PROBABILITY PART A: Introduction

Probability concepts build on some concepts covered in Set Theory notes. Refer back as needed.

EXPERIMENTAL PROBABILITY:

Refers to outcomes from events that actually occurred. For example, if you flip a coin four times and it lands on heads three times and tails one time, then the experimental probability that it lands on heads is:

$$P(desired\ outcome) = \frac{\#\ of\ times\ desired\ outcome\ occurs}{\#\ of\ experiments\ run}$$

$$= \frac{landed\ on\ heads\ 3\ times}{coin\ was\ flipped\ 4\ times} = \frac{3}{4} = 0.75$$

("Desired outcome" refers to the outcome you care about for answering the question)

The experiment indicates that there is a 75% chance a coin will land on heads when you flip it.

But wait, does that seem right to you? When you flip a coin you likely think there is a 50% chance it will land on either side. You're already thinking of the theoretical probability!

THEORETICAL PROBABILITY:

Refers to the outcome we would expect if we were to do the experiments. For example, when you flip a coin you expect there to be a 50% chance it lands on either side.

$$P(desired\ outcome) = \frac{\#\ of\ desired\ outcomes}{total\ \#\ of\ possible\ outcomes}$$

If we assume a coin will be flipped four times, we can use theoretical probability to **PREDICT** how many times it will land on heads. The theoretical probability of it landing on heads is 50%, so we can fill in what we know in the equation and solve for the predicted number of heads!

$$P(heads) = \frac{\#\ of\ heads}{\#\ of\ times\ coin\ will\ be\ tossed}$$

$$0.50 = \frac{\#\ of heads}{4}$$

$$0.50\ \times\ 4 = \#\ of\ heads = 2$$

We would **predict** that if you flip a coin four times, it will land on heads 2 times.

<u>EXPERIMENTAL VS. THEORETICAL</u>

Does it seem strange that in this example our theoretical probability was different than our experimental? Why does theoretical probability matter if it doesn't match what actually happened?

If you were to continue the experiment and flip more coins, the number of times it lands on heads would become closer and closer to the number of times it lands on tails.

The more times you conduct an experiment, the closer your outcomes will match the theoretical probability. If you kept flipping coins an infinite number of times, eventually 50% of the outcomes would be heads.

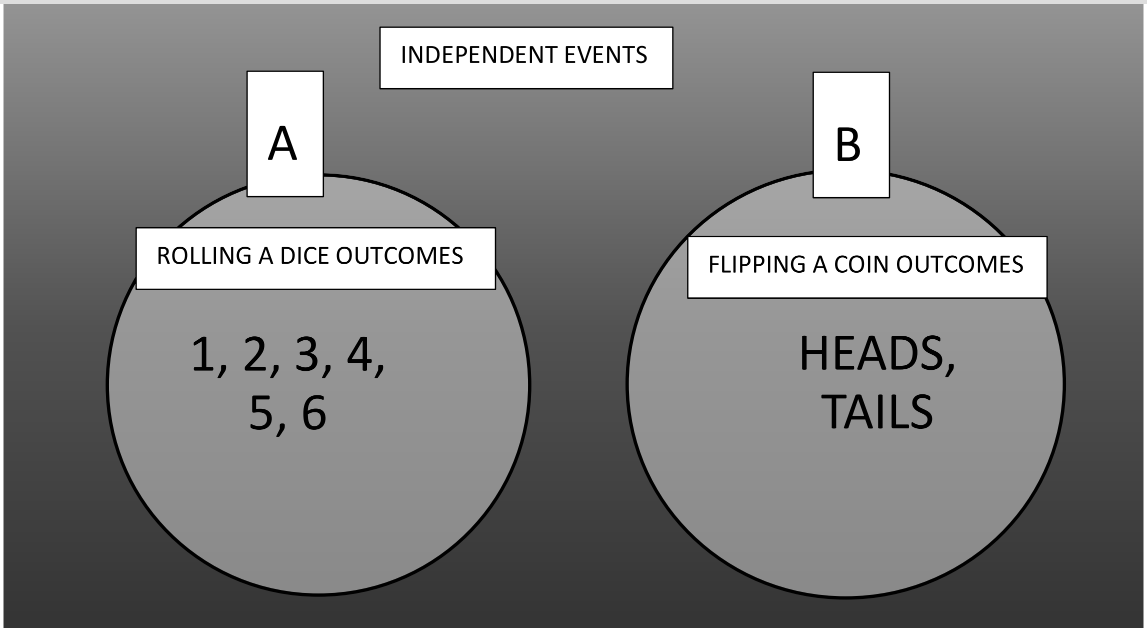
That is why theoretical probability is important. It can predict the ***most likely*** outcome of an experiment.

