SET THEORY NOTES

READY… SET…. GO!

TERM	MEANING
SET THEORY	Complicated sounding way of using math to describe groups of things and numbers
SET THEORY NOTATION	The math version of emojis, #mathspeak, ROFL, SMH, WTF…etc, that are used to describe the groups of things/numbers
UNIVERSAL SET -→ U	All the things related to the groups you're looking at. Ie, if you study dogs and you're investigating what dogs are fluffy and what dogs drool a lot… your universal set has all the dogs that are fluffy, drool a lot, plus all the other non-fluffy, non-drooling dogs. A cat would not be in your universal set.
ELEMENT -→ ∈, ∉	A distinct object within a set. May be a single item or one set may be an element within another set.
EMPTY SET (AKA NULL SET) → ϕ	A special type of set – the empty set has no elements

NOTATION:

Set name | Elements of the set

$$S = \{1, 4, 7, 10\}$$

(A set named S has the elements 1, 4, 7 and 10 in it)

$4 \in S$ 4 is an <u>element</u> of the set, S

$6 \notin S$ 6 is NOT an <u>element</u> of the set, S

EMPTY SET: $\phi = \{\ \ \}$

SET THEORY	NOTES
SUBSET → ⊆	A set that contains some or all of the elements of another set (similar to "less than or equal to" symbol, ≤)
PROPER SUBSET→ ⊂	A set that contains some of the elements of another set (similar to "less than" symbol, <)

Remember the empty set has no elements in it → this means that every set contains the empty set. The empty set is a <u>subset</u> of every set.

S = {1, 4, 7, 10, {9}}

A = {1, 4, 10} and **B = {1, 4, 7, 10, {9}}**

A and B are both subsets of S, but only A is a proper subset of S. B has all the same elements, so it cannot be a proper subset.

$\boldsymbol{\phi} \subseteq \boldsymbol{S}$ The null set is a subset of S.

$\boldsymbol{A} \subseteq \boldsymbol{S}$ A is a subset of S.

$\boldsymbol{B} \subseteq \boldsymbol{S}$ A is a subset of S.

$\boldsymbol{B} \subset \boldsymbol{S}$ B is a proper subset of S.

Where it gets weird:

The set {9} is an element in set S: $\{\mathbf{9}\} \in \boldsymbol{S}$

BUT $\mathbf{9} \notin \boldsymbol{S}$

Pretend you're handing out Halloween candy from a bucket. If you have a pack of skittles in the bucket, it is an element of your bucket. BUT since the skittles are INSIDE a pack, you can't give just one skittle to a trick or treater. So:

