{-3} \cdot f^3 =$
5. $(3c)^{-2} =$

EXPONENTS REVIEW & PRACTICE PROBLEMS

An exponent on numbers that are being divided (power of a fraction):

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Change the expression by applying the above exponent rule:

1. $\left(\frac{5}{3}\right)^2 =$
2. $\left(\frac{x}{y}\right)^m =$
3. $\frac{5^{15}}{3^{15}7^2} =$
4. $\left(\frac{4}{xy}\right)^7 =$
5. $\frac{3^{-3}}{a^{-3} \cdot b^{-3}} =$

A power of a power…

$$(a^m)^n = a^{m \times n}$$

Simplify:

1. $(5^2)^4 =$
2. $(7^9)^{-2} =$
3. $(x^3)^2 =$
4. $(a^3 \cdot b^2)^4 =$
5. $(3^4 \cdot x^m)^n =$

EXPONENTS REVIEW & PRACTICE PROBLEMS

Fractional exponents:

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

- Remember, when an exponent has a fraction of ***1/n***, it is describing the n^th root of our base.
  - Eg. $\mathbf{25^{\frac{1}{2}} = \sqrt{25}, \textit{or } 27^{\frac{1}{3}} = \sqrt[3]{27} \ldots}$
- When we have an exponent of ***m/n***, that means we want to take the ***n^th root*** of our base and put it to the power of ***m***. Or put Our base to the power of ***m*** and then take the ***n^th root*** of THAT. Our rule says that the order we do them in doesn't matter.

1. True or False: $\sqrt{x^3} = x^{\frac{3}{2}}$
2. Solve: $8^{2/3} =$
3. Write with an exponent as a fraction: $\sqrt[5]{x^7} =$
4. Write with an exponent as a fraction: $\left(\sqrt[3]{27}\right)^2 =$
5. Solve: $\left(\sqrt[3]{15}\right)^3 =$

EXPONENTS REVIEW & PRACTICE PROBLEMS

Altogether now …

Simplify:

1. $7^8 \cdot 7^2 \cdot 7^3 \cdot 7 \cdot 7^{-5} =$
2. $y^{\frac{1}{2}} \cdot y^3 \cdot \frac{z^3}{z^2} =$
3. $(5x^2y)^2 + \frac{6c}{2c^2} =$
4. $(3a^8)^2 + (2a^4)^4 =$
5. $\left(\frac{8x^5y^3}{y}\right)^{-2} =$

Solutions:

Solve:

1. $4^3 = 4 \times 4 \times 4 = 64$
2. $(-5)^2 = (-5) \times (-5) = 25$
3. $-5^2 = -(5 \times 5) = -25$ *Using BEDMAS this is like -1×5^2, where we deal with our EXPONENT first, and then have our negative sign in front. Question 2 is different because the BRACKETS indicate that it is the full number "-5" that we are multiplying together. .

4. $7^{-2} = \frac{1}{7\times7} = \frac{1}{49}$
5. $2^2 \times 3^4 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 324$
6. $2^3 \times 4^{-2} = \frac{2\times2\times2}{4\times4} = \frac{4\times2}{4\times4} = \frac{2}{4} = \frac{1}{2} = 0.5$

Simplify:

Multiplying numbers with exponents that have the <u>same base</u>:

$$\boldsymbol{a^m \times a^n = a^{m+n}}$$

1. $4^3 \times 4^2 = 4^{3+2} = 4^5$
2. $7^6 \times 7^4 \times 7 \times 6 = 7^{6+4+1} \times 6 = 7^{11} \times 6$ *Only the exponents that have MATCHING bases can be added together. When a number has no exponent, it is the same as that number having an exponent of 1.
3. $9^8 \times 9^{-2} = 9^{8+(-2)} = 9^{8-2} = 9^6$
4. $x^3 \times x^2 \times y^7 = x^{3+2} \times y^7 = x^5y^7$
5. $a^5 \cdot a^{-2} \cdot b \cdot c^{-4} \cdot c = a^{5-2} \cdot b \cdot c^{-4+1} = a^3 \cdot b \cdot c^{-3}$

Dividing numbers with exponents that have the <u>same base</u>:

$$\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$$

Simplify:

1. $\frac{13^5}{13^3} = 13^{5-3} = 13^2$

2. $\frac{5^{23}}{5^2 6^3} = \frac{5^{23-2}}{6^3} = \frac{5^{21}}{6^3}$
3. $\frac{x^7}{x^4} = x^{7-4} = x^3$
4. $\frac{5x^3}{5^{-2}x^2} = 5^{1-(-2)} \cdot x^{3-2} = 5^{1+2} \cdot x^1 = 5^3 \cdot x$ *the numbers with the same base can each be combined.
5. $\frac{4^6}{4^3 4^2} = 4^{6-3-2} = 4^1 = 4$