TERM	MEANING
EXPERIMENTAL PROBABILITY	Refers to outcomes from events that actually occurred. For example, if you flip a coin 4 times and it lands on heads 3 times and tails 1 time, then the experimental probability that it lands on heads is 3/4. $P(outcome) = \frac{\#\ of\ times\ outcome\ occurred}{\#\ of\ times\ experiment\ was\ tested}$
THEORETICAL PROBABILITY	Refers to the outcomes that we would expect from events without them actually occurring. For example, if you flip a coin 4 times, you would expect it to land on heads half the time. $P(desired\ outcome) = \frac{\#\ of\ desired\ outcomes}{total\ \#\ of\ possible\ outcomes}$

Why study probability? Probability methods are the closest you will get to being able to predict the future using math. It will also make you a better gambler. Disclaimer: I do not recommend counting cards at a casino.

Before we get to the fun stuff, like code breaking safes and understanding Vegas odds, there's some slightly less exciting things you will need to understand.

Some events are described as **INDEPENDENT** of one another. This means that one thing happening does not impact the likelihood of the other thing happening. For example, if you flip a coin and then roll a dice, the outcome of your coin flip does not impact your dice roll.

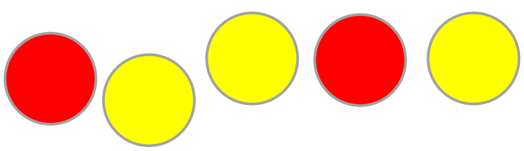


If events are **DEPENDENT**, then the outcome of one event will influence the outcome of the next event.

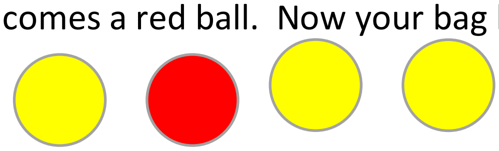
EXAMPLE 1

There is a bag with 5 balls in it. Three are yellow and two are red.

The ‘events’ of drawing one ball and then drawing a second ball are dependent.

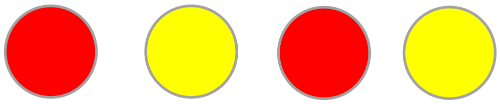


The probability of drawing a red ball **first** is 2/5. Let’s say you reach in and out comes a red ball. Now your bag looks like this:



The probability of **drawing a red ball second** is now 1/4.

If you had drawn a yellow ball first, then when you went to draw a second ball, your bag would have looked like this:



Now, the probability of **drawing a red ball second is** 2/4. The probability of drawing a red ball second changed; it depends on the outcome of what ball you drew first. These two events are **DEPENDENT.**

The **FUNDAMENTAL COUNTING PRINCIPLE** can be used when you have multiple **INDEPENDENT** events. It tells you how many different outcomes are possible, considering all of the events. When you hear the word 'and' connecting different events, you will likely be able to apply this principle.

The number of outcomes = (# Event 1's possible outcomes x # of Event 2's possible outcomes x # Event 3's possible outcomes….etc….)

<u>EXAMPLE 2</u>

You roll a dice and flip a coin. What is the probability of you rolling a 3 and the coin landing on heads?

Solution: Rolling a dice and flipping a coin are two different events with independent outcomes. This means we can use the **Fundamental Counting Principle** to determine the total number of possible outcomes. See the Venn Diagram previously shown.

