Individual Round

Absolute Value

Fall 2022 Tournament

Solutions

Full name: _____

Email: _____

Put all answers in the following spaces. Guessing is not penalized. Work or answers written on other pages will not be graded. Submit only this page. Do not flip to the following pages until told to by your proctor.

1: _____ **6**: _____

2: _____ **7**: _____

3: _____ **8**: _____

4: _____ **9**: _____

5: _____ **10**: _____

Tie-breaker value: _____

The following should only be filled out by graders:

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	5	5	5	10	10	10	10	15	15	15	100
Score:											

Tie-Breaker

To break ties, please write the largest prime number you can in the "Tie-breaker value" section of your solution sheet. It is recommended you spend time on this **after** solving all the math questions you can.

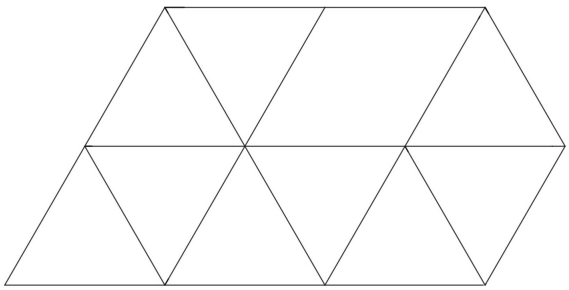
For specific details as to how the tie-breaker works:

1. The tie-breaker only determines ranking between those with the exact same score on the Individual round.
2. Those with a larger prime number will be placed higher.
3. Those whose tie-breaker value is not prime will be placed lower than all those whose tie-breaker value is prime.
4. Those who do not fill in the tie-breaker will be placed last.
5. Placement among participants with the same tie-breaker value, those whose tie-breaker value is composite, or those who leave the section blank will be determined randomly.

Enjoy the problems!

1. (5 points) Barney (the big purple dinosaur) has 30 ice-cream cones. He is very hungry and eats 6. His lovely friend Clifford (the big red dog) eats twice that amount. How many cones does Barney have left?

2. (5 points) How many triangles are inscribed in the following figure?



3. (5 points) Compute:

$$\left(\frac{338^2}{337 \cdot 338}\right) \cdot \left(\frac{339^2}{338 \cdot 339}\right) \cdot \left(\frac{340^2}{339 \cdot 340}\right) \cdots \left(\frac{2022^2}{2021 \cdot 2022}\right)$$

4. (10 points) A rectangular prism has volume 144. Its length is twice its width. Its length is also 2 less than its height. Its length, width, and height are all integers. Calculate the sum: Length + Width + Height.

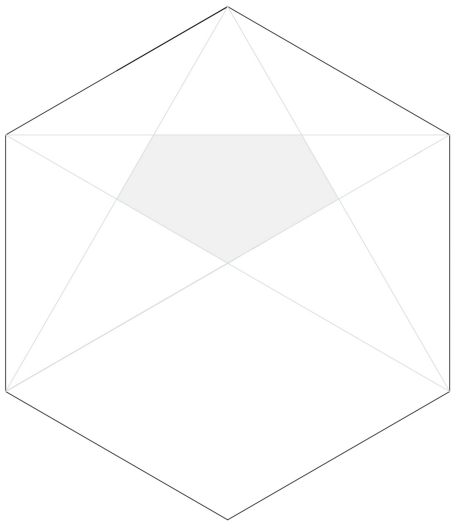
5. (10 points) An analog clock has a minute and an hour hand. How many times between 8:00 AM and 8:00 PM are the two hands perfectly aligned?

6. (10 points) Alice runs from her house to Bob's house to visit him. The path is 2 miles long, but Alice walks the second mile. She walks at half the speed she runs, and she makes the journey in 30 minutes. Alice and Bob collectively travel back to Alice's house at Alice's running pace. Including both journeys, what is Alice's average speed, in miles per hour?

7. (10 points) Take 2 different numbers from the set $\{-3, -2, -1, 0, 1, 2\}$. What is the expected value of their product?

8. (15 points) Values a and b are picked at random from the interval between 0 and 1. The odds that $\frac{3}{4} < b + \frac{1}{4} < a$ are 1 in what?

9. (15 points) Below is a regular hexagon with area 18. Evaluate the area of the shaded region.



10. (15 points) Julian picks marbles out of a bag in a random order. In the bag are 4 red marbles, 6 green marbles, and 10 blue marbles. Once he picks out all marbles of a certain color, he plays with those marbles, then continues to pick marbles out of his bag. Calculate the chance that he plays with his red marbles first, green marbles second, and blue marbles last. Express your answer as a percentage.
Hint: What is the chance he plays with the blue marbles last?