RATIONAL EXPRESSIONS SOLUTIONS

1. **ANSWER:**

$$x^2 - 2x - 35 \neq 0$$

$$(x - 7)(x + 5) \neq 0$$

$$\boldsymbol{x \neq 7 \; and \; x \neq -5}$$

2. **ANSWER:**

$1 + 2x^2 \neq 0$, but since any number 'x' will be positive or zero when it is squared, there are no values of 'x' for which the denominator would equal zero.

There are no non-permissible values for this expression.

3. **ANSWER:**

a) Distance [meters] = Speed [meters / second] x Time [seconds]

Her typical speed:

$$18\ m = Speed \times t$$

$$S = \frac{18\ m}{t}$$

When she hears the treats, it takes her 4 less seconds to get to the kitchen:

$$18\ m = Speed' \times (t - 4)$$

$$S' = \frac{18\ m}{t - 4}$$

b) $S = \frac{18\ m}{t}, t \neq 0$. Also t > 0, as we cannot have negative time.
$S = \frac{18\ m}{t-4}, t \neq 4$. Also t > 4, as we cannot have negative time.

The denominators cannot be zero because that would mean that she travelled some distance in zero amount of time, which is impossible.

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4. **ANSWER:**

$$Non-permissible\ values: x^2(3x^2-27) \neq 0$$
$$3x^2 \neq 27$$
$$x^2 \neq 9$$
$$Thus: x \neq 0, 3, or -3$$

*Remember that when you take the square root of a number the answer can be either positive or negative.

$$GCF: \frac{3x^2 \times 3}{3x^2(x^2-9)}$$
$$Divide\ by\ GCF: \frac{3}{x^2-9}$$

5. **ANSWER:**

Start by factoring:

$$\frac{3x+3}{15x^2-5x} \times \frac{3x-1}{x^2-1} = \frac{3(x+1)}{5x(3x-1)} \times \frac{3x-1}{(x+1)(x+1)}$$

State non-permissible values:

$$5x \neq 0,\ therefore\ x\ \neq 0$$

$$3x-1 \neq 0, therefore\ x\ \neq \frac{1}{3}$$

$$(x+1) \neq 0\ and\ (x-1) \neq 0, therefore\ x\ \neq \pm 1$$

Cancel the matching terms: $\frac{3(x+1)}{5x(3x-1)} \times \frac{3x-1}{(x+1)(x+1)}$

$$= \frac{3}{5x(x+1)}$$

$$= \frac{\mathbf{3}}{\mathbf{5x^2+5x}}, \mathbf{x \neq 0, \frac{1}{3}, 1, -1}$$

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6. ANSWER:

Factor:

$$\frac{4x^2+4xy}{2}\times\frac{x-y}{(x^2-y^2)}=\frac{4x(x+y)}{2}\times\frac{(x-y)}{(x-y)(x+y)}$$

Determine Non-permissible values for 'x':

$$(x-y)\neq 0, therefore\ x\ \neq y$$
$$(x+y)\neq 0, therefore\ x\ \neq -y$$

Cancel the matching terms: $\frac{4x(x+y)}{2}\times\frac{(x-y)}{(x-y)(x+y)}$

$$=\frac{4x}{2}$$
$$=2x, x\neq \pm y$$

7. ANSWER:

Factor: $\frac{7(x+1)}{(x-5)}\div\frac{5}{(x-5)(x+7)}$

NPV: $x\neq 5, x\neq -7$

Flip: $\frac{7(x+1)}{(x-5)}\times\frac{(x-5)(x+7)}{5}$

Check for new NPV: Bottom of the fraction is now 5. This does not introduce any new NPV for *x*.

Multiply and cancel the matching terms:

$$=\frac{7(x+1)}{(x-5)}\times\frac{(x-5)(x+7)}{5}$$

$$=\frac{7(x+1)(x+7)}{5}$$

$$=\frac{7x^2+56x+49}{5}, x\neq 5, -7$$

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8. ANSWER:

Factor: $\frac{9x^2+18x+9}{x+1} \div \frac{3x}{x-6} = \frac{3(x+1)(x+1)}{(x+1)} \div \frac{3x}{(x-6)}$

NPV: $(x+1) \neq 0 \; and \; (x-6) \neq 0$, so $x \neq -1, 6$

Flip: $= \frac{(3x+3)(3x+3)}{(x+1)} \times \frac{(x-6)}{3x}$

Check for new NPV: Bottom of the fraction is now $3x$.