Event A:	Event B:
6 possible outcomes	2 possible outcomes

Total # of possible outcomes = 6 x 2 = 12

Rolling a 3 and flipping a heads is **one** possible outcome of the experiment. The probability is:

$$P(three\ \&\ heads) = \frac{\#\ of\ desired\ outcomes}{total\ \#\ of\ possible\ outcomes} = \frac{1}{12}$$

P(three & heads) = 1/12 or 8.33%

USEUFL STUFF:

As different problem types come up, there are a couple things that will be useful to remember to help you solve them:

CARDS:

- There are 52 cards in a deck. These are divided into 4 groups – called suits
- 2 of the suits are red (hearts and diamonds) and 2 of the suits are black (spades and clubs)
- Each suit has 13 cards (52/4 suits = 13 cards)
- Each suit has 3 face cards (King, Queen, Jack)
- Knowing this you will be able to figure out possible outcomes for probability questions
- EG. a) How many face cards in a deck of cards? (12) How many red 6's? (2) b) If you have only red cards shuffled together, what is the probability of drawing a red king? (26 red cards, 2 red kings, P = 2/26 = 1/13 or 7.69%)

LETTERS: There are 26 letters. 5 vowels (A, E I, O, U) and 21 consonants (Y is a consonant unless the question tells you otherwise)

DIGITS: There are 10 different single digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

DICE:

- Unless the dice is described as being different in the problem statement, a typical dice has **6 sides**, numbered 1 to 6.
- If you roll two dice, each dice roll is **independent**

Four common problem solving strategies are to use **outcome tables**, **decision trees**, **adjacent lines** where you fill in the blanks, and **Venn Diagrams**. When there are a large number of possible outcomes it could become time consuming or impractical to be writing all possible combinations in a table or decision tree – but all of these problem solving strategies are important tools in your toolbelt that will help you understand the problem.

EXAMPLE 3

a) You roll 2 dice. What is the probability of rolling a 6 and a 3?
b) You roll 2 dice. What is the probability of rolling a total less than 5?

a)

DICE	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

The table above shows all the possible outcomes of rolling two dice. One dice is shown in red and the other in black. There are 2 possible outcomes where one dice is a 6 and one dice is a 3 out of a possible 36 total different outcomes.

$$P(rolling\ 3\ and\ 6) = \frac{2}{36} = \frac{1}{18} = 5.56\%$$

b) A table can once again be used. The outcomes with sums less than 5 are highlighted.

DICE	1	2	3	4	5	6
1	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7
2	2 + 1 = 3	2 + 2 = 4	2 + 3 = 5	2 + 4 = 6	2 + 5 = 7	2 + 6 = 8
3	3 + 1 = 4	3 + 2 = 5	3 + 3 = 6	3 + 4 = 7	3 + 5 = 8	3 + 6 = 9
4	4 + 1 = 5	4 + 2 = 6	4 + 3 = 7	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10
5	5 + 1 = 6	5 + 2 = 7	5 + 3 = 8	5 + 4 = 9	5 + 5 = 10	5 + 6 = 11
6	6 + 1 = 7	6 + 2 = 8	6 + 3 = 9	6 + 4 = 10	6 + 5 = 11	6 + 6 = 12