An exponent on numbers that are being multiplied (power of a product):

$$(\boldsymbol{ab})^{\boldsymbol{m}} = \boldsymbol{a}^{\boldsymbol{m}}\boldsymbol{b}^{\boldsymbol{m}}$$

Change the expression by applying the above exponent rule:

1. $(5 \times 3)^6 = 5^6 \times 3^6$
2. $(a \cdot b \cdot c)^{-2} = a^{-2} \cdot b^{-2} \cdot c^{-2}$
3. $3^4 \times 5^4 \times x^2 = (3 \times 5)^4 \times x^2 = 15^4 \cdot x^2$ *Only terms that have the SAME exponent can be combined with the product rule
4. $d^3 \cdot e^{-3} \cdot f^3 = (d \cdot f)^3 \cdot e^{-3}$
5. $(3c)^{-2} = 3^{-2} \times c^{-2}$

An exponent on numbers that are being divided (power of a fraction):

$$\left(\frac{\boldsymbol{a}}{\boldsymbol{b}}\right)^{\boldsymbol{m}} = \frac{\boldsymbol{a}^{\boldsymbol{m}}}{\boldsymbol{b}^{\boldsymbol{m}}}$$

EXPONENTS REVIEW & PRACTICE PROBLEMS

Change the expression by applying the above exponent rule:

1. $\left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2}$
2. $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ *Your exponent may be a variable. Treat it the same way you would treat a number.
3. $\frac{5^{15}}{3^{15}7^2} = \left(\frac{5}{3}\right)^{15} \cdot \frac{1}{7^2}$
4. $\left(\frac{4}{xy}\right)^7 = \frac{4^7}{x^7y^7}$
5. $\frac{3^{-3}}{a^{-3} \cdot b^{-3}} = \left(\frac{3}{a \cdot b}\right)^{-3}$

A power of a power…

$$(\boldsymbol{a^m})^{\boldsymbol{n}} = \boldsymbol{a^{m \times n}}$$

Simplify:

1. $(5^2)^4 = 5^{2\times4} = 5^8$
2. $(7^9)^{-2} = 7^{9\times(-2)} = 7^{-18}$
3. $(x^3)^2 = x^{3\times2} = x^6$
4. $(a^3 \cdot b^2)^4 = a^{3\times4} \cdot b^{2\times4} = a^{12} \cdot b^8$ *If you have more than one term, your outermost exponent will multiply each of your inner exponents.
5. $(3^4 \cdot x^m)^n = 3^{4n} \cdot x^{mn}$

Fractional exponents:

$$\boldsymbol{a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m}$$

1. True or False: $\sqrt{x^3} = x^{\frac{3}{2}}$ **TRUE**
2. Solve: $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$
3. Write with an exponent as a fraction: $\sqrt[5]{x^7} = x^{7/5}$
4. Write with an exponent as a fraction: $\left(\sqrt[3]{27}\right)^2 = 27^{2/3}$
5. Solve: $\left(\sqrt[3]{15}\right)^3 = 15^{3/3} = 15^1 = 15$

Altogether now …

Simplify:

1. $7^8 \cdot 7^2 \cdot 7^3 \cdot 7 \cdot 7^{-5} = 7^{8+2+3+1+(-5)} = 7^9$
2. $y^{\frac{1}{2}} \cdot y^3 \cdot \frac{z^3}{z^2} = y^{\left(\frac{1}{2}\right)\times 3} \cdot z^{3-2} = y^{\frac{3}{2}} \cdot z\ Also\ acceptable\ are\!: \sqrt{y^3} \cdot z\ OR\ \left(\sqrt{y}\right)^3 \cdot z$
3. $(5x^2y)^2 + \frac{6c}{2c^2} = (5^2) \cdot x^{2\times 2} \cdot y^2 + \left(\frac{6}{3}\right) \cdot c^{1-2} = 25x^4y^2 + 2c^{-1}$ *Note that the rules apply to each term in the addition, but because our final terms are still not “like terms” we cannot combine them.
4. $(3a^8)^2 + (2a^4)^4 = 3^2a^{8\times 2} + 2^4a^{4\times 4} = 9a^{16} + 16a^{16} = 25a^{16}$ *We can combine our addition terms because they are “like terms”; our variables have the same base and exponent.

EXPONENTS REVIEW & PRACTICE PROBLEMS

5. $\left(\frac{8x^5y^3}{y}\right)^{-2} = (8 \cdot x^5 \cdot y^{3-1})^{-2} = (8 \cdot x^5 \cdot y^2)^{-2} = 8^{-2} \cdot x^{5\times(-2)} \cdot y^{2\times(-2)} = \frac{x^{-10}y^{-4}}{64}$ *Using BEDMAS, we simplify inside the brackets before we get to the outer exponent.

CONGRATULATIONS! As a reward for making it to the end of this worksheet, here is a cat joke:

What is a cat's favourite cereal?

MICE KRISPIES!