Bucket of candy = {chocolate bar, lollipop, pack of skittles}

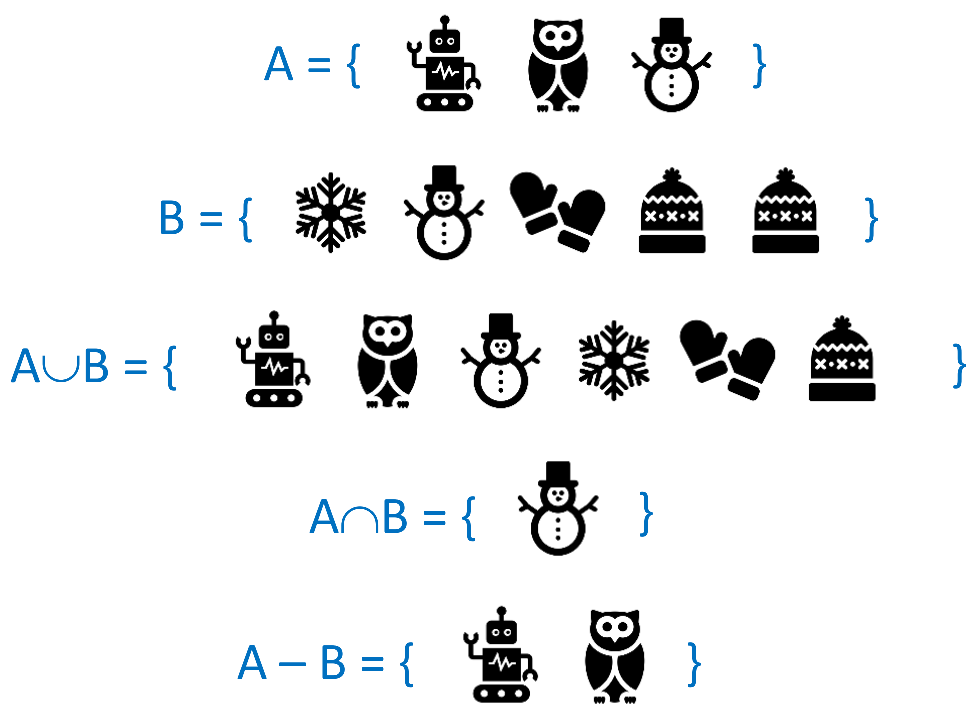
Pack of skittles ∈ Bucket of candy

One skittle ∉ Bucket of candy

One skittle ∈ Pack of skittles

SET THEORY	NOTES
UNION → ∪	**A set containing any item in either or both groups. Eg. A∪B has any element in set A, B or both A and B**
INTERSECTION → ∩	Elements that are found in both sets. Eg., A∩B has only elements that fit in **both** set A and set B
DIFFERENCE/RELATIVE COMPLIMENT → ie, A-B	Elements that are in one set, but not also in the other Eg. A-B is every element in A, minus those that are in A and B
CARDINALITY → |S|	The number of distinct elements inside a set

Example:

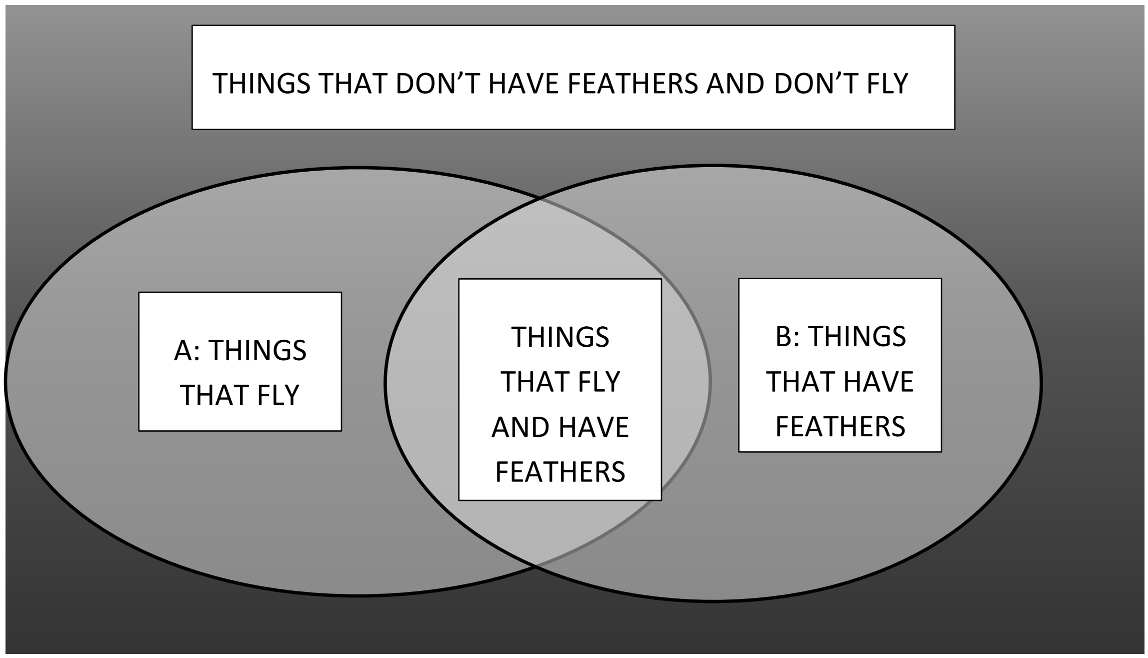


|A| = 3 There are 3 unique elements in Set A.

|B| = 4 There are 4 unique elements in Set B. The two toques are the same, so they are counted only once.

SET THEORY	NOTES
COMPLIEENT → line, or ‘, ie $\overline{S}$ or S’	**Every element within the universal set that is NOT in the set. Eg. A’ is every element outside of set A.**
VENN DIAGRAM	**An illustration that can be used to show the relationships between sets**

EXAMPLE: The Venn diagram illustrates a universal set containing set (things that fly) and set B (things that have feathers).