$$P(rolling\ total\ less\ than\ 5) = \frac{6}{36} = \frac{1}{6} = 16.67\%$$

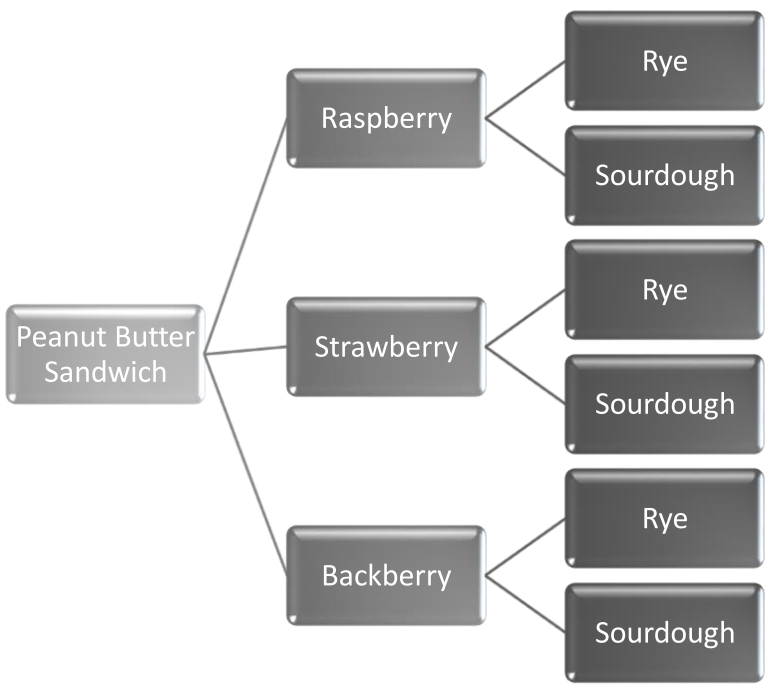
When solving problems with multiple independent events, the Fundamental Counting Principle can be applied using decision trees or adjacent lines.

EXAMPLE 4

Susan likes to have a peanut butter sandwich evert day. To keep things interesting, each day she chooses between rye and sourdough bread. She also chooses between raspberry, strawberry and blackberry jam.

a) How many different sandwiches could she make?
b) If she is equally as likely to pick any type of sandwich, what is the probability she will make a sandwich on rye bread with strawberry jam?

a) A decision tree, outcome table and horizontal lines can all be used to show how many different sandwiches she could make.



A decision tree shows each decision that she makes and all the possible outcomes in making her sandwich. She can either choose her jam or her bread first – the order doesn't matter in this problem. If she chooses her jam first, then after choosing her jam she must choose her bread. At the end, you can follow the path from her final choice back to the trunk of the tree to read out the sandwich that she made. The number of different sandwiches she could make is equal to the number of branches at the end – 6.

An outcome table could also be used to show all 6 sandwiches:

	Raspberry	**Strawberry**	**Blackberry**
Rye	rye with raspberry jam	rye with strawberry jam	rye with blackberry jam
Sourdough	sourdough with raspberry jam	sourdough with strawberry jam	sourdough with blackberry jam

One more way to solve this problem is with the Fundamental Counting Principle and "filling in the blanks".

Remember, the Fundamental Counting Principle says that:

The number of outcomes = (# Event 1's possible outcomes x # of Event 2's possible outcomes x # Event 3's possible outcomes….etc….)

Think of each event as a decision or an action and draw one line for each, i.e. one for the choice of jam and one for the choice of bread:

_____ x _____

Then fill in the blanks with the number of choices/outcomes available for each:

_3_ x _2_ = 6 possible sandwiches.

We used three different strategies to come to the same answer. So, at the beginning of a problem how do you know which strategy to use? It takes practice, but think about which one will help you best visualize the problem and how long it will take. For example, making a table may take a long time, but can sometimes be worth the work – especially if you have a problem with multiple parts.

b) There are 6 types of sandwich, so the probability she makes one specific type:

$$P(rye\ with\ strawberry) = \frac{\#\ of\ desired\ outcomes}{total\ \#\ of\ possible\ outcomes} = \frac{1}{6} = 16.67\%$$

TERM	MEANING
INDEPENDENT EVENTS	The outcome of one event does not impact the outcome of another event occurring.
DEPENDENT EVENTS	The outcome of one event will influence the probability of the second event occurring. Eg. if you have a bag of Skittles and you blindly pick out 5 green ones in a row, then there are fewer green ones left in the bag. The likelihood of drawing another green one is less than when the bag was full.
FUNDAMENTAL COUNTING PRINCIPLE	Used to determine how many outcomes are possible when multiple independent events occur. Eg. If event ‘A’ has 6 outcomes, and event ‘B’ has 2 outcomes, then there are 6x2 possible outcomes. (# of outcomes = # of A’s outcomes x # B’s outcomes)