Chaotic Friendship Round

Absolute Value

Fall 2022 Tournament

Solutions

Team name: _____

Names of team members: _____

> Put all answers in the following spaces. Guessing is not penalized. Work or answers written on other pages will not be graded. Submit only this page. Do not flip to the following pages until told to by your proctor.

1: _____ **7**: _____

2: _____ **8**: _____

3: _____ **9**: _____

4: _____ **10 (a)**: _____

5: _____ **10 (b)**: _____

6 (a): _____ **10 (c)**: _____

6 (b): _____ **11**: _____

6 (c): _____ **12**: _____

For grader's only:

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	5	5	5	5	5	15	5	5	5	15	5	5	80
Score:													

1. (5 points) Tyler is 10 years old. In 10 years, Nick will be twice as old as Tyler. What is Nick's current age?

2. (5 points) Quick flashback to Game of 24 (but here's an extra hard one):

   Using only 1's, what is the least number of 1's needed to construct the number 24? You can use some extra tools: Addition, Subtraction, Multiplication, Division, Exponents, Square roots, Factorials—where $x! = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot 2 \cdot 1$—all trigonometric functions, and concatenation. *For instance, you can construct the number 12 using three 1's:*

   $$11 + 1$$

3. (5 points) Brendon the beetle walks in a path that resembles a spiral according to the following rules: He starts at the coordinates (0, 0) facing north (the positive y direction). Every day, he walks one more mile than he did the previous day then rotates the direction he is facing 90 degrees clockwise. He walks 1 mile (where 1 mile is 1 unit) on the first day. For example, at the end of the first day he ends at position (0, 1) by walking 1 mile north. Similarly, he is positioned at (2, 1) at the end of the second day by walking an additional 2 miles east and (2, -2) at the end of the third day by walking an additional 3 miles south. What is his position at the end of the twentieth day? *(Maybe draw a graph?)*

4. (5 points) The following proof contradicts:

Step 1: Let $a = b$, where a is a positive real number.
Step 2: $a^2 = ba$
Step 3: $a^2 - b^2 = ba - b^2$
Step 4: $(a + b)(a - b) = b(a - b)$
Step 5: $\frac{(a+b)(a-b)}{a-b} = \frac{b(a-b)}{a-b}$
Step 6: $a + b = b$
Step 7: $a = 0$ (contradicts with line 1)

Which step contains the mistake that makes the proof contradicting? *(Siri's Hint: Imagine that you have zero cookies and you split them evenly among zero friends? How many cookies does each person get? See? It doesn't make sense. And Cookie Monster is sad that there are no cookies, and you are sad that you have no friends.)*

5. (5 points) You are at a math competition (maybe it is called Absolute Value?) with two other friends and one enemy. How many ways are there for the four of you to sit in a straight line such that you are never sitting directly next to your enemy?

6. The next question might take some time, unless you know the trick! They are all Yes/No questions.

   (a) (5 points) Is 130813095 divisible by 5?

   (b) (5 points) Is 130813095 divisible by 3?

   (c) (5 points) Is 130813095 divisible by 9?

7. (5 points) Compute the largest prime factor of 97.

8. (5 points) Bobby wants to arrive at Absolute Value math competition 5 minutes before it begins, so he starts traveling 20 minutes before the start of the competition at a pace of 60 miles per hour. After traveling a bit, he hits a bit of traffic. He travels at half his typical speed while in traffic. He eventually starts moving at 60 miles per hour again and ends up arriving to the Absolute Value math competition 5 minutes after it starts. How long was Bobby stuck in traffic?

9. (5 points) You stack two standard 52-card decks on top of each other and shuffle thoroughly. Then you draw the top two cards. What is the probability that the two cards are identical, written as a reduced fraction?

10. This one's a hard one (write all fractions in reduced form):

   (a) (5 points) What is the probability that you flip a coin four times and it lands Heads, Tails, Heads, Tails?

   (b) (5 points) What is the probability that you flip a coin four times and it lands Heads, Heads, Heads, Heads?

   (c) (5 points) What is the probability that you flip a coin four times and it lands on heads two times?

11. (5 points) Two people in your group play x consecutive games of rock, paper, scissors. It just so happens each of the x games has a definite winner—that is, there are no ties. What is the probability that one of the players wins every game? *Use x in your final answer, rather than the number of people in your group.*

12. (5 points) Assume that n is a positive integer. What value of n satisfies the the following statement:

$$n(n - 1)(n - 2) \cdots (2)(1) = n^3 - n$$