$$3x \neq 0, therefore \; x \neq 0$$

Multiply and cancel the matching terms:

$$= \frac{3(x+1)(x+1)}{(x+1)} \times \frac{(x-6)}{3x}$$

$$= (x+1) \times \frac{(x-6)}{x}$$

$$= \frac{x^2-5x-6}{x}, x \neq -1, 0, 6$$

9. ANSWER:

$$\frac{3x-1}{x^2+6x} + \frac{1}{x} = \frac{3x-1}{x(x+6)} + \frac{1}{x}$$

NPV: $x \neq 0, -6$

$$\frac{3x-1}{x(x+6)} + \frac{1}{x} \times \frac{(x+6)}{(x+6)}$$

$$\frac{3x-1}{x(x+6)} + \frac{(x+6)}{x(x+6)} = \frac{3x-1+x+6}{x(x+6)}$$

$$\frac{4x+5}{x(x+6)}, x \neq -6, 0$$

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10. ANSWER:

$$\frac{3z}{5z+15}+\frac{z}{z+3}=\frac{3z}{5(z+3)}+\frac{z}{z+3}$$

NPV: $z \neq -3$. Then make the bottoms match by multiplying by 1:

$$\frac{3z}{5(z+3)}+\frac{z}{z+3}\times\frac{5}{5}=\frac{3z}{5(z+3)}+\frac{5z}{5(z+3)}$$

$$\frac{8z}{5(z+3)}, z \neq -3$$

11. ANSWER:

$$\frac{x}{x^2-9}-\frac{3}{x+3}$$

Factor if needed:

$$\frac{x}{(x+3)(x-3)}-\frac{3}{x+3}$$

NPV: $x \neq 3, -3$ Then make the bottoms match by multiplying by 1:

$$=\frac{x}{(x+3)(x-3)}-\frac{(x-3)}{(x-3)}\times\frac{3}{x+3}$$

$$=\frac{x-(3x-9)}{(x+3)(x-3)}$$

$$=\frac{-2x+9}{(x+3)(x-3)}, x \neq 3, -3$$

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12. ANSWER:

$$\frac{3x+2}{x-1}-\frac{2x-2}{2x-1}$$

NPV: $x \neq 1, \frac{1}{2}$. Then make the bottoms match by multiplying by LCD:

$$=\frac{3x+2}{x-1}\times\frac{(2x-1)}{(2x-1)}-\frac{2x-2}{2x-1}\times\frac{(x-1)}{(x-1)}$$

$$=\frac{(3x+2)(2x-1)-(2x-2)(x-1)}{(x-1)(2x-1)}$$

$$=\frac{(6x^2-3x+4x-2)-(2x^2-2x-2x+2)}{(x-1)(2x-1)}$$

$$=\frac{(6x^2+x-2)-(2x^2-4x+2)}{(x-1)(2x-1)}$$

$$=\frac{6x^2+x-2-2x^2+4x-2}{(x-1)(2x-1)}$$

$$=\frac{4x^2+5x-4}{(x-1)(2x-1)}, x \neq 1, \frac{1}{2}$$

13. ANSWER:

$$\frac{2}{4-m}=3$$

NPV: $m \neq 4$. Then make the bottoms match by multiplying by LCD:

$$\frac{2}{4-m}\times(4-m)=3\times(4-m)$$

Simplify and solve:

$$2=12-3m$$

$$3m=10$$

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$$m = \frac{10}{3}$$

Check: $m \neq NPV$ → OK. And

$$\frac{2}{4 - \left(\frac{10}{3}\right)} = \frac{2}{\frac{12}{3} - \frac{10}{3}} = \frac{2}{\frac{2}{3}} = 3$$

Therefore, m = 10/3 is a valid solution.

14. ANSWER:

$$\frac{5x}{10 - 5x} + \frac{3}{x - 2} = 2$$

Factor:

$$\frac{5x}{-5(x - 2)} + \frac{3}{x - 2} = 2$$

NPV: $x \neq 2$

Now note we can cancel the two 5's in the first expression which leaves us with:

$$\frac{-x}{x - 2} + \frac{3}{x - 2} = 2$$

And now we can see that the two denominators are the same, so we can combine these expressions!

$$\frac{3 - x}{x - 2} = 2$$

This looks much nicer to use for our next steps.

$$\frac{3 - x}{x - 2} \times (x - 2) = 2 \times (x - 2)$$

$$3 - x = 2x - 4$$
$$3x - 7 = 0$$
$$x = \frac{7}{3}$$

Check: $x \neq NPV$ → OK. And

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$$\frac{5\left(\frac{7}{3}\right)}{10-5\left(\frac{7}{3}\right)}+\frac{3}{\left(\frac{7}{3}\right)-2}=-7+9=2$$

Therefore, x = 7/3 is a valid solution.