From the Venn diagram, we could say that the following are true:

Universal set, U = {Everything within the rectangle}

A bird has feathers **and** can fly: A bird ∈ B and A bird ∈ A

Therefore: A bird ∈ **(**A∩B)

A plane ∈B but **A plane ∉A.**

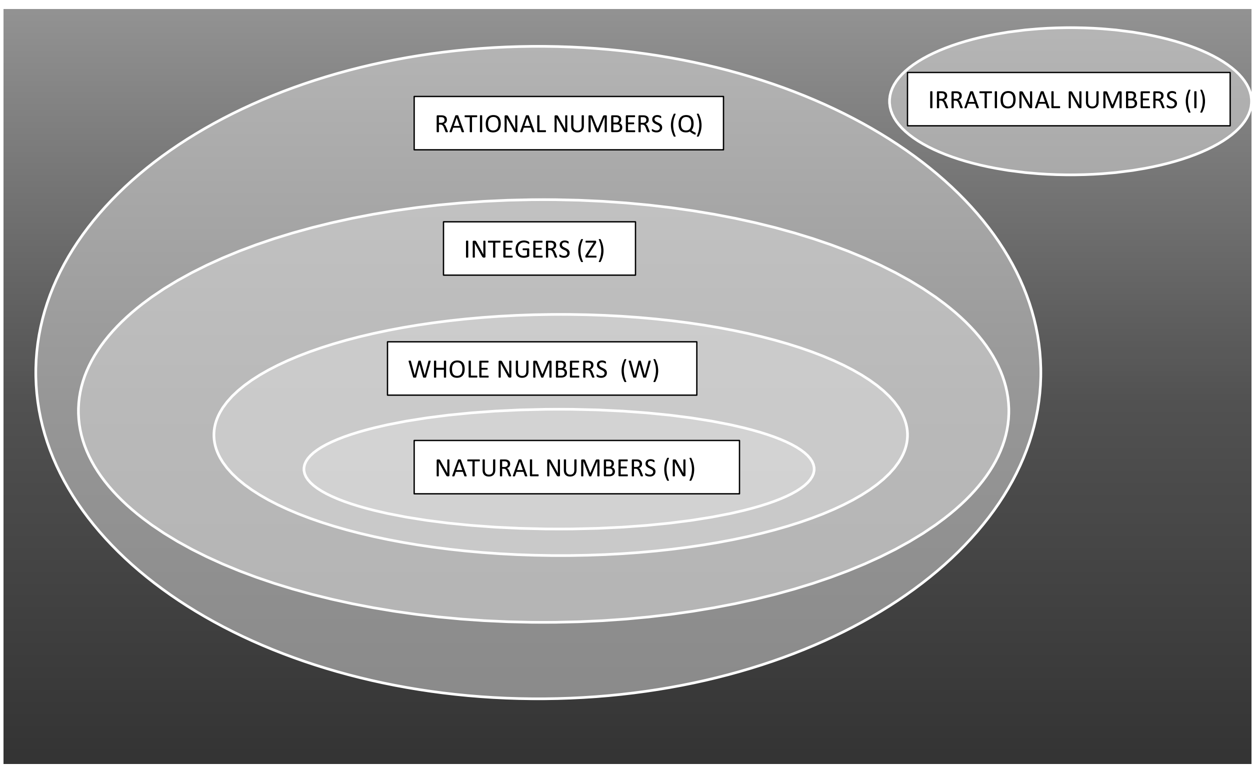
A’ = everything in the universal set that cannot fly.

B – A = Things that have feathers, not including things that can also fly

SET THEORY NOTES

Here's where things start to get a bit more "mathy". Set theory is used to describe different types of numbers.

NATURAL NUMBERS → N	Positive whole numbers that you could physically count. Does not include zero.
WHOLE NUMBERS → W	Positive numbers, not including fractions. Does include zero.
INTEGERS → Z	Any positive or negative number, not including any fractions or decimals.
RATIONAL NUMBERS → Q	Any number that can be written as a fraction. The decimal form of that number included.
IRATIONAL NUMBERS → I (P or $\overline{Q}$sometimes used)	A number that cannot be written as a fraction
REAL NUMBERS → R	Any number that you could find on a number line. Universal set containing all of the above. Does not include imaginary numbers. Which exist, but are not real. (I know, sometimes math doesn't make any sense.)

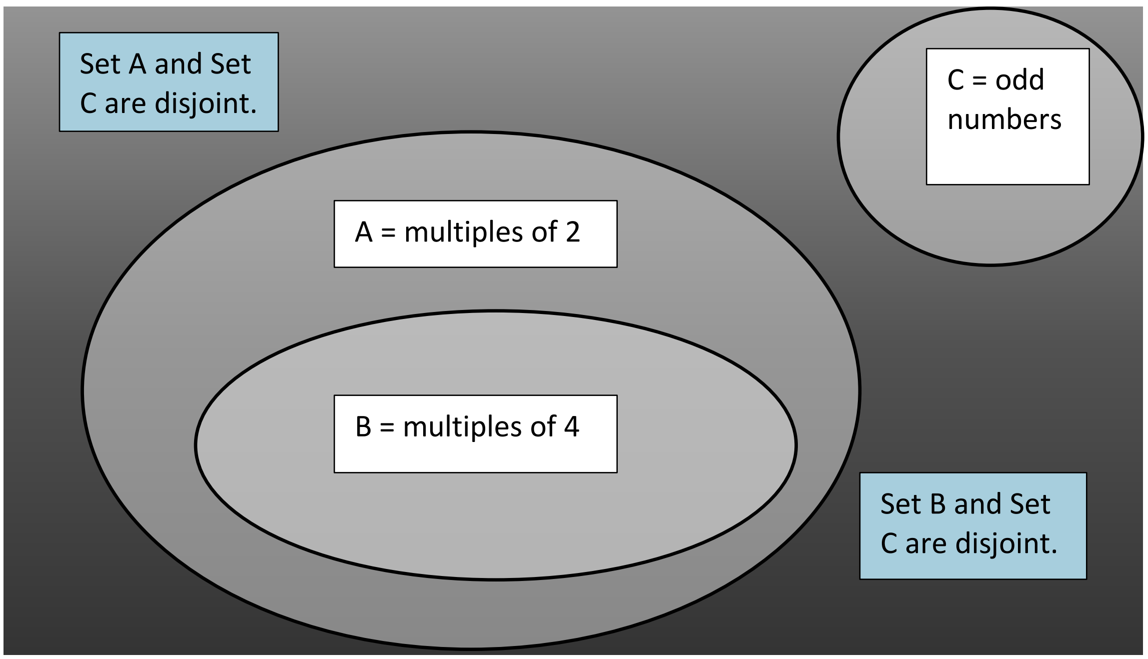


VENN DIAGRAM OF REAL NUMBERS

SET THEORY	NOTES
DISJOINT (Mutually Exclusive)	Sets that have no elements in common. Eg. A and be are disjoint if $\lvert A \cap B \rvert = 0$
SUCH THAT → $\mid$	Notation used to describe elements in sets. Eg. $S = \{x \mid x = 3n, n \in Z\}$ means that S is a set of all integers that are multiples of 3, since n must be an integer.

EXAMPLE:

Given – the rectangle represents a universal set of all whole numbers, W.



Set A and Set S are disjoint. Set B and Set C are disjoint.

n is always a whole number

$$A = \{x \mid x = 2n, n \in W\}$$

A contains elements, x, such that x is always equal to 2 times 'n'

"A is a set of elements, x, such that x is an even whole number."

$B \subset A$ → Set B is a proper subset of Set A.

$A - B$ = Numbers that are multiples of 2 but not multiples of 4.

EXAMPLES:

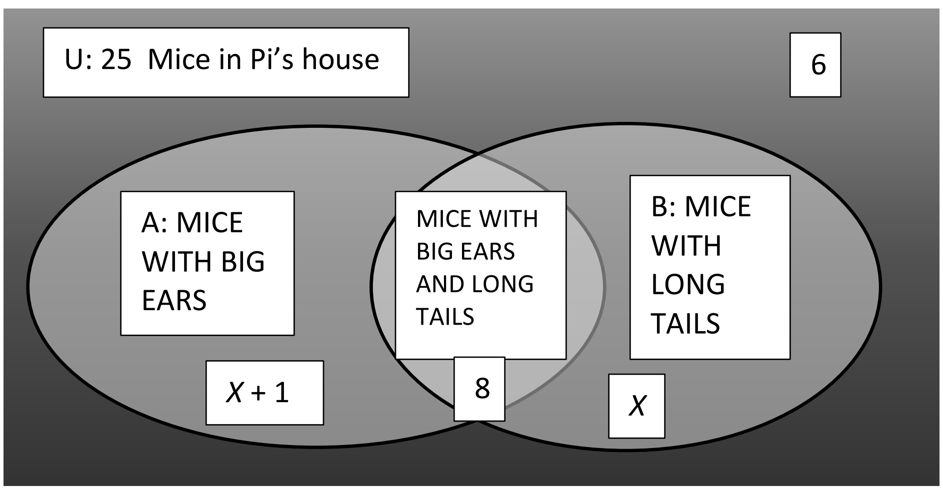
EXAMPLE 1:

There are 25 toy mice in Pi's house. There are 8 mice that have BOTH big ears and long tails. 6 mice have NEITHER big ears or long tails. There is 1 more mouse with only big ears than there are mice with only long tails.

a) How many mice have **only** long tails?
b) How many do not have **both** big ears and long tails?

ANSWER:

Start by drawing a Venn diagram:



From the problem statement, we are told:

$|U| = 25$

$|A \cap B| = 8$

$|(A \cup B)'| = 6$

$|B - A| = x$

$|A - B| = x + 1$

SET THEORY NOTES

a) We are asked to find $|B - A| = ?$

ANSWER: $|B - A| = 5$, Five mice have only long tails, with no big ears.

SOLUTION: Using set theory, we know:

$$|U| = |B - A| + |A - B| + |A \cap B| + |(A \cup B)'|$$

$$25 = x + x + 1 + 8 + 6$$

Collect like terms: $25 = 2x + 15$

Get the x terms alone on one side of the "=" symbol: - 15 from each side

$$25 - 15 = 2x + 15 - 15$$

$$10 = 2x$$

Divide both sides by 2 to solve for x: $10/2 = (2x)/2$

$$\boldsymbol{X = 5}$$

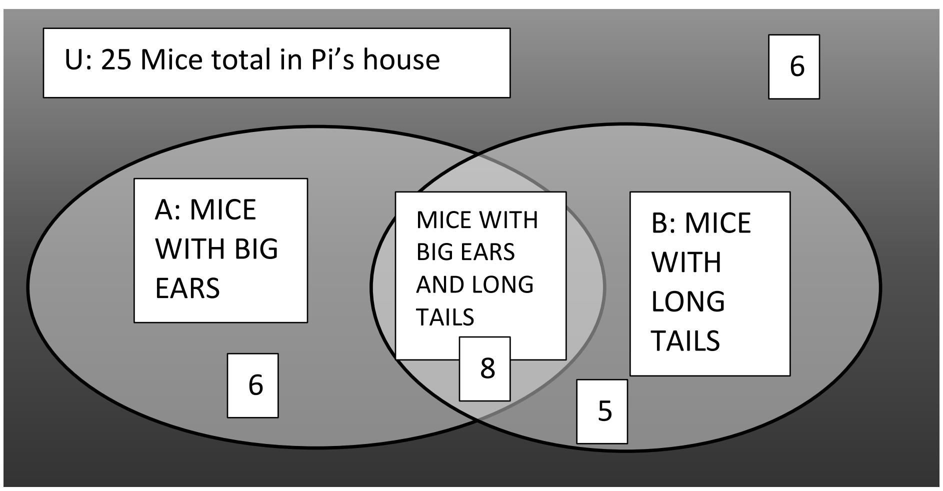
b) **We are asked to find:** $|(A \cap B)'| = ?$

ANSWER: $|(A \cap B)'| = 17$

SOLUTION: Use what we found in part A to finish filling in the Venn diagram.

$|A - B| = x + 1$

$|A - B| = 5 + 1 = 6$



$|(A \cap B)'| = 6 + 5 + 6 = 17$

OR, we know $|(A \cap B)'|$ is the number of all the mice in the universal set that are outside of $|(A \cap B)|$, so:

$|U| - |(A \cap B)| = 25 - 8 = 17$

EXAMPLE 2:

There are two sets of numbers. Both are subsets of a universal set of whole numbers, W.

A = {Factors of 16)

B = {Prime numbers less than 16}

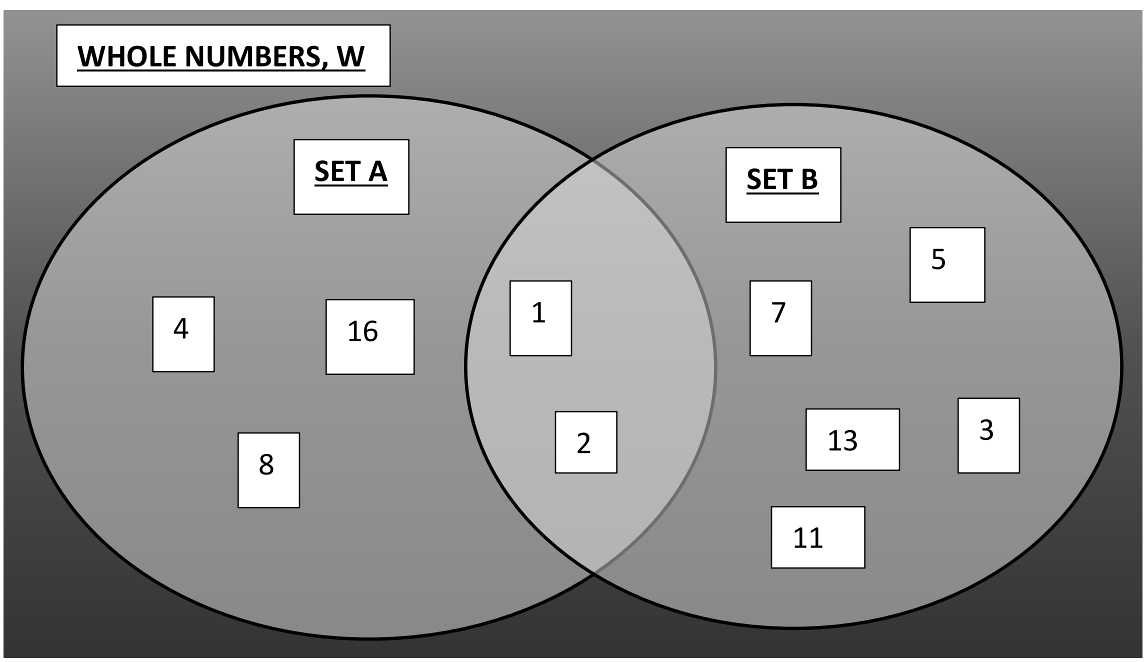
a) Find the intersection of A and B.
b) Find the union of A and B.
c) Find the set of numbers B – A

SOLUTION: Start by writing out the sets.

A = {1, 2, 4, 8, 16}

B = {1, 2, 3, 5, 7, 11, 13}

In a Venn diagram this would look like:



a) A∩B = {1, 2}
b) A∪B = {1, 2, 3, 4, 5, 7, 8, 11, 13, 16}
c) B – A = {3, 5, 7, 11, 13}

SET THEORY	NOTES
INCLUSION-EXCLUSION PRINCIPLE	$\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert - \lvert A \cap B \rvert$ **The number of elements in the union of A and B is equal to the sum of the elements in set A and Set B, less the number of elements in both sets A and B.**

Inclusion-Exclusion Principle for three sets, A, B, and C:

$$\lvert A \cup B \cup C \rvert = \lvert A \rvert + \lvert B \rvert + \lvert C \rvert - \lvert A \cap B \rvert - \lvert A \cap C \rvert \, \lvert - \lvert B \cap C \rvert + \lvert A \cap B \cap C \rvert$$

EXAMPLE:

- 10 children have lollipops (L) and chocolate bars (C).
- 4 children have a lollipop and a chocolate bar.
- 7 children have a lollipop.

How many children have a chocolate bar?

SOLUTION: Start by writing out what we know from the Inclusion-Exclusion Principle. Then fill in the information we are provided.

$\lvert L \cup C \rvert = \lvert L \rvert + \lvert C \rvert - \lvert L \cap C \rvert$
$10 = 7 + \lvert C \rvert - 4$
$\lvert C \rvert = 10 – 7 + 4$
$\lvert \mathbf{C} \rvert = \mathbf{7}$

Seven children have a chocolate